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Building pathways for Mathematics classes from an inclusive perspective through the Universal Design for Learning approach

Construyendo caminos para las clases de Matemáticas desde una perspectiva inclusiva a través del enfoque de Diseño Universal para el Aprendizaje

Construire des parcours pour les cours de mathématiques dans une perspective inclusive grâce à l'approche Universal Design for Learning

Construindo caminhos para aulas de Matemática na perspectiva inclusiva por meio da abordagem do Desenho Universal para Aprendizagem

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Abstract

With a view to promoting inclusive mathematics education, it is essential to adopt new paradigms that eliminate or reduce barriers, allowing for inclusive didactic practices. This research aims to analyze the didactic processes of a teacher-researcher weaving relationships that bring the UDL closer to inclusive mathematics education, with the intention of enriching the academic experience and promoting a full life. The methodology used was qualitative,

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through pedagogical intervention with didactic practices applied to 28 participants, children aged between 7 and 10 years, from a class of the 3rd year of Elementary School I, in a municipal school in Araucária (PR). Nineteen math meetings were held, in which 16 practical activities were applied. In this proposal, five practical activities on the Brazilian Monetary System will be presented and analyzed. Through the analysis of the observations and the theoretical basis, the dialogues and situations that occurred in contact with the students were confronted. So far, it is possible to state that planning based on the development of mathematics activities from an inclusive perspective, based on the UDL, made knowledge accessible to all students, especially those with greater learning difficulties. In this way, planning can significantly contribute to the interaction and accessibility of different types of knowledge, highlighting the importance of mathematical knowledge in this process.

Keywords: Universal Design for Learning; Mathematics; Inclusion; Didactic practices; Elementary School.

Resumen

Con miras a promover la educación matemática inclusiva, es fundamental adoptar nuevos paradigmas que eliminen o reduzcan las barreras, permitiendo prácticas didácticas inclusivas. Esta investigación tiene como objetivo analizar los procesos didácticos de un docente-investigador tejiendo relaciones que acercan la UDL a la educación matemática inclusiva, con la intención de enriquecer la experiencia académica y promover una vida plena. La metodología utilizada fue cualitativa, a través de intervención pedagógica con prácticas didácticas aplicadas a 28 participantes, niños con edades entre 7 y 10 años, de una clase de 3º año de la Escuela Básica I, en una escuela municipal de Araucária (PR). Se realizaron diecinueve encuentros de matemáticas, en los que se aplicaron 16 actividades prácticas. En esta propuesta, serán presentadas y analizadas cinco actividades prácticas sobre el Sistema Monetario Brasileño. A través del análisis de las observaciones y la base teórica, se confrontaron los diálogos y situaciones que ocurrieron en el contacto con los estudiantes. Hasta el momento, es posible afirmar que la planificación basada en el desarrollo de actividades matemáticas desde una perspectiva inclusiva, basada en la UDL, hizo accesible el conocimiento a todos los estudiantes, especialmente a aquellos con mayores dificultades de aprendizaje. De esta forma, la planificación puede contribuir significativamente a la interacción y accesibilidad de diferentes tipos de conocimiento, destacando la importancia del conocimiento matemático en este proceso.

Palabras clave: Diseño Universal para el Aprendizaje; Matemáticas; Inclusión; prácticas didácticas; Enseñanza fundamental.

Résumé

En vue de promouvoir une éducation inclusive en mathématiques, il est essentiel d'adopter de nouveaux paradigmes qui éliminent ou réduisent les obstacles, permettant des pratiques didactiques inclusives. Cette recherche vise à analyser les processus didactiques d'un enseignant-chercheur tissant des relations qui rapprochent l'UDL de l'éducation inclusive en mathématiques, avec l'intention d'enrichir l'expérience académique et de favoriser une vie épanouie. La méthodologie utilisée était qualitative, à travers une intervention pédagogique avec des pratiques didactiques appliquées à 28 participants, des enfants âgés de 7 à 10 ans, d'une classe de 3e année de l'école élémentaire I, dans une école municipale d'Araucária (PR). Dix-neuf réunions de mathématiques ont eu lieu, au cours desquelles 16 activités pratiques ont été appliquées. Dans cette proposition, cinq activités pratiques sur le système monétaire brésilien seront présentées et analysées. A travers l'analyse des observations et des bases théoriques, les dialogues et les situations qui se sont déroulés au contact des étudiants ont été confrontés. Jusqu'à présent, il est possible d'affirmer qu'une planification basée sur le développement d'activités mathématiques dans une perspective inclusive, basée sur l'UDL, a rendu les connaissances accessibles à tous les élèves, en particulier ceux ayant de plus grandes difficultés d'apprentissage. De cette manière, la planification peut contribuer de manière significative à l'interaction et à l'accessibilité de différents types de connaissances, soulignant l'importance des connaissances mathématiques dans ce processus.

Mots-clés: Conception universelle pour l'apprentissage , Mathématiques; Inclusion, Pratiques didactiques, École primaire.

Resumo

Com vistas a promover uma educação inclusiva em matemática, é essencial adotar novos paradigmas que eliminem ou reduzam as barreiras, permitindo práticas didáticas inclusivas. Esta pesquisa tem como objetivo analisar processos didáticos de uma professora-pesquisadora tecendo relações que aproximam o DUA da educação matemática inclusiva, com a intenção de enriquecer a experiência acadêmica e promover uma vida plena. A metodologia empregada é qualitativa, por meio de intervenção pedagógica com práticas didáticas aplicadas a 28 participantes, crianças com idades entre 7 e 10 anos, de uma turma do 3º ano do Ensino Fundamental I, em uma escola municipal de Araucária (PR). Foram realizados 19 encontros de matemática, nos quais foram aplicadas 16 atividades práticas. Nesta proposta, serão apresentadas e analisadas cinco atividades práticas sobre o Sistema Monetário Brasileiro. Por

meio da análise das observações e do embasamento teórico, foram confrontados os diálogos e situações ocorridas no contato com os estudantes. Até o momento, é possível afirmar que um planejamento baseado no desenvolvimento de atividades de matemática na perspectiva inclusiva, fundamentadas no DUA, tornou o conhecimento acessível a todos os estudantes, especialmente aqueles que apresentaram maiores dificuldades de aprendizagem. Dessa forma, o planejamento pode contribuir de maneira significativa na interação e na acessibilidade de diversos conhecimentos, destacando a importância dos conhecimentos matemáticos nesse processo.

Palavras-chave: Desenho Universal para Aprendizagem; Matemática; Inclusão; Práticas didáticas; Ensino Fundamental.

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Despite progress in promoting educational inclusion, ensuring this right remains a challenge. It becomes essential to transcend such aspects and documents, offering approaches and solutions that cater to the individual needs of students, which includes relationships of affection, solidarity, understanding, and a sense of belonging in school spaces. For inclusive practice, it is necessary for the teacher to understand the diversity of learning styles and use differentiated pedagogical resources that overcome the pedagogical barriers faced by students (A.R.T. Góes & Costa, 2022).

Teaching and learning practices in mathematics for historically marginalized groups in the school context have aimed to overcome the exclusion and underestimation faced by these individuals in conventional math education. When these discussions began, materials were adapted for specific audiences, such as people with disabilities. However, inclusion should be considered for all students, regardless of whether they belong to a specific audience, as each individual has their own way of learning and demonstrating their learning (A.R.T. Góes & Costa, 2022), which presents a challenge for educators who encounter resistance in all areas of lifelong development and learning (Viana & Manrique, 2019). Differences should not be ignored or dismissed; on the contrary, they should be validated through differentiated pedagogical practices, promoting access to knowledge for all students (Nogueira, 2020).

