EMP Educação Matemática Pesquisa
ISSN 1983-3156
Qualis A1
http://dx.doi.org/10.23925/1983-3156.2023v25i4p190-213

Access to mathematical knowledge in inclusive classes: the differentiation of teaching from tasks structured in legitimizing variables of deaf students

Acceso al saber matemático en clases inclusivas: la diferenciación de la enseñanza a partir de tareas estructuradas en variables legitimadoras de alumnos sordos

L'accès aux savoirs mathématiques dans les classes inclusives: la différenciation de l'enseignement des tâches structurées en variables légitimantes des élèves sourds

O acesso ao saber matemático em turmas inclusivas: a diferenciação do ensino a partir de tarefas estruturadas em variáveis legitimantes de estudantes surdos

Nadjanara Ana Basso Morás[1]
Secretaria de Estado da Educação do Paraná (Seed-PR) e Secretaria Municipal da Educação de Foz do Iguaçu (Smed/ Foz do Iguaçu-PR)
Doutora em Educação Matemática
http://orcid.org/0000-0002-8683-4289

Clélia Maria Ignatius Nogueira[2]
Universidade Estadual do Oeste do Paraná (Unioeste)
Doutora em Educação
https://orcid.org/0000-0003-0200-2061

Luiz Marcio Santos Farias[3]
Universidade Federal da Bahia (UFBA)
Doutor em Didática das Ciências e Matemática
http://orcid.org/0000-0002-2374-3873

Abstract

This text proposes to discuss access to mathematical knowledge "problems of additive structures with natural numbers", in inclusive classrooms based on tasks structured in variables that legitimize the differences of deaf students. Therefore, it is based on Vergnaud's Theory of Conceptual Fields, for the deepening of studies referring to the mathematical knowledge considered; on Chevallard's Anthropological Theory of Didactics, for the development of research; and it conjectures that T4TEL is a possibility for the implementation of access to mathematical knowledge for students with different specific needs, in the same school environment. For the development of the investigation, the Conceptual Field of Additive Structures established by Vergnaud was considered, as the Epistemological Reference Model,

[1] nadjanara.moras@escola.pr.gov.br.
[2] voclelia@gmail.com.
[3] lmsfarias@ufba.br.

to develop a didactic device based on T4TEL, considering didactic variables that considered the differences of deaf students. The device was validated in two specialized institutions for deaf students, and it was later implemented in a class with hearing and deaf students. Among the conclusions, it is highlighted that, supported by T4TEL and structured in legitimizing variables, it is possible to generate a set of tasks that can contribute not only to deaf and hearing students having access, simultaneously, to the mathematical knowledge studied, but also, that considering variables that legitimize differences when developing tasks makes them potentially inclusive.

Keywords: Didactics of mathematics, Inclusive mathematics education, Problems of additive structures, Deaf people, Differentiating teaching.

Resumen

Este texto propone discutir el acceso al conocimiento matemático 'problemas de estructuras aditivas con números naturales', en clases inclusivas a partir de tareas estructuradas en variables legitimadoras de las diferencias de los alumnos sordos. Para ello, se basa en la Teoría de los Campos Conceptuales de Vergnaud, para la profundización de los estudios relacionados con el conocimiento matemático considerado, en la Teoría Antropológica de la Didáctica de Chevallard, para el desarrollo de la investigación y, conjetura ser el T4TEL una posibilidad para la realización del acceso al conocimiento matemático para alumnos con diferentes necesidades específicas, en el mismo espacio escolar. Para llevar a cabo la investigación, se utilizó el Campo Conceptual de Estructuras Aditivas establecido por Vergnaud como Modelo Epistemológico de Referência para desarrollar una herramienta didáctica basada en el T4TEL, teniendo en cuenta variables didácticas que atienden a las diferencias de los alumnos sordos. El dispositivo se validó en dos instituciones especializadas para alumnos sordos y, a continuación, se puso en práctica en una clase con alumnos oyentes y sordos. Entre las conclusiones, se destaca que, apoyado en el T4TEL y estructurado en variables legitimadoras, es posible generar un conjunto de tareas que pueden contribuir no sólo para que alumnos sordos y oyentes tengan acceso, simultáneamente, al conocimiento matemático estudiado, sino también que considerar variables legitimadoras de las diferencias al elaborar las tareas, las hacen potencialmente inclusivas.

Palabras clave: Didáctica de las Matemáticas, Educación Matemática Inclusiva, Problemas de estructuras aditivas, Personas sordas, Enseñanza diferenciada.

