

The Pedagogical Residency Program as a formative and mobilization space for Didactic-Mathematical Knowledge: Theoretical and methodological perspectives

El Programa de Residência Pedagógica como espacio de formación y movilización del Conocimiento Didáctico-Matemático: Perspectivas teóricas y metodológicas

Le Programme de Résidence Pédagogique comme espace de formation et de mobilisation des Connaissances Didactiques-Mathématiques: perspectives théoriques et méthodologiques

O Programa Residência Pedagógica como espaço formativo e de mobilização do Conhecimento Didático-Matemático: Perspectivas teóricas e metodológicas

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Abstract

In this article we assume the perspective of the meta-analytical study to present discussions about theoretical and methodological perspectives that contribute to the understanding of the Pedagogical Residency Program as a formative space and mobilization of Didactic-Mathematical Knowledge. For this purpose, a set of five master's theses defended in the Postgraduate Program in Mathematics Education at the Federal University of Ouro Preto, linked to the Center for Studies, Research, and Formative Practices for Teachers who teach Mathematics, was considered the corpus of analysis, led by the authors of this article. Data were organized and analyzed based on three analytical categories: i) DMK: Interactional Ecological Facets; ii) Didactic Suitability; and iii) Formation of Teacher Educators. To analyze them, we sought theoretical support in studies that deal with teacher education and Didactic-Mathematical Knowledge. In the theoretical dimension, research has been concerned with reflecting on the theoretical construct of the OSA, particularly regarding the facets of the DMK

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and the didactic suitability criteria. The theoretical deepening and the contribution, with its advances, to the research are fundamental for new investigative horizons to emerge. Regarding the methodological dimension, investigations within the scope of NEPEFEM have shown the importance of using the criteria and components proposed by the DMK model in data production and reflection on teaching practice. In addition, the studies analyzed here indicate the need to improve and adapt them to the reality of mathematics teacher education in Brazil, especially when these researches turn to public policies, as is the case of the PRP.

Keywords: Pedagogical Residency Program, Didactic-Mathematical knowledge, Didactic suitability, Preceptor, Formation of mathematics teachers.

Resumen

En este artículo asumimos la perspectiva del estudio metaanalítico para presentar discusiones sobre perspectivas teóricas y metodológicas que contribuyan a la comprensión del Programa de Residencia Pedagógica como espacio de formación y movilización de saberes didáctico-matemáticos. Para ello, se consideró como corpus de análisis un conjunto de cinco Disertaciones de Maestría defendidas en el Programa de Posgrado en Educación Matemática de la Universidad Federal de Ouro Preto, vinculada al Núcleo de Estudios, Investigación y Prácticas de Formación de Profesores que enseñan Matemática, dirigido por los autores de este artículo. Los datos fueron organizados y analizados en base a tres categorías analíticas, a saber: i) MDL: facetas ecológicas interactivas; ii) Idoneidad Didáctica y iii) Formación de formadores. Para analizarlos, buscamos apoyo teórico en estudios que versan sobre la Formación del Profesorado y el Conocimiento Didáctico-Matemático. En la dimensión teórica, la investigación se ha preocupado por reflexionar sobre el constructo teórico de la EOS, en particular, en lo que se refiere a las facetas del MDL y los Criterios de Adecuación Didáctica. Entendemos que la profundización teórica, así como el aporte con sus avances, desde las investigaciones, es fundamental para que surjan nuevos horizontes investigativos. En cuanto a la dimensión metodológica, las investigaciones en el ámbito del NEPEFEM han mostrado la importancia de utilizar los criterios y componentes propuestos por el modelo CDM en los procesos de producción de datos y reflexión sobre la práctica docente. Además, los estudios aquí analizados indican la necesidad de perfeccionarlos y adaptarlos a la realidad de la formación de profesores de Matemática en Brasil, especialmente cuando esas investigaciones se vuelcan a políticas públicas, como es el caso del PRP.

Palabras clave: Programa Residencia Pedagógica, Conocimiento Didáctico-Matemático, Idoneidad didáctica, Preceptor, Formación de profesores de matemáticas.

Résumé

Dans cet article, nous adoptons la perspective de l'étude méta-analytique pour présenter des discussions sur les perspectives théoriques et méthodologiques qui contribuent à la compréhension du programme de résidence pédagogique en tant qu'espace de formation et de mobilisation des connaissances didactiques et mathématiques. À cette fin, un ensemble de cinq mémoires de maîtrise soutenus dans le programme d'études supérieures en enseignement des mathématiques à l'Université fédérale d'Ouro Preto, liés au noyau d'études, de recherche et de pratiques de formation des enseignants qui enseignent les mathématiques, a été considéré comme le corpus d'analyse, menée par les auteurs de cet article. Les données ont été organisées et analysées selon trois catégories analytiques, à savoir: i) MDP: facettes écologiques interactionnelles ; ii) Aptitude didactique et iii) Formation des formateurs. Pour les analyser, nous avons recherché un support théorique dans des études qui traitent de la formation des enseignants et des connaissances didactiques et mathématiques. Dans la dimension théorique, la recherche s'est attachée à réfléchir sur la construction théorique de l'EOS, en particulier en ce qui concerne les facettes du MDP et les Critères d'adéquation didactique. Nous comprenons que l'approfondissement théorique, ainsi que l'apport avec ses avancées, des recherches, est fondamental pour que de nouveaux horizons d'investigation émergent. En ce qui concerne la dimension méthodologique, les investigations menées dans le cadre de NEPEFEM ont montré l'importance d'utiliser les critères et composants, proposés par le modèle CDM, dans les processus de production de données et de réflexion sur la pratique enseignante. De plus, les études analysées ici indiquent la nécessité de les améliorer et de les adapter à la réalité de la formation des professeurs de mathématiques au Brésil, surtout lorsque ces recherches se tournent vers les politiques publiques, comme c'est le cas du PRP.

Mots-clés: Programme Résidence Pédagogique, Connaissances Didactiques-Mathématiques, Aptitude didactique, Précepteur, Formation des professeurs de mathématiques.

Resumo

Este artigo assume a perspectiva do estudo meta-analítico para apresentar discussões acerca de perspectivas teóricas e metodológicas que contribuam para o entendimento do Programa Residência Pedagógica como espaço formativo e de mobilização do conhecimento didático-matemático. Para tanto, considerou-se como corpus de análise um conjunto de cinco dissertações de mestrado, defendidas no Programa de Pós-Graduação em Educação Matemática da Universidade Federal de Ouro Preto, vinculadas ao Núcleo de Estudos, Pesquisas e Práticas de Formação de Professores que ensinam Matemática, liderado pelos autores do presente artigo.

