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Representations of a sector graph for teaching statistics to blind students

Representaciones de un gráfico circular para estudiantes ciegos en la enseñanza de la estadística

Représentations d'un diagramme circulaire pour les étudiants aveugles dans l'enseignement des statistiques

Representações de um gráfico de setores para alunos cegos no ensino de estatística

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Abstract

The objective of this article is to demonstrate the proficiency of a congenitally blind student in tactually interpreting accessible representations of a sector graph. The graph depicted in this study was extracted from a mathematics textbook in ink, subsequently rendered as a tactile graph generated with the *Braille Fácil* program, *MONET* software, and in manually crafted format using several materials, including EVA, Golden Beads, and various textures. The study focused on the examination of statistical graphical representations and the development of accessible materials for students with visual impairments, including those who are blind or have limited vision. More specifically, our research was centered on the creation of graphical-tactile materials. The study employed a qualitative methodology and utilized task-based interview techniques. Through this methodological tool, we delineated the profile of the participant and examined his approach to interpreting the tactile aspects of the graphic representations offered.

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The findings of the study indicate that the ability to comprehend a graph is not acquired spontaneously. Rather, it occurs in a sequential manner, allowing students to systematically process information, establish connections, and ultimately gain meaningful knowledge.

Keywords: Visual impairment; Sector graph; Pie charts; Accessible representations; Statistics teaching.

Resumem

Este artículo tiene como objetivo presentar cómo un estudiante con ceguera congénita puede realizar la lectura táctil de algunas representaciones de un gráfico circular accesible. El gráfico que se presenta en este trabajo fue tomado de un libro de texto de matemáticas en tinta y representado por medio de gráficos táctiles producidos por el programa *Braille Fácil* , por el software MONET y elaborados artesanalmente utilizando diferentes materiales, como EVA, cubos de material dorado y varios tipos. de texturas Nuestro estudio se basó en trabajos e investigaciones relacionadas con las representaciones gráficas estadísticas y la elaboración de materiales accesibles para estudiantes ciegos y con baja visión, en particular, grafo-táctiles. La metodología utilizada en este artículo es cualitativa y utilizamos técnicas de entrevista por tareas. A través de este instrumento metodológico, describimos el perfil del entrevistado y la forma en que realizó la lectura táctil de las representaciones gráficas presentadas. En los resultados obtenidos, notamos que la interpretación de un gráfico no se aprende espontáneamente; procede por etapas para que el estudiante pueda procesar la información con el fin de conectarlas y adquirir conocimientos que sean significativos para él.

Palabras clave: Discapacidad visual; Gráficos circulares; Representaciones accesibles; Enseñanza de la estadística.

Résumé

Cet article vise à présenter comment un étudiant aveugle congénital peut réaliser la lecture tactile de certaines représentations d’un graphique de secteurs accessibles. Le graphique présenté dans ce travail a été pris d’un manuel de mathématiques à l’encre et représenté par des graphiques tactiles produits par le programme Braille Easy, par le logiciel MONET et à l’aide de différents artefacts matériels, tels que l’EVA, les cubes de matière dorée et divers types de textures. Notre étude a été basée sur des travaux et des recherches liés aux représentations graphiques statistiques et à la fabrication de matériaux accessibles aux étudiants aveugles et malvoyants, en particulier les graphes tactiles. La méthodologie utilisée dans cet article est qualitative et nous avons utilisé les techniques d'interview basées sur des tâches. À travers cet outil méthodologique, nous avons décrit le profil de l'interviewé et la façon dont il a effectué la lecture tactile des représentations graphiques présentées. Dans les résultats obtenus, nous avons remarqué que l'interprétation d'un graphique n'est pas apprise spontanément ; il suit des étapes afin que l'étudiant puisse traiter les informations afin de les connecter et d'acquérir des connaissances qui lui sont significatives.

Mots-clés : Déficience visuelle, Graphiques de secteurs, Représentations accessibles, Enseignement de la statistique.