Résumé

Ce texte vise à discuter l'accès au savoir mathématique " problèmes de structures additives avec les entiers naturels " en classe d'inclusion, à partir de tâches structurées autour de variables qui légitiment les différences des élèves sourds. Pour ce faire, il s'appuie sur la théorie des champs conceptuels de Vergnaud, pour approfondir les études sur les connaissances mathématiques considérées, sur la théorie anthropologique du didactique de Chevallard, pour le développement de l'enquête, et conjecture que la T4TEL est une possibilité de rendre l'accès aux connaissances mathématiques efficace pour les élèves ayant des besoins spécifiques différents, dans le même espace scolaire. Pour mener à bien la recherche, le champ conceptuel des structures additives établi par Vergnaud a été utilisé comme modèle de référence épistémologique pour développer un dispositif didactique basé sur la T4TEL, en considérant des variables didactiques qui prennent en compte les différences des élèves sourds. Le dispositif a été validé dans deux institutions spécialisées pour les étudiants sourds, puis mis en œuvre dans une classe composée d'étudiants sourds et entendants. L'une des conclusions est que, en s'appuyant sur la T4TEL et en s'appuyant sur des variables de légitimation, il est possible de générer un ensemble de tâches qui peuvent contribuer non seulement à ce que les étudiants sourds et entendants aient un accès simultané aux connaissances mathématiques étudiées, mais aussi à ce que la prise en compte des variables de légitimation des différences lors de la conception des tâches les rende potentiellement plus inclusives.

Mots-clés : Didactique des mathématiques, Enseignement inclusif des mathématiques, Problèmes de structures additives, Personnes sourdes, Enseignement différencié.

Resumo

Ce texte vise à discuter l'accès au savoir mathématique " problèmes de structures additives avec les entiers naturels " en classe d'inclusion, à partir de tâches structurées autour de variables qui légitiment les différences des élèves sourds. Pour ce faire, il s'appuie sur la théorie des champs conceptuels de Vergnaud, pour approfondir les études sur les connaissances mathématiques considérées, sur la théorie anthropologique du didactique de Chevallard, pour le développement de l'enquête, et conjecture que la T4TEL est une possibilité de rendre l'accès aux connaissances mathématiques efficace pour les élèves ayant des besoins spécifiques différents, dans le même espace scolaire. Pour mener à bien la recherche, le champ conceptuel des structures additives établi par Vergnaud a été utilisé comme modèle de référence épistémologique pour développer un dispositif didactique basé sur la T4TEL, en considérant des variables didactiques qui

prennent en compte les différences des élèves sourds. Le dispositif a été validé dans deux institutions spécialisées pour les étudiants sourds, puis mis en œuvre dans une classe composée d'étudiants sourds et entendants. L'une des conclusions est que, en s'appuyant sur la T4TEL et en s'appuyant sur des variables de légitimation, il est possible de générer un ensemble de tâches qui peuvent contribuer non seulement à ce que les étudiants sourds et entendants aient un accès simultané aux connaissances mathématiques étudiées, mais aussi à ce que la prise en compte des variables de légitimation des différences lors de la conception des tâches les rende potentiellement plus inclusives.

Palavras-chave: Didática da matemática, Educação matemática inclusiva, Problemas de estruturas aditivas, Surdos, Diferenciar o ensino.

Access to mathematical knowledge in inclusive classrooms: differentiation of teaching through tasks structured in legitimizing variables for deaf students

According to data from the School Census, released by the National Institute of Educational Studies and Research Anísio Teixeira, in 2008, out of 700,000 students with disabilities enrolled in the Brazilian education network, 378,000 were enrolled in regular schools. The number of enrollments in Special Education reached 1,500,000 students in 2022, a 29.3% increase compared to 2018. The majority of these enrollments, 65.5%, are in Elementary Education (Brasil, 2023).

This significant increase in students supported by Special Education in regular classrooms brings with it numerous challenges to ensure their effective access to knowledge mediated by the class teacher, not solely relying on support professionals, when available. However, it is precisely these challenges that call us to constantly rethink the school system, aligning it with the new demands arising from the fundamental human characteristic: diversity.

Inclusion as an unconditional and non-transferable right has brought about the need to rethink schools. All educational institutions have been called upon to reconsider the conceptual foundations that, until the 1960s, did not include practices designed to accommodate the diversity of students, considering their abilities, diverse knowledge, and formative and accessibility needs.

In this space, unquestionably for everyone, we still experience discomfort in the face of differences. An inclusive school is a place where diversity is essential, requiring an understanding of each difference. This is the school we aspire to build in Brazil, despite all the difficulties and uncertainties inherent in processes of change, such as those promoted by inclusive education, especially for students supported by Special Education.

For inclusive education to be effective, there is a need for a structure that comprises human, material, and physical resources, but nothing compares to the role of the teaching staff. It is up to the teacher to organize the *milieu* (Brousseau, 1975), that is, the physical environment, educational resources, methodological strategies and, most importantly, to develop tasks proposed in the classroom to facilitate students' access to knowledge, whether supported by Special Education or not.