> A planning with a focus on equity also demands a clear commitment to reversing the historical exclusion that marginalizes groups – such as indigenous peoples and the populations of quilombo communities and other Afro-descendants – and individuals who couldn't attend or complete their schooling at the appropriate age. It equally requires a commitment to students with disabilities, recognizing the need for inclusive pedagogical practices and curriculum differentiation, as established in the Brazilian Law of Inclusion of Persons with Disabilities – Law No. 13.146 (2015) (Brazil, 2017, p. 16, our translation).

In this regard, promoting equity and inclusion is associated with recognizing and valuing students' cultural knowledge, experiences, and perspectives. To achieve this, there is a need to employ sensitive and contextualized strategies that enable the construction of mathematical meanings. Diverse and technological equipment and materials are required to meet the students' needs, stimulating the potential of each individual, fostering the development of reasoning, problem-solving skills, critical thinking, and enhancing their confidence and autonomy to confront and overcome daily challenges (Takinaga, 2016).

An approach within the classroom context that aims to develop the indicated aspects is the Universal Design for Learning (UDL). This methodological approach has been gaining prominence as an ally in the inclusion process for equitable education, as can be seen in the works of Bachmann (2020), Cassano (2022), Coelho and A.R.T. Góes (2021), Muzzio et al. (2022), Velasco and Barbosa (2022), where they demonstrate the elimination of attitudinal and pedagogical barriers.

This approach is not rigid, inflexible, or closed; it is guiding, "[...] it is not about a pedagogical preference or a teaching model, but rather an emphasis on the need to renew practices" (Zerbato & Mendes, 2018, p. 150, our translation), providing considerations for the teacher to plan their classes to meet the needs of all students.

With the aim of contributing to Inclusive Mathematics Education, this text aims to analyze didactic processes of a teacher-researcher (the first author of this text), establishing connections that bring UDL closer to inclusive mathematics education. It involves a pedagogical intervention in a 3rd-grade class of Elementary School I, in a school within the municipal network of Araucária/PR, in the metropolitan region of Curitiba/PR, Brazil. The curriculum content addressed is the Brazilian Monetary System (BMS), which holds social relevance, facilitating academic breadth for a full life while minimizing barriers faced by teachers and students who encounter mathematical challenges, both in the educational process and in inclusion, particularly in the teaching of mathematics in an inclusive perspective.

Mathematics Education from an Inclusive Perspective

Mathematics is present in various fields and professions, such as science, technology, engineering, finance, and many others. It provides the foundations for understanding and solving problems, contributing to the development of logical reasoning, critical analysis, and informed decision-making. However,

> understanding mathematical learning is not a simple task, especially when we consider the relationships between this process and the subjectivity of the individuals involved, given that the understandings are plural, both in the possibilities of topics to be considered and in conceptual approaches. (Morais & Peixoto, 2022, p. 560, our translation).

Furthermore, authors A.R.T. Góes and H.C. Góes (2015) express that mathematics plays an important role in people's lives because it is present in various everyday contexts, such as at home, in the workplace, in commerce, in studies, and more.

In the school context, for example, the subject of mathematics encompasses a wide range of concepts related to numbers, algebra, geometry, quantities and measurements, probability, and statistics. Through the learning of these concepts, students can acquire solutions and strategies to solve problems that are connected to their reality. Additionally, by developing mathematical communication skills, we are able to overcome geographical, temporal, linguistic, and social barriers (Rodrigues et al., 2023).

According to the Common National Curriculum Base (CNCB)[5], (Brazil, 2017, p. 266, our translate), mathematics "[...] ensures that students recognize that mathematical knowledge is fundamental for understanding and acting in the world." The integration of mathematics by the student in the school context allows them to acquire knowledge related to and understood through real-world problems. "Another relevant aspect of this approach is the fact that curriculum content emerges from the real situation to address a specific question, giving meaning to what is learned" (Littig et al., 2020, p. 315-316, our translation).

Furthermore, mathematics can be integrated into other areas of knowledge, providing tools and methods for interpreting and analyzing data, understanding patterns and relationships, modeling real-world phenomena, and making evidence-based decisions. However, it is not enough to know mathematics to teach mathematics; it is necessary to understand, both theoretically and experientially, the processes of teaching and learning this science in different educational contexts and at all school levels. This also encompasses the initial and ongoing teacher training, the development of educational materials, and the study of the daily school environment (Nogueira et al., 2022).

By overcoming various barriers in mathematics, we enter the field of inclusive mathematics education, which seeks to recognize the importance of making the teaching and learning of mathematics accessible and relevant to all students, regardless of their abilities or specialized educational needs. However, unfortunately,

> The logic of exclusion is undeniably one of the most significant forms of violence against people with disabilities. The school, the place of socialization par excellence, still too often practices the exclusion of students with special educational needs, either explicitly or, more insidiously, implicitly, by "tolerating" their presence in the classrooms. Seeking to minimize this situation, Inclusion or inclusion strategies – a relatively new concept in some countries, such as Brazil, for example – appear in different instances, especially in Education (Nogueira, et al. 2020, p. 185, our translation).

[5] Where it reads Common National Curriculum Base or CNCB was translated by the authors of this text.

As stated in the CNCB (Brazil, 2017, p. 14), the school environment should be a "[...] space for learning and inclusive democracy, strengthening itself through the coercive practice of non-discrimination, anti-prejudice, and respect for differences and diversities." It involves understanding the true meaning of inclusion, providing affection, integration, opportunities, and support. "In this movement, mathematics education will be established in an inclusive perspective, being redesigned to promote a school centered on ethics, respect for individualities, and diversity" (Viana & Manrique, 2019, p. 664, our translation). Furthermore, it is necessary to promote students' understanding and motivation to learn, even though the school may not always recognize the variety of mathematical expression forms; its unique codes and languages play an important role in the social inclusion of all individuals (Rodrigues et al., 2023).

Teaching Mathematics from an inclusive perspective requires overcoming daily barriers, developing a sensitive understanding of the diversity of individuals, seeking stimuli to work with the remaining senses so that content assimilation happens naturally. Silva and Diáz-Urdaneta (2021, p. 5, our translation) offer possibilities to reflect on how to equip our students, aiming to provide "opportunities in the social sphere and in society's dynamics to seek, inquire, organize, represent, and publicize this information, focusing on critical, reflective, and mathematical approaches." Furthermore, to think about mathematics for all, firstly,

> we must equip our students and teachers with an awareness of mathematics and the proper use of mathematical, technological, and, above all, inclusive tools so that they can solve problems and other teaching situations present in society based on well-formulated strategies throughout the journey (Silva & Diáz-Urdaneta, 2021, p. 51, our translation).

By providing different resources for students, it will contribute to the promotion of inclusive mathematics education, offering various learning opportunities to all students who require resources to facilitate the enhancement of mathematical concepts (Silva & Diáz-Urdaneta, 2021). For example, expanding the representation of mathematical concepts through the use of visual resources, such as graphs and diagrams; tactile resources, like concrete models; auditory resources, such as sound representations; and digital resources, like interactive tools. With this, the teacher can "provide students with means to develop concepts, promoting significant advancements in the teaching and learning process" (Borges et al., 2022, p. 395, our translation).

In this perspective, inclusive mathematics education aims to provide all students with the development, application, and understanding of mathematical concepts. It values diversity and encourages a culture of collaborative and inclusive learning. In this context, it is necessary to adopt a flexible methodological approach, taking into consideration that each student has their own particularities. This way, we understand that there is no standard student profile, and we can observe these aspects in the UDL model.

