

Development of mathematical thinking in students with intellectual disabilities

Desarrollo del pensamiento matemático en estudiantes con discapacidad intelectual

Développement de la pensée mathématique chez les élèves présentant une déficience intellectuelle

Desenvolvimento do pensamento matemático em estudantes com deficiência intelectual

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Abstract

We discuss the development of mathematical thinking in students with intellectual disability (ID). The study is based on the cultural-historical theory and the developmental teaching theory. We seek to answer the question: How can we contribute to mobilizing mathematical thinking in students with ID? To answer the guiding question, we conducted a didactic experiment in which three students diagnosed with ID who attended the specialized educational service (AEE, in the Portuguese acronym) in the opposite shift to their regular schooling participated. We worked on the task of *the unfinished house*, through which it was possible to identify signs of mobilization and action of students' mathematical thinking. We emphasize that if we use in pedagogical actions the principle of education that develops, we promote in students with intellectual disabilities possibilities to create and consolidate different zones of proximal development (ZDP) and thus enhance the learning and development of mathematical concepts.

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Keywords: Mathematical concepts, Cultural-historical theory, Developmental teaching, Inclusive education, Specialized educational care.

Resumen

Se analiza el desarrollo del pensamiento matemático en alumnos con discapacidad intelectual (DI). El estudio se basa en la teoría histórico-cultural y en la teoría del desarrollo de la enseñanza. Buscamos responder a la pregunta: ¿Cómo podemos contribuir a la movilización del pensamiento matemático en alumnos con DI? Para responder a la pregunta guía, realizamos un experimento didáctico en el que participaron tres alumnos diagnosticados con DI que asistían a la atención educativa especializada (AEE) en turno contrario al de su escolarización ordinaria. Trabajamos con la tarea la casa sin terminar, a través de la cual fue posible identificar señales de movilización y acción del pensamiento matemático por parte de los alumnos participantes. Destacamos que si utilizamos en las acciones pedagógicas el principio de la educación que se desarrolla, promovemos en los alumnos con discapacidad intelectual posibilidades de crear y consolidar diferentes zonas de desarrollo próximo (ZDP) y así potenciar el aprendizaje y desarrollo de conceptos matemáticos.

Palabras clave: Conceptos matemáticos, Teoría histórico-cultural, Enseñanza evolutiva, Educación inclusiva, Atención educativa especializada.

Résumé

Nous examinons le développement de la pensée mathématique chez les élèves présentant une déficience intellectuelle (DI). L'étude est basée sur la théorie historico-culturelle et sur la théorie de l'enseignement développemental. Nous cherchons à répondre à la question suivante : Comment pouvons-nous contribuer à la mobilisation de la pensée mathématique chez les élèves présentant une déficience intellectuelle ? Pour répondre à la question directrice, nous avons mené une expérience didactique à laquelle ont participé trois élèves diagnostiqués avec une DI qui ont fréquenté le Service d'éducation spécialisée (SEA) à contre-courant de leur scolarisation régulière. Nous avons travaillé avec la tâche de la maison inachevée, à travers laquelle il a été possible d'identifier des signes de mobilisation et d'action de la pensée mathématique par les élèves participants. Nous soulignons que si nous utilisons, dans les actions pédagogiques, le principe de l'éducation qui se développe, nous favorisons chez les élèves présentant des déficiences intellectuelles les possibilités de créer et de consolider différentes zones de développement proximal (ZDP) et d'améliorer ainsi l'apprentissage et le développement des concepts mathématiques.

Mots-clés : Concepts mathématiques, Théorie historico-culturelle, Enseignement développemental, Education inclusive, Attention éducative spécialisée.

Resumo

Discutimos sobre o desenvolvimento do pensamento matemático em estudantes com deficiência intelectual (DI). O estudo baseia-se na teoria histórico-cultural e na teoria do ensino desenvolvimental. Buscamos responder o questionamento: De que forma podemos contribuir com a mobilização do pensamento matemático em estudantes com DI? Para responder à questão norteadora, realizamos um experimento didático formativo do qual participaram três estudantes com diagnóstico de DI que frequentavam o atendimento educacional especializado (AEE) em turno inverso à escolarização comum. Trabalhamos com a tarefa “A casa inacabada”, por meio da qual foi possível identificar indícios de mobilização e ação do pensamento matemático por parte dos estudantes participantes. Destacamos que se utilizarmos nas ações pedagógicas o princípio da educação que desenvolve, promovemos nos estudantes com deficiência intelectual possibilidades de criar e consolidar diferentes zonas de desenvolvimento proximal (ZDP) e com isso potencializamos a aprendizagem e o desenvolvimento de conceitos matemáticos.

Palavras-chave: Conceitos matemáticos, Teoria histórico-cultural, Ensino desenvolvimental, Educação inclusiva, Atendimento educacional especializado.