With such responsibilities, teacher training, whether initial or continuing, is a frequent topic of studies and research, such as those carried out within the scope of the Working Group – Teacher Training of the Brazilian Society of Mathematics Education, GT7, but it is not the focus of this article. Our focus is on the organization of a school milieu, specifically in the development of tasks that address the specific needs of each student, i.e., in differentiating teaching, which, according to the French educator Philippe Perrenoud (2000, p. 9), means "adapting the pedagogical action to the learner" without, however, "renouncing to instruct them, nor abandoning essential objectives. Differentiating is, therefore, to strive for the attenuation of inequalities within the school, while simultaneously working to elevate the quality of education.".

How can we differentiate teaching for students supported by Special Education and, at the same time, promote access to knowledge for other students? Therefore, this article presents part of an investigation based on the assumption that activities carried out in the classroom promote inclusion, proposing a solution to the teacher's challenge of designing tasks with inclusive potential in classrooms with students supported by Special Education.

The path to the conducted research

The conducted research aimed to establish a method for the effective access to mathematical knowledge, specifically in the context of "additive structures problems" for both deaf and hearing students, within the same school environment, legitimizing their differences. Legitimizing differences in the classroom not only involves respecting them but also recognizing, considering, and, above all, valuing them in the planning of teaching actions.

In the case of deaf students, the focus of this work, an example of legitimizing their differences is the consideration of visual aspects in the presentation of mathematical task statements, as evidenced in research conducted by the Study and Research Group on Deafness and Mathematics Teaching (*Grupo de Estudos e Pesquisas em Surdez e Ensino de Matemática*, in Portuguese, GEPSEM), by researchers Nogueira and Soares (2018); Soares, Nogueira and Borges (2018), and Nogueira and Borges (2019).

These researchers studied different ways of presenting statements related to mathematical knowledge[4], particularly "additive structures problems". These studies are grounded in Vergnaud (2014), in the Theory of Conceptual Fields, considering the epistemology of mathematical knowledge presented by the author and the different ways of representing mathematical knowledge.

Nogueira and Soares (2019) carried out an experiment in which deaf students solved problems similar to those proposed by Magina *et al*. (2001, p. 20), but whose statements were

[4] We use Chevallard's (1992) definition for mathematical knowledge; a particular category of objects whose characteristic is that they can be learned and taught but cannot be known without having been learned. In this investigation, we understand it as a word with a similar meaning to "object of knowledge", previously known as content, and which concerns the subjects covered throughout each curricular component.

presented in different ways, including "Portuguese in the written form, considering the grammar of the Brazilian Sign Language (Libras)" (interlanguage[5]); "Portuguese in the written form and an illustration" and "Portuguese in the written form and a scheme". The research identified that with the inclusion of visual aspects in the statements of mathematical tasks, the performance of deaf students was similar to that of hearing students of the same age and educational level, who were collaborators in the research conducted by Magina *et al*. (2001).

When experimenting with the sequence of tasks in the classroom, in addition to deaf students there was a hearing child[6] with an age and educational level compatible with the other research collaborators, who was also invited to carry out the tasks. Nogueira and Soares (2019) highlight in their analysis that this child was interested in the different forms of presentation included in the statements. This situation caught the attention of researchers and ours, leading us to reflect on the possibility that adaptations and concerns for students supported by Special Education could be beneficial for other students.

Based on the satisfactory results obtained, the researchers asked whether Mathematics teachers would be able to present problem statements using the three ways established by Nogueira and Soares (2019). Thus, Soares, Nogueira, and Borges (2018)[7] investigated whether Mathematics undergraduates could adapt problem statements following the proposed model. In the same direction, Nogueira and Borges (2019) conducted an investigation with Pedagogy undergraduate students. The results of both studies indicated that the future teachers algorithmized visual resources, with a focus on problem answers, at the expense of visual adaptation and inconsistencies between different representations.

From the realization that future teachers are not equipped to think about tasks in interlanguage (Portuguese and Libras) and with visual support, the natural question arose: how to provide tools to teachers to formulate problems whose statements are accessible to both deaf and hearing students simultaneously, within the same school environment? For this, it would be necessary to model the tasks, without, however, failing to consider the institutions, the

---

[5] "Interlanguage" is considered the writing of sentences or texts in which the lexicon of one language and part of the syntax of another are used. In this investigation, it is characterized by short and clear sentences; by sentences that use the names of the subjects to reintroduce them, avoiding the use of pronouns; by sentences without articles, prepositions, and conjunctions; by sentences without unnecessary information to understand the task; and by phrases that avoid terms that generate an ambiguous interpretation.