Resumo

Este artigo tem como objetivo apresentar como um aluno portador de cegueira congênita pode realizar a leitura tátil de algumas representações de um gráfico de setores acessíveis. O gráfico apresentado neste trabalho foi retirado de um livro didático de matemática em tinta e representado por meio de grafo-táteis produzidos pelo programa *Braille Fácil*, pelo software MONET e artesanalmente utilizando diferentes artefatos e materiais, tais como EVA, cubinhos do material dourado e diversos tipos de texturas. Nosso estudo se baseou em trabalhos e pesquisas relacionadas às representações gráficas estatísticas e à confecção de materiais acessíveis para alunos cegos e com visão subnormal, em especial, grafo-táteis. A metodologia utilizada neste artigo tem caráter qualitativo e fizemos uso das técnicas de entrevistas baseadas em tarefas. Por meio deste instrumento metodológico, descrevemos o perfil do entrevistado e a maneira como realizou a leitura tátil das representações gráficas apresentadas. Nos resultados obtidos, percebemos que a interpretação de um gráfico não é aprendida espontaneamente; segue em etapas para que o aluno possa processar as informações de modo a conectá-las e adquirir um conhecimento que seja significativo para ele.

Palavras-chave: Deficiência visual; Gráficos de setores, Representações acessíveis, Ensino de estatística.

Introduction

The mathematics curriculum in Brazil (2018) incorporates the teaching of statistics throughout elementary education, spanning from early childhood to high school. Statistical analyses conducted at various educational levels, particularly in the early years of elementary school, requires the use of reading skills, interpretation abilities, and comprehension of tables and graphs. According to Gal (2002; 2019), it is crucial to foster the acquisition of skills that involve interpretation, questioning, and transmission of statistical information among students. This is imperative in order for students to form well-founded judgments and make informed decisions based on the statistical data given.

The Common National Core Curriculum (*Base Nacional Comum Curricular – BNCC*) (Brazil, 2018), recommends instructional approaches for statistics that consistently incorporate tasks that demand the development of skills related to statistical data analysis and graphical interpretation. Nevertheless, it is essential to consider how these guidelines might be effectively implemented in the teaching and learning process for blind students, considering that visual perception is typically relied upon for accessing and comprehending graphic elements.

In elementary education mathematics lessons it is common to observe the introduction of statistical concepts through the use of written materials on the board, accompanied by verbal explanation delivered by the teacher. While this approach may be viable for sighted students, accessing information this way is impracticable or even impossible for the visually impaired due to their inability to perceive written material on the board. Thus, tactile representations of visual content become essential, even more if tables or graphs are displayed. According to Souza (2022, p. 8), even though "spoken language is essential for development, language developed through tactile representation of visual content is also very important for blind students".

The right to attend regular education classes must be guaranteed to visually impaired students, as well as the conditions that facilitate substantial learning advancement, as "one of the principles of inclusive education is that school must be a hospitable space, conducive to the learning of all individuals" (Correia & Cazorla, 2021, p. 2).

In light of the foregoing, the objective of this article is to present tactile mediation strategies for teaching visually impaired students, derived from an excerpt from the primary author's doctoral dissertation (Santos, 2022). Three accessible resources, originated from a sector graph in a mathematics textbook, were presented to a blind student. After handing them to the student, we instructed him to thoroughly review each of the resources, so that he could use one of them to answer the questions proposed in the activity found in the book which included the graph in question.

This article discusses the process of reading a sector graph through the sense of touch. The sector graph used for this study was taken from a mathematics textbook for fifth-grade students (Rocha, 2014). This work aims to disseminate information regarding sector graph representations specifically designed for visually impaired learners. The objective is to enhance the teaching and learning experience for such individuals.

Bases for Research

Zucherato and Freitas (2011) assert that a graph has a synthetic and informative character. According to those authors, a graphic representation must be dynamic; not a ready-made, static device; rather it should encourage the reader to engage with the information being conveyed. According to Passini (2007) apud Zucherato and Freitas (2011, p. 30), the use of graphs as visual resources enables the reader to comprehend the material efficiently by "presenting data in a logical manner that conforms to its fundamental essence. It is a universally intelligible language that enables the individual to *see* information. It is the evolution of reading proficiency."