Os dados foram organizados e analisados a partir de três categorias analíticas, a saber: i) CDM: Facetas Ecológica Interacional; ii) Idoneidade didática e iii) Formação de formadores. Para analisá-los, buscou-se respaldo teórico em estudos que versam sobre a Formação de Professores e sobre o conhecimento didático-matemático. Na dimensão teórica, as pesquisas têm se ocupado de refletir o construto teórico do EOS, em especial, o que concerne às facetas do CDM e aos critérios de adequação didática. O aprofundamento teórico, bem como a contribuição com seus avanços, a partir das pesquisas, é fundamental para que surjam novos horizontes investigativos. No que tange à dimensão metodológica, as investigações no âmbito do NEPEFEM têm mostrado a importância de utilizar critérios e componentes, propostos pelo modelo CDM, nos processos de produção de dados e de reflexão sobre a prática docente. Ademais, os estudos aqui analisados sinalizam a necessidade de aperfeiçoá-los e adaptá-los para a realidade da formação de professores de matemática no Brasil, em especial, quando essas pesquisas se voltam para as políticas públicas, como é o caso do PRP.

Palavras-chave: Programa Residência Pedagógica, Conhecimento Didático-Matemático, Idoneidade didática, Preceptor, Formação de professores de matemática.

The Pedagogical Residency Program as a formative and mobilization space for Didactic-Mathematical Knowledge: Theoretical and methodological perspectives

In 2020, after a modality change process with the Coordination for the Improvement of Higher Education Personnel (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - CAPES), the Postgraduate Program in Mathematics Education (Programa de Pós-Graduação em Educação Matemática - PPGEDMAT) began to offer the academic master's degree. As a result, the authors of this article founded the Center for Studies, Research, and Formative Practices for Teachers who teach Mathematics (Núcleo de Estudos, Pesquisas e Práticas de Formação de Professores que Ensinam Matemática - NEPEFEM). Since then, NEPEFEM has guided teachers' education based on the repercussions of existing programs and public policies in the country.

Considering the authors' trajectory and professional performance, the Pedagogical Residency Program (PRP), established in 2018 through Ordinance N. 38, was assumed as an investigative context of NEPEFEM. We started from the premise that "PRP is a relatively new program with still many issues to be investigated" (Tinti & Silva, 2020, p. 168). Furthermore, in our institutions, we took over the position of Institutional Coordinators of the PRP, which brings us closer and mobilizes us to problematize teacher education in the interface with this program.

In this process, we seek theoretical support in studies that deal with teacher education and in the perspective of the onto-semiotic approach to cognition and mathematical instruction (OSA), in particular, discussions related to Didactic-Mathematical Knowledge (DMK), as it can be "a basic theoretical repertoire for the planning, execution, and evaluation of formative actions in the field of initial and continuing education of mathematics teachers" (Silva & Tinti, 2021, p. 23).

Thus, the PRP has been the focus of several studies within the scope of NEPEFEM. In this article, we will resort to those done by Faria (2023), Frade (2022), Horta (2023), Martins (2022), and Santos (2023) to discuss theoretical and methodological perspectives that contribute to the understanding of the Pedagogical Residency Program as a formative space and mobilization of Didactic-Mathematical Knowledge.

To do so, we will initially contextualize the PRP and the DMK. Next, we will analyze the studies to respond to the proposed objective.

The Pedagogical Residency Program and mathematics teachers' education

The PRP was instituted through GAB Ordinance N. 38³, dated February 28, 2018. In 2019, CAPES published the regulation of the PRP and the Institutional Scholarship Program for Teaching Initiation (Programa Institucional de Bolsa de Iniciação à Docência - PIBID). However, after completing the first two notices, there was a need to reformulate this regulation to dissociate the two programs. Thus, on April 26, 2022, CAPES released a new regulation for the PRP.

The PRP is implemented through public notices. Three public notices of the referred program have been published: Public Notices n. 06/2018, n. 01/2020 and n. 24/2022. Table 1 illustrates the total quotas for residents offered in each of these notices:

Table 1.

Distribution of resident quotas by PRP notices

| Public Notice PRP | Quotas for Residents | Growth Rate |
|-------------------------|----------------------|-------------|
| N. 06/2018 | 45 000 | - |
| n. 01/2020 | 30 096 | -33.12% |
| n. 24/2022 | 30 840 | 2.47% |
| Cumulative Total | 105 936 | - |

As we can see, the PRP could contribute to the education of more than 106 000 graduates, given that there may have been exchanges of scholarship holders over the 18 months of validity of each public notice. However, there was a significant reduction (-33.12%) in total resident quotas from the first to the second edition. In 2022, the number increased slightly.

Tinti et al. (2021), in a study on Public Notice n. 01/2020, presented an overview of the distribution of quotas (scholarships) of that notice. The authors point out that of the 30 096 residents' quotas offered, 55.42% were allocated to residents in priority areas and 45.58% in general areas. These authors also point out the presence of the PRP in all Brazilian states and in approximately 20% of the Brazilian municipalities, which explains the scope and importance of the PRP for initial teacher education in Brazil.

When we analyzed the data related to Notice n. 24/2022, we can see that not all the HEIs covered could implement the quotas achieved for different reasons, among which, for example, the mismatch between the academic calendars of the HEIs and the calendar proposed in Notice

3 Available at: <https://www.gov.br/capes/pt-br/centrais-de-conteudo/28022018-portaria-n-38-institui-rp-pdf>

n. 24/2022, given that many HEIs are still facing delays in their academic calendars due to the Covid-19 pandemic.

However, with the change in the federal government and, consequently, of the presidency of CAPES, some actions indicate a new look at the PIBID and PRP Programs. Recognizing the importance of the PRP and the need to strengthen it, the Ministry of Education, through the President of CAPES, in an unprecedented way, published a second call to fill the requested resident scholarship quotas that were not implemented within the scope of the institutional projects approved through CAPES notice mentioned above. Table 2 shows the distribution of these quotas by region.

Table 2.

Distribution by regions of the quotas provided for in the second call of Notice n. 24/2022.

| Region | Total quotas | % |
|------------------|--------------|------------|
| Southeast Region | 2,175 | 38.67 |
| Northeast Region | 2,055 | 36.53 |
| South | 810 | 14.40 |
| North | 360 | 6.40 |
| Midwest | 225 | 4.00 |
| Total | 5,625 | 100 |

Public Notice n. 24/2022 is in force and is expected to close in March 2024. However, as PRP management data are not disclosed on the Capes site, it is impossible to accurately measure its scope or the number of people involved in institutional projects.

The PRP has been constituted as an object of study by researchers at different levels: course conclusion works (Souza & Tinti, 2022), scientific initiation, master's degrees and independent studies that often reflect on the experience of those involved with the PRP and contributions to teacher education and the quality improvement of basic education.