Development of mathematical thinking in students with intellectual disabilities

Historically, people with intellectual disabilities (ID) were segregated from the regular school context, due to the then-current conception of society regarding the non-learning of people with such specificity. Before the creation and implementation of public policies, this scenario began to change and is still on the path of transformation. We can cite the National Policy on Special Education from the Perspective of Inclusive Education [Política Nacional de Educação Especial na Perspectiva da Educação Inclusiva] (Brasil, 2008) and the Statute of Persons with Disabilities [Estatuto da Pessoa com Deficiência] (Brasil, 2015). Intellectual disability is defined in the Brazilian context as a long-term intellectual impediment (Brazil, 2009), and this type of disability, in particular, challenges schools to reflect on their teaching methods, the appropriation of scientific knowledge, being a discussion of mathematical pedagogical practices that aim to be inclusive.

Students with intellectual disabilities can be mistakenly seen as incapable of learning mathematics, which can cause the creation of practices that are limited to copying numerals and carrying out meaningless mathematical operations. Such tasks are usually reductionist regarding mathematical concepts (Shimazaki & Pacheco, 2010). Therefore, few tasks offered put the mind into action. When we consider that one of the tasks of the Specialized Educational Care (SEC) program [Atendimento Educacional Especializado – AEE] is to offer students with intellectual disabilities the possibility of developing higher psychological functions in a complementary way, we believe it is necessary to provide them with mathematical tasks to boost their intellectual processes.

Teaching mathematical concepts according to formal logic is related to functions already formed in the student (Davídov, 2007), offering them already present opportunities. However, pedagogical practices aimed at the dialectical essence presuppose the principle of education that provides students with possibilities for improvement. Thus, education directs content through actions that influence development or, as Vygotsky (2008) taught us, act and enable zones of proximal development in students.

Given these initial reflections, in this article, we present results from a doctoral research work conducted by one of the authors under the supervision of the other two. The guiding question is: How can we contribute to mobilizing mathematical thinking in students with ID? We organized this text through discussions related to mathematical thinking from a historical-cultural perspective. Then, we discussed the methodological procedures adopted, the results, and the discussions on the data collected.

Reflection on mathematical thinking from a historical-cultural perspective

Mathematical thinking can be understood as a particular type of thinking resulting from the interaction between subjects, mediated by instruments and signs created and appropriated by people. What differentiates this thinking from other thinking and other areas of knowledge is precisely the mathematical concepts (arithmetic, geometric, and algebraic) taught that develop thinking. Mathematical thinking can be expressed through the first language or a language specific to mathematics and be driven through study activity –the main activity developed at school age.

Davídov (2020) considers that the essence of the study activity is the tasks and actions developed. Each task is made up of six study actions, and each study action is made up of several tasks. Davídov (2008) presents individual mathematical tasks by dialectical logic.

Rosa (2012, p. 72) indicates that the tone adopted by all the tasks proposed by Davídov meets the principle of developmental character, i.e., “[...] it does not allow the child to expose only what they can observe spontaneously, it also gives them the opportunity, with the teacher’s guidance, to direct attention to aspects that would otherwise go unnoticed”.

In our research, we adopted tasks presented by the Elkonin-Davídov system in textbooks produced for schools, for which we used the teacher’s manual (Горбов, Микулина, & Савельева, 2008). The theoretical references present in the textbooks of the Elkonin-Davídov system are based on historical-cultural theory, activity theory, and developmental teaching theory. We highlight that this system “was designed to meet the level corresponding to primary education, that is, children aged 6 to 10” (Cunha, 2019, p.181). To materialize the Elkonin-Davidov system, theorists carried out extensive experimental research and considered for the production of the proposed tasks, mainly the child’s level of psychic development, which at the age of 6 to 10 years corresponds to the study activity (Cunha, 2019). The level of psychic development refers to issues of periodization of human ontogenesis. We mention the main stages of this periodization as the baby’s emotional communication, manipulative object activity, role play, study activity, personal, intimate communication, and work activity (Elkonin, 1987).

For this present study, we selected tasks from the book for the first grade of elementary school because Vasily Vasilovich Davídov (1930-1998), a theorist who formulated the theory of developmental teaching, is one of its authors (Cunha, 2019). In this text, we present one of the tasks called “The unfinished house,” which refers to the properties of objects: color, shape, and size, based on equality and inequality concepts. Such concepts are elementary for acquiring

the theoretical notion of numbers, which is in the process of appropriation and development by students with intellectual disabilities participating in this research.

Davídov (1988) states that the task of contemporary schools is to teach students to orient themselves independently with input from scientific knowledge. To achieve this, the school must teach students how to think, which means that teaching must be organized to boost development.

The scholar also states that any school education somehow develops students' intellectual abilities; however, the traditional school is organized to create a specific type of thinking in its students: empirical thinking. In the author's words: "Empirical thinking has its specific types of generalization and abstraction, its peculiar procedures for forming concepts, which precisely hinder children's full assimilation of the theoretical content of knowledge that increasingly penetrates current school" (Davídov, 1988, p. 5).