We consider it relevant for this article to address the definition and characteristics of the sector graph, which is presented in this text. The sector graph, also called pie chart, consists of a circle, where each sector proportionally represents the frequencies of values of a statistical variable in relation to its percentages. For instance, let us consider the graph depicted in Figure 1 which illustrates the frequency of people who express support football teams.

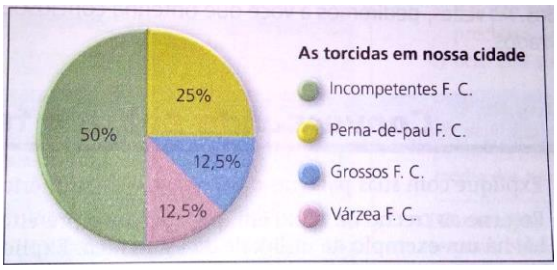

Figure 1.

Sector graph titled "Fans in our city" (Santos, 2022, p. 30)

The graph displays the frequencies of the variable *fans* for different clubs (*Incompetentes F. C., Perna-de-pau F. C., Grossos F. C., Várzea F. C.*) as percentages, which are represented by sectors in the graph, whose areas are equivalent to their respective relative frequencies.

The process of constructing graphs requires care so that the information being conveyed is clear and succinct. To ensure effective comprehension of graphic information, individuals who create them must “exercise caution throughout the entire process, starting with data collection until the development of tables and graphic representations” (Peixoto & Cruz, 2011, p. 129).

Peixoto and Cruz (2011) state that information will be adequately represented graphically when the most suitable type of graphic is selected to do so; otherwise, the readers’ comprehension of the text and graphics may be compromised. For example, the bar graph is considered the most practical tool to represent absolute frequencies of variables in research studies. Line graphs are commonly employed to illustrate the temporal progression of phenomena, while sector graphs are most suitable for comparing specific components of a survey with the entire dataset.

It is essential to modify the graphic materials utilized in the classroom so as to foster student engagement and facilitate interpretations, questions, and reflections regarding the information conveyed through such visual aids. Teachers must therefore provide graphic interpretation and reading tasks that facilitate the development of students' comprehension of the information communicated.

Peixoto and Cruz (2011) state that graphs are resources that encode data visually. Therefore, it is necessary for the teacher to provide students with graphic representations which facilitate the development of skills in interpreting tables and graphs related to their own daily lives. This enables them to acquire graphic comprehension and reflect on the information contained in the visual resources. In this sense, researchers highlight the importance of literacy or graphic literacy of elementary school students, especially in the initial years.

> There is a latent urgency to promote graphic literacy among students, so that they can develop the ability to analyze and interpret information critically, thereby establishing their own reading proficiency. However, this literacy must begin in the early years of elementary education. (Peixoto & Cruz, 2011, pp. 152-153)

Those authors concluded their research by highlighting the significance of incorporating graphs into the classroom, as they are also present in the majority of the curricular components taught as per the BNCC (Brazil, 2018); as well as being a means of representation constantly appropriated by the media. Additionally, the authors emphasize that graphic literacy instruction should begin in early childhood, as the task of interpreting graphs should correspond to the cognitive development stages of students. This is due to the complex nature of the task and the need for guidance; therefore, the school is responsible for fulfilling this responsibility. By the time a student reaches their senior year of high school, they should be able to comprehend each meaning represented in a graph (Peixoto & Cruz, 2011, p. 153).

In this same context, the BNCC (Brazil, 2018) proposes that:

> reading, interpreting and creating tables and graphs play a fundamental role, in the same manner that a written text produced for communication of data, as it is necessary to understand that the text must summarize or justify the conclusions. (Brazil, 2018, p. 275).

Hence, it is our contention that the inclusion of graphic resources is essential, not just for students with visual impairments, but for all students at large, as it enables them to access the informational content conveyed through graphics. Cerqueira and Ferreira (2000) argue that the use of teaching materials by educators within the classroom is essential, as it serves to enhance students' comprehension of the content, hence facilitating and promoting the teaching and learning process.