Following the example of this movement, in 2022, we organized a thematic issue in the journal *Educação Matemática Pesquisa* called "Mathematics Teacher Education at the Interface with the Pedagogical Residency Program" (Tinti & Silva, 2022)⁴. The issue brought 18 articles presenting different investigative focuses and reflections on how mathematics teacher education has taken place in the context of immersion in the school context provided

4 Available at: <https://revistas.pucsp.br/index.php/emp/issue/view/2716>. Accessed on: Apr, 20, 2023.

by the PRP. Among those articles, two were produced by members of the NEPEFEM (Martins et al., 2022; Silva et al., 2022).

In general terms, based on this thematic issue, we can point out that these have been the main points of discussion involving the PRP: mathematics teachers' education in contexts of interdisciplinary subprojects; the implementation of innovative practices (such as mathematical modeling, the use of technology and inclusive education); reflections on the experiences lived within the PRP mathematics subprojects; discussions about professional identity; and, finally, studies on the PRP and the mobilization of knowledge. These last two foci were assumed by the studies we propose to analyze. Thus, below, we will discuss briefly the Didactic-Mathematical Knowledge.

Theoretical perspectives

Over the last few years, nationally and internationally important investigations have been carried out on the initial and continuing teacher education, especially in mathematics (Ortiz & Alsina, 2017; Ponte, 2006). Among the different factors contributing to this scenario, curricular changes in basic education stand out. These changes require well-prepared teachers and a dialogue between teacher education and contemporary challenges (Font, 2011; Carpes & Bisognin; 2021).

Among the different approaches to mathematics teacher education, those that refer to knowledge have been highlighted because “[...] to improve students' mathematical education, we must pay special attention to the teacher's knowledge” (Ortiz & Alsina, 2017, p. 82). Different theoretical discussions have focused on the knowledge necessary for mathematics teachers (Ball et al., 2008; Hill et al., 2008; Pino-Fan & Godino, 2015; Schoenfeld & Kilpatrick, 2008). Based on these and other theoretical discussions, in the context of the onto-semiotic approach to cognition and mathematical instruction (OSA)⁵, Godino (2009) establishes important categories to analyze mathematics teachers' knowledge. Such categories constitute a model endowed with facets and levels for didactic analysis, as shown in Figure 1:

5 According to Godino (2009, p. 20) “The OSA is a theoretical framework that proposes to articulate different points of view and theoretical notions about mathematical knowledge, its teaching and learning. A global perspective is adopted, taking into account the different dimensions involved and the interactions between them.”

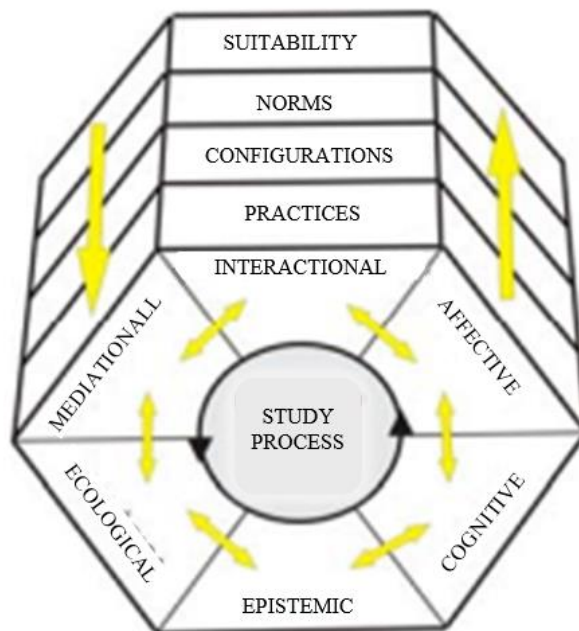


Figure 1.

Categories of mathematics teacher's Didactic-Mathematical Knowledge (Godino, 2009, p. 21, our translation)

In this model, the facets are defined, according to Godino (2009), as follows:

- *Epistemic*: refers to the mathematical knowledge related to the institutional context in which the study process takes place and the distribution in time of the different components of the content (problems, languages, procedures, definitions, properties, arguments).
- *Cognitive*: this concerns the students' personal knowledge and the assessment of their learning progression.
- *Affective*: relates to the affective states (attitudes, emotions, beliefs, values) of each student towards mathematical objects and the teaching and learning process.
- *Mediational*: addresses technological resources and means and the organization of time for different actions and processes.
- *Interactional*: comprises patterns of interaction between teacher and students, student-students, students and curricular materials, and sequencing oriented towards the fixation and negotiation of meanings.
- *Ecological*: encompasses the system of relations with the social, political, economic, and other surroundings that directly impact the process of teaching and learning mathematics in the classroom.

Figure 2 shows the levels of didactic analysis proposed by Godino (2009):

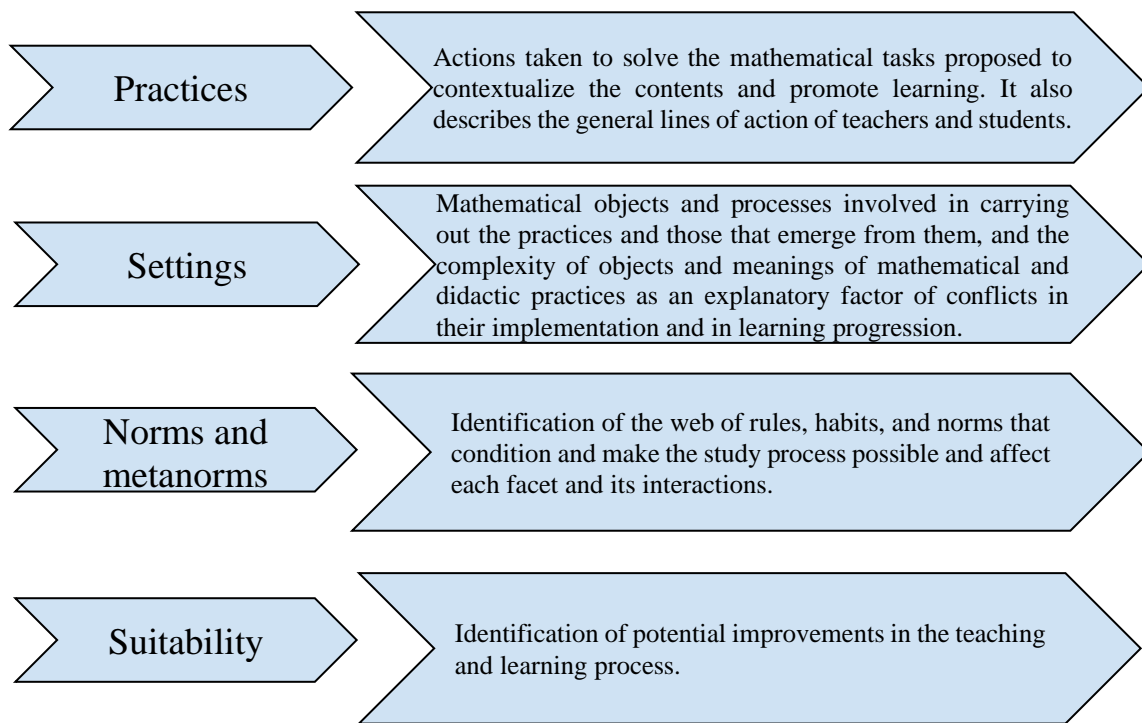


Figure 2.