Universal Design for Learning

Universal Design for Learning (UDL) has its origins in the ideas of Universal Design (UD), beginning with its recognition and development. In the 1960s in the United States, UD started to be recognized and developed by architects and designers when they began to question the approach of designing spaces exclusively for people without disabilities, leading to discussions about accessibility and the rights of people with disabilities, particularly due to the high number of prominent war veterans who were disabled after World War II (A.R.T. Góes & Costa, 2022). However, it was during the 1980s and the early 1990s that the concept of UD began to gain widespread accessibility, especially with the proposal of architect Ronald Mace, who used a wheelchair and an artificial ventilator. This highlights the growing importance of accessibility among construction professionals (Cassano, et al. 2022).

Universal Design has in its conception, "[...] the design of products and environments that can be used by everyone to the greatest extent possible without the need for adaptation or specialized design for people with disabilities" (Carletto & Cambiaghi, 2007, p. 10, our translation), meaning it does not aim to create a new science or style, but rather enhance the perception when designing environments and products to make them usable by all people (A.R.T. Góes & Costa, 2022).

It is evident that this conception, which deals with designs for society's everyday life, would have its influence in the school environment, as

> [...] the school, as an integral part of society, cannot lag behind in such discussions, not only about physical access, suitable furniture, and equipment but also about the need to expand and promote accessibility for all students to learning (Cassano et al., 2022, p. 5, our translation).

With this, in the 1990s, the concept of UDL emerged, originating from the discussions of researchers at the Center for Applied Special Technology (CAST) to develop a digital book that would be accessible to all users. These discussions led to the understanding that it is

necessary to eliminate or reduce methodological barriers to learning in order to ensure an equitable approach that can be applied to all students, but at the same time, flexible enough to meet the specific educational needs of each one (A.R.T. Góes & Costa, 2022). This approach provides a diverse set of possibilities that enhance and strengthen student learning, regardless of whether they have disabilities or not, promoting the construction of knowledge in a universalized manner (A.R.T. Góes & Costa, 2022).

UDL is recognized as an important alternative to achieve inclusive education as it prioritizes the provision of instructional resources that enable students to have different modes of learning and demonstrating their knowledge. In this way, it does not follow a fixed model, as it is crucial to understand the individuals involved to determine the best way to serve them (A.R.T. Góes & Costa, 2022), allowing the teacher the necessary flexibility to plan their actions, reducing methodological barriers, including the development of objectives, methods, materials, resources, and assessment methods aimed at promoting knowledge construction for all students. With UDL and the creation of flexible proposals from the outset, it provides customizable options that allow all students to progress from their individual starting points, rather than being limited by pre-established assumptions, all through their learning networks (Sebastián-Heredero, 2020).

UDL has in its conception three distinct brain networks that are related to each of the principles of this methodological approach (Cast, 2018). The first network is the affective one, which seeks to identify and promote students' interests, encouraging motivation and engagement in learning; thus, it is related to the principle of Engagement. The second is the recognition network, related to the principle of Representation, aiming to offer students various ways to perceive the presented information and transform it into practical and applicable knowledge in their personal experiences. The third network is the strategic one, whose goal is to enable students to initiate their actions in the educational context through planning and organizing their activities, related to the principle of Action and Expression (Cast, 2018).

In the principle of Engagement, students are motivated to solve problem situations through differentiated activities, taking into account the affective dimension of learning. It is important to highlight the diversity of work, as both curious and uninterested and resistant students are encouraged to learn specific concepts during teaching actions (A.R.T. Góes & Costa, 2022). The guidelines of this principle include options to stimulate student interest, provide support for effort and persistence, and promote self-regulation (Cast, 2018).

In the second principle of UDL, called Representation, the diversity of students who have different difficulties and learn in various ways for knowledge construction is considered.

Through it, the goal is to provide a variety of learning opportunities, increasing the possibility of connections between concepts (A.R.T. Góes & Costa, 2022). Therefore, the guidelines for this principle involve options for perception, language use, mathematical operations and symbols, and understanding (Cast, 2018).

In the third principle of UDL, Action and Expression, students are encouraged to explore information in learning environments and to use their prior knowledge to solve problems. Through it, the importance of expressing knowledge through writing, oral communication, and other forms of communication is recognized (A.R.T. Góes & Costa, 2022). The guidelines for this principle cover options for physical activities, expression and communication, and executive functions (Cast, 2018).

The principles and guidelines of UDL reflect different levels of student learning.

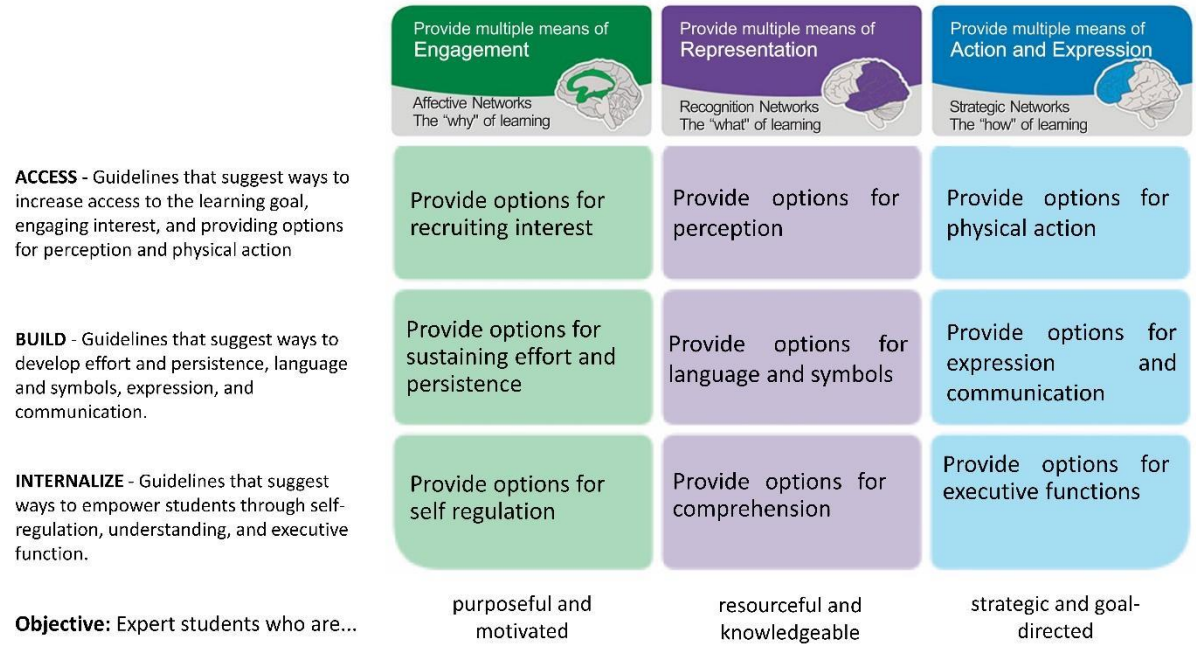


Figura 1.

Principles and Guidelines of Universal Design for Learning (CAST, 2018, adapted by authors, 2023)

In Figure 1, the top guidelines indicate ways of accessing learning, the middle guidelines are related to constructing learning, and the bottom guidelines point out ways to internalize acquired knowledge. In instructional processes that are able to meet the Engagement guidelines, dedicated and motivated students are provided; by adhering to the Representation guidelines,

resourceful and experienced students are obtained; and by following the guidelines of Action and Expression, students become strategic and goal-directed (Coelho & A.R.T. Góes, 2021).

Each guideline includes checkpoints that seek tools and resources that can cater to all students, allowing for the construction of knowledge and the association of content with their everyday experiences. This results in motivated students with extensive knowledge and strategic skills to solve problems that arise in their lives (Cast, 2018). Another point is that there is no obligation to use all UDL guidelines and checkpoints in a single learning opportunity; it will depend on the objectives and goals set (Cast, 2018).