When dealing with visually impaired students, the teacher must evaluate the way they expresses themselves verbally, the teaching resources and methodology to be used, and previously acquired knowledge to decide which materials should be made accessible. From a Vygotskyan point of view, Fernandes and Healy (2009) discussed alternatives to educate teachers and equip institutions that deal with blind students, and realized that:

> deprivation of any of the means for accessing the socio-historical culture in which individuals are immersed does not fundamentally restrict human potential. Instead, it underscores the need for devising alternative channels to facilitate such access which deviate from the conventional methods described in the literature and are generally associated with learners who are regarded as typical. (Fernandes & Healy, 2009, p. 3).

The primary objectives of the authors' study were to examine the appropriateness of certain tests for students with visual impairments and to foster self-reflection among those responsible for designing assessment instruments. After analyzing the results, they concluded that the use of material instruments helps both the evaluation process and learning itself. We consider the research of those authors relevant, as it provides substantial insights into the difficulties associated with ensuring the accessibility of visual resources for visually impaired students. They also observed that:

> In order for learners to effectively compare their perceptions with the elements in their repertoire of multimodal representations, it is necessary to establish a connection between the fragmented information gathered during tactile exploration and the overall context. In contrast to the visual system, which enables the simultaneous perception of

> information, the tactile system affords a progressive and sequential experience of information. (Fernandes & Healy, 2009, p. 13).

Aligned with this perspective and taking into consideration the substantial use of visual resources in the field of statistics, Marson et al. (2013) conducted a theoretical investigation that examined the factors that should be considered while teaching statistics to visually impaired (VI) students. The researchers emphasize that a crucial aspect is the fact that statistical concepts are predominantly presented through the use of tables and graphical representations.

According to those authors, gestures made by the teacher, the use of graphic resources and the use of visual tools during a lesson are decisive in teaching of concepts and the meaning of abstract mathematical objects. Therefore, it is essential for educators who teach visually impaired students to critically examine their predominantly visual teaching methods, since blind learners rely on sensory experiences, such as touch or other non-visual senses, to signify the learning process in which they are engaged.

For Marson et al. (2013) the greatest challenge faced by visually impaired learners in educational settings is the substantial reliance on visual resources such as textbooks, class notes, and blackboards, severely limiting the access of those students to school content, due to the lack of accessibility of such teaching tools.

The researchers highlighted a significant challenge faced by visually impaired students in their access to statistics textbooks, which is attributed to the extensive use of graphs and tables that illustrate statistical concepts. According to the authors, the process of adapting visual content originally intended for sighted students to make it accessible for visually impaired students is a highly intricate task which demands commitment and thoughtful consideration on the part of teachers.

According to Marson et al. (2013), the most effective method to support visually impaired students in their learning is to create an extensive range of easily accessible educational materials. The writers conclude their recommendations by asserting that:

> It is important to keep in mind that teaching is individualized for each student and requires careful formative reflection in search of the most effective combination of teaching techniques and tools." (Marson et al., 2013, p. 24)

Methodology

A qualitative approach was employed in this study, utilizing task-based interviews as the primary data collection technique (Goldin, 2000). The author provides a definition of task-based interviews, which entail the participation of at least one respondent and an interviewer.

Such interviews comprise an interactive exchange centered around one or more tasks, such as questions, problems, or activities, as outlined in a pre-established script. In this study, the researcher employs the method proposed by Goldin (2000) to investigate the mathematical cognition and learning processes of the participant through the use of interviews. By focusing attention onto the task executed by the participant, namely through the use of statistical visual aids, the interviewer places emphasis on the actions undertaken by the visually impaired student during the performance of such tasks.

One of the methodological characteristics involved in this type of interview is

> the need to consider research proposals, which include exploratory investigation, description, inference or analysis techniques; development of conjectures; investigation or tests to raise hypotheses. (Fernandes, 2008, p. 73)

The structure of the interview enables the researcher to intervene during the execution of the task. Such intervention constitutes the methodology employed in the research. Goldin (2000) argues that interventions undertaken during activities can naturally lead to outcomes that differ from those that would likely be achieved without the researcher's interference.