Levels of didactic analysis - adapted from Godino's (2009) ideas

Such categories are closely linked to the teaching and learning process of mathematics, in particular, to the planning process, performance in the classroom and assessment and/or reflection on the practice that can be synthesized in the three dimensions:

- ❖ Mathematics: refers to the specific knowledge necessary for the teacher to teach the contents, i.e., the teacher mobilizes concepts, notions, and problem solving, taking into account the organization of the contents throughout the institutional curriculum (Pino-Fan & Godino, 2015).
- ❖ Didactics: concerns knowledge about the planning, execution, and evaluation of the process of teaching and learning mathematics.
- ❖ Didactic-mathematical goal: it is related to praxis, i.e., a practice endowed with reflection.

In this sense,

For each of the facets created in the context of the DMK's didactic dimension, as part of the development of the theoretical framework to which we adhere (OSA), suitability criteria to allow teachers to reflect on their own practice and identify potential improvements were provided (Pino-Fan & Godino, 2015, p.103).

The didactic suitability is embodied, according to Godino et al. (2007), into six components, as follows:

- *Epistemic suitability*: relates to the degree of representativeness of the implemented (or intended) institutional meanings in terms of a reference meaning placed in the curriculum. Furthermore, it makes it possible to reflect on the quality of the mathematics taught (concepts, rules, problems, language, properties, ...).
- *Cognitive suitability*: expresses the degree to which the intended/implemented meanings are in the students' zone of potential development and the proximity of the personal meanings achieved to the intended/implemented meanings. In particular, attention is paid to students' prior knowledge of the content studied.
- *Interactional suitability*: deals with a teaching and learning process that will be more appropriate from the interactional point of view if the configurations and trajectories allow, on the one hand, to identify potential semiotic conflicts and, on the other hand, resolve conflicts that occur during the instruction process. In this context, interactions between different actors (teachers, students, administrators, institutions, curriculum materials, ...) are fundamental to the educational process.
- *Mediational suitability*: concerns the degree of availability and suitability of material and temporal resources necessary to develop the teaching process. Thus, it is up to the teacher to select technologies, playful and/or manipulative resources, books and others, and manage teaching time and the learning process.
- *Affective suitability*: means students' degree of involvement (interest, motivation, ...) with the study process. Affective suitability is associated with factors that depend on the institution and those that basically depend on the student and his/her previous school history. It must be considered that there is a cultural motivation for resistance to mathematics, which culminates in the fact that many students carry these beliefs to school.
- *Ecological suitability*: refers to the degree of conformity of the study process with the education project of the center, school, and society and the environmental constraints in which it develops. The curriculum needs to be understood as a place of dialogue with social, political, economic, and cultural contexts.

Undoubtedly, in view of the above, according to Pino-Fan and Godino (2015), the components of the DMK, together with the tools for analysis and reflection on practice, are essential to describe and characterize in detail teachers' initial knowledge during the

preliminary, planning, implementation, and evaluation study phases of the mathematics teaching and learning process. In this way, this approach must be considered promising, both theoretically and methodologically, to reflect the implementation and development of public policies in the context of the degree in mathematics.

Therefore, taking into account teacher education as a space of complexity, NEPEFEM has undertaken studies and critical reflections aimed at understanding the knowledge necessary for teaching and, at the same time, has proposed reflections on educational practice in mathematics, particularly on planning, implementing, reflecting, communicating, and disseminating the experienced repercussions.

Methodology

This article discusses theoretical and methodological perspectives that contribute to understanding the Pedagogical Residence Program as a formative and mobilization space for Didactic-Mathematical Knowledge. We assume the viewpoint of a meta-analytical type qualitative study which, according to Fiorentini and Lorenzato (2006, p. 103), is “a systematic review of other studies, aiming to carry out a critical evaluation of them and/or produce new results or syntheses from the confrontation of these studies transcending those previously obtained” (p. 103).

To compose this meta-synthesis, we considered a set of research carried out within the scope of the Center for Studies, Research, and Formative Practices for Teachers who teach Mathematics (NEPEFEM)⁶, linked to the Postgraduate Program in Mathematics Education (PPGEDMAT) at the Federal University of Ouro Preto (UFOP). These are studies by Faria (2023), Frade (2022), Horta (2023), Martins (2022), and Santos (2023).

In addition to being a production of a research group led by the first and second authors, their choice is justified by the fact that they defend the Pedagogical Residency Program as a formative space. It also connects different theoretical approaches that deal with mathematics teacher education, among them knowledge and professional identity.

Data were organized and analyzed based on three analytical categories: i) DMK: Interactional Ecological Facets; ii) Didactic Suitability; and iii) Teachers' Educators' Formation. Below, we present the analysis of each.

⁶ <https://ppgedmat.ufop.br/nepefem>

Discussion and analysis

We grouped the five analyzed master's theses into the indicated categories, as shown in Table 3.

Table 3.

Analytical categories considered in the article

| Category | Research |
|--------------------------------------|-------------------------------|
| Educators' formation | Faria (2023) |
| DMK: Interactional Ecological Facets | Horta (2023) & Santos (2023) |
| Didactic suitability | Frade (2022) & Martins (2022) |

Below, we will present each of the studies, highlighting the theoretical and methodological perspectives considered in each.

a) DMK: Interactional Ecological Facets

Research by Horta (2023) and Santos (2023) aimed to discuss the ecological and interactional facets of the Didactic-Mathematical Knowledge of residents and preceptors. Considering the actions developed by the PRP – 2020 edition – within the scope of a federal institution located in the state of Minas Gerais, the researchers sought to highlight the mobilization of knowledge for the educational practices developed in the interrelationship between the educational institution and basic education schools. In this context, the authors' mobilization for the investigative process undertaken is in line with the current perspectives of teacher education, as “reflecting one's own practice constitutes an opportunity to (re)evaluate the teaching and learning process, especially on the relations between contents and contexts in which educational institutions are inserted (Silva et al., 2022, p. 328).

To better characterize the studies in question, we present them in Table 4:

Table 4.