Davídov (1988, p. 6) indicates that the schools' development of empirical thinking is one of the causes that lead education to weakly influence students' psychic development, as empirical thinking "originates and can more or less develop outside of school, and its sources are linked to people's daily lives." The author emphasizes that proper education must be oriented towards developing students' theoretical thinking: "Contemporary knowledge presupposes that the human being dominates the process of origin and development of things, through theoretical thinking, which studies and describes dialectical logic" (Davídov, 1988, p. 6). Regarding empirical thinking, the theorist states that "[...]empirical thinking and concepts consider objects as constant and finished, while theoretical thinking and concepts analyze the processes of their development" (Davídov, 1992, p. 7).

In the author's sense, schools must teach students to think theoretically. And if we are talking about students, this includes students with disabilities, for whom the school should also aim to develop theoretical thinking. Davídov (1992) also draws attention to the fact that theoretical thinking should not be confused with abstract thinking since "The essence of theoretical thinking consists of and is a special procedure with which an individual focuses on understanding things and events through the analysis of the conditions of its origin and development" (Davídov, 1988, p. 6). Thus, he argues that students begin to think theoretically when they study concepts from this perspective.

The main objective of teaching mathematics, according to Davidov (1988), is to teach students, until the end of elementary school, the concept of real numbers, "[...] whose basis is the concept of magnitude" (p. 211). In this way, "[...] natural and real numbers are a particular aspect of a general mathematical object, the concept of magnitude. He proposes that the child

first becomes familiar with this general object and, later, studies the particular cases of its manifestation” (Rosa, 2012, p.30).

Since the first grade of elementary school, the theoretical concept of a real number to which continuous quantities are taken as meanings –instead of the natural number which is traditionally taught by making associations between objects and through numerical writing– must be prioritized, as Damázio, Rosa, and Euzébio (2011) argue.

The basis of real numbers, relationships between quantities of the same kind, is constituted, according to Souza (2013, p.149), “[...] in units of numerical singularities such as natural, integers, rational, and irrational numbers”. The relationship between quantities encompasses length, area, capacity, mass, volume, and degree, among others, and it is from the relationship between quantities that “children begin and develop the process of representation at their levels: object, graphic, and literal” (Souza, 2013, p. 149).

For the process of appropriating the concept of numbers, understood as a relationship between quantities, we begin working with some essential skills for students to understand the relationships between magnitudes. Such skills refer to the properties of objects and figures (color, shape, size, position) and the relations of equality and inequality to compare the concepts.

Then, in the study task system, students analyze the object material and identify its characteristics and relationships. At this point, they do not count or learn to identify numbers, as in traditional teaching. They first “[...] assimilate knowledge about magnitudes in great detail” (Daviđov, 1982, p. 431). Students establish relationships and comparisons and highlight the parameters of magnitudes in objects and figures, relating and specifying equalities and inequalities (Rosa & Damázio, 2012, p. 83).

Next, as Rosa and Damázio (2012) state, students write down the variations in magnitudes with greater, lesser, addition, and subtraction signs. Knowledge of variations between magnitudes allows them to solve “[...] the most diverse problems related to the need to consider the moment of ‘equilibrium’ and the conditions for its maintenance” (Daviđov, 1982, p. 432). Students also learn how to form and write equations. Thus, “if $a < b$, from inequality, we must move to equality: $a + x = b$. The direction of variation of magnitudes is determined by the conditions of the problem (if $a > b$, $a - x = b$) when it is required to equate a in relation to b ” (Daviđov, 1982, p. 433). Next, the number is introduced “as a particular case of representing the general relationships between magnitudes, i.e., when one of them is taken as a unit of measurement to calculate the other.” (Rosa & Damazio, 2012, p. 86). The number line is also introduced.

Davídov (1988) proposes the movement of thought from the general to the particular. “The general, in turn, is the diverse relationships between the quantities represented algebraically and geometrically. The particular is the point of arrival, expressed in the value of the magnitude measurement through a number, that is, in its arithmetic significance” (Rosa & Damázio, 2012, p.90).

Quantification is inserted at this stage. “The tasks no longer emphasize procedures for directly comparing magnitudes. To solve them, it is necessary to use a unit of measurement” (Rosa & Damázio, 2012, p. 94). The comparison takes place in the search for how many times the unit of measurement fits into the magnitude to be measured, revealing the universal relationship of the object studied based on the relationship of multiplicity and divisibility.

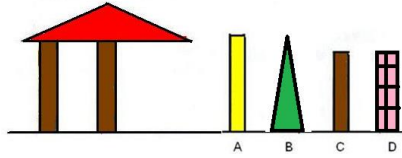
The first concept to be taught that appears in the textbook prepared by Davídov and collaborators (ДАВЫДОВ et al., 1997) is the development of thinking related to the recognition of the characteristics of objects and figures through relations of equality and inequality. The first characteristics refer to color, shape, and size. We develop this thinking with our students during the teaching offered at SEC.