UDL ensures access for all students to the teaching process, regardless of their abilities, needs, and individual characteristics. It adopts a pedagogical approach that values the diversity of students as an opportunity for teaching strategies, flexibly planning to provide various resources in the same teaching unit so that all students can understand and actively participate in learning. Understanding the essence of UDL leads us to the next section, which presents the methodological pathways of this research.

The methodological pathways of the research

The research has a qualitative approach (Lüdke & André, 2020) and is of the pedagogical intervention type, as it is "[...] intended to produce advancements, improvements, in the learning processes of the subjects, and the subsequent evaluation of the effects of these interferences" (Damiani et al., 2013, p. 58). Thus, it aims to analyze didactic processes of a teacher-researcher, weaving connections that relate DUA to inclusive mathematics education.

These processes are excerpts from Stellfeld's research (2023), developed and implemented in a 3rd-grade class of Elementary School I, consisting of 28 participants from a municipal school in the city of Araucária/PR. As this is a research involving human subjects, it was approved by the Ethics Committee on Research with Human Beings at the Federal University of Paraná, protocol CAAE no. 59276522.0.0000.0214, and opinion no. 5.545.178 dated July 26, 2022.

Among the research participants, there is a child with Autism Spectrum Disorder (ASD), Arthur (8 years old), and another child undergoing an investigation for possible signs of Intellectual Disability (ID), Pedro (9 years old). It is worth noting that, to maintain the anonymity of the participants, all names are fictitious. Another important aspect to highlight about the participants is that a little over half of the children are repeating the school year and

have learning difficulties. Consequently, the age range of the children is between 7 and 10 years old.

The mathematical content covered is the Brazilian Monetary System (SBM), as suggested in the Araucária Curriculum Organization document (2019)[6] for the third quarter of the academic year. To achieve this, the learning objectives are related to solving and formulating problems involving comparison and the.

> [...] equivalence of monetary values in the Brazilian system in situations of buying, selling, and exchanging; Understanding historical aspects related to the Brazilian Monetary System; Comprehending the various contexts in which money is used through the reading of texts that circulate in commerce, purchase and sale situations, field research, exchange of experiences among peers, and other situations; Recognizing and establishing exchange relationships between the banknotes and coins that circulate in Brazil, solving and formulating problems involving the Brazilian Monetary System; Learning and using words related to the context of commerce: on credit, in cash, discounts and surcharges, change, installments, credit, debt, profit, loss, check, credit card, bank slips, and more (Araucária, 2019, p. 586, our translation).

Furthermore, the development of activities was based on the principles of inclusion and guided by the guidelines of the DUA, promoting inclusive education and encouraging students to reflect on an inclusive society.

In total, 19 meetings were held, and 16 practical intervention activities were implemented, grouped into two stages. The first stage, consisting of seven meetings with four activities, included presenting the research to the entire school community, including administrators, educators, staff, and students' caregivers, sensitizing the participants to respect for others, promoting empathy, and fostering harmonious coexistence with differences. Specifically with the students, activities such as watching videos depicting situations experienced by people with disabilities, exploring Braille books, tactile games, obstacle courses, among others, were conducted to eliminate prejudices and stereotypes, promoting an understanding of human diversity.

The second stage, the pedagogical intervention in the classroom, consisted of 12 meetings and 12 activities. In addition to addressing the mathematical concepts outlined in the municipality's official education documents, it explored paths towards an inclusive society. Among the activities was a group discussion about the purpose and use of banknotes and coins, in which students had the opportunity to handle real-sized banknotes and coins, as well as copies

[6]Where it reads Araucária Curriculum Organization document (2019) was translated by the authors of this text.

of checks, promissory notes, credit/debit card machines, calculators, and bank cards. In an effort to familiarize them with various modes of communication, these objects were presented with captions, Braille printing, and Libras symbolism. Videos about the manufacturing of Brazilian banknotes and coins were shown to the students, sparking dialogues related to economics and Financial Mathematics Education.

The activities described up to this point summarize the intervention conducted in the first six meetings and activities. However, for this text, we have focused on describing in detail and analyzing the next five meetings and activities, totaling 11 class hours of 50 minutes each, as expressed in Table 1. For each named theme, the DUA guidelines present in the planning are indicated, along with a brief description of the activity.

Table 1.

Stage of Practical Application (Stellfeld, 2023, p. 112, our translation*)*

Let's talk about inclusion and the market with accessibility?	
Principle of Engagement Guidelines: Provide options to stimulate interest; provide options to sustain effort and persistence; provide options for self-regulation.	Discussing inclusion and the creation of an accessible market; questioning what an accessible market that serves both people with and without disabilities would be like – what an ideal market would be, what it should have, appropriate heights, aisles, packaging, identification methods, among other aspects.
Shall we build a market?	
Principle of Representation Guidelines: Provide options for perception; provide options for languages and symbols; provide options for comprehension.	Engage students in sorting packaging by categories (cleaning, food, cold products, hygiene), comparing prices between product brochures from different markets, creating a shopping list of items they would like to buy at the markets, organizing products to build an accessible market with features like Braille prices, Libras signage, textures, shelf heights, aisle spaces, etc. Encourage discussion about what else needs to be included to make the setup of the market as inclusive as possible.
Today is shopping day!	
Principle of Representation Guidelines: Provide options for perception; provide options for languages and symbols. **Principle of Action and Expression Guidelines:** Provide options for physical action; provide options for expression and communication.	Divide the packaging into two markets, one inclusive and the other not; conduct shopping; assign roles to market employees: cashier, packer, salespeople, among others; distribute play money to the students; after shopping and rotating the "employees," calculate the expenses of each one and check whether they saved money or spent it all.
Shall we talk and record?	

Principle of Engagement Guidelines: Provide options to stimulate interest; provide options to sustain effort and persistence; provide options for self-regulation. **Principle of Action and Expression Guidelines:** Provide options for physical action; provide options for expression and communication; provide options for executive functions.	Engage in a discussion with the students about their shopping experiences in the market, whether they saved or spent all their money, what they bought, how they felt, and what they would do differently. Discuss whether the market was inclusive, what might have been missing, and what they would add or change, among other questions. Afterward, present problem-solving scenarios on the board for them to complete activities on loose sheets and in their notebooks.
Time to express what they have learned.	
Principle of Representation Guidelines: Provide options for languages and symbols; provide options for comprehension. **Principle of Action and Expression Guidelines:** Provide options for physical action; provide options for expression and communication; provide options for executive functions.	Split the students into groups and provide different materials on the tables for them to express themselves and demonstrate their understanding of the BMS. They can do this through music, videos, posters, games, activity sheets, and more.

In an activity following those indicated in Table 1, an extracurricular assignment was requested involving the students and their caregivers. For this assignment, they could make use of video, documentaries, photos, images, text, posters, or any other resource they deemed suitable to synthesize the knowledge from the meetings and activities. To conclude, they conducted an exhibition of their work.

The results obtained in the five meetings and activities presented in this text are discussed in the next section. In the data produced through observation and audio and video recordings by the teacher-researcher, we aim to highlight the contributions of the DUA in a process that goes beyond mathematics teaching and learning from an inclusive perspective.

Results: evidence of learning beyond mathematics

The planned activities allowed the establishment of connections with the lived context, making learning meaningful and applicable throughout life. Therefore, it is important to emphasize that there is intent in this analysis to uncover pathways for lifelong mathematical learning, with concepts and ideas that students will encounter in their daily lives.