Authors, general and specific objectives and research questions (Horta, 2023, p. 29; Santos, 2023, p. 30)

| Author | General objective | Specific objectives | Research question |
|--------------|--|--|--|
| Horta (2023) | <i>To investigate the interactional facet of the Didactic-Mathematical</i> | <i>Identify preceptor/resident interactions within the scope of the PRP.</i> | <i>How is the interactional facet of the didactic-mathematical</i> |
| | | <i>Identify interactions between residents within</i> | |

| | | | |
|----------------------|---|--|---|
| | <i>Knowledge from the link between the preceptor and the residents within the scope of a mathematics subproject of the Pedagogical Residency Program.</i> | <i>the scope of the PRP</i> | <i>knowledge presented from the link between the preceptor and residents within the scope of a mathematics subproject of the Pedagogical Residency Program?</i> |
| | | <i>Verify if the interactions between the preceptor and residents foster the development of autonomy in the context of the PRP.</i> | |
| | | <i>Understand how the preceptor and residents evaluate the actions developed and guided by interactions within the PRP scope.</i> | |
| | | | |
| <i>Santos (2023)</i> | <i>To study the elements of the ecological facet of the Didactic-Mathematical Knowledge mobilized by undergraduates participating in the Pedagogical Residency Program.</i> | <i>Investigate whether the pedagogical proposals are suited to the curricular bases.</i> | <i>What elements of the ecological facet of the Didactic-Mathematical Knowledge are mobilized by participants of the Pedagogical Residency Program?</i> |
| | | <i>To analyze whether the actions developed considered the social, professional, and cultural peculiarities of the public served and the use of technology-oriented resources.</i> | |
| | | <i>Identify possible elements that may indicate traces of the construction of knowledge based on democratic bases and encouragement of critical and reflective thinking.</i> | |
| | | <i>Check whether relationships were established between mathematical content and other areas of knowledge in the analyzed lesson plans.</i> | |

The authors' methodological choices were based on a qualitative approach, using different instruments for data production. In this way, they understand the importance of scientific rigor; however, they highlight in their research the dynamism of this research perspective:

We are aware that the research is carried out by an individual who, before assuming the identity of a researcher, has one's experiences, impressions of the world, and unique repertoire. Thus, the researcher is a producer of knowledge based on what is investigated, but at no time losing sight of the possible implications of one's view as a political and social subject inserted in a sociocultural system. (Santos, 2023, p. 31)

This research was developed amid the Covid-19 pandemic, which impacted the investigations carried out. Methodological refinements were continuous, and the dialogue between research planning and sanitary conditions was constant.

Therefore, to understand the context of the development of PRP actions, the authors began observing the meetings held by the Program's actors. Then, they constructed a questionnaire and applied it via email to the 19 participants in the PRP mathematics subproject and to two basic education teachers (preceptors). This instrument aimed to raise the group's main characteristics, motivations for participating in the PRP, contexts, experiences, and practices. As these were collaborative and cooperative investigations, the researchers worked together to meet the proposed objectives, avoiding the maximum impact on the development of

actions planned by the PRP actors; so, a single questionnaire was applied, designed in stages in both studies.

In the last stage, they interviewed residents and the preceptor to learn about the process experienced: their understanding of interactions and the contexts for which they carried out their lesson plans –due to the nature of his research, which was concerned with investigating the issue of the development of the ecological facet, Santos (2023) analyzed two lesson plans constructed by residents.

Horta's results (2023) were discussed in four categories, represented in Figure 3.

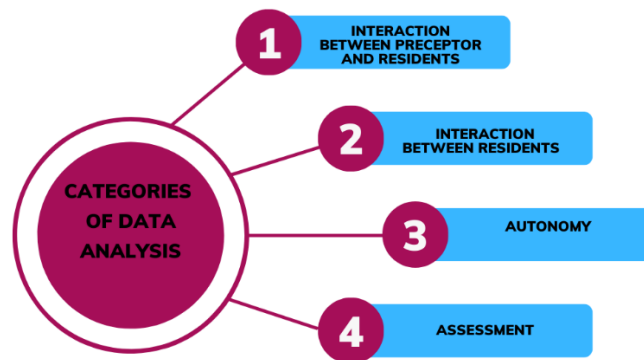


Figure 3.

Data analysis categories (Horta, 2023, p. 47)

To the author, the construction of categories was implemented from the reading, analysis, and interpretation of the components and indicators of interactional didactic suitability, proposed in the context of the onto-semiotic approach (OSA), which describe the indicators of the components that substantiate the teaching and learning process, based on the interactions between teachers and students; between students and students; in autonomy and formative assessment. From these categories, 17 subcategories, called descriptors, emerged.

We present an analytical category by Horta (2023) to exemplify the context of the results obtained.

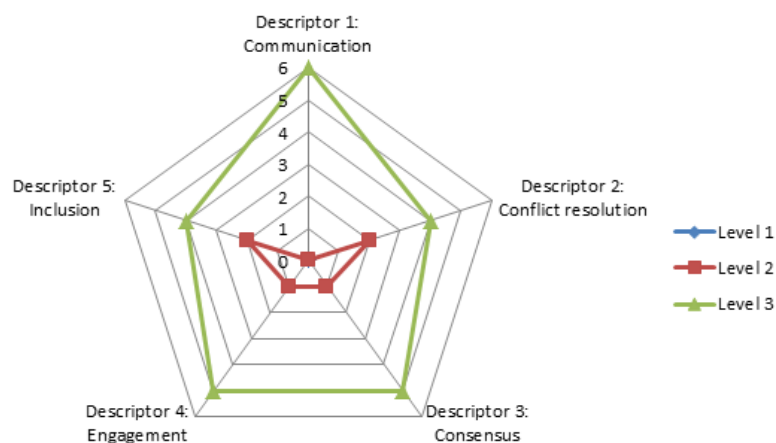


Figure 4.

Indicator levels of the interaction preceptor/residents category (Horta, 2023, p. 96)

According to Horta (2023), the data analyzed regarding descriptor 1, communication, allow us to point out that communication in the context of the PRP actually happens and that it meets the proposal of building knowledge based on practice, in which residents have the opportunity to interact with the entire process of their education as a prospective mathematics teacher. The author also emphasizes that the data show that the preceptor recognizes and resolves the residents' conflicts that eventually occur in the process, searches for consensus through plausible and coherent arguments with the demands of the program, and engages with the activities of the PRP, despite the limitations imposed by the pandemic.

Furthermore, Horta (2023) points out that the data analyzed concerning descriptor inclusion point to a preceptor's practice tangential to the inclusion of residents in the PRP dynamics, which favors the construction of knowledge and strengthens the professional identity of prospective teachers.