[6] Sister of a deaf student who was present at the time to learn Libras.

[7] The research by Soares, Nogueira, and Borges was conducted after the completion of the investigation carried out by Nogueira and Soares in 2017. The results of Soares, Nogueira, and Borges were published in the Proceedings of SIPEM in 2018, thus predating the publication of the article containing the results of Nogueira and Soares in 2019.

contexts and, mainly, the specificities of the students involved. Since the T4TEL task generator[8] allows for task modeling concerning the considered mathematical knowledge while respecting the diversity of students and the context through the adoption of suitable didactic variables, we hypothesize that this is the answer to our research question.

The conducted research

The objective of this work is to discuss access to mathematical knowledge, specifically "problems of additive structures with natural numbers", in inclusive classrooms, using a tool to generate tasks initially structured in legitimizing variables for deaf students. To achieve this objective, we consider the concept of disability established in Decree No. 6,949 (BRASIL, 2009)[9], which aims to modify the environment to minimize barriers and discussions of Didactics of Mathematics that consider the students, the knowledge in question, and the teacher as the object of study, simultaneously. We chose Gérard Vergnaud's Theory of Conceptual Fields for an in-depth study of the mathematical knowledge under consideration, Yves Chevallard's Anthropological Theory of the Didactic for the development of the research, and we conjectured that T4TEL[10] is a possibility for the effective access to mathematical knowledge for both deaf and hearing students in the same school environment.

Similar to the research by Nogueira and Soares (2019), the investigation reported here also had an additional gain. The implementation of the task sequence designed to meet signing deaf students could not be carried out at the initially planned period and in the inclusive school due to the pandemic. When it could finally be carried out at another school, the tasks needed to be adapted, since the deaf student enrolled there was not a Libras user, in addition to the presence in the class of a student with low vision. Thus, the device built from T4TEL needed to be modified to meet these new differences and proved to be effective for the teacher to propose potentially inclusive tasks.

Anthropological Theory of Didactics and T4TEL

According to Bosch and Chevallard (1999), the Anthropological Theory of Didactics

[8] The T4TEL model aims to structure a set of types of tasks specific to school content. It is not a template or platform.

[9] The decree establishes that "[...] people with disabilities are those who have long-term impairments of a physical, mental, intellectual or sensory nature, which, in interaction with various barriers, can obstruct their full and effective participation in society on equal terms with other people" (BRASIL, 2009, Art. 1°).

[10] T4TEL – T4 in reference to the praxeological quartet: task, technique, technology, and theory; and Technology Enhanced Learning TEL. T4TEL aims to enable the structuring of a set of types of tasks specific to a given school content, it is not a template or platform.

considers all mathematical activity and the knowledge that emerges from it in terms of mathematical organization. For these authors, a mathematical organization has its origin in the analysis carried out by teachers of official educational documents[11], from which the mathematical knowledge chosen to be taught emerges. From this, the teacher begins to determine which types of tasks will support the teaching and learning processes of this knowledge, bringing with them the other praxeological components (technique, technology, and theory) (Bosch & Chevallard, 1999).

In turn, a didactic organization arises with the intention of implementing or guiding a certain mathematical organization, enabling its (re)construction or transposition. According to Bosch and Chevallard (1999), we cannot expect the (re)construction, during a study process, of a mathematical organization to be organized by itself in a unique way. However, for researchers, certain situations will necessarily be present, quantitatively and qualitatively, in any study path, even if in a heterogeneous manner.

A mathematical organization and a didactic organization can be implemented in an institution through the T4TEL framework, introduced by Chaachoua and Bessot (2018). T4TEL is part of the Anthropological Theory of Didactics by extending the praxeological approach through the introduction of the notions of variables and personal praxeology[12].

The objective of introducing variables into the T4TEL structure is to organize a set of specific knowledge situations, characterized by a restricted set of relevant variables. For Chaachoua and Bessot (2018, p. 120), the notion of variables "appears primarily as a methodological tool in a modeling process, associated with the a priori analysis of a particular or fundamental situation".

The first function of a variable is to generate types of tasks and subtypes of tasks considering the values of the variables that depend on the subject, mathematical knowledge, and the institution in question (Chaachoua & Bessot, 2018). In T4TEL, a task type T is described by an action verb and a complement, T = (action verb, complement). The action verb characterizes the types of tasks, such as: "calculate", "add", "subtract", among others. The complement is defined according to the level of granularity (specifics), from specific to generic (for example, "calculate the sum of two numbers" is more generic than the task type "calculate the sum of two natural numbers with measures in the tens place") (Chaachoua & Bessot, 2018).

