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History as a cognitive agent of comprehensive mathematical learning

La historia como agente cognitivo del aprendizaje matemático comprensivo

L'histoire en tant qu'agent cognitif de l'apprentissage global des mathématiques

História como um agente cognitivo de aprendizagem matemática compreensiva

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Abstract

This essay aims to discuss the importance of the historical investigation of mathematics as a cognitive agent in the comprehensive learning of school mathematics. Initially, I make conjectures regarding creativity as a human ability according to its cognitive dynamics in the acts of knowing, understanding, and explaining. Throughout the essay, I discuss and reflect on principles that support the use of historical investigative practices in mathematics teaching, considering that teachers need to appropriate historical mathematical facts to promote a mediation that provides conceptual reorganizations around the mathematical objects that students will construct during their school learning. These are investigative, problematizing, and foundational principles of mathematics teaching and learning, which take historical mathematical facts as cognitive agents in mathematics teaching and as triggers of student learning. Finally, I mention themes that can be agents for teaching and learning mathematics, evoking conceptual and pedagogical functions for teaching mathematics based on historical mathematical methods.

Keywords: History of Mathematics. Cognitive agency. Historical investigation. Mathematics teaching. Mathematics learning.

Resumen

Este ensayo tiene como objetivo discutir la importancia de la investigación histórica de las matemáticas como agente de la cognición en el aprendizaje integral de las matemáticas escolares. Inicialmente, conjeturo sobre la creatividad como una habilidad humana en su dinámica cognitiva para conocer, comprender y explicar. A lo largo del ensayo, discuto y

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reflexiono sobre principios que sustentan el uso de prácticas investigativas históricas en la enseñanza de las matemáticas, considerando que los docentes necesitan apropiarse de hechos matemáticos históricos para promover ensamblajes que proporcionen reorganizaciones conceptuales en torno a los objetos matemáticos que los estudiantes construirán durante su vida escolar. Se trata de principios investigativos, problematizadores y fundacionales de la enseñanza y el aprendizaje de las matemáticas, que toman los hechos matemáticos históricos como agentes cognitivos en la enseñanza de las matemáticas y como detonantes del aprendizaje de los estudiantes. Finalmente, menciono temas que pueden ser agentes para la enseñanza y el aprendizaje de las matemáticas, evocando funciones conceptuales y pedagógicas para la enseñanza de las matemáticas a partir de métodos matemáticos históricos.

Palabras clave: Historia de las Matemáticas, Agente cognitivo, Investigación histórica, Enseñanza de las matemáticas, Aprendizaje de las matemáticas.

Résumé

Cet article essayiste vise à discuter de l'importance de la recherche historique en mathématiques en tant qu'agent cognitif dans l'apprentissage global des mathématiques à l'école. Dans un premier temps, j'émets des hypothèses sur la créativité en tant que capacité humaine en fonction de sa dynamique cognitive dans les actes de connaissance, de compréhension et d'explication. Tout au long de l'essai, je discute et réfléchis aux principes qui soutiennent l'utilisation des pratiques d'investigation historique dans l'enseignement des mathématiques, considérant que les enseignants doivent s'approprier les faits mathématiques historiques pour promouvoir l'agence qui fournit des réorganisations conceptuelles autour des objets mathématiques que les élèves construiront au cours de leur vie scolaire. Il s'agit des principes d'investigation, de problématisation et d'ancrage de l'enseignement et de l'apprentissage des mathématiques, qui considèrent les faits mathématiques historiques comme des agents cognitifs dans l'enseignement des mathématiques et comme des déclencheurs de l'apprentissage des élèves. Enfin, je mentionne des thèmes qui peuvent être des agents de l'enseignement et de l'apprentissage des mathématiques, évoquant des fonctions conceptuelles et pédagogiques pour enseigner les mathématiques à partir de méthodes mathématiques historiques.

Mots-clés : Histoire des mathématiques, agent cognitif, enquête historique, enseignement des mathématiques, apprentissage des mathématiques.

Resumo

Este artigo ensaístico tem como objetivo discutir a importância da investigação histórica da matemática como um agente de cognição na aprendizagem compreensiva da matemática escolar. Inicialmente, faço conjecturas sobre a criatividade como uma habilidade humana de acordo com sua dinâmica cognitiva nos atos de conhecer, compreender e explicar. Ao longo do ensaio, discuto e reflito sobre princípios que sustentam o uso de práticas investigativas históricas no ensino de matemática, considerando que os professores precisam se apropriar de fatos matemáticos históricos para promover agenciamentos que proporcionem reorganizações conceituais em torno dos objetos matemáticos que os alunos irão construir durante sua vida escolar. Estes são princípios investigativos, problematizadores e fundamentadores do ensino e da aprendizagem matemática, que tomam os fatos matemáticos históricos como agentes cognitivos no ensino de matemática e como acionadores da aprendizagem dos estudantes. Por fim, menciono temas que podem ser agentes do ensino e da aprendizagem matemática, evocadores de funções conceituais e pedagógicas para se ensinar matemática a partir de métodos matemáticos históricos.

Palavras-chave: História da Matemática, Agente cognitivo, Investigação histórica, Ensino de matemática, Aprendizagem matemática.

History as a cognitive agent of comprehensive mathematical learning

> One cannot say that something is without saying what it is. When reflecting on the facts, we are already relating them to concepts, and it is certainly not indifferent to know what these concepts are (Friedrich Schlegel, 1975, cited by Reinhart Koselleck, 2006)

The opening epigraph of this article raises reflections regarding the theme to be addressed in the following pages, in which this article is structured in the form of an essay. My intention is to establish answers to a question that is frequently manifested in educational environments, regarding the creative act or the processes of knowledge production and promotion of learning in its creative sense, that is, to understand and explain what one wants. This question leads me to think about how I understand this act of creativity, which constitutes an inherent human ability in their cognitive dynamics to know, understand and explain.