We work on such mathematical relationships because everything can be compared, but there is something essential. We teach the concepts of color, shape, and size so that students can understand and compare them through distinct characteristics and arrive at a fundamental one, namely, magnitude.

The tasks of the planned teaching system were adapted and used from Горбов, Микулина, and Савельева (2008), the guidance manual for first-grade teachers. In this system of study tasks, we address special tasks. In this text, we work with the task entitled “The unfinished house,” which presents the essence of the study activity from a dialectical perspective based on the properties of objects: color, shape, and size, and concepts of equality and inequality. Table 1 briefly describes the objective of the study task, as well as the material used to teach it.

Table 1.

Task developed (Горбов; Микулина; Савельева, 2008, p.3)

PARTICULAR TASK			
Properties of objects and figures: Color, shape, and size, through the concepts of equality and inequality			
Task	<p>The unfinished house</p> 	<p>Objective</p> <ul style="list-style-type: none"> - Place the student in investigative action; - Boost mathematical thinking about equality and inequality; - Choose the column that completes the unfinished house, justifying your choices regarding the color, shape, and size of the columns. 	<p>Material used:</p> <p>Expanded manipulatives - used with the three students</p>

Faced with this task, the concern is to work on conceptual notions and promote in students essential and necessary conditions to represent and compare quantities; that is, to encourage the appropriation of characteristics that allow establishing relationships between figures and objects such as color, shape, and size (Gorbov, Микулина, & Савельева, 2008). In this task, the objective is to form investigative action in students. The formulation of questions is crucial, both those asked by the teacher to the student and questions asked by the student to the teacher and himself (Gorbov, Микулина, & Савельева, 2008).

Methodology

Initially, we forwarded the research to the ethics committee of the Federal Technological University of Paraná – CAAE n. 49158521.9.0000.5547, Opinion n. 4.993.569, which was approved. In this qualitative research, we decided to develop a formative didactic experiment. We had the participation of three students with a diagnosis of intellectual disability, as characterized in Tables 2, 3, and 4 below:

Table 2.

Characterization of student 01 (Noronha, 2022, p. 56)

Student 01 (E1)
<p>Student 01 (E1) is seven years old and attends the first grade of elementary school. He started early childhood education when he was eight months old. E1 has Down syndrome and intellectual disability. He recognizes colors and a few letters –and names them– and relates some numerals to quantities. He speaks with phoneme changes.</p>

Table 3.

Characterization of student 02 (Noronha, 2022, p. 56)

Student 02 (E2)
Student 02 (E2) is nine years old and attends the fourth grade of elementary school. He started schooling in 2017. E2 has Down syndrome and intellectual disability. His orality is affected; he changes phonemes and usually does not use speech as a means of communication. He recognizes a few colors, numbers, and letters.

Table 4.

Characterization of student 03 (Noronha, 2022, p. 56)

Student 03 (E3)
Student 03 (E3) is ten years old and attends the fifth grade of elementary school. He recognizes a few colors and letters and can count a little. He started school in 2017, presents echolalia, has low vision, and has a clinical report of intellectual disability.

During the task development, students were constantly encouraged to register their understanding. Besides registers, task development was also filmed for later transcription of the dialogues established between the teacher/researcher and students.

Sanitary measures to prevent and contain the COVID-19 pandemic were adopted during the services according to the guidelines passed on by the school, which followed the guidelines of the World Health Organization (WHO) and the Ministry of Health.

We analyzed the data produced and organized based on isolates, episodes, and scenes (Caraça, 1998; Moura, 2005). Isolates consist of episodes, which, in turn, consist of scenes. Caraça (1998) understands isolates as a reality cut-out that the observer highlights and analyzes from the totality of phenomena, “In the impossibility of embracing in a single blow the entirety of the Universe, the observer cuts out, highlights, from this totality, a set of beings and facts, abstracting from all others that are related to it” (Caraça, 1998, p. 112). The author calls this an isolate, “[...] an isolate is, therefore, a section of reality arbitrarily cut” (Caraça, 1998, p.112).

Moura (2005, p.276) indicates that episodes “[...] may be written or spoken phrases, gestures, and actions that constitute scenes that can reveal interdependence between the elements of a formative action.” The author highlights that the episodes are not linear because “[...] it may be that the statement made by a participant in an activity does not have an immediate impact on the other subjects of the community. This impact may be revealed at another time [...]” According to Caraça (1998) and Moura (2005), the facts and dialogues that occurred during the study task are exposed in the scenes. With support from historical-cultural

theories and developmental teaching, we move on to discuss and reflect on the isolate, the episode, and the scenes in Table 5:

Table 5.