Considering the notion of granularity, Chaachoua and Bessot (2018) presented the

[11] Such as laws, decrees, curricula, programs and school manuals, among others.
[12] We use the notion of personal praxeology, developed by Chaachoua and Bessot (2018), as a difference between the personal relationship and the institutional relationship of a student related to a studied knowledge.

notions of Task Type Generator and variable system. A Task Type Generator (TG) is defined by a task type and a system of variables, and it can be described as follows: TG = [action verb, fixed complement; variable system]. The action verb and the fixed complement identify the type of tasks, and the variable system comprises the variables and the values they can receive within the domain of a discipline in a specific institution.

Thus, to model the system of variables we consider the epistemological, institutional, and didactic perspectives. The epistemological perspective of variables understands that the "division of the values of a variable is such that changing a value modifies the range of possible techniques for a type of task" (Chaachoua & Bessot, 2018, p. 124-125). To illustrate this perspective, we present the task type T1 = (Calculate the sum of two natural numbers with the first measure in the tens place and the second measure in the units place). There is an economical technique for solving this type of task, overcounting, in which the student starts counting from the largest measure, that is, in the tens place; performs the overcount with the second measure, that is, by the units place; and represents the resolution. This technique is less relevant, for example, for two numbers with measures in the tens and hundreds place because it requires a higher cost and is prone to error.

In an institution, there will always be conditions and restrictions that will not only restrict the type of task, but also the possible values of an epistemological variable of a type of institutional task. In the early years of Elementary School, for example, for T2 = (Calculate the sum of two numbers), the numbers involved, most of the time, are Natural numbers, and the measures are restricted to units, tens, and hundreds places. A variable and its institutional values model explicit or implicit conditions and constraints on the levels of co-determination under which a praxeology exists or can exist in a particular institution. An example of institutional values is the measures of numbers being worked on. In the 3rd year of Elementary School, following the conditions of this institution, we work with natural numbers up to the hundreds place.

A didactic variable is one that is within an institution and potentially available to the teacher. This can enrich, a posteriori, the values of the didactic variables considering the students' personal praxeologies, that is, it can refine the values of these variables through a subsequent analysis of the knowledge already acquired by the students. Regarding didactic variables, in this investigation, we considered in the modeling[13] of the values attributed to the

[13] Although the term "modeling" is not found in the Portuguese language, it is frequently used in works based on Didactics of Mathematics in Brazil. Its use is to explain its difference with "Mathematical Modeling", a trend in

variables the praxeologies that the student already knows regarding the mathematical knowledge studied and the differences of each student present in the classroom. We believe that by assigning values to variables, to legitimize[14] the differences of each student from a social perspective of disability[15], it is possible to promote equity of access to the studied knowledge.

The second function of a variable is to characterize the scope of the techniques (Chaachoua & Bessot, 2018). Outside its scope, the technique may fail; it can be applied, but there will be a risk of error. For example: the successive counting technique can be applied to T = (calculate the sum of two integers). If applied to large numbers, it is very likely to fail. Therefore, the scope of a technique is the set of types of tasks in which it is reliable because it allows these types of tasks to be carried out with little risk of failure or at a reasonable cost.

The third and final function of a variable is the notion of personal praxeology (Chaachoua & Bessot, 2018). This notion is important for diagnosing the learning trajectories of students in a given institution, for the inclusion of the cognitive subject and error as objects of study in the Anthropological Theory of Didactics. Researchers understand that the notion of personal praxeology expands the use of the praxeological quartet, considering the description of errors both at the level of the student's techniques and technologies.

In this article, we address the role of these variables associated with tasks and techniques, along with their developments, considering the variables as tools that will enable students to access mathematical knowledge: "problems involving additive structures with natural numbers".

Mathematics Education. Thus, we use "modeling" to describe and interpret the conditions of existence of mathematical knowledge in an institution. We emphasize that we do not use it to create models.

[14] Legitimizing in this text means recognizing, considering, and valuing differences.

[15] From a social perspective, the disabled person is not responsible for the oppression suffered, but rather society, which fails to foresee and incorporate diversity (Diniz, 2007).

Locus schools, investigated institutions, research collaborators, and the mathematical knowledge studied

The locus schools are a bilingual school for the deaf and a regular school. The former has Brazilian Sign Language (Libras) as the language of instruction, and Portuguese in written form as a second language; the latter has Portuguese as the language of instruction in both oral and written forms. The institutions investigated in this study are a 3rd grade class in Elementary School and a 2nd stage of Youth and Adult Education class in a bilingual school for the deaf; as well as a 3rd grade class in Elementary Education in a regular school. Depending on the school level, the deaf students who participated in the research were, at the time of the research, in the literacy process, while the hearing students were in the process of learning to read and write.[16]

However, we highlight that, while listeners have already mastered the Portuguese language in oral form, deaf students are still in the process of acquiring Libras. This is because, according to Gomes (2010, p. 35), more than 90% of deaf children are born to hearing parents and, therefore, do not naturally acquire their language in the family environment. They come to school with a homemade sign communication, very close to mime, so their first formal contact with Libras occurs at school. In other words, deaf children acquire their first language while learning Portuguese in the written form.