My questioning restlessness leads to two questions: what's the reason, and what's the purpose? In the wake of these questions, it appears that, throughout the 19th and 20th centuries, several scholars of this field discussed the notion of creativity, emphasizing that it is an essential human skill that is continuously exercised because this act constitutes a cognitive dynamic that was and remains to be fundamental for the development of the potential of those who study, learn and produce knowledge in any socio-cultural, scientific and technical field[2].

Studies I have carried out since 1993 have shown me that research related to the use of history in Mathematics teaching has potentially multiplied since the last decade of the twentieth century (from 1990 onwards), thus meaning a diversity of principles, methods, and modalities of didactic approaches employed by teachers to structure the focus given to the thematic units regarding the teaching of Mathematics. In this regard, I highlight that the discussions and reflections presented throughout the sections of this essay are supported by fundamental principles concerning the uses of the history of Mathematics for classroom teaching through investigation, already defended by several authors in previous publications, since the 1990s.

It is my understanding that investigative ability is essential for human intellectual autonomy in that it develops innovative/creative thinking, which constitutes a primordial strategy for us to learn to lead our lives in a process of constant emancipatory learning. Furthermore, I start from the assumption that the search for dynamics for the production of new knowledge can point to ways that enrich educational processes that might favor the growth of

[2] In this regard, consult Poincaré (1920, 2010); Hadamard (1944, 2009); Boirel (1961, 1966); Moles (1970, 1998, 2007, 2012); Csikszentmihaly (2006), mentioned in the references at the end of the article.

those who produce knowledge at school, so as to lead a process of learning and cognitive production that is always pleasant and innovative, that never allows for the crystallization of the rigidity of practices and concepts, but rather creates the possibility of implementing thinking strategies, through a dynamic that imprints a constant interest for renovating and breathing fresh air into the ideas of those who learn.

Even though, in the course of this essay, it becomes clear that the discussion presented here is not intended to deal directly with the histories of mathematical creation and its creative processes, my intention is to explore a few ways in which several intellectuals such as philosophers, researchers, scientists, or other related professionals got involved in the search for solutions to problems that challenged them and helped them organize dynamics of combinations between different kinds of knowledge produced previously so that they could point out solutions to the new problems that arose.

What is understood as mathematical creativity

Concerning the typologies related to the intellectuals I mentioned in the previous paragraph, they rely on Daniel J. Boorstin's discussions in three books that make up his thematic trilogy: Os Descobridores (1994), Os Criadores (1995), and Os Investigadores (2003), whose main focus is the history of human creativity of getting to know its own world and itself in search of procedural historical explanations regarding the activation of human cognition around the sociocultural construction of reality. To these ideas, I also added the updated reflections of A. C. Grayling (2021) regarding the frontiers of knowledge in the relationship between science, history, and mind.

In *Os descobridores*, Boorstin (1994) based his reflections on the history of human culture and its process of creating explanations for its existence in the world to demonstrate the human need to gain knowledge so as to comprehend what is "beyond" what we can imagine. In *Os Criadores*(1995) the author deepens his historical elaboration related to human creativity in order to situate such creative processes around branches of knowledge such as religion, magic, alchemy, science, and art. He ends the trilogy with *Os investigadores* (2003), showing that this history has seen humans permanently searching for an understanding of themselves, of the world they inhabit, and of a new world yet to be invented, without neglecting the aspects mentioned in the three books, emphasizing more the aspects marked by the scientific spirit in production and knowledge.

When expanding discussions on this topic, the updated approach established by Grayling (2021), in the book *As Fronteiras do Conhecimento*, the author argues that technologies have existed before science, as ways of making science emerge to create means of representing the world. His text emphasizes how the writing of stories describes the birth of humanity, the problems of comprehensive relations between the past and its projection in the present-future[3], organized through the mind and the materialized records starting from the development of human consciousness regarding observation, representation, and the future projection of scientific and non-scientific facts in constant transformation.

Based on epistemological approaches such as those mentioned above, I justify the direction chosen for my studies and research, which have been following these processes, which I consider as a continuum of creative movements, carried out by mathematicians from different fields throughout history who intended to produce solutions that explained challenging mathematical facts, thus generating new knowledge, that is, new mathematical explanations for old facts or for the evidence of new facts.

Thus, my proposal is an invitation for readers to reflect on the processes operationalized by human thought in order to appropriate and explain mathematical objects, mathematical practices, and their relationship with the sociocultural context at all times and in all spaces, and its main objective is to answer the following question: how these ways of being of mathematical thought and mathematical practices were and still are captured by processes of mathematical cognition in all their dimensions. This proposal also involves constantly asking how these investigative actions are processed. Do these processes have a single dynamic or are they a combination of multiple dynamics of mathematical cultures, moving about amid cognitive exercises to develop this understanding having the history of mathematics as a starting point?

In an attempt to answer the questions posed in the previous paragraph, I believe it is necessary to install a procedural movement of historical understanding of mathematics that must be practiced in the school environment. However, the school context is often permeated with gaps, perhaps due to the fragmented or incomplete training of mathematics teachers, which may point toward a lack of knowledge regarding the historical development of mathematical ideas expressed as concepts, properties, and epistemic relationships. Furthermore, we can clearly see the need for those involved in the process to appropriate the cognitive processes

[3] I have adopted present-future as an expression meaning that the present is instantaneous, that is, it is infinitesimal, and soon it becomes past, and will move forward into the future. It's only the boundary between the past and the future. I adopt the notion of time established by Henri Bergson (1891), when he asserts that real time is succession, continuity, change, memory, and creation. By definition, the past is that which no longer is, the future is that which is yet to be, and the present is what it is. But the present moment, when it is perceived, has already passed away.

arising from the assemblages[4], offered by historical mathematical facts in order to bring about new appropriations and cognitive reorganizations around the mathematical objects that must be constructed by students throughout their school journey.

Still regarding the questions raised earlier, I reflect based on all the years of studies, research, and my teaching experience, intending to understand and explain that, if teachers face difficulties to elaborate and clarify explanatory arguments related to certain subjects during their teaching activities in order to expand in more detail the guidance given to help students achieve comprehensive learning, that happens because, often due to their lack of knowledge about the historical-epistemological development of the Mathematics contents they have to teach. During their classes, one finds evidence that perhaps these teachers do not fully master the historical conceptual compositions of mathematics, that is, they do not feel comfortable organizing a sequential historical movement (MSH)[5] that demonstrates how certain school mathematical themes were developed over time and in multiple spaces[6].