Isolate, episode, and scenes (Noronha, 2022, p. 70)

Isolate	Episode	Scenes
Principle of education that develops in mathematics teaching.	It encourages the creation and consolidation of distinct levels of zones of proximal development in students with intellectual disabilities.	Scene 1: The unfinished house by student E1
		Scene 2: The unfinished house by student E2
		Scene 3: The unfinished house by student E3

We report and analyze below the task proposed to the students, which aimed to mobilize mathematical thinking and observe their development level. For that, we consider the level that it is potentially possible for students to reach, i.e., what they can do with the teacher's help and what they will do independently in the future.

Principle of developing education: The process of developing mathematical thinking in students with intellectual disabilities

In the first task studied, there is an unfinished house (Chart 1) with a missing column, and some columns are different in color and shape. Students must choose the column that completes the house. It is possible that they quickly choose the correct column, but the important thing in this task is that the teacher discusses the wrong variants, so the teacher must suggest other columns. According to Rosa (2012, p. 72), "Observing each of the proposed pillars and expressing their peculiarities and similarities with the others allows students to construct arguments that justify the option for the one considered correct." According to the author, even if the established criteria are color and shape, with the proposed task, they can "[...] expand these limits to other properties, such as size and surface" (Rosa, 2012, p. 72).

We observed three scenes related to the task by students E1, E2, and E3 to verify their level of development and mathematical thinking movements. We started with the scene with student E1:

Scene 1: The unfinished house, student E1

The teacher⁴ explained the task to the student and indicated that there would be a house and that he should choose a column to complete it. Then, she distributed the manipulative pieces for the task on the table, as we can see in the interaction established between her and E1:

Scene 1: The unfinished house, student E1

(01) E1: A house?	(17) E1: Because it isn't the same...
(02) Teacher: It's a house! Look, this is the roof of the house (puts the roof on the table). Here, there is a piece of the house (teacher puts a brown column on the house), here there is the other piece of the house (puts the other brown column on the house). Now, we will have to choose which column here (teacher places the brown, red, green, and yellow columns on the table) completes the house.	(18) Teacher That's it! because it's not the same. What is the difference?
(03) E1: (Immediately points to the brown column)	(19) E1: It's red.
(04) Teacher This one? Put it in the house so we can see it.	(20) Teacher: It's red, very good! What if we put this one here? (puts the green column)
(05) E1: (Places the column in the house)	(21) E1: No! It's green!
(06) Teacher: Why do you think it is this one?	(22) Teacher: And what color are these columns? (points to the other columns)
(07) E1: (gives a thumbs up with his hand)	(23) E1: Brown.
(08) Teacher: Why?	(24) Teacher: And if we put this one (places the yellow column in the house)
(09) E1: Because this one... (points to the brown columns, wanting to indicate that the one he chose is the same as the other columns.)	(25) E1: No either, it's yellow!
(10) Teacher: Because this... is like the other?	(26) Teacher: And is this one (points to the yellow one) bigger or smaller than these? (points to the brown one)
(11) E1: (Shakes his head)	(27) E1: Bigger.
(12) Teacher: Are they the same? (points to the brown columns)	(28) Teacher: Is it the same size?
(13) E1: (Nods)	(29) E1: No! This is it! (takes the brown column again and places it in the house)
(14) Teacher: What if we put this one here (changes the brown column chosen by the student and puts the red one)	(30) Teacher: Very well, you made the little house. Now, let's draw it?
(15) E1: Makes a negative sign with his hand	(31) E1: Yes. (takes his materials and starts drawing. Student records the task in the form of a drawing).
(16) Teacher: Why can't it be that?	(32) Teacher: (Observes the student as he records the unfinished house). So, E1, this is right, just like you did, this is the column that fits into the house (points to the brown column), it fits because of its color and shape.
	(33) E1: (nods and continues drawing)

Source: Noronha (2022)

We found that E1 immediately indicated that the column that completed the house was the brown one (Scene 1 - line 03). He realized that this column was the correct one because it was the same color as the columns supporting the house (Scene 1 - line 09). When asked about the other column possibilities, E1 was emphatic in stating that they were incorrect, as they were of other colors (Scene 1 - lines 17, 19, 21, 23, and 25). In line 26, the teacher asked about the size of the yellow column, whether it was bigger or smaller than the brown column. E1 also noticed the difference in size between the columns (Scene 1 - line 27).

⁴ When reporting to the teacher, we highlight the researcher in her research and teaching activity in developing the proposed study.

We emphasize that perceptions constitute the highest mental processes and are part of the formation of human thought. Vygotsky (2010) argues that human perception becomes an increasingly complex process throughout human development, moving further and further away from the physiological determinations of sensory organs, despite continuing to be based on physical organs.

Perception is explained via mediation and the origin of higher psychological functions. “For example, human vision perceives light, hearing allows the perception of sounds, touch allows the perception of textures and sensations are defined by the characteristics of the human species compared to the perceptive apparatus” (Noronha, 2017, p. 68). This perception, which is initially direct, becomes mediated by culture through internalizing different concepts (Vygotsky, 2010). Thus, when we look at a given object, we do not visualize isolated physical attributes, but rather, we look at it from situations in which culture and language are present.