Regarding the level of partial inclusion, the moments in which some less participating residents were not called by the preceptor to engage more effectively in the debates and activities are highlighted. Another issue in this context concerns the very difficulty of that online interaction, with all the problems surrounding it: connection difficulties and lack of technical improvement, among others, in the context of the previously described pandemic.

Santos's (2023) research, carried out collaboratively with Horta (2023), focused on investigating the elements of the ecological facet of the Didactic-Mathematical Knowledge mobilized by undergraduate students participating in the Pedagogical Residency Program. Her investigation focused on the aspects dealing with the organization and how the PRP was situated

in the social, political, economic, and cultural context. To this end, the author organized the analysis into the following categories, arranged in Figure 5:

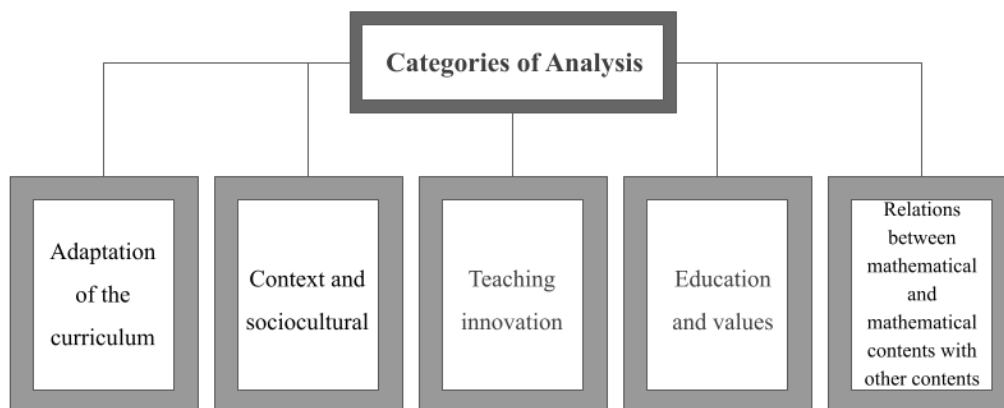


Figure 5.

Analysis categories (Prepared by Santos (2023) based on Godino (2011))

According to Santos (2023):

The idea for constructing those categories was based on the five components and indicators of ecological didactic suitability proposed by Godino (2011). The author lists, in this perspective, the components: adaptation to the curriculum, which addresses the suitability of the curriculum to the established guidelines; socio-professional and cultural adaptation; openness to didactic innovation, which deals with the possibilities of innovation in the didactic perspective, related to the socio-professional of the students involved in the teaching process; education in values, which tends to the issues associated to democratic values; and intra and interdisciplinary connections, which are connected to the possibilities of interlocution between contents. (p. 43)

By way of illustration, we highlight Santos's (2023) discussions on how residents got to know the field schools, as it was a moment of remote teaching. In this regard, the author points out:

Residents got to know the school through documents and the preceptor's lens, which may have compromised the perception of those prospective teachers of the school's reality. Therefore, there may be a construction of a perception of the reality of the idealized school since documents such as the PPP present ideal proposals for the school. What actually happens daily may differ from this idealization. Thus, it must also be agreed that knowing the school through what a subject describes, in this case, the preceptor, is to build an idea of this environment from the perspective of the other. This gaze can be dubious because the vision of a teacher who has worked in education for a while may not be the same as prospective teachers who are still building their impressions of this space. (p. 99)

This fact shows the complexity of teacher education in the context of the pandemic, as both the PRP planning activities and those of the regency were carried out remotely. To

illustrate this complexity, we present the discussions by Santos (2023) on the curriculum adaptation category.

According to the author, the category emerged from the component of ecological suitability found in Godino (2011), which presents as an indicator for analysis the observance of curricular guidelines for the implementation and evaluation of mathematical contents. Thus, this category focused on analyzing whether, in the investigated lesson plans, there is compliance with the proposed curricula for implementation in the public education network in the state of Minas Gerais.

A point highlighted by Santos (2023) was the signaling of the use of Plan of Tutored Studies (Plano de Estudos Tutorados - PET), which, at the time of data collection, was implemented in the state network of Minas Gerais as curricular material for remote classes. Mobilized by knowing how residents planned their classes, the researcher accompanied the group at this stage to clarify questions such as: How would residents plan their classes? How would the curricular proposals and the reality of basic education students articulate? How would they organize the contents in the face of social, cultural, political, and economic realities?

The results show crucial data, including the difficulties the residents and preceptors find in adapting the curriculum to mathematics teaching because, in their class planning, almost all official curriculum resources have been almost completely reproduced.

In short, research by Horta (2023) and Santos (2023) contribute to the theoretical construct of the onto-semiotic approach in the DMK aspect, as they discuss teacher education's doing, its practices, and specificities regarding its development context and the relationships established in that space. Such research favors establishing a link between theoretical development and spaces for the initial and continuing education of mathematics teachers, especially within the scope of the PRP.

b) Didactic suitability

Throughout the development of the research by Frade (2022) and Martins (2022), Public Notice n. 01/20207 of the PRP was in force. Item 4.2 of the announcement defined that among the 138 hours that made up the modules, residents should dedicate 12 hours to preparing the lesson plans. Such prescription, inserted in the list of activities that make up the teaching practice, was considered a starting point for the development of research by Frade (2022) and Martins (2022). The researchers envisioned the possibility of taking advantage of this demand to prepare lesson plans to structure the data production moments for their research. However,

7 <https://www.gov.br/capes/pt-br/centrais-de-conteudo/06012020-edital-1-2020-residencia-pedagogica-pdf>

what would be the subject that would be addressed in these lesson plans? What didactic-methodological strategies could be adopted? How to develop this planning in the pandemic context?

There were many questions, but one desire: to address the perspective of active methodologies. Added to this desire was the recognition of the need to promote

teacher education spaces where active methodologies can be problematized and investigated not in an imposing, technical, or market-oriented way but in an investigative and collaborative perspective, making teachers feel confident and free to learn, create, adapt, experiment, and develop practices with active methodologies in the contexts in which they are inserted. (Souza & Tinti, 2021, p. 402)

Thus, Frade (2022) and Martins (2022) chose to jointly carry out a mini-course with residents to promote reflections and experiences with two approaches from the list of active methodologies. Frade (2022) focused on a group work approach, while Martins (2022) opted for gamification. Below, we present a summary of the proposal for this mini-course:

We prepared a mini-course as follows: in the first meeting, we presented the principles of some active methodologies, especially group work and gamification. In the second meeting, the participants were asked to divide into two groups, a group of four people and a pair, one of which produced the lesson plan from the perspective of group work and the other from the perspective of gamification. In the third meeting, the groups developed the practical activity of preparing a lesson plan and reflected on their knowledge. (Frade, 2022, p. 59)

So, the corpus of data from the two analyzed surveys included: audio and video recordings of the meetings, answers to the two proposed questionnaires, and the lesson plan the participants prepared. To analyze them, Frade (2022) and Martins (2022) considered the components and indicators of didactic suitability⁸ as a theoretical lens. In all, 22 components and 44 indicators were considered.