That methodological choice was made in order to determine a variety of understandings regarding the cognitive processes of the participant in the research, particularly in relation to mathematical knowledge, notably statistical objects. The empirical data analyzed focus on the actions and interactional processes of the blind student participating in the research (Goldin, 2000). Hence, the data was subjected to analysis wherein the reading process conducted by the student participating in the investigation was described and elucidated.

An interview was conducted with a fifth-year elementary school student who had congenital blindness. The interview lasted for one hour. Within this timeframe, we presented the participant with two sector graphs, one generated using the *MONET* software and the other manually crafted. Additionally, we provided a graph from a braille textbook produced by *Benjamin Constant Institute*, which was created using the *Braille Fácil* program. The objective of this study was to utilize three distinct forms of tactile graphic representations in order to enable the organization of reflections and promote discussions regarding the possibilities of each tool.

The interview

In the first part of the interview, we asked questions that allowed us to evaluate the profile and prior knowledge of the participant. For the purpose of preserving confidentiality we will refer to the participant as “Shaun”. Shaun and his mother reported that he lost his vision when he was four years old, therefore his condition is characterized as congenital blindness. The statistical visual resources utilized in the activities were of particular interest and relevance to Shaun. This is because Shaun does not possess any visual memory of events that occurred prior to his loss of vision. Thus, it can be inferred that his current memories are primarily derived from auditory, tactile, olfactory, and gustatory experiences.

In 2019, at the time of the interview, Shaun was 11 years old and lived in the municipality of Itaboraí (RJ), approximately 58.8 km from the *Benjamin Constant Institute* (IBC). The student reported that prior to attending IBC, he had been enrolled in a daycare center. Shaun joined early childhood education in 2013 and completed his whole primary education at IBC. This suggests that the majority of the knowledge obtained by the participant, up to the time of the interview, was acquired during his tenure at IBC.

In the interview, the student mentioned having missed a few classes due to “transportation issues”. However, Shaun attended every class during the observations for the study. When asked about what he learned in statistics, he reported that he did not know how to read a bar graph, regardless of the accessible material.

In the second part of the interview, the discussion shifted towards guiding the participant regarding the interpretation of graphical representations within the statistical resource. Initially, the student was asked if he knew what the shape of a pizza pie was. The participant responded by making a circular movement with his right hand (Figure 2). At this point, the participant was informed that the graph being explored was called a pie chart or sector graph, as it has a circular shape and was divided into sectors that can be compared to slices in a pizza.