I start from the premise that the mastery of these historical conceptual processes in relation to the school Mathematics that is to be taught can grant teachers possibilities of inserting this movement in their teaching work in the classroom since this is an approach that I have been dealing with since 1993 and deepening over time, throughout the studies and research I carried out, as well as in my work as an advisor for thesis and dissertations at the postgraduate level. It is an investigative, interpretative, and comprehensive movement regarding the historical creative processes of mathematicians, which currently requires researchers and teachers to develop new cognitive exercises so that they can establish creative dynamics to be inserted in teaching, with the aim of composing and implementing methodological teaching strategies that can help students achieve mathematical learning.

It was with these intentions that, for more than two decades, I developed studies and research with the objective of experimenting with a multitude of didactic strategies that could be associated with the investigative, problematizing, and fundamental principles of mathematics teaching and learning established from the historical-epistemological development of Mathematics. Building on these experiences and reflections, I reinvented principles and methods that converged to create new principles to build *a history that functions as a cognitive trigger in mathematical learning, as a cognitive reorganizer in mathematical*

[4] The term *agency* comes from the discussions established by Ludwik Fleck (2010) about the genesis and development of a scientific fact., when he clarifies aspects regarding the relations between subject and object in the production processes of scientific knowledge.
[5] On MSH, see Mendes (2021b; 2023).
[6] Regarding this matter, see Mendes (2022a, p. 65-71).

learning, and as an agent of cognition in the teaching of mathematics and mathematical cognition in the classroom.

These expressions have a conceptual trajectory that began at the end of my doctoral studies, after my reflections on the readings and interpretations of historical sources and their pedagogical functions for teaching Mathematics. At that time, I interpreted that such sources were agents for conducting the process of teaching and learning Mathematics, as they were made up of arguments and philosophical positions about the relationship between history and Mathematics teaching (Mendes, 2021a, 2022).

However, before adopting any position, either favorable or unfavorable to the use of history in teaching Mathematics, like any of the forms proposed until then, it was important to understand what pedagogical purposes were intended when using history to teach Mathematics that could lead me to achieve a Mathematics teaching that would manage the students' cognition to achieve comprehensive learning. This concern arose due to the position that Mathematics occupies in official teaching curricula, as well as its insertion in textbooks used in the classroom, since, as we know, most official teaching proposals and textbooks have Mathematics as preconceived knowledge, without any historical, social, and cultural context that presupposes its construction.

The expression *history as an agent for conducting* the process of teaching and learning Mathematics was already included in the first edition of a pioneering book on this theme (MENDES, 2001a). At that moment, the ideas emerged as a conceptual embryo that matured and structured itself over two decades of studies, research, and theoretical reflections. In this sense, the conceptual notion underwent a process of theoretical deepening and, subsequently, was stated under three names: *1) history as a cognitive trigger in the teaching of Mathematics; 2) history as a cognitive and didactic reorganizer in Mathematics teaching, and, finally, 3) history as an agent of cognition in Mathematics learning*.

After deepening studies on the subject and based on exultant conclusive reflections of research I carried out, I defined it as a *history that functions as an agent of cognition in Mathematics Education*, since it meant betting on a way of proposing a movement for handling the thinking of students through the teacher's actions, guided by historical mathematical facts, which bring with them a set of cognitive actions historically produced by society, culture and, in general, by mathematicians who represented these collective socio-cognitive movements.

However, at this moment, it is worth describing what I mean by these three conceptual expressions that have been transformed over the years, based on the didactic experiences that involved research and practices related to the uses of the history of Mathematics in teaching and that, after my reflections, allowed me to enunciate the characteristics of a kind of history that I consider more adequate for Mathematics teaching. First, however, I must explain the conceptual meaning adopted to address concepts such as *activator, reorganizer, and agent,* as well as relate them to the concept of cognition and to different types of teaching and learning.

However, the central focus of such meaning lies on the mediation processes which refer to the expression whose central meaning is implicit in the title of this article, when associated with the concept of history (of Mathematics) and Mathematics Education (action and education through Mathematics) when it becomes a school subject, that is, how school culture forms the thoughts and actions of a collective, based on the styles of thought already established in each thought collective, as stated by Ludwik Fleck (2010) in the book Genesis and development of a scientific fact[7].

In this book, Fleck discusses the emergence and development of a scientific fact in its reflections and propositions regarding what he called style of thought and collective thought or thought collective, in the production of knowledge, which were the precursor concepts used to explain the ways in which scientific ideas change over time. That means the ideas that were later institutionalized by the sociology of science in terms of scientific community, paradigm, and normal science, as proposed by Thomas Kuhn (1996) and of épistème[8], as enunciated by Michel Foucault (2002)), as well as the ideas defended by Bruno Latour (2000, 2013) in the books *Science in Action* and *We were Never Modern* and by Michel Serres (2008), in the book *Ramos*, when addressing the concepts of mother science, daughter science and the new epistemological inventions that indicate the scientific ramifications that correspond to Kuhn's thinking regarding pre-paradigmatic science and normal science.

The three questions I refer to at the beginning of this article are expressed in the following sections and are reflected in the discussions here presented and converge to the meaning of its title when it deals with the cognitive agency of students to accomplish a mathematics education in a broader sense. wide. The term agency here is understood in the

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[7] This book was originally published in German under the title Entstehung und Entwicklung einer wissenschaftlichen Tatsache: Einführung in die Lehre vom Denkstil und Denkkollektiv (1935) and later in English under the title (1979), edited by T.J. Trenn and R.K. Merton, with a preface by Thomas Kuhn. The Brazilian publication is from 2010, mentioned at the end of this article.