Perception is not a psychological function that acts alone, in isolation; on the contrary, it acts in a system that involves contrasting functions. When we perceive objects or elements of the real world, we base ourselves on previously experienced situations, on acquired knowledge, on established social interactions, and internalized psychological functions. In the case of the task “The unfinished house,” E1, upon noticing the difference in colors and the size of the columns, based it on the knowledge he had already acquired, and interactions established with the teacher. In this way, E1 did not perceive colors or size as a pile of sensory information but rather from a complete reality (Vygotsky, 2010).

The teacher led the discussion that the column did not fit because of its shape, because of its color, or because it had neither the correct shape nor the correct color, and that only column C fit both because of its shape and color (Scene 1 - line 32). E1 recorded the unfinished house in the form of a drawing (Figure 1).



Figure 1.

E1's record (Noronha, 2022, p. 77)

(34) E1: (When painting the roof with the red pencil, he realizes that the color was orange) Wow, it's orange! (continues painting). Here is the room!

(35) Teacher: Is it a room?

(36) E1: Yes (paints it blue)! And here is Mom and Dad's room (paints it orange)

In the drawing of the unfinished house (Figure 1), E1 recorded the perceptions about the figure he observed and manipulated –roof and columns– and recorded the colors correctly, except for the roof, but realized that the color was not the right one (Scene 1- line 34). We found that E1 managed to achieve the objective of the task, which was to identify the correct column and justify his choices. Student E1 showed his real level of development by identifying the colors and indicating that the yellow column was larger than the brown one. He also presented his level of potential development, which indicates that soon he will be able to express comparison relationships of other quantities, since in the real moment he performs size comparisons. This demonstration, however, was only possible through the task provided by the teacher, through her intentional action through which E1 demonstrated understanding of these characteristics.

The teaching activity –the teacher's main activity– requires intentional and systematic action on the teacher's part. It is a complex activity that requires understandings that encompass: the concept being taught, the learning processes and development of students' mathematical thinking, the development of students with intellectual disabilities, the diversity in the ways of appropriating concepts, the form of interaction established, the potential, and the action and consolidation of successive zones of proximal development.

We can understand E2's level of development given the interactions made possible by the teacher, as shown in Scene 2 below. In the same way that the teacher explained the task to E1 and started by placing the manipulable parts –roof and columns– on the table:

Scene 2: The unfinished house, student E2

Scene 2 – The unfinished house, student E2

- | | |
|------|--|
| (01) | Teacher: We're going to get a house! Look here, this is the roof (teacher places the roof on the table), now, I'm going to place a column (places the brown column) and now another column (places the other brown column) |
| (02) | E2: (Observes the teacher's action) |
| (03) | Teacher: Now, you must choose which of these columns we should place there (distributes the other columns across the table). Which one goes? |
| (04) | E2: (Student takes the brown piece and moves it to the correct location). |
| (05) | Teacher Very good, and why this piece? |
| (06) | E2: (Student disperses and starts playing with the pieces) |
| (07) | Teacher Why this piece? |
| (08) | E2: (Student remains absent-minded, playing with the pieces) |
| (09) | Teacher And if we swapped it (exchanges the brown piece for the yellow piece), would it work? |
| (10) | E2: (He doesn't answer, but swaps the pieces, takes out the yellow one and puts the brown one in its place) |
| (11) | Teacher: What if we put this one here? (places the red piece), would it work? |

- | | |
|------|--|
| (12) | <i>E2: (He does not answer, seems uninterested)</i> |
| (13) | <i>Teacher: What about we put this piece in (places the green piece), would it work?</i> |
| (14) | <i>E2: (Once again, he doesn't answer but changes the pieces, takes out the green one and puts the brown one in)</i> |
| (15) | <i>Teacher: Well done! This piece fits both in shape and color. Look, what is the color of this piece? (points to the red piece)</i> |
| (16) | <i>E2: Red.</i> |
| (17) | <i>Teacher: And the one you picked up?</i> |
| (18) | <i>E2: Green.</i> |
| (19) | <i>Teacher: And this other one (points to the yellow piece)</i> |
| (20) | <i>E2: Yellow.</i> |
| (21) | <i>Teacher: And this one (points to the brown columns)</i> |
| (22) | <i>E2: Brown.</i> |
| (23) | <i>Teacher: Well done! Let's draw the house now?</i> |
| (24) | <i>E2: (He agrees and starts recording the task)</i> |

Source: Noronha (2022)

Initially, E2 chose the brown piece and moved it to the unfinished house (Scene 2 - line 40). The teacher started asking why the student chose the correct piece (Scene 2 - lines 05, 07, and 09). E2 seemed absent-minded, started playing with the other pieces available, and did not answer when the teacher talked to him (Scene 2 - lines 06 and 08). She insisted on the questions, but E2 hardly expressed herself orally, answering more through gestures and movement of the pieces (lines 10, 12, and 14). As she has restricted oral communication, the teacher insisted that E2 used this means of communication/speech, asking questions about the colors of the pieces (Scene 2 - lines 15, 17, 19, and 21). E2 then used verbal expression to answer the questions (Scene 2 - lines 16, 18, 20, and 22). At this point, we must highlight that oral communication and verbal thinking may be impaired in students with intellectual disabilities. In such cases, the teacher must consider other means of communication, as they also transmit concepts students have already internalized or that are in the process of internalizing.