From a methodological point of view, we highlight the structure of one of the proposed questionnaires. The researchers considered the perspective of components and indicators of didactic suitability to promote a movement of reflection on the practice of preparing the lesson plan. This proposition was structured as questions aligned with the components and indicators of didactic suitability. Thus, there was no need for the researchers to meet with the residents to explain the theoretical perspective assumed in the research. Given the complexity of the chosen theoretical perspective, this action was configured as a methodological innovation.

⁸ The term “didactic suitability criteria” can be translated into “didactic adequacy criteria”, however, we will use the term used in the research analyzed.

In addition, we emphasize the theoretical-methodological rigor and the meticulous process of analysis of each of the components and indicators of the epistemic dimensions; interactional; cognitive; affective; *mediational*; and ecological aspects of didactic suitability. Frade (2022) and Martins (2022) presented summary tables and graphs to represent each of the dimensions of suitability evidenced in the process of preparing the lesson plan by the residents, as illustrated in Figure 6.

| Component | Level |
|--|--------|
| Prior knowledge (take into account the same elements of epistemic suitability) | High |
| Curricular adaptation | Middle |
| Learning | High |

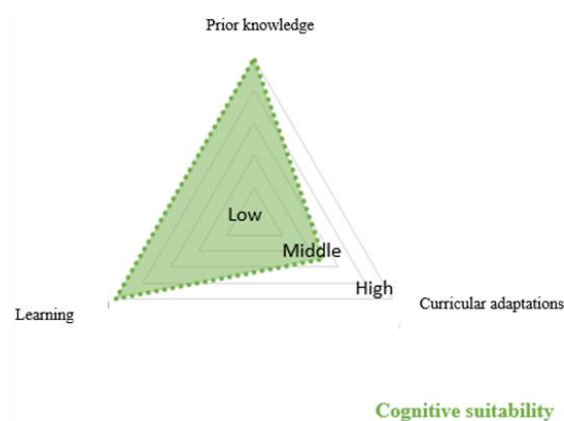
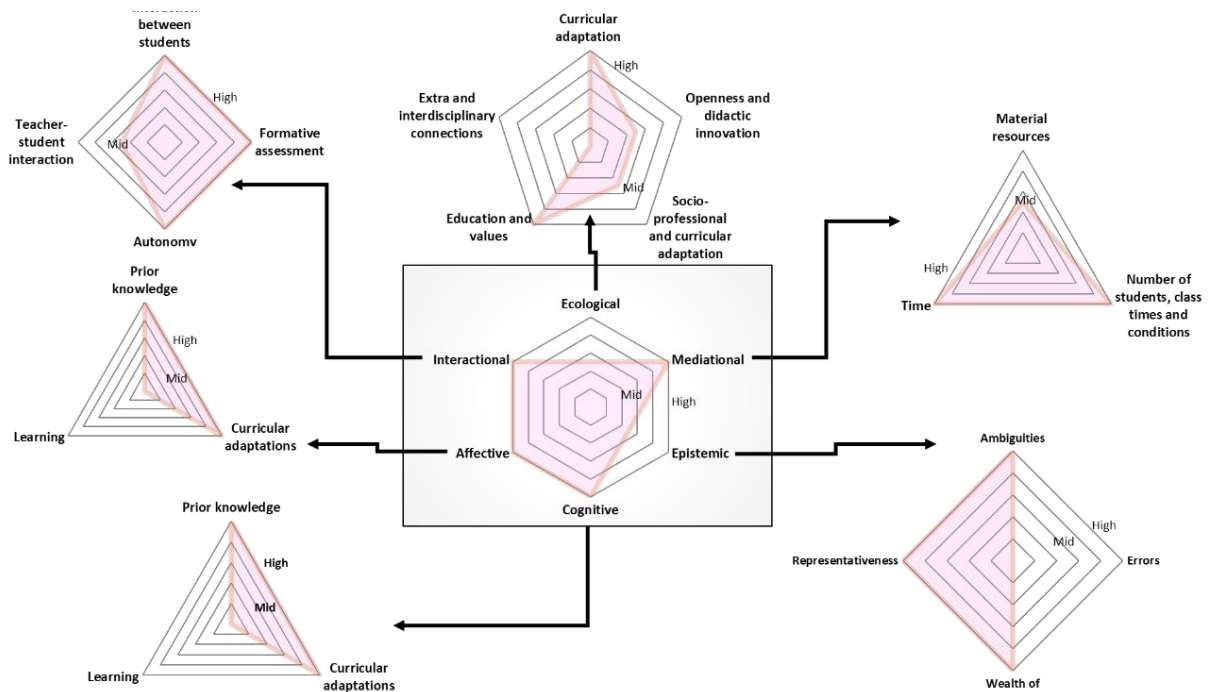


Figure 6.

Summary of the analysis of the levels of cognitive suitability evidenced in the process of elaborating the lesson plan by the residents (Adapted by de Martins (2022, pp. 95-96))

At the end of this process, the researchers also synthesized the entire analytical process. Figure 7 shows the synthesis created by Frade (2022).



Picture 7.

Suitability criteria mobilized in the preparation of the lesson plan (Frade, 2022, p. 107)

The reflective, analytical process consisted of a fundamental construct for analyzing the practices carried out by the participants and highlighting the importance of parameters to support the teacher in planning the mathematical content teaching.

When investigating the planning process for implementing group work and gamification approaches, Frade (2022) and Martins (2022) show that structuring such active methodologies can favor the mobilization of didactic suitability criteria. In Table 5, we summarize these approximations.

Table 5.