[8] From the ancient Greek ἐπιστήμη, it corresponds to the term épistème, meaning knowledge. In philosophy, it refers to scientific knowledge as a principled system of understanding, sometimes contrasted with empiricism. Specifically in ancient Greek philosophy it means know-how, corresponding to techne (τεχνική , meaning technical knowledge (Durozoi; Roussel, 1999). In Foucaultian philosophy it means the scope of historical-cultural ordering of discourses independently and below the possible criteria of scientificity of scientific discourse itself, through which the limits of what is accepted as true knowledge at a given time are established (FOUCAULT, 2002). Bunge (2012) refers to the subject by mentioning that the term corresponds to the relations between knowledge, truth, and belief; what is common and different between ordinary, scientific, and technological knowledge.

sense of an action of mediating the subject's interactions with objects of knowledge in order to establish relationships of common interest.

Thus, a mediation presupposes the existence of an agent, that is, the author of the action; one that promotes interaction in order to lead the subject to appropriate the object of knowledge. This agent can be animate or inanimate, as is the case of another living physical subject or some object of historical culture such as material artifacts, manuscripts, etc. However, its insertions and uses by the subject of the process of knowledge acquisition lend it its character as an agent in cognitive action.

In view of what was exposed in the previous sections of this article, I move on to the three sections, below, with the itinerary that constitutes a cognitive agency and its implications in the processes of teaching and learning mathematics, mainly in the ways of promoting activities that provoke the cognition of the one who learns through cognitive learning triggers, cognitive reorganizers, and cognition agents in mathematics teaching.

These conceptual expressions are based on the concept of learning as understanding. Such expressions manifest a synonymic characterization of comprehensive learning, described by Richard Skemp (1976; 2023) as relational understanding and reiterated later by him (SKEMP, 1980; 1993) when addressing aspects related to intuitive and reflective intelligence, referring to habitual learning and resistance of concepts such as cultural heritage in learning.

Such ideas made me interpret the spiral historical structure of the epistemological development of Mathematics as a schematic combination originated from the combinations of many concepts —either mathematical concepts or otherwise—in continuous connections, through which it was essential to establish actions of recurrence to previously instituted mathematical concepts, mainly, because we consider that when each individual of a thought collective constructs new concepts, they establish a network between previously established concepts, which are managed to structure new explanations related to everything that already exists in the world.

Likewise, these expressions are also associated with my interpretation of the concept of learning as conceived by Kieran Egan (2002) in his book The Educated Mind, when he presents a discussion of five types of understanding: mythical, romantic, philosophical, ironic and somatic understanding, to design a possibility for the procedural trajectory of the institution and thought strategies in favor of the production of knowledge, giving rise to expressions of languages to represent the world through scales and types of conceptual organs, which made possible the emergence of these five modes of understanding that are directly related to the historical moments that constituted the epistemology of Mathematics or of another field of knowledge.

It is also in these terms that we converge to what Margarida Knobbe proposes when she explains her reflections in this regard, in the book *O que é compreender*, when she asserts that

> (...) trying to understand something only through reason leads to a state of inertia, indifference [and that] we don't get tired of trying to understand because it is in this way that, when we exhaust our own thoughts, we need to feel the influx of opinions others, even if we do not follow their impulse (Knobbe, 2014, p. 71).

Therefore, the five manifestations of understanding enunciated by Egan (2002), can enable us to have a dialogue with Knobbe's indications (2014) in search of a proposition articulated between history and understanding as a process of knowledge production and learning. Along these lines, the author reiterates that understanding is characterized by its manifestation as a way of expressing a knowledge that creates harmony, situated at the gates of pleasure and scientific analysis, that is, as self-socio-knowledge, and presupposes the sharing of a feeling of truth. Language is the means through which occurs the interlocutors' agreement on the understanding of the cause (Knobbe, 2014, p.73).

Therefore, I can argue that the insertions of Skemp, Egan, and Knobbe regarding the concept of comprehension allowed me interpretations through which I conjectured that the cognitive process that mediates comprehensive learning has no delimitation, that is, it is infinite, and develops in multiple directions and dimensions, thus characterizing the diversity of complementary and correlated learning in terms of cognitive drives used by the human mind in search of interpreting and understanding situations that may explain different facts and events, as I will discuss below.

On history as a cognitive activator in mathematics learning

Regarding the expression history as a cognitive activator[9] in mathematical learning, first, we have to start with the term *activator*, which can be understood as that which puts into action or into practice, which causes something to start working. It is a word referring to the act of provoking thought, triggering mental synapses in the elaboration of thoughts that express senses and meanings to the objects that one intends to give existence.

Thus, it means activating the cerebral sectors that connect mathematical facts extracted from the readings of historical texts concerning the conceptual development of Mathematics,

[9] The term activator refers to the concept of complex method as a strategy, in: MORIIN, Edgar. O método 3. O conhecimento do conhecimento. Porto Alegre: editora Sulina, 1999.

that is, the épistèmes that correlate epistemological paths that constitute the historical construction of mathematical theories, in order to provoke interconnections that involve mathematics from the past and the present, intending to give meaning to the process of apprehension and appropriation of information by students, in their movements of incorporation of styles of thought that one intends for them to appropriate and incorporate into their cognitive exercise.

This movement around cognitive activation through history underlies an excerpt extracted from Maia (2015) when he addresses relationships between history, science, and language, expressing that one of the greatest challenges, both in the research in the field of history and in science studies, is to learn the historical character that involves the past and the present-future of its objects. To argue in this regard, the author emphasizes that:

> Human activities occur in a scenario that is historically constituted, that is, the perceptions that one has in a given time and place are always produced from others that preceded and that will be replaced by them. Any new understanding of something always starts from the previous understanding of it– this is the notion of historical becoming (MAIA, 2015, p. 16).

This movement of transformation of the understanding of the facts, from the connections between what is already established and what it intends to establish, leads us to compose the meaning for the concept of history as a cognitive activator, although still needing more explanatory details, among which we can mention the importance of the facts already internalized in the individual who learns in favor of the dynamics of their enrollment in a new adventure that involves their access to new knowledge, that is, to the process of appropriation of information and its interpretations.

Therefore, to have a more detailed approach to this process of activating human cognition through the exploration of historical facts, first, we must clarify the notes resulting from investigations on cognition, which are currently part of the body of knowledge of cognitive sciences, as the first studies referring to cognitive aptitudes or competencies in the classic sense of the term (reasoning, language, perception, and action).