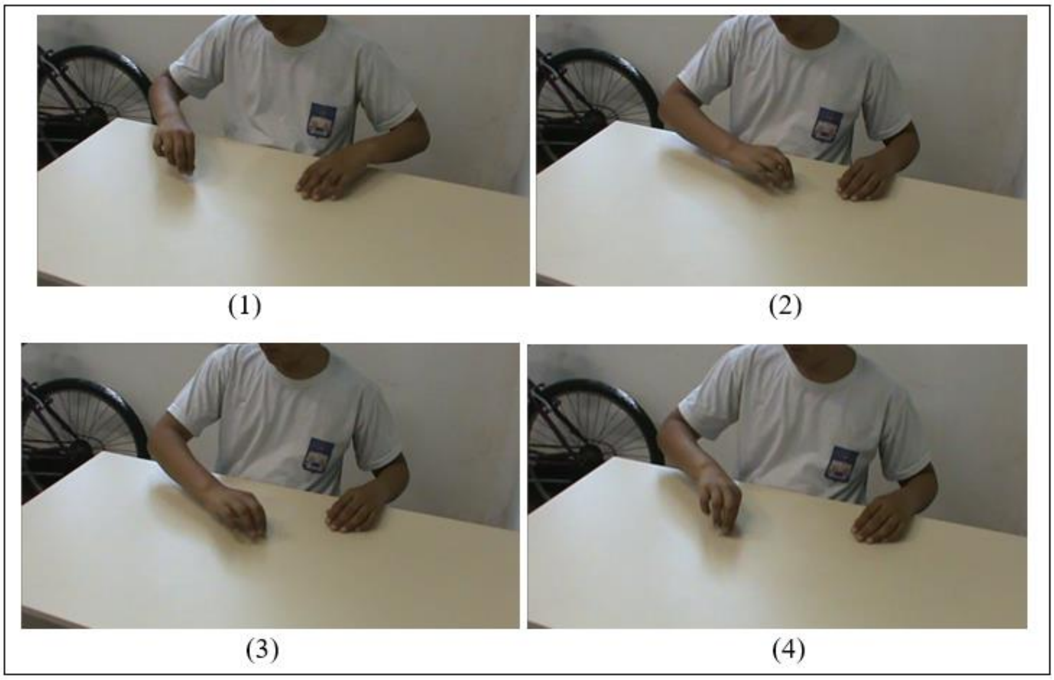

Figure 2.

(1), (2), (3) and (4) depict a circular movement performed by the student with his right hand on the table (Authors)

Upon beginning the proposed activity, we instructed the visually impaired student on how to interpret (read) the graph from a textbook, which was rendered in ink (Figure 3), created through the use of the *Braille Fácil* program (Figure 4). Subsequently, we facilitated the reading process by presenting the graph that was produced using the MONET software (Figure 5). Finally, we directed the student to read the handcrafted graphic developed by the research team (Figure 6), that is, tactile material not produced using a braille printer. During this meeting, we introduced essential statistical concepts to the student, as we were unsure whether any statistics content had been presented to him in elementary school.

Subsequently, we offered provided instructions for reading by presenting the graph that had been generated using the MONET software (Figure 5). Finally, we invited the participant to examine the manually crafted visual representation created by the researchers (Figure 6), which is a tangible medium produced without the use of a braille printer. During the meeting, we presented the student with an overview of basic statistical principles introduction regarding fundamental statistical concepts, as we lacked information regarding the inclusion of statistics in his elementary school curriculum.

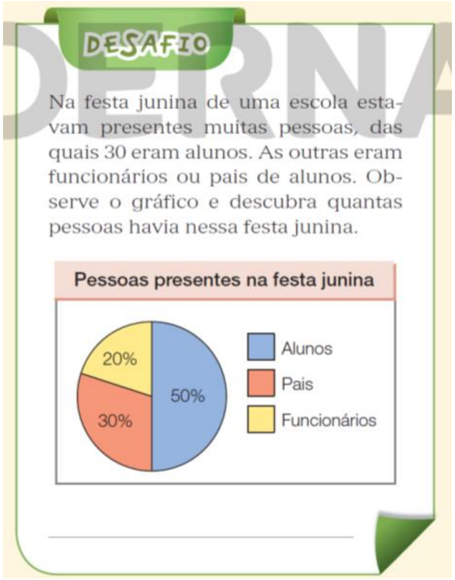

Figure 3.

Ink book activity. (Rocha, 2014, p. 227)

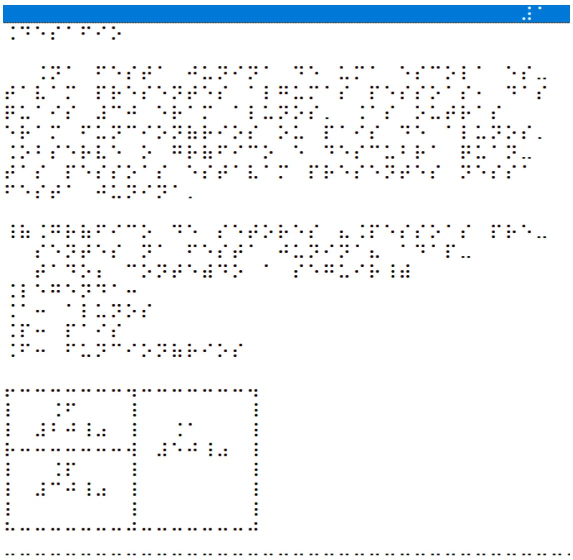

Figure 4.

Book activity in accessible ink produced in Easy Braille. (Benjamin Constant Institute)

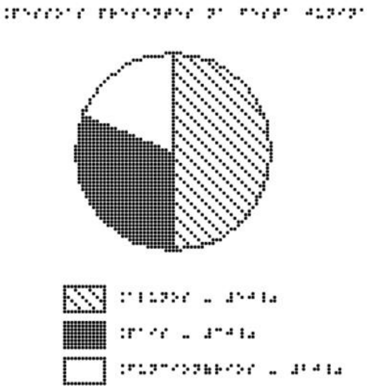

Figure 5.