At the beginning of the discussion in the activity 'Let's Talk about Inclusion and the Accessible Market,' the teacher-researcher seized the opportunity to apply mathematical

concepts in creating an accessible environment. Students were questioned about what should not be missing in this environment, how products should be classified, and what the ideal characteristics of the aisles are. Additionally, students were invited to think about how the flooring could be designed, considering measurements, proportions, and mathematical patterns. The class also discussed the best way to design checkout counters for purchases, taking into account mathematical aspects such as space, geometry, and efficiency. In parallel, students analyzed whether the packaging brought from home was inclusive, using mathematical concepts to assess dimensions, capacities, and ergonomics. This activity provided the practical application of mathematics related to the creation of an inclusive and accessible environment, aligning with the principles of UD (A.R.T. Góes & Costa, 2022).

This activity proposal aimed to engage students by sparking their interest and motivating them to actively participate in the learning process, to connect it with their reality. Thus, this activity is grounded in the principle of engagement in UDL, providing options to stimulate interest, maintain effort and persistence, and promote self-regulation. It is also possible to observe the principle of representation, which seeks to provide options for perception, symbols, and understanding.

To highlight the achieved objectives, we present the statements of students that reflect the understanding of the other participants, as they engaged themselves, emphasizing the importance of reference points to facilitate spatial location and enable all individuals to find products independently.

Table 2

Reports on Inclusion and the Accessible Market (Stellfeld, 2023, p. 185, our translation)[7]

Student	Students' accounts of what an inclusive market should be like.
Fabiana (8 years old)	*I think everything has to be organized, teacher. Each product needs to be together: cleaning with cleaning, food with food, cookies with cookies. It also needs to have large prices because my grandmother can't see those tiny letters, you know!*
Valentina (9 years old)	*There should be people in every place to help the elderly and those who can't see. It also needs to have wide aisles and low shelves, teacher. Because neither I nor my mom can reach those products up there. It's so high!*
Patrícia (10 years old)	*It's important to have some benches for older people to sit and wait in the checkout line because it takes a long time, and people get tired of standing. There also needs to be someone to help with paying for the purchases and packing them.*
Fábio (9 years old)	

[7] The transcriptions of the students' speeches presented in *italics* in the text were translated by the authors.

Do you know that video we watched? The one with the different flooring where a blind person goes through with a cane? It has to be in the accessible market too!

In addition to the UDL aspects already mentioned, it is possible to observe learning related to concepts that are included in the Mathematics curriculum, such as organization, categorization, spatial orientation, size, proportion, and spatial representation and visualization. There are also concepts that are learned for life, such as awareness of inclusion and the need to create accessible spaces for people with visual impairments and other different abilities (Carletto & Cambiaghi, 2007).

In the activity "Let's Build a Market," the teacher-researcher addressed concepts such as counting, categorization by categories (cleaning, food, cold items, hygiene), price comparison through manipulation in different market flyers, and the organization of shopping lists. (Figure 2)

Figure 2.

Price comparison in market flyers (collection of the first author).

When handling market flyers, Camila creates her list and states *"Teacher, this paper you gave me has only a few lines, just five spaces to put the products I'm going to buy. It won't fit here, can I make more lines on the back of the sheet and add more products?".* The ability to improvise and adjust available resources is important in mathematical learning as it involves spatial thinking, counting, and planning skills (Araucária, 2019).

The analysis and reflection on the flyers provided valuable insights into issues involving quantity, economy, abundance, and scarcity, clearly highlighting the economic vulnerability that pervades our society, as detailed in Table 3.

Table 3.

Reports regarding students' reflection on social vulnerability (Stellfeld, 2023, p. 184).

Student	Reflections on financial education
João (9 years old)	*You know, teacher, in my house, we always lack many things. I wish we had more food at home: cookies, milk for my brother, a video game. But my mom says the money she earns hardly covers anything.*
Gabriel (9 years old)	*My mom collects recyclable materials to sell. During the vacations, I help her by picking up cans on the street to buy things for us to eat and help my dad pay the bills too.*

In activities like this (Table 3), the teacher has the opportunity to understand the students' life situations, which promotes empathy and inclusion, while also developing socio-environmental skills, such as awareness of the importance of recycling and the pursuit of sustainable solutions (Araucária, 2019).

While handling product packaging, the teacher-researcher requested that they observe characteristics such as weight, capacity, and texture, offering multiple forms of representation and action, providing flexible access to knowledge and promoting the understanding and expression of the students, highlighting aspects of UDL. In addition, the activity aimed to develop mathematical skills and promote critical analysis, decision-making, and financial management. In this way, it allowed students to relate the product characteristics to their personal experiences, developing a deeper understanding of the mathematical concepts involved, resulting in statements like Camila's (8 years old): *"This one with rice is bigger and heavier than the one with beans. I know because I've helped my mom store them in the pantry, and I had to drag the rice one because I couldn't carry it!"*

When proposing the construction of an accessible market, the students were encouraged to indicate prices in Braille and Libras, using textures and adjusting the height of shelves and aisle spaces. Thus, strategies to make packaging accessible were proposed by the students, highlighting the aspects of UDL by emphasizing the importance of providing different forms of information representation, allowing students to understand and engage with the content in different ways (Cast, 2018). Thiago (9 years old) suggests: "*Teacher, what if we put that little square where we put the phone on top, and it goes straight to a website? Then the person can know everything that's written on the product.".* Daniel (10 years old) adds: *"Exactly! Then*

people who can't see can listen on their phones! What's it called, teacher? Look, there's one here on this milk carton!".

With these statements, the teacher-researcher demonstrates what happens when you read the QR code on a milk carton. This sparked enthusiasm in Lucas (10 years old) "*How cool, teacher! Can I try it? Look, it goes to this place on the internet, and we can learn about the products!".* The teacher-researcher's action is one of the skills that educators should possess in the context of the UDL approach, being open to unexpected situations and adapting their planning based on the uncertainties present in the classroom. The action was not foreseen by the teacher, but she saw the opportunity to go beyond mathematical concepts by demonstrating technology as a learning resource and information source, promoting independence, and ensuring equitable access for all consumers.

The handling of the packaging brings up comments heard in the previous activity, such as the issue of printing in very small letters and access to information, demonstrating an understanding of the elimination of barriers faced by some individuals. Fábio (9 years old) mentions the difficulty of reading small letters: *"These letters are very tiny; I can't read what's written on this packaging!*". Camila (8 years old) suggests that the packaging should be more accessible: *"It also needs to have the price in Braille, like the one you showed, teacher!".*

The UDL approach promotes student participation, as clearly expressed in the above expression and communication, contributing to their active engagement, which at another time fostered the learning of geometric shapes, the ability to relate them to real objects, and the development of mathematical language, stimulating cognitive development and critical thinking among students, as per the concepts attributed to the teacher-researcher's curriculum planning based on Araucária Curriculum (Araucária, 2019). The understanding of the students can be observed in Table 4.

Table 4.

Reports from the students regarding the identification of units of measurement (Stellfeld, 2023, p. 178)

Students	Identification of products in different units of measurement
Helena (9 years old)	*Teacher, milk, soda, and juice are sold by the liter.*
Rafaela (9 years old)	*Rice, beans, and sugar are sold by the kilogram, and popcorn and coffee by the gram.*
Tatiana (9 years old)	*Look, teacher, that die over there looks like a cube, that box looks like a rectangular prism, that one looks like a sphere, and this can here looks like a cylinder because it rolls.*

In the statements of Helena and Rafaela (Table 4), when they identify products sold in different units of measurement, there is stimulation for learning about these concepts and their application in everyday life. This shows that they are acquiring practical knowledge related to units of measurement and their relevance in real-life situations. On the other hand, the statements of Raquel and Tatiana relating packaging to spatial geometric shapes, such as a cube, rectangular prism, sphere, and cylinder, demonstrate that students are developing observation, comparison, and naming skills, as indicated in the Araucária Curriculum in the subject of Mathematics (Araucária, 2019). This contributes to the development of students' spatial thinking, helping them understand the relationships between shapes and objects in space in mathematical concepts (Araucária, 2019). By relating abstract concepts of geometric shapes to tangible everyday objects, students are building a more solid understanding of mathematical concepts and their practical application (A. R. T. Góes & H. C. Góes, 2015). Furthermore, the active participation of students in the dialogue demonstrates their interest in the terminology and vocabulary of geometry.