We infer that E2 recognized the colors the teacher used and named them, because he noticed the difference in colors but did not notice the difference in the shapes and size of the columns. In his record (Figure 2), we observed that E2 is in the scribbling phase, a phase Vygotsky (2009) defines as the beginning of graphism, the first representation of drawings made by children. The author states that “The first scribbles are followed by naming through verbal language. The lines on the paper thus constitute the beginnings of a graphic narration” (Vygotsky, 2009, p. 109). Thus, the words that initially explain and identify the scribbles acquire other functions in the child’s development.

In drawing, the verbal form of language is essential, as in play, to carry out the task, since “[...] it accompanies the child’s actions and production, giving it meaning. Narrative and dramatization are intrinsically related to the gesture that marks the movement of the young

children's body movement and actions on paper" (Vygotsky, 2009, p. 109). In his drawings, despite being a stage above his age (10 years), E2 used the colors used in the task; he used red and brown to represent the house and green and yellow for the columns that were not useful.

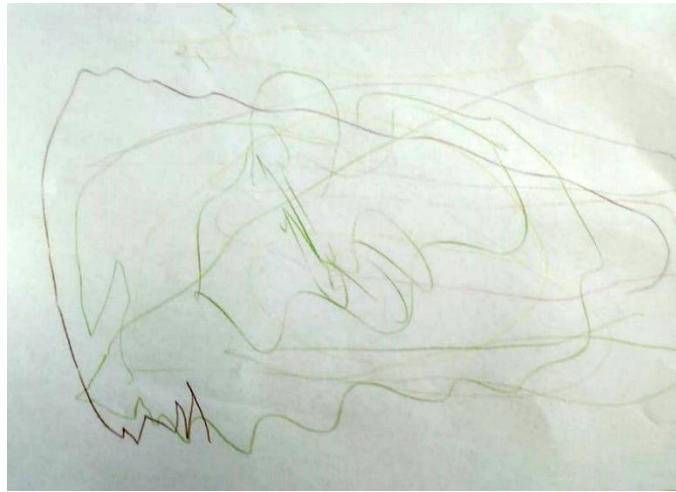


Figure 2

E2's register (Noronha, 2022, p. 80)

E2's concentration time was short, he was easily distracted by the pieces or any other interference that caused him to divert his attention, making it necessary for the teacher to insist so that she could understand how the student was thinking. E2 had difficulty expressing himself and restricted vocabulary; he used the form of communication he had or that was easiest for him, that is, gestures and isolated words.

E2 used his native language in a restricted way to express his understanding of the proposed task: gestures and isolated words prevailed. It was challenging for E2 to express his thoughts, as he demonstrated, based on the movement of the pieces (Scene 2 - lines 40 and 46), that he understood which column completed the house, but he did not express what he had understood in words.

According to Vygotsky (2008), verbal thinking is a complex higher psychological function; it is the expression of thinking in words and demonstrates the ability to unite language with thought. The author believes that the meaning of the words is responsible for this union. In E2's case, the expression of verbal thought is a process that was still developing in his ZPD.

As the teacher did for E1 and E2, she also explained the task to E3, and it is possible to observe in Scene 3, transcribed below, how the interaction with the teacher during the study task was.

Scene 3: The unfinished house, student E3

Scene 3 – The unfinished house referring to E3

- (01) *Teacher: (Teacher takes the roof of the house and puts it on the table) This here is the roof, the roof of the house! I'm going to place one column here (place one column under the roof) and the other column here (place the other column under the roof). Now, E3 will have to choose between these columns that the teacher She will place it here on the side the one we will use to complete the house (the teacher distributes the other columns on the table). Can you see the columns?*
- (02) *E3: [I] can do it (takes the brown column and puts it in the house).*
- (03) *Teacher: Why this column?*
- (04) *E3: Why is it red?*
- (05) *Teacher: What's red, is it the roof and columns?*
- (06) *E3: Brown.*
- (07) *Teacher: That's it! Is brown! And if we changed it, we put this one here (places the yellow column). Would the little house work?*
- (08) *E3: Yes.*
- (09) *Teacher: Would it?*
- (10) *E3: It wouldn't work.*
- (11) *Teacher: Why?*
- (12) *E3: It's yellow.*
- (13) *Teacher: It's yellow! And if we put this one (places the red column), would it complete the little house?*
- (14) *E3: No.*
- (15) *Teacher No, right! Why?*
- (16) *E3: It's red.*
- (17) *Teacher: It's red, very good! And what color are the others?*
- (18) *E3: They are brown.*
- (19) *Teacher: And if we put this one here (places the green column), could it be or not?*
- (20) *E3: No!*
- (21) *Teacher: Why?*
- (22) *E3: Why?*
- (23) *Teacher: Why can't it be that one?*
- (24) *E3: It's green.*
- (25) *Teacher: That's it! It is green; and what color are the other columns?*
- (26) *E3: They are brown.*
- (27) *Teacher: Well done! So, what is the correct column?*
- (28) *E3: (The student takes the brown column and places it in the appropriate place)*
- (29) *Teacher: Well done! So, now, we have completed the little house. So, now, let's draw?*
- (30) *E3: Let's draw.*
- (31) *(Teacher gives a sheet to the student to record).*