Likewise, it is necessary to enter the studies that focused on the constitutions and material realizations of different types of neurophysiological (in Biology) or electronic mechanisms, and even mechanical (in artificial intelligence) mechanisms, and eventually, the studies that characterized the modes of functioning of the activities of these mechanisms until we are able to describe them as revealing processes in elementary operations, and then model them in terms of properties that make them formal, as described and analyzed by Georges Vignaux (1995).

Therefore, it is based on these groupings of studies that denote the movements of the cognitive sciences around the development of human thought that I believe it to be possible to establish correlations involving the development of mathematical ideas throughout human history, their systematic organization, and representation in the form of ideographic writing[10]. Such writing was present in different modes of expression and communication, such as cave paintings, graphic records on the bones of various animals, clay tablets from Mesopotamia, manuscripts produced before the creation of the press, and the various books produced and inserted in the context of the relations between society, cognition, and culture, as a variety of writing supports invented to express thought through different codes.

Regarding the invention of the most varied supports for recording writing as an expanded memory of the human brain, which was historically established through the history of books, we can mention comments by René Salles (1986), when he addresses essential aspects about the 5000 years of a history of the written forms that resulted in books as a memory of human writing in all its forms of expression and communication of thought and orality about what was intended to be explained from the individual and collective understanding through a process of information socialization.

Along these lines, Salles (1986) considers that the movement for the invention of writing marks the prehistory of the modern book, based on the most varied supports established by human society to give names, structure, and function to the natural and artificial things that shaped society's organization on the planet.

The same movement is manifested by Georges Jean (2002) regarding human memory through writing in *A escrita: memória dos homens*, when he asserts that 20,000 years before our Era, humans were already making their first drawings, although we have archaeological confirmation that it was only 17,000 years later that writing appeared in its most organized form as ideographic writing, with indications of the creation of the first representation codes through ideograms (written signs), which later originated the signs of writing in different human cultures

[10] Ideographic writing is a way of representing language through the use of special drawings called ideograms or symbols that represent the ideas you want to communicate, that is, its principle is to represent the meaning of things. Over time, the drawings were exchanged for symbols that made it possible to represent more complex words of communication and the writing was continuously modified. For more details, see *Writing: memory of men*, by Georges Jean (2002); *History of Writing*, by Steven Roger Fisher (2009); or *5000 ans d'históire du livre* by René Salles (1986), *History of the Alphabet* by John Man (2002) and *Dynamic ideography* by Pierre Levy (1997), mentioned in the references at the end of this article.

on the planet. Thus, the first stories of human memories began to be recorded in the form we know today.

To this end, Jean (2002) reiterates that the writing system did not emerge in a few millennia, but during a long space-time history that is confused and intertwined with human history itself. One of the ways of making of this history makes manifest the presence of ideographic writings that contain indications of negotiations, whose record supports were clay tablets from the Mesopotamia region. However, ideographic writing expresses cultural sociodynamics and seems to be much older than this period.

It is by admitting this creative cultural sociodynamics[11] that we understand it to be possible for the history of Mathematics to potentially exercise a movement that activates students' understanding of the mathematical knowledge that they are expected to learn during school activities, using the history of the conceptual development of Mathematics as a basis for supporting knowledge for such learning. It is a mental architecture, concerning artificial intelligence, which involves three components, as Vignaux (1995) points out when he asserts that:

> The basis of the knowledge that contains all the information related to the domain we are dealing with [the object of knowledge] and that is written in a language of knowledge representation;
> The basis of facts (working memory), which contains the data of the problem to be dealt with and which, by keeping a record of the reasoning produced, plays the role of an auxiliary memory;
> Finally, the inference engine, that is, the program destined to use the data and heuristics of the knowledge basis to solve the different problems related to this information and these data (Vignaux, 1995, p. 30-31, sic).

The artificial intelligence I mentioned earlier was already being discussed since the 1990s (Vignaux, 1991, 1995) as a movement of researchers in the cognitive, mathematical and computational sciences around the construction of a model of representations of human behavior and capabilities, which has been expanding since the latter half of the twentieth century, intending to show how these three components, that Vignaux deals with, make up a system of interactions in order to attribute meaning to a set of diverse conceptual organizations that have expanded over time and went on to compose the diversity of historical documents recording human interpretations of historical objects, which have constituted —and still do— the defining épistèmes of the pillars of theories of knowledge that have been socially disseminated and activated whenever they were necessary.

[11] On the sociodynamics of creative culture, see Moles (2007, 2012).

Perhaps due to this understanding, Vignaux (1995) adopted the three terms – knowledge basis, fact basis, inference engine - to offer ways of interpretive triangulation of each movement around cognitive activations, in order to establish relations between the historical triad subject-object-context in the production of knowledge. Currently, the even greater expansion of artificial intelligence through computer networks has led scholars in this field to expand the network of explanations and conceptual expression of the subject, to the extent that the need has arisen for other environments to represent the interpretations of phenomena and historical facts, as in the case of interactive computer networks.

Springing from this need, the concept of ideographic writing was extended to the creation of the concept of dynamic ideography, proposed by Pierre Levy (1991, 1997), in view of the possibility of constituting an adequate environment for the exercise of an artificial imagination that would project the capacities human creative expressions through an image-thought, whose support would be the computer, with the function of processing ideographic writings through new dynamics as a renewal of the already existing forms of language, but with the insertion of constitutive processes of the creation of each object of knowledge.

Thus, as emphasized by Pierre Levy (1991, 1997) communication would occur through the expanded use of graphics in all its states and thus this new form of ideography (now dynamic) would constitute an intellectual technology, whose central role would be to express processes related to the functioning of human imagination, and increasingly aggregating, in an integrative way, the three poles of communication, namely oral, written, and digital (LEVY, 1993, 2004).