In the activity "Today is Shopping Day!" the students participated in the creation of two distinct markets, one with accessibility features and the other without. In the accessible market, they designed features with Libras, Braille, and QR codes, as well as spacious areas and tactile flooring, which were not present in the other project. This initiative aimed to highlight the importance of inclusion, sensitivity, and understanding of the perspectives and feelings of others by putting themselves in their shoes. To achieve this, students took on roles of individuals belonging to priority groups such as the blind, physically disabled, individuals with ASD, pregnant women, mothers with babies in strollers, obese individuals, and the elderly. This initiative underscored the importance of mathematics as an essential tool for creating inclusive spaces, as they were able to explore mathematical concepts when designing features like Libras, Braille, and QR codes, dimensioning spacious areas, and planning the arrangement of products on the shelves.

In order to convey the speech of each of the children who experienced these roles, we present the summary in Table 5.

Table 5.

Reports from the students regarding the practical application of the activities

(Stellfeld, 2023, p. 187)

Priority student	Reports about the moment in the non-accessible market	Reports about the moment in the accessible market
Blind individual João (9 years old)	*Wow! I couldn't get through here. It's too tight, I'm knocking everything over, and I can't grab anything. How difficult! I think I won't participate anymore. I need help.*	*That's cool, teacher! It would be so great if there were people in the markets to help those who are short, blind, or elderly. Because I, being small, can't even reach properly when I go to the market. My mom helps me.*
Deaf and mute individual Arthur (8 years old)	*Guys, my friend at the checkout isn't understanding me so I can pay for the groceries! I'm getting annoyed.*	*Oh, that's great! There's someone here who knows sign language, and now I'll be able to pay for my purchases and take everything home.*
Obese individual Fábio (9 years old)	*Wow, I can't get through here. It's too tight; I'm knocking everything over, and I can't grab anything.*	*Now it's good! Wow, there's plenty of space; I can walk, turn around, and people are helping me with the bags.*
Individual with a baby in a stroller Camila (8 years old)	*I give up on shopping at this market! I can't pass with my baby stroller, and the products are all mixed up, and the shelves are too high!*	*I can also pass with my stroller, reach the shelves, and there are people to help. At the checkout, there are people assisting as well.*
Elderly individual Thiago (9 years old)	*How frustrating, teacher. I felt bad. It reminded me of my grandpa; he used to say he didn't like to go out because people would keep bumping into him. They even knocked him over once.*	*I think in this market, he would definitely come. Everything is organized, it's spacious, and there are people to help. And there are even benches to wait.*
Individual with a physical disability Juliana (9 years old)	*I didn't like passing through there at all. There was no one to help me pick up the products. I had to put my bag in my mouth and pick up the products with my feet.*	*In this other one, it's much better because there were people to help, the shelves are low, and I could reach everything.*
Individual with autism Felipe (8 years old)	*Teacher, I did manage to buy some things here. I put some items in the bag, but there was a moment when I got lost. I couldn't see the price, and I got nervous at the checkout.*	*I liked this other one. Did you see how much stuff I got? I filled the bag, but I didn't have enough money to buy everything I wanted.*

Overall, the statements presented in Table 5 demonstrate that the students have gained knowledge related to the lack of accessibility and inclusion in the market environment since they experienced situations by putting themselves in the shoes of individuals with the indicated disabilities. The children's statements highlight the importance of mathematics in creating inclusive and accessible markets. Through mathematics, it is possible to plan spacious areas

and shelves organized at suitable heights, ensuring circulation and access to products for blind individuals, those with physical disabilities, the elderly, and obese individuals. Furthermore, the understanding and use of mathematical language in sign language (Libras) enable communication and payment for deaf individuals.

Mathematics is also present in the planning of layouts, rest benches, and assistive devices, considering dimensions, proportions, and user comfort. Additionally, mathematical skills such as price calculations, values, and financial management are essential for a satisfactory checkout experience, especially for autistic individuals. Thus, mathematics plays a crucial role in ensuring inclusion, accessibility, and equity of opportunities in markets. The experiences described in the reports in Table 5 highlight issues such as tight spaces, mobility difficulties, high shelves, mixed products, lack of assistance, and inadequate communication (Brazil, 2015).

These situations exemplify how the principles, guidelines, and checkpoints of Universal Design for Learning (UDL) can serve as a foundation for pedagogical methodologies and practices to create an inclusive environment and promote the active participation of all students. By considering individual needs and providing appropriate resources, such as communication in sign language (Libras), flexible spaces, adapted shelves, and staff support, it is ensured that all students can engage in the teaching and learning process and express themselves. These practices promote equity and inclusion, fostering the success of all students, regardless of their individual characteristics (A.R.T. Góes & Costa, 2022), highlighting the importance of considering the needs of different individuals in creating inclusive environments. In this perspective, mathematics education becomes more comprehensive and, above all, inclusive, as it considers the possibility of building a different path for its exploration, potentially sparking new interests and perspectives (Muzzio, 2022, p. 62, our translation).

Students had the opportunity to develop their problem-solving skills related to BMS values in buying and selling situations. They also used manipulable resources like calculators and collaborated with their peers, enabling teamwork. They mentioned, *"I have 50.00 reais and want to buy these products here. I added it up on the calculator, and it came to 35.00 reais. How much do you have to give me in change?"* (Thiago, 9 years old). Rodrigo (10 years old) used the calculator and replied, *"I'll give you 15.00 in change. You bought a small amount of products, and you'll still have some money left to buy more!"*

When initiating the Let's Talk and Record activity, the teacher-researcher facilitated a discussion about the acquisition of products and the amount spent by the participants, highlighting the principle of engagement in UDL and guidelines that pertain to providing

options to stimulate interest, maintain effort and persistence, and support self-regulation. Additionally, this aligns with the principle of action and expression, a guideline concerning providing options for physical action, expression, communication, and executive functions.

Regarding the acquisition of products and the amount spent by the participants, we highlight the following statements presented in Table 6.

Table 6

Students' Accounts Regarding Purchase Payments (Stellfeld, 2023, p.189)

Students	Student report
Carla (9 years old)	*Teacher, the money you gave me wasn't enough to buy everything I wanted because when I went to pay, I had to remove a lot of things!*
Camila (8 years old)	*What I found most challenging was doing the math with the money I had and figuring out if I could buy everything when I was choosing things from the shelves. I just kept picking things up.*

From the statements, it's evident that the students aren't performing calculations to check if the provided amount was sufficient to buy the products. However, they are developing decision-making skills, an understanding of quantity, counting, calculation, and comprehension of the value of money, which align with the learning objectives outlined in the Araucária Curricular Organization (Araucária, 2019, our translate). As a result, the activity offers a practical opportunity to apply these mathematical concepts in the context of everyday life, fostering a meaningful, real-life learning experience.