Source: Noronha (2022)

E3's level of development can be understood by observing his gestures and how he interacts verbally with the teacher. Student E3 has echolalia and repeats the teacher's words immediately after the teacher pronounces them. The teacher was concerned about whether the pieces of the unfinished house were accessible to E3, questioning whether he could see them (Scene 3 - line 01) since E3 has low vision and an intellectual disability –specifically for E3, the tasks were conducted with enlarged parts or images. With E3's statement that he could see the pieces (Scene 3 - line 02), the teacher began the questions (Scene 3 - line 03). At first, in line 08, E3 stated that the house would be complete if he replaced the brown column with the yellow one, but he realized his mistake (Scene 3 - line 10) and justified his way of thinking (Scene 3 - line 12). The teacher, when exchanging the other columns that were not useful (Scene 3 - lines 13, 15, 17, 19, 21, 23, and 25), asked E3 about the possibility of exchanging, and he

answered justifying his choices, pointing out that it could not be the other columns, given that they had different colors (Scene 3 - lines 12, 16, 18, 20, 24, 26, and 28). Below is E3's record of the unfinished house (Figure 3).



Figure 3.

E3's register (Noronha, 2022, p. 83)

We verified that E3 used the colors correctly. He used red, brown, green, and yellow to highlight the house and the columns, but he still did not draw the way he had observed.

The three students –E1, E2, and E3– demonstrated their understanding of the correct column differently: E1 and E3 expressed themselves through speech and gestures, while E2 used gestures. Gestures are also ways for the student to communicate their understanding of the task performed, which is why we consider movements, intentions, and thoughts expressed through gestures. Having the house and columns in manipulative material, not just the drawing on paper, helped the students because they could manipulate the pieces and place them in the correct place. Thus, even when oral communication was limited, as in the case of E2, he could express his mathematical thinking. The use of mediating instruments (manipulative materials) was essential for students to express their understandings; it is as if these instruments functioned as a support for learning or a means of compensating for intellectual disability.

The important thing about the task was that the teacher discussed the wrong variants, and because of that, she suggested other columns. The intention was for students to begin asking questions, questioning mathematical situations by asking the teacher and themselves (Why doesn't this column fit? Why can't this be the column that completes the house?) We can infer that thinking related to the characteristics of objects and figures is in the student's zone of proximal development, as they could perceive the differences between colors and express this understanding. With the teacher's help, shape and size are characteristics to be revisited during other tasks, since, with help, in collaboration, students can do more than they would do alone (Vygotsky, 2010). Still,

[...] not infinitely more, but only within certain limits, rigorously determined by the state of their development and their intellectual potential. In collaboration, children show to be stronger and more intelligent than when working alone. They project themselves to the level of the intellectual difficulties that they solve, but there is always a distance strictly determined by law that conditions the divergence between their intelligence occupied in the work that they perform alone and their intelligence in working collaboratively. (Vygotsky, 2010, p. 329)

The way of internalizing concepts is unique; the ZPD is different for each student. In the task taught, the students needed help from the teacher through the questions they asked and the notes they made. This possibility for the teacher to interfere in the development of students is essential during the study activity. Once again, the role of the teacher stands out because, with his intentional action, they boost student learning by considering not only what students could do alone at the moment but also what they did with help, and which, in the future, they will do independently.

Final Remarks

The mathematical thinking of students with intellectual disabilities must be enhanced with tasks that allow them to put their thinking into action. This presupposes that the proposed tasks cannot be meaningless tasks or be reductionist in the sense that the mathematical concept taught does not encompass the essence of the concept, but only its appearance.

In this study, we proposed that students with intellectual disabilities would have their higher psychological functions boosted by offering them a mathematical task planned from a dialectical perspective in the Elkonin-Davidov teaching system. The Russian teaching material was chosen because schools based on formal logic have not been able to promote intellectual development in their students fully.

The task “The Unfinished House” seems simple, but it encompasses issues beyond the immediate appearance, such as relationships between equality and inequality. We understand that with appropriate interactions and tasks that put thinking in motion, students with ID can learn and develop more intensely, appropriating mathematical concepts, in the case of this text, even elementary ones and that thinking can be gradually driven.

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