Approaches to didactic suitability in Frade and Martins

| Didactic suitability | Friar (2022) - Group work | Martins (2022) - Gamification |
|-----------------------------|--|---|
| Epistemic | This approach can boost knowledge construction. In group discussions, the error can be problematized, and discussions can minimize possible ambiguities. | Gamification as a methodology values the adaptation of tasks to the development of students. The error can be problematized, and the discussions can minimize possible ambiguities. |
| Cognitive | The approach assumes problem solving and provides opportunities for students to use multiple skills. In addition, it can address important concepts, | Within a gamified activity, the players' achievements contribute to the beginning of new cycles, increasing complexity and difficulty and impacting the individual |

| | | |
|---------------|--|---|
| | requires individual responsibility, positive interdependence, and includes criteria for assessing learning. | emotionally and cognitively. The proposed activities can favor the expansion and reinforcement of conceptual understandings. |
| Interactional | It is an approach that values interaction and cooperation among students and favors the construction of their autonomy. The proposed activities can be considered in the formative evaluation process. | Gamification addresses both socialization and collaboration. It favors the construction of students' autonomy. The proposed activities can be considered in the formative evaluation process. |
| Affective | The approach can generate students' interest. It anticipates the participation of all students by assigning each one a role in the group. It can favor the use of mathematics in everyday life. | The approach can generate students' interest. Gamification as a teaching method tends to contribute to students' emotions, as it encourages them to persevere and overcome challenges. |
| Mediational | It is an approach within the perspective of active methodologies that foresees the use of different resources, time organization, and group structuring. | It is an approach within the perspective of active methodologies. Its use in teaching and learning processes can contribute to reducing the time for understanding concepts. |
| Ecological | For the composition of the groups and definition of the proposed tasks, the teacher needs to know the characteristics of the class. It favors inter and interdisciplinary connections. Its use can be considered a didactic innovation. It promotes the development of democratic values and the construction of critical thinking | The activities proposed in gamification can be aligned with the curricular guidelines and can favor inter- and interdisciplinary connections. Its use can be considered as a didactic innovation. |

In general terms, Frade (2022) indicated that “this structuring contributed to bringing some of these components closer to a high level, even considering that residents already assumed that students had already been prepared to work in groups” (p. 113). In the same direction, Martins (2022) pointed out that “gamification as a methodology contributed to a better adaptation of the interactional and affective criteria” (p. 120).

c) Teachers' educators' formation

The structure of the PRP contemplates the participation of university professors (advisors), undergraduate students (residents) and basic education teachers (preceptors). In Article 9 of GAB Ordinance N. 82⁹, regulating the PRP, the preceptor's role is defined as follows: “Teacher of the basic education school responsible for monitoring and guiding residents in the activities carried out in the field school”. However, is the preceptor's role

⁹ https://www.gov.br/capes/pt-br/centrais-de-conteudo/documentos/diretoria-de-educacao-basica/28042022_Portaria_1691648_SEI_CAPES_1689649_Portaria_GAB_82.pdf

limited to “monitoring” and “guiding” the residents? Wouldn’t they also be responsible for training the prospective teachers?

Such questions guided Faria’s research (2023), which investigated preceptors’ performance in the Pedagogical Residency Program to understand its repercussions on the movement towards the constitution of the identity of mathematics teacher educators. In that research, semi-structured interviews were used as a data production instrument. Two preceptors who worked in the mathematics subproject of the UFOP’s Pedagogical Residency Program were interviewed.

Unlike the other studies analyzed here, Faria (2023) considered as a theoretical assumption the characterization of the professional identity of teachers who teach mathematics, proposed by Cyrino (2017). This characterization considers that the following elements must be considered for the constitution of the PI: emotions, beliefs, and conceptions; knowledge; self-knowledge; autonomy (vulnerability and the sense of agency); and political commitment.

In theoretical terms, the analysis of Faria’s research (2023) reveals that there is an approximation between the characterization of the professional identity of teachers who teach mathematics, proposed by Cyrino (2017), and the identity of the educator who trains teachers who teach mathematics, given the duality of simultaneous roles they play as preceptors. The author points out that “the interviews allow us to perceive moments that refer us to the teacher’s identity when looking at their basic education students, and others that refer to the teachers’ educator’s identity, when looking at the residents” (Faria, 2023, p. 78). In this context, Faria (2023) emphasizes that participation in the PRP enabled the school teacher to take on a new role (preceptor) that triggered a shift towards the constitution of his/her identity as a teacher educator.

Throughout the research, the author emphasizes the importance of the preceptors’ participation in all PRP activities and reveals that this action collaborates with constituting their identity.

The research data show how much the PRP provides opportunities for mathematics teacher preceptors to develop tasks that allow them to reflect on the residents’ education, interpreting and enabling them to have direct contact with their future professional practice, i.e., with teaching practices, interaction directly with the school environment, and with decision-making. It was possible to identify through the interviews –even with some vulnerabilities imposed due to the pandemic and the singularities that each of the preceptors carries about the use of some of the technological tools– that there were positive manifestations for the preceptors, which allowed them to constitute their own education during their participation in the program. (Faria, 2023, pp. 89-90)

In methodological terms, we highlight the construction of the script for the interviews. We also observed an adherence to the proposal by Cyrino (2017) and with the preceptor's performance.

The analysis of Faria's research (2023) reveals the importance of the preceptor being recognized as a teacher educator, including both in the Program's regulations and in implementing the PRP in the institutions. We agree with this point and defend that the educators' formation can be guided, problematized, and triggered in the interactions between advisors and preceptors, given the experience that advisors have with teacher education. Sharing this experience with preceptors can help them recognize and construct their professional identity.

Final considerations

NEPEFEM has become a space for debates and reflections on the education of mathematics teachers, in which the collaborative and cooperative aspect of the investigations undertaken by the leaders and the group members has been developed. In this context, the investigations carried out and in progress have enabled us to discuss theoretical and methodological perspectives that help to understand the Pedagogical Residency Program as a formative space where Didactic-Mathematical Knowledge is mobilized.

In the theoretical dimension, researchers have focused on reflecting on the theoretical construct of the OSA, in particular concerning the facets of the DMK and the didactic suitability criteria. Theoretical deepening and the contribution with its advances, based on research, are fundamental for the emergence of new investigative horizons. Regarding the methodological dimension, investigations within the scope of NEPEFEM have shown the importance of using the criteria and components proposed by the DMK model in data production and reflection on teaching practice. In addition, the analyzed studies indicate the need to improve and adapt them to the reality of mathematics teacher education in Brazil, especially when these works turn to public policies, as is the case of the PRP.

The implementation of such a public policy requires more straightforward guidelines for teachers' educators so that they can plan actions, considering the development of mathematical knowledge, resources and means for teaching, evaluation processes, relationships that are established, and the context where formative and intervention actions are implemented in the scope of basic education. Furthermore, it has been a historic challenge to place undergraduate degrees in dialogue with the educational contexts that constitute the future field of professional activity for teachers.

Overcoming this challenge requires rethinking the curriculum arrangements of courses to allow institutions, teacher educators, and prospective teachers to (re)organize planning actions, pedagogical practice, and evaluation of educational processes, respecting the contexts in which they are inserted. Illustrating the above, we highlight the practices of preparing the lesson plan within the scope of the investigations mentioned. In this context, it is essential to highlight how the suitability criteria make it possible to reflect on the knowledge necessary for teaching mathematical content.

Finally, the future perspectives of NEPEFEM aim at the development of collaborative actions and research that contribute to highlight the repercussions of the PRP for the education of mathematics teachers based on the speech of graduates of the Program.

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