Once again, we encounter evidence of cognitive activation processes as the basis of the dynamic ideography proposed by Pierre Levy, made evident in the emphasis on human imaginative thinking, expanded by the artificial domain of the computational environment, thus considering the expansion of cognitive dynamics from the brain to the machine. This movement became strongly necessary and emerged at the end of the 1980s, whose first theoretical models discussed would already appear in 1991, in the publication of the first edition of the French version of Levy's book, which dealt with the proposition of his model of problematization and explanation of the object that would be expanded in the following decades.

In this regard, we reiterate the implications of this movement in the relations of cognitive activation through history in different research supports in search of understanding, because, as already highlighted by Maia (2015), as the subject is dynamic and historical, the processes of cognitive activation involve the three pillars mentioned by Vignaux (1995) as supporters of these activating processes, according to the position of each one at the moment they trigger the synapses provoked by each historical fact taken as a cognitive exercise in search of learning. Starting there, it may become feasible to practice exercises of establishing interconnection involving past and present, so that students can reach an understanding of the epistemic movement around concepts that triggered the constitution of new concepts, and therefore the expansion of objects of knowledge.

However, we must also have a clear vision of the connection movement of the épistèmes that involve the student's cognitive drive to understand the development of mathematical concepts, properties, and relationships throughout human history. It is a dynamic that intends to appropriate school mathematics and therefore requires cognitive reorganization exercises. Therefore, I felt the need to rethink the concept of cognitive activator, as will be discussed in the following section.

History as a cognitive reorganizer in mathematical learning

With a slightly different character from the movement described in the previous section, the action that underlies the term reorganizer refers to the fact of taking what is already available and attributing a new configuration or disposition to the history of Mathematics according to the interests of each individual, bearing in mind what they want to recompose. It is a matter of giving a new meaning to what already exists, often modifying its previous functions and other times assigning to it new representations to reach the desired objective.

It is with this understanding that the meaning given to history as a cognitive reorganizer in mathematical learning corresponds to providing, to both teachers and students, the possibility of renewing and expanding their understanding regarding certain mathematical concepts, properties, and relationships that were constructed throughout the historical-epistemological development of Mathematics through creative processes that have the potential to expand cognitive exercises and can promote a relational understanding, as advocated by Richard Skemp (1978) when he emphasizes the importance of expanding schemes related to the representation of a mathematical concept for the constitution of this type of understanding.

The adoption of the history of Mathematics in the teaching of school Mathematics in this process of cognitive reorganization involves the teachers' actions when planning their teaching activities, as well as the students' actions in the tasks proposed by their teachers and in the comprehension-explanation of the connections that will allow them to identify, reflect on, and organize forms of representation that indicate their —the students'— cognitive development in relation to the topic presented by the teacher, so as to give feedback on the degree of achievement of the objectives planned to help the students to accomplish their learning.

Therefore, this movement around history as a cognitive reorganizer can strengthen the development of student creativity in favor of their learning, as well as provide opportunities for innovative exercises to benefit the multiplicity of mathematical interpretations and representations of the subjects investigated in a historical text originating from primary, secondary or other sources not directly related to Mathematics, such as the histories of religions, of art in general, or the histories of other sciences, techniques or socio-cultural practices in general.

As an example of history as a cognitive reorganizer in mathematical learning, we can mention historical episodes related to the development of mathematical concepts. Such episodes are understood as clippings of historical facts that one examines to understand and explain the historical-epistemological development of a mathematical topic, aiming to enable comprehensive learning processes of the topic, as well as reflections on the influences it suffers, and its implications for the expansion of the field of Mathematics in its current state. Depending on the degree of depth, it is necessary to organize sets of historical episodes, which, if they are systematized in a coherent way, according to the organizational logic of the contents of the teaching programs and textbooks adopted by the school, can constitute a historical didactic sequence (SHD) to be used when teaching mathematical contents.

A suitable example we can use to clarify this subject is the concept of function, because when several historical episodes related to this theme are reorganized in logical historical stages by a researcher or by a teacher, they generally tend to indicate processes of conceptual establishment of the theme, from the interpretations of the Historians regarding the “proto-functional”[12] expressions represented on the tablets identified in Babylonian sociocultural practices (c. 2000 BC), in the representations of the Pythagoreans (c. 6th century BC), in relationships that involved problematizations on the chords of the circumference in the Almagest of Ptolemy (c. 3rd century), in the representation of the laws of nature by Oresme around 1350, in the systems of representations of Galileo, Leibniz, and Descartes, in the 17th century, in the conceptual formulations of D'Alembert, Euler, and Lagrange in the eighteenth century, even in the most up-to-date formulations established in the nineteenth century, by Dirichlet, Lobatchevsky, and Cauchy, in order to compose a panorama of conceptual

[12] The use of this term refers to practices and diverse representations that establish relations between embryonic ideas on the subject, which later gave rise to the first notions about the concept of function, whose epistemic trajectories were formalized and structured more appropriately from the 17th century onwards.

development that brought in its essence a structural matrix that denotes a process of reconfiguration of the epistème related to the subject.

In this regard, Mendes (2019, 2021b) asserts that the history of the development of the concept of the function shows the exercise of creative actions (conceptual, constructive), characterized by functional models, such as the logarithmic function originated in the works of John Napier (1550- 1817) and moved towards algorithms that involved studies on infinitesimals, originated by the method of John Wallis in his *Arithmetica Infinitorum*, implying new studies on the mathematical object of function, established by the principles and laws of variations, consolidated in the works of René Descartes (1637) on geometric curves and the algebraic functions that represented such curves, Isaac Newton's physical explorations and Leibniz's logical systematization, expanding the concept of the function by resuming embryos planted by Archimedes, Oresme, and Cavalieri. It was, therefore, a matter related to the continuity of these activities, which allowed the establishment of the algebraic analysis produced mainly in the 18th century, in the works of Daniel Bernoulli (1700-1782) and Leonhard Euler (1707-1783), and which later opened spaces for the phenomenon of multiform functions.