To deepen the study of the mathematical knowledge under investigation, we relied on the Theory of Conceptual Fields. In this theory, Vergnaud (2014) identified six basic relationships in the study of additive structures, from which it is possible to formulate addition and subtraction tasks in elementary arithmetic. These tasks may involve, for their resolution, ternary schemes (three measures involved) or quaternary schemes (four measures). Due to the conditions (mathematical knowledge) of the institutions investigated, we limited ourselves to the fundamental ternary schemes of these six categories:

> First category: two measures are composed to result in a third measure. Second category: a transformation operates on a measure to result in another measure. Third category: a relationship links two measures. Fourth category: two transformations are composed to bring about a transformation. Fifth category: a transformation operates on a relative state (a relationship) to result in a relative state. Sixth category: two relative states (relationships) are composed to result in a relative state (Vergnaud, 2014, p. 200).

[16] Literacy for the deaf and literacy for hearing individuals are distinct processes, and teaching strategies, as well as educational resources, need to be adapted to the specific needs of each group. Literacy for the deaf often requires a bilingual approach, which involves teaching sign language and literacy in written language. Literacy for hearing students generally follows traditional methods of teaching reading and writing in the mother tongue.

The researcher establishes as the conceptual field of additive structures the set of situations whose treatment involves one or more additions or subtractions and the set of concepts and theorems that allow analyzing such situations as mathematical tasks. The six categories of addition and subtraction situations are designed based on three ideas: composition, transformation, and comparison.

Our initial goal was to discuss the different tasks relating to the six categories demonstrated by Vergnaud (2014), focusing only on natural numbers, as this is the focus of the investigation. However, when consulting documents such as the Brazilian National Common Core Curriculum and the Curriculum of the Paraná State Network, we found that the fourth, fifth, and sixth categories were not considered in these documents for the institutions under investigation.

Considering the information in the documents that guide the teaching of mathematics in the state of Paraná, we restricted the study to the first three categories (which address the ideas of composition, transformation, and comparison between measures). One conjecture we made regarding the absence of the other categories at this level of education is related to the increasing order of complexity of the situations: the higher the category level, the more difficult the situations can be considered.

Studies carried out

For the development of the didactic device proposed as an alternative to teachers for the development of potentially inclusive tasks regarding "problems of additive structures", we rely on the following studies:

1) In the historical and epistemological study of the mathematical knowledge studied, we emphasize the conditions and restrictions of the existence of mathematical and didactic organizations with mathematical knowledge in a school context that aims to be inclusive. We study the evolution of these organizations over time and the possible evolution of didactic organizations so that they can legitimize the differences of each of the students present in the classroom, considering, in the statements of the types of tasks, legitimizing variables. Legitimating variables are those that legitimize the differences of students in the classroom and contribute to ensuring that all students have simultaneous access to the mathematical knowledge studied.

2) In the study of official educational documents (such as the National Common Curricular Base, Curriculum of the Paraná State Network, textbooks, and school manuals,

among others) that structure the investigated institutions regarding the studied knowledge, we identified the types of tasks that exist in the mathematical knowledge studied and the variables included in the presentation of these types of tasks. For this, we conducted an institutional analysis according to Henriques *et al*. (2012) in the three investigated institutions.

3) In the study on Vergnaud's Theory of Conceptual Fields, we researched the epistemology of the mathematical knowledge studied. We identify the types of tasks that exist, the epistemological variables related to this knowledge and the didactic variables that contribute to access to knowledge by hearing students.

4) In the study of Inclusive Mathematics Education, we researched the teaching of Mathematics for deaf students and identified possible legitimizing variables for deaf students[17].

Through these studies on the mathematical knowledge studied, considering the conditions and restrictions imposed by the investigated institutions, we identified 14 types of tasks – in the first three categories presented by Vergnaud (2014). Among the conditions imposed by the institutions, we highlight: the type of number - natural; meanings of composition, transformation, and comparison - between measures.

- First category:

  T_{11} = (Calculate, the result of the composition of two or more measures).
  T_{12} = (Calculate, a measure that combines with another known measure, knowing the resulting value of the composition).