When asked about their experience of going through both markets, the statements of João (9 years old) who was blindfolded stand out:

Table 7

Student's Account Regarding His Experience in the Accessible and Non-Accessible Markets (Stellfeld, 2023, p.187)

Statements from the student João, representing a blind individual
Teacher, I found it difficult to go through that first market (non-inclusive) because it was very cramped. I couldn't find the products, didn't know where I was, and I kept bumping my cane into everything. I couldn't even find the exit or reach the cashier, and nobody helped me! It must be so hard for people who can't see to go to the market!" (João, 9 years old).
In that other market, it was easier for me. Pedro helped me with the shopping. He held onto my arm, and there was that thing on the floor that I could walk over. I knew where I was going. There were also people who talked to me at the places where I picked up things. I bought a lot of stuff. (João, 9 years old).

These experiences (Table 7) allowed João to understand the importance of accessibility and inclusion in public spaces, promoting the interest, participation, and engagement of all students, regardless of their abilities or individual characteristics. In this way, the activity addresses the UDL guideline on providing options for self-regulation as it enables students to reflect on their experiences and express their emotions and perspectives.

Figure 5.

Accessible Market: Students guiding the representative of the blind person (collection of the first author).

To the students who were at the cash registers, they were asked about their feelings regarding the calculations they performed while serving the customers. Rodrigo (10 years old) was at the cash register of the non-accessible market, and he now shares his experience. Below, we present his account in Table 8.

Table 8.

Student's Account Regarding His Experience in the Non-Accessible Market

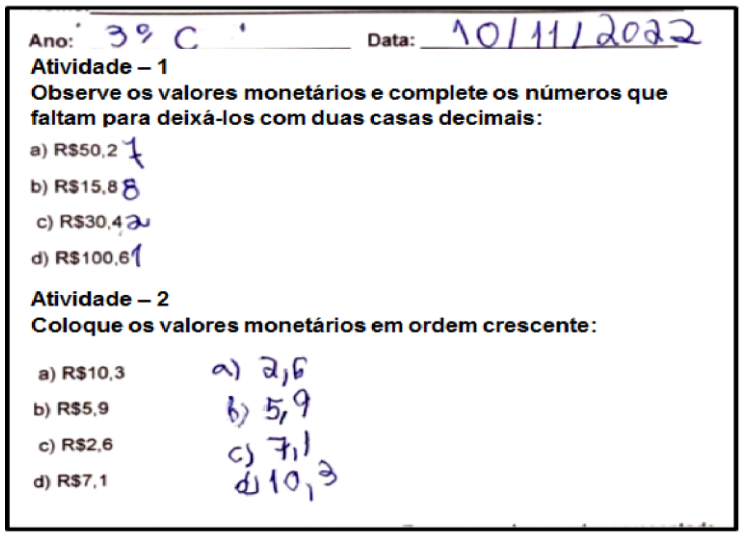
(collection of the first author)

Statement from the student Rodrigo, representing the cashier in the non-accessible market
Wow, teacher! It was very difficult because I was all alone there to serve the customers, do the calculations, pack everything, and the line was quite long! I didn't understand what the one who couldn't speak meant. And the one who couldn't see didn't know where he had put the money. I had to give him the change. How difficult, you know, teacher? I kept thinking afterward: can't someone deceive the blind person when taking their money and giving change?" (Rodrigo, 10 years old).

The student highlights (Table 8) the need for appropriate communication resources and support in real activities for employees, as well as the importance of creating a safe and reliable environment for all customers. These are crucial considerations to ensure a satisfactory and inclusive experience for all individuals involved, emphasizing the UDL principle of action and expression, promoting the development of cognitive skills, and valuing inclusion.

Subsequently, the teacher-researcher provided the students with a recording activity related to the products they purchased, along with three problem-solving situations involving the use of the BMS, addition, and subtraction calculations using monetary notation.

During an activity, the students encountered incomplete and incorrect price values that needed to be correctly filled in. The objective was to help them understand that prices in commerce are represented with two decimal places and emphasize the importance of paying attention to details, especially concerning the placement of the decimal point.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Atividade – 1

Observe os valores monetários e complete os números que faltam para deixá-los com duas casas decimais:

a) R$50,2 7

b) R$15,8 8

c) R$30,4 2

d) R$100,6 1

Atividade – 2

Coloque os valores monetários em ordem crescente:

a) R$10,3 — a) 2,6

b) R$5,9 — b) 5,9

c) R$2,6 — c) 7,1

d) R$7,1 — d) 10,3

Figure 6.

Math Activity (1 and 2) - BMS (Stellfeld, 2023, p. 190)

The activity presented in Figure 6, proposed by the teacher-researcher, aligns with the principles of UDL by promoting student engagement, representation, and action in the context of mathematics and the BMS. It also encourages critical thinking, creativity, and perseverance as students tackle challenges.

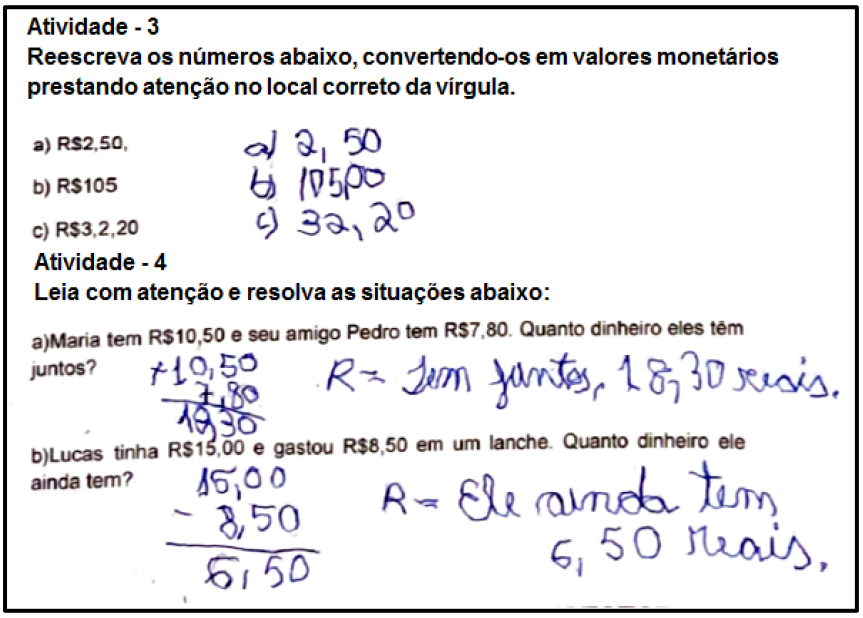

Figure 7.

Math Activity (3 and 4) – BMS (Stellfeld, 2023, p. 191)

Juliana demonstrates the ability to perceive and identify the teacher-researcher's proposal by stating, *"Teacher, this activity here is wrong. There's a comma after the last number, after the zero (R$2, 50,)."* The skill of spotting errors and paying attention to details is crucial for solving problem situations and understanding mathematical concepts, such as the use of currency symbols and addition and subtraction operations (Araucária, 2019).

On the other hand, Marcelo (10 years old) mentioned that he had difficulties with performing monetary additions and subtractions: *"I can't do the math, teacher. Can I use a calculator?"* In response, the teacher-researcher approved Marcelo's request and offered the calculator as a support for all who needed it, demonstrating a flexible and inclusive approach that aims to address the individual needs of the students. This approach is in line with the UDL principles (Cast, 2018) and is consistent with Bachmann's (2020) assertions about the role of teachers as transformative agents in the inclusion process, with the responsibility to commit to education for all, with the premise of eliminating physical and attitudinal barriers.

Finally, in the Time to Express What We've Learned! activity, students were able to apply their knowledge of the BMS through a variety of activities, including memory games and dominoes in Libras and Braille.

> [...] Educational games, in addition to their rules and game descriptions, bring knowledge, cognitive practices, and socio-emotional skills that contribute to the full exercise of citizenship. The development of educational resources by students, with the support of the teacher, according to the specific needs of the participants, is a valuable approach (Venturini, 2021, p. 10, our translation).