To the extent that this movement of building a chain of conceptual meaning and multiple representations of a mathematical theme is problematized through the exploration of episodes extracted from historical texts, in the form of historical investigation, it becomes possible for students to situate and reorganize their thinking concerning the subject and, with the teacher's support, broaden their understanding of what they are supposed to learn, because, as Fleck points out, "when one more domain of expertise (knowledge) is systematically elaborated and rich in details and relationships with other domains, the smaller are the differences of opinion" (FLECK, 2010, p. 50). This is the same justification given by Skemp in his network of schematic connections to expand relational understanding, especially when the schemas are associated with ways of understanding and explaining the established and replaced épistèmes over time and historical spaces.

In view of what was discussed in these two sections, I believe that I have explained the context in which the resumption of aspects arising from my experiences and reflections on the ways of incorporating history in the teaching of Mathematics became gradually viable, until reaching an understanding that led me to suggest the incorporation of history as an agent of cognition in the teaching of Mathematics, which would make it possible to establish, in teaching, situations that mediate investigative, problematizing, interpretive, and creative practices to be incorporated in teaching, as I will discuss in the following section.

History of Mathematics as an agent of cognition in Mathematics teaching

In this section, my discussion continues to be supported by theoretical foundations such as those arising from Fleck's propositions (2010), from which I adopted the term *agent*, when considering an aspect addressed by the author when he discusses the historical dynamics of activity-passivity in the production of scientific knowledge when he criticizes the classic subject-object dichotomy inserted in the confrontation between realism and relativism, concerning the process of understanding and explaining human cognition. In this sense, I interpret that in Fleck's dynamics the production of knowledge focuses on the active-passive relationship, considering that subject and object are independent entities that provide distinct understanding and explanation for the conflict of realism *versus* relativism, given that both subject and object are agents of the historical movement which are articulated with each other.

It is a sociological notion about science, that founded the concept of mediation as a result of human intentions and decisions that must be amplified and expanded toward animate or inanimate things, in favor of a cognitive activity that promotes movements around an understanding of conceptual development, and which constitute the pillars of the organization of scientific knowledge, as in the case of the different kinds of academic and school mathematics produced in diverse sociocultural contexts, that are socially transformed, expanded and disseminated in diverse sociocultural environments such as schools.

In order to make clear the justification that the history of Mathematics constitutes a mediating device for the materialization of the cognitive act on the part of the teacher and, consequently, involved in the student’s learning experiences, one can make use of historical documents that are considered primary or secondary sources, as well as the historical records expressed in the most diverse artifacts, instruments, tools and other objects, of material or immaterial culture, that elicit questions that can establish symbolic-material mediation in the sense of sharpening thought, or the elaboration of conjectures and partially conclusive enunciations, followed by their multiplicity of representations that can be adopted in Mathematics teaching, in order to promote comprehensive student learning.

To process the mediation, I consider that all objects that comprise the historical context in which the mathematical facts are configured must be taken as individual or collective agents, considering that they make up a collection whose elements are interconnected to give meaning and expanded meaning to what one investigates, aiming to reach an understanding through cognition. Thus, the investigation of historical sources can be situated as a memorial device socially established in documents and other sources of crystallized knowledge which can mediate the teacher's ways of teaching and the learning processes in a dynamic and reflective way that the students will focus on.

In this case, history exerts an agency that produces senses and meanings through cognitive exercises that articulate brain synapses that create a memorialistic field in which relations between the past and the present arise, with regards to mathematical facts, which affect and transform the students' processes of understanding regarding the epistemological development of Mathematics. Such a mediation is not defined as a single intentional human act but becomes an action that produces some effects that need to be explored according to the teacher's interests in their Mathematics classes.

In my reflections, I adopted the term *to affect* in this text, according to the concept established by Jacques Derrida (1999), when he argues that the motivation for the subject of knowledge (for learning) to "be affected" does not refer to an innate quality of the agent in relation to the mediation, since it —that is, the motivation to be affected; in other words, to develop affection—depends on the learning resulting from previous experiences and on the agent's thinking style, as emphasized by Fleck (2010), thus influencing the configuration of the traits of affection caused by the agent, to originate types of decodification and representation of the understanding reached by the student regarding the historical and epistemological development of Mathematics.

These traits of affection that Derrida (1999) refers to appear as a vestige of something, before its meaning, that is, a sensory perception that is not yet significant, as mentioned by Maia (2015), when he asserts that:

> The signification process will occur in the writing in which the agent is inscribed (their Fleckian collective). This provides a harmony between these authors that feeds the notion that the undefined sensory impressions are a portrait of the invasion of a hypothetical reality, outside of language, in the historically constituted reality. Between the imagined "real" as such and the historical reality verified in practice, there is the mediation of language (Maia, 2015, p. 46).

Regarding mediation through language, it is important to highlight that the writing of history (historiography) as a representation of realities, is constituted by a set of discursive traits that involve symbolic agents re7presented by historical sources that lead us to write the history (of Mathematics) and, consequently, reinvent these symbolic agents, now as sources of a history mobilized around an object of learning, that is, a new symbolic agent that triggers the cognitive processes to apprehend the reality that one intends that the student incorporates.

Another important aspect to be addressed in this section refers to the impacts that affect individuals cognitively, that history as an agent of cognition can cause in the form of practical activities, that is, the ways in which cognitive dynamics are organized for mathematical learning when students experience experimental and manipulative practices, with the use of historical artifacts or prototypes of these artifacts, or with the exploration of simulacra in the form of dynamic ideographies, which represent mathematical experiences historically written or documented, now located in virtual environments through software or computational applications.

It is an activity that involves at the same time mental and material actions simultaneously, since these updated and situated historical practices promote interactivity that connects the mind in an integrative way, exercises that strengthen mathematical skills such as counting, measuring, estimating, combining, graphing, sizing spaces, etc.

Final notes and further remarks

As I have already mentioned throughout this article, it was with the intention of finding ways that could offer teachers teaching strategies towards a learning of mathematical culture in its epistemic aspects suitable for a relational understanding, that, over more than two decades I have developed studies and research with the intention of experimenting with a multiplicity of didactic strategies that could be associated with the investigative, problematizing and fundamental principles of teaching and learning mathematics with the historical-epistemological development of Mathematics. Based on these experiences and reflections, I established principles and methods that converged to create foundations and teaching methods that would support the use of history as a cognitive activator in mathematical learning, as a cognitive reorganizer in mathematical learning, and as a cognition agent in mathematics teaching. and mathematical cognition in the classroom.