Sector graph produced in MONET of the book activity in ink. (Authors)

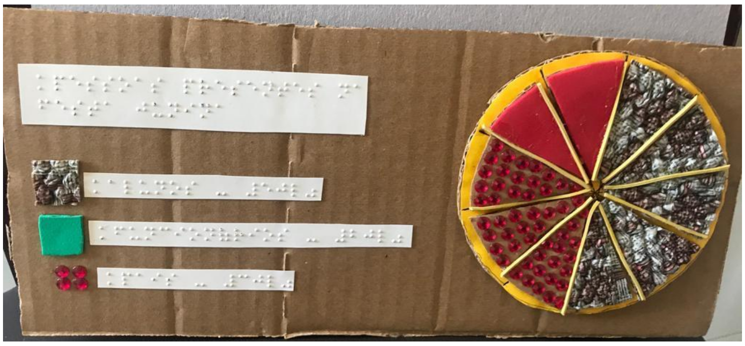

Figure 6.

Handmade sector graph produced by the researchers derived from the book activity in ink. (Authors)

At first, we provided Shaun with the graph shown in Figure 4 and instructed him to examine the content without any interference. Upon initial interaction with the graph, we clarified that the activity sheet started with a statement, which was subsequently followed by a caption. Subsequently, we requested that he examined both the statement and the caption. It was evident to the researchers that Shaun read both passages very rapidly.

After reading the statement and caption referring to the graph presented, Shaun began exploring the statistical resource in a random manner, unable to understand the information contained in it. Thus, we had to intervene and guide Shaun's hands (Figure 7), in order to explain to him that the sector graph he was trying to read was transformed into a rectangle partitioned internally.

Figure 7.

The researcher places Shaun's left hand fingers on one side of the rectangle. (Authors)

After the clarification regarding the rectangular form of the sector graph, we allowed the student to explore it so that he could perceive its internal details. Afterwards the participant became familiar with the graphical representation, we asked about the number of divisions in the rectangle and information contained in each partition. The student provided accurate answers, stating that each part corresponded to the percentages of the qualitative values of the activity.

At this point, we asked if the participant had learned percentages in his mathematics classes and he replied that he had not. Therefore, the researcher intervened and explained to the student what the percentage would be and how to use it to perform calculations. The student seemed to have good knowledge of fractions, as the researcher was able to explain the content in question through the idea of dividing a whole object into 100 equal parts. After that, we explained to the participant that, to calculate the value corresponding to a certain percentage of a number, one should multiply it by the numerator of the number and divide the value obtained by one hundred.

The researcher found that the student was able to understand the explanations, when he began analyzing the graph to perform the necessary percentage calculations to solve the task. Upon solving the task, Shaun was presented to the graph in Figure 5. He began reading the title of the representation, and subsequently read the accompanying caption. He offered the following remark.

Shaun: I think I know; this one must be the same as the one with the cars and dolls.

From the student's words, it became evident that he observed a correlation between the various textures in the caption and the distinct qualitative values depicted in the graph. We asked whether he was able to explain the meaning behind each texture, and he effortlessly gestured with his hand to identify the precise location of each qualitative value on the graph. When the researcher inquired about the segment of the graph that depicted the number of students, he instantly pointed out the corresponding area.

It was noted that the student tactually examined the graph by running his hands over the sheet to read its description. Subsequently, he explored the visual representation in order to find specific information of interest, as depicted in Figure 8.

Figure 8.

(1) Shaun reads the graph caption / (2) Shaun explores the sector graph produced with MONET. (Authors)

Upon completion of the reading, we retrieved the graph presented and provided the participant with the hand-crafted graph (Figure 6). The student again began reading the resource by its title. It was observed that the participant exhibited a greater rate of graphic reading speed in comparison to the preceding task. Upon reading the caption, he promptly identified the value associated with each texture in the graph. Figure 9 shows the participant exploring the caption of the hand-crafted graph.