Furthermore, they engaged in the manipulation of bills and coins, recognizing the exchange relationships and value comparisons of Brazilian banknotes and coins. Some students demonstrated skills in solving problem situations involving addition, subtraction, and division, like João (8 years old): *"If I buy two of these notebooks here, for 5.00 reais each, it will be 10.00 reais. I'll give one to my friend, and the other will be for me. I'll pay you with this 20.00 reais bill. So, the change will be 10.00 reais. See, I don't need a calculator; I'm good at math!"* Other students used different methods to arrive at their answers, such as mental calculations, calculators, and manipulative materials available: *"Can I use the calculator, teacher?"* (Juliana, 9 years old). *"Can I use the popsicle sticks or the pencils, teacher?"* (Arthur, 8 years old).

As a result, they were able to distinguish the meanings of addition and subtraction, understanding the concept of equality to create various mathematical sentences and solve problems related to additive situations.

Figure 8.

Games for the students' action and expression. (Stellfeld, 2023, p.193-194)

As highlights of the events during the activity, we present the statements of Arthur, Juliana, and Thiago in Table 9.

Table 9.

Action and Expression of the Students Regarding Mathematical Operations.

(Stellfeld, 2023, p.194)

Students	Statements from the students regarding mathematical operations
Arthur (8 years old)	*Great job with the mathematical operations! You are demonstrating a good understanding of the values of Brazilian banknotes and coins and how to perform calculations involving these values. Keep practicing, and your math skills will continue to develop. If you have more questions or mathematical challenges, feel free to ask!*
Juliana (9 years old)	*I managed to add up the parts of the monetary domino game. I needed a calculator, but I was able to win the game. I also helped Carlos with the calculations.*
Thiago (9 years old)	*Teacher, I managed to buy some toys here. I saw that the toy car cost 10.00 Brazilian reais, and I had 20.00, so the change that Paulo had to give me was 10.00.*

In summary, the activities allowed students to experience and understand mathematical concepts in the context of everyday life, promoting inclusion, critical thinking, and the development of socio-environmental skills. The approach based on UDL involved engagement, representation, and connection between the content, facilitating meaningful and lasting learning.

Analytics

The following analyses presented below highlight the relevant aspects for the development of students' mathematical learning through a UDL approach.

The dialogue and the described situations evidence the active involvement of students in mathematical learning, providing meaningful contexts for the application of mathematical knowledge, promoting more engaged and motivating learning through activities such as simulations and practical buying and selling scenarios.

Through the principle of *Representation*, multiple forms of numerical representation were utilized when offering different resources such as calculators, sticks, and paper records. The UDL approach provided students with opportunities to explore and understand mathematical concepts in various ways, strengthening their comprehension of numbers,

operations, and mathematical relationships, as well as promoting the development of calculation and problem-solving skills.

Verbal expression and enthusiasm are aspects present in the *Action and Expression* principle of UDL, where by sharing their ideas, achievements, and victories, students indicate that they are developing skills that encompass mathematical communication. The ability to verbally express mathematical concepts and strategies is fundamental for the development of understanding and conceptual clarity. Furthermore, the enthusiasm demonstrated by the students suggests that they are engaged and motivated in regards to mathematical learning, which can contribute to greater involvement and depth in the study of this discipline.

Multiple forms of *Action and Expression* by the students were evident in the practice, as predicted by the *Representation* principle of UDL, providing students with the opportunity to offer different responses to their learning. This allows them to apply their mathematical knowledge in practical and meaningful ways. By calculating change, making purchases, or assisting their peers, they are exercising calculation skills, logical reasoning, and problem-solving strategies. These opportunities for applying knowledge in real-life contexts strengthen their understanding of mathematical concepts, promoting learning.

There is evidence of the *Action and Expression* principle when the described experiences highlight the development of mathematical skills such as calculation, understanding of monetary value, logical reasoning, and problem-solving. Additionally, the students also demonstrated cross-cutting competencies such as collaboration, communication, and autonomy, which are essential for mathematical learning and the overall development of students. These cross-cutting competencies not only enrich mathematical learning but are also valuable in other areas of learning and life.

Thus, we emphasize the relevance of UDL for the development of students' mathematical learning, contributing to the construction of a solid foundation of mathematical knowledge, the development of essential skills, and the strengthening of cross-cutting competencies necessary for success in mathematics and beyond, such as promoting accessibility and inclusion. It is evident that the approach taken is inclusive and diverse, creating a conducive learning environment where all students could actively participate because "[...] when other means of content presentation are offered, students are able to learn" (Ribeiro, 2019, p. 77), reducing learning barriers and facilitating access to the curriculum.

The strategies and approaches described and analyzed aim to ensure accessibility and meet the individual needs of students, contributing to providing equitable opportunities for all to learn and actively participate in mathematical activities, regardless of their abilities,

individual characteristics, or specific conditions. It's also important to highlight that the adopted strategies not only seek to ensure physical accessibility and the availability of resources but also to promote the value of diversity, respect for differences, and the development of an inclusive culture within the learning environment. The focus is on creating an environment that welcomes and respects everyone, encouraging collaboration, mutual understanding, and active participation by all students, regardless of their individual characteristics.

Considerations

This study aimed to analyze the didactic processes of a teacher-researcher by establishing connections that bridge UDL with inclusive mathematics education, addressing concepts related to the BMS.

Through the analysis of students' statements and the proposed activities, it is evident that mathematical learning can be enriched when diversity is valued and differences are respected. Inclusion in the context of mathematics is not just about ensuring physical accessibility or providing exclusive resources for those who need them, but about creating an environment that welcomes and promotes the active participation of all students, regardless of their abilities, individual characteristics, or specific conditions.

The use of inclusive pedagogical strategies such as simulations, practical situations, and the support of resources like calculators provides meaningful contexts for the application of mathematical knowledge, stimulating engagement, motivation, and the development of essential mathematical skills, including calculations, problem-solving, and logical reasoning.

Furthermore, the experiences shared by the students emphasized that the "seed of inclusion" has been planted, as they highlight the importance of providing appropriate support and assistance, whether through the presence of qualified individuals such as sign language interpreters or other professionals who ensure the understanding and involvement of everyone.

It is crucial to emphasize that inclusion in mathematical learning goes beyond the theoretical and methodological aspects of this discipline, encompassing the building of an inclusive culture based on the appreciation of diversity and the promotion of respect and collaboration among students, which will be reflected in society.

The planning and execution based on the UDL approach had a positive impact on the teaching and learning process, as students understood and deepened their knowledge, overcame

obstacles, and promoted interaction and knowledge accessibility, especially for those with learning difficulties, stimulating engagement.

The activities developed regarding the BMS allowed students to recognize and identify the different values of coins and banknotes, practice addition and subtraction when making transactions in the little market, develop social skills such as communication, negotiation, and conflict resolution, understand the importance of money and how it can be used to acquire goods and services, learn about different products, their prices, and measurements, and use these measurements to weigh or measure products.

By sharing our experience and the achieved results, we hope to inspire other educators to adopt inclusive approaches in mathematics education and create learning environments that value diversity and meet individual students' needs based on the UDL approach. It is essential for educators, researchers, and other professionals in the field of education to continue to explore and implement inclusive approaches in mathematical learning, always seeking to improve pedagogical practices and promote an inclusive and enriching educational environment for all students. Inclusion plays an essential role in recognizing and empowering marginalized individuals, overcoming social barriers, and creating an environment that fosters different possibilities, opportunities, and full respect for human rights.

With this, together, we can build a present-day education in which all students have equitable opportunities and can flourish academically and with confidence, having their multidimensions considered, thus becoming sensitive, ethical, and compassionate global citizens.

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