After carrying out experimental exercises based on these founding principles, I organized a theoretical-practical model called *history as a didactic and conceptual mediator* (HMDC), which presupposes the incorporation of the modalities of using history in the teaching of Mathematics already mentioned in this article, involving information directly extracted or adapted from the history of Mathematics, texts and historical problems extracted or adapted from primary sources, and finally the possibility of using digital technologies for the production and didactic use of videos and other applications having primary or secondary historical sources as a starting point. In most of these exercises, the main basis has always been the development of thematic research projects on the history of Mathematics.

The proposal to materialize this didactic model in classroom practice should follow certain guidelines, such as the identification and selection of some historical themes to be investigated and then planning, executing, and evaluating actions that involve the development of historical investigations of the selected themes, through individual or collective research with students in the classroom or through actions carried out outside the classroom.

Afterwards, the teacher should ask the students to work out the elaboration of didactic situations that involve the established problematizations and present such situations in the classroom, in the form of seminars that promote discussions, analyses, and syntheses of each topic addressed, followed by a qualitative analysis of the investigative process carried out, looking forward to a possible reorientation of the didactic approach to school contents based on the investigation carried out.

The constitution of the various episodes of the written and rewritten history of Mathematics, according to the epistemological lens of each researcher who writes them, causes the emergence of some echoes regarding the creative dynamics of each historiographical Mathematics, that is, of traits that characterize the creative processes in the historical development of Mathematics.

Thus, I reiterate the considerations established since the opening parts of this essay, when considering that when one intends to understand the production of Mathematics, one must dive deep into the epistemologies contained in the written histories of this Mathematics or in other fields directly or indirectly related to this branch of knowledge, for it is a kind of creation that occurs through a conceptual historical process of continuous flow characterized by a *historical sequential movement* (MSH) that is constantly expanding, which is processed discontinuously (does not obey a chronological or spatial movement, that is, it is disordered and does not present an organized sequence), which happens in different ways and in different places, with the involvement of different groups, who are not always studying the same problem, and, especially, not in the same way.

Regarding this subject, I highlight some of these echoes such as for example, the historical sequential movements (MSH) related to the irrational, to the problems about squaring the curves and their conceptual movements, to the creative processes regarding Cavalieri's indivisibles and the trajectory of the concept of variables, functions and differential and integral calculus, as well as complex numbers, the conceptual trajectory of analytic geometry before and after the studies of René Descartes, the method of fluxions of Isaac Newton and Colin Maclaurin, the composition of the field of string trigonometry, semi-strings, flat and spherical triangles, algebraic representations in forms of writing, in different historical periods, non-

Euclidean geometries created by problematization or just imagined, among other imaginations or mathematical creations (Mendes, 2020).

Given the above, I consider that historical sequential movements (MSH) must be constituted by a logical organization that overcomes the space-time discontinuities that configure the creation of each mathematical theme, as is the case of the historical-epistemological development of irrational numbers. These are exercises of (re)writing the histories of this conceptual development to shape these sequences. Depending on the objectives and issues to be dealt with in the classroom, teachers must add to each sequence, and to the ordering of the teaching programs, as an explanatory complement for the understanding of the school contents that are planned for each school level.

Equally, the MSH must be connected to the thematic units of the textbooks adopted by the teacher for the construction of a tripartite model towards a comprehensive or relational learning of the subject that one intends the students to appropriate. It means, therefore, that the history of investigated Mathematics (HMI) presents itself as an activator of mathematical cognition in its constructive process, to then be structured as a practice that enables a cognitive reorganization of the student's mathematical awareness when it is associated with school mathematical knowledge (CME) established in the teaching program and materialized through the mediation of the student's cognition to correlate historical knowledge, systematic ordering based on this teaching program and materialized in the mathematical approach contained in the textbook (MLD).

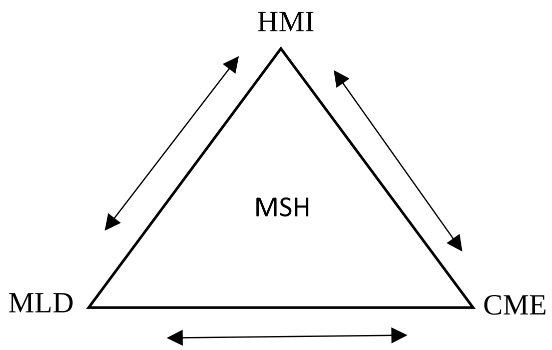

Figure 1.

A representation of the Historical Sequential Movement (MSH) in its interconnections involving the history of Mathematics, Mathematics teaching program and Mathematics textbooks.

According to the representation shown in Figure 1, we have an operationalization configuration involving the sequential historical movement (MSH) in its integrated connections between the history of Mathematics, the Mathematics teaching program, and the Mathematics textbooks. It is a continuous process of intramathematical[13] integration that associates information on the historical conceptual (epistemological) development of Mathematics in its complementary insertion to the explanations of the syllabus of Mathematics that should be materialized in the textbook.

To make use of it, the teacher needs to develop dialogic integration skills with the three forms of knowledge organization, based on an initial historical investigation and the practice of connecting the investigated history to the school contents presented in the teaching program and in the textbook adopted by the school, as long as each part of the historical sequence is inserted in adequate contexts of the lesson plan, which need conceptual clarification regarding the epistemic movement of the theme addressed. It is through the exploration of this epistemic movement established by the MSH, incorporated into the approaches to mathematics contained in the textbook, that the school mathematical contents (CME) can be mediated for the student's cognitive exercise of comprehensive learning in the moments when the teachers explore the history of Mathematics as a cognition agent in the teaching of Mathematics.

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[13] The term intramathematical corresponds to the contexts of interconnections that involve concepts, properties and relationships established within the area of knowledge itself; in this case, mathematics. These interconnections were developed throughout the history of the development of Mathematics and became the basis of the process of axiomatization of mathematical theories over time.

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