Figure 9.

Shaun's fingers positioned over the title of the handmade graphic. (Authors)

During his tactile exploration, Shaun noticed that the graph was divided into several sectors, as illustrated by the following statement.

Shaun: There are a lot of little triangles.

We explained to the participant that the "triangles" he perceived by touch were actually the sectors of the circle. After that, we assisted the student by physically guiding his hands to trace the curve of one of the sections on the graph using his fingers. This was done to demonstrate to him that the shape of the arc differed from that of a straight line segment (Figure 10).

Figure 10.

The researcher guides Shaun's left hand over the arc of one of the sectors of the hand-crafted graph. (Authors)

Finally, we asked the participant the final questions of this first part of the second session, as shown in the following dialogue:

Researcher: Which of the three graphics did you like the most?

Shaun: I liked all of them. They are all good.

Researcher: Which one would you use to answer more questions in the activity, if applicable?

Shaun: All of them. They are all good.

It is significant to mention that we instructed the participant to initially examine the graph by exploring each of its components, in order to facilitate his subsequent overall comprehension of the graph (Fernandes & Healy, 2009).

Furthermore, we found that the suggested reading method had a positive impact on the participant, as it assisted him in solving the graph-related task. However, it remains uncertain whether such strategies may effectively facilitate tactile reading for other children, as the instruction for each learner "is personalized and necessitates meticulous formative contemplation to determine the optimal combination of teaching methodologies and resources" (Marson et al., 2013, p.24).

Despite our persistent inquiry regarding the student's preference for one of the three graphs, his answer remained the same, indicating a clear comprehension of the information presented in the visual representations, after our initial intervention.

Final Considerations

The interview activities yielded results that provided insights into the participant's ability to comprehend statistical visual resources through tactile reading. These findings also shed light onto the possibilities of such representations for congenitally blind students.

Throughout the task, we came to the realization that the ability for interpreting graphs is not acquired spontaneously; rather, it involves sequential stages in the student's cognitive development "therefore, requires guidance [...] so that the student is able to comprehend various meanings represented in the graph." (Peixoto & Cruz, 2011, p. 153).

The gestures made by the student provided a clearer understanding of the method adopted for tactile reading of the representations. In general, Shaun interpreted the sector graphics as follows: 1st) he examined the title and explored the captions referring to the sectors of the graph; 2nd) he positioned his fingers over the sectors in order to determine their sizes and the categories each of them represented.

The process of interpreting figures (graphs and tables) by touch can be challenging for individuals with visual impairments who are not familiar with such pictorial elements, as they are designed to convey information quickly and concisely to sighted individuals. Therefore, representing statistical visual resources for blind students poses a significant challenge, as

> learners must correlate fragmented information gathered during tactile exploration with their repertoire of multimodal representations in order to compare the elements perceived with the whole. Unlike the visual system which allows for the simultaneous processing of information, the tactile system only allows for a gradual, sequential processing of information. (Fernandes & Healy, 2009, p. 13).

In line with other researchers, we advocate for the development of statistical proficiency that provides individuals with the ability to engage in critical analysis of data presented in statistical studies.

In general, we observed that representations developed for students with visual impairments must be carefully thought out and analyzed so that they are as effective as possible for their learning (Marson et al., 2013). The representations shown in this article were inspired by workshops we participated in at IBC (Tactile Graphics Elaboration Workshop and Mathematics Workshop) and by conversations with educators from the Institute.

This study is restricted to the tactile interpretation of sector graphs by a single congenitally blind student attending a specialized education institution. Due to the time required to complete the doctorate of the first author and the pandemic caused by covid-19, interviews with other visually impaired students attending IBC or regular classrooms were not feasible. In the future, we aim to conduct research regarding mathematics textbooks in Braille for different levels of education, such as, mathematics textbooks spanning the three years of high school. Moreover, our focus may be directed towards other mathematical content such as the study of functions and geometric shapes.

In conclusion, we anticipate that this article will help improve inclusive mathematics education by supporting the teaching and learning of visually impaired students. Furthermore, it will serve as a catalyst for future research in this area.

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