- Second category:

  T_{21} = (Calculate, the final state (measure) resulting from the (positive) transformation of a known initial state (measure).
  T_{22} = (Calculate, the final state (measure) resulting from the (negative) transformation of a known initial state (measure).
  T_{23} = (Calculate, the transformation occurred on an initial state (measure) to result in a final state (measure) with (final state > initial state)).
  T_{24} = (Calculate, the transformation occurred on an initial state (measure) to result in a final state (measure) with (final state < initial state)).
  T_{25} = (Calculate, the initial state (measure) that was transformed (positively) and resulted in a known final state (measure).
  T_{26} = (Calculate, the initial state (measure) that was transformed (negatively) and resulted in a known final state (measure).

- Third category:

  T_{31} = (Calculate, the referred of a comparison of measures with a positive relationship).
  T_{32} = (Calculate, the referred of a comparison of measures with a negative relationship).

[17] These variables were chosen taking into account the research carried out in the Study and Research Group on Deafness and Mathematics Teaching (GEPSEM).

T_{33} = (Calculate, the comparison relationship between two measures with (referred < referent)).
T_{34} = (Calculate, the comparison relationship between two measures with (referred > referent)).
T_{35} = (Calculate, the referent of a comparison of measures (addition)).
T_{36} = (Calculate, the referent of a comparison of measures (subtraction)).

To identify the variables and constitute the system of variables, we considered the notion of variable presented by Chaachoua and Bessot (2018), which is a methodological tool in the modeling process that allows students to access the mathematical knowledge studied. Furthermore, the variable system is fundamental when it comes to generating task types and it is, in this specific case[18], strictly related to the research question. Thus, we developed the variable system, as described below:

- Variables and values assigned to the variables identified in the didactic booklet[19] used by regular school:

$V_{1/2}$[20] = Natural language/Writing (Portuguese in the oral form[21], Portuguese in the written form).

- Variables and values assigned to variables identified in studies regarding problems involving additive structures:

$V_{1/2}$ = Natural language/Writing (Portuguese in the oral form, Portuguese in the written form).
V_3 = Size of the first measure m_1 ($m_1 \in N \mid 0 < m_1 > 100$).
V_4 = Size of the second measure m_2 ($m_2 \in N \mid 0 < m_2 > 100$).
V_5 = Information presentation (information in the temporal order of the facts reported, information provided in disorder, reverse order).
V_6 = Type of theme (common themes from the student's daily life, uncommon themes from the student's daily life).
V_7 = Visual support (scheme to establish a relationship between the solution and the numerical data, scheme to establish the relationship between the solution and the type of task).

- Variables and values attributed to variables identified in studies in the field of Inclusive Mathematics Education:

[18] The generation of tasks can be related to an investigation, as is the case in this study, or it can also be related in an exclusively didactic way.
[19] The didactic booklet used in this study was prepared by the Municipal Department of Education of Foz do Iguaçu to provide information, lessons, and exercises to help students learn about mathematical knowledge. This booklet is used as a complementary resource and as part of a formal teaching program.
[20] The variable numbers were placed randomly. They do not correspond to the order in which the variables were identified. As the system of variables was developed simultaneously in studies 2, 3 and 4, at times, variables 1 and 2 appear together.
[21] We consider "Portuguese in the oral form" as a value attributed to the variable "natural language", since it is the language of instruction in regular schools.

$V_{1/2}$ = Natural language/Writing (Portuguese in the written form, interlanguages, Portuguese in the written form (showing one sentence per line), Libras).
V_7 = Visual support (scheme, illustration).

Analysis of the data produced

The Anthropological Theory of Didactics and T4TEL were essential for the development of this investigation, since we were able to model the types of tasks, the variables and the values attributed to them. Initially, we present the modeling we carried out in the official educational document, a didactic booklet used in regular schools. As shown in Figure 1:

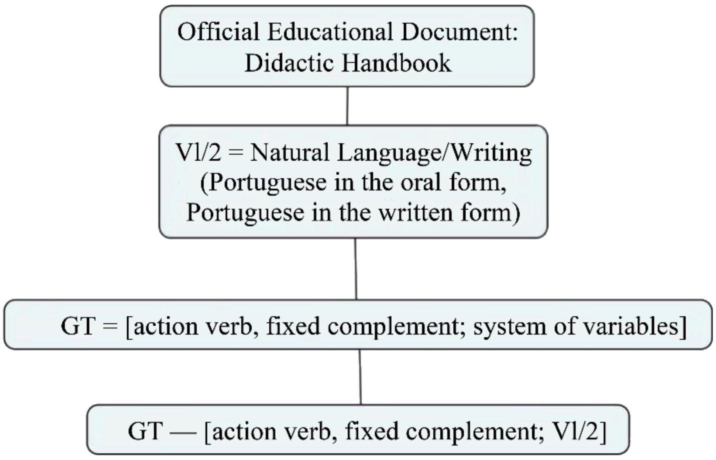

Figure 1.

Official educational document

By modeling the didactic booklet used in regular schools, we observed that, most of the time, the task type statements only contemplate the variable Portuguese language in the written form. Considering the different types of students in the classroom, having only this variable in the task type statements can hinder access to mathematical knowledge for some students present in the classroom, such as those not familiar with Portuguese in its formal form and those who are not fluent in this language, like deaf students.

Regarding the contemplation of different variables in the statements of types of tasks (different representations), Vergnaud (2014) considers that a concept can have different representations. In this sense, for the researcher, some representations are accessible, and we can perceive them, thus producing important indicators for students' access to knowledge, such as natural language, schemes, diagrams, among others.

In order to deepen studies on problems involving additive structures, we also modeled the types of tasks and variables structured in some elements of Vergnaud's Conceptual Field

Theory (2014), as shown in Figure 2:

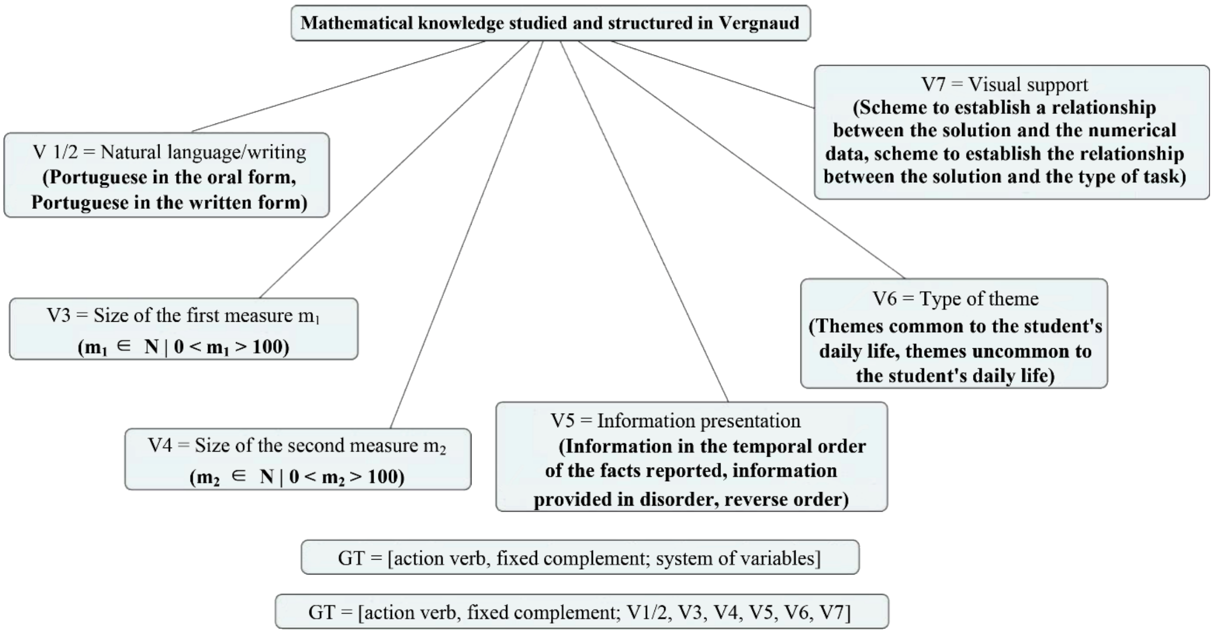

Figure 2.

Mathematical knowledge studied and structured in Vergnaud

When modeling the mathematical knowledge studied with the support of Conceptual Field Theory (2014), we identified seven variables. The seven variables and their respective values are presented in Figure 2. When studying the variables identified in this modeling, we identified important data that allowed us to question, supported by Vergnaud (2014), part of the teaching structure that is present in most schools that they aim to be inclusive, since the author draws attention to the presentation of information in the statements of task types. For him, the complexity of a problem can increase if the order of pertinent information is inverted or if this information is given out of order and, even more so, if it is embedded among other information. According to the researcher, at school, especially in the early years of Elementary School, it is common for task statements to provide only the necessary and sufficient information for their resolution.

Another example we can present concerns the domain of relationships involved in task type statements. According to Vergnaud (2014), saying “they lost 6 dollars” is not necessarily equivalent to “they have 6 dollars less”. Although static ternary relations and transformations can be placed in the same sagittal form,[22] or algebraic, the student in the first years of Elementary School does not capture a static relationship between two elements in the same way

[22] Sagittal: scheme established by Vergnaud, in which arrows are used to represent binary relationships (which connect two elements to each other).

(Vergnaud, 2014). We consider that the form of the relationship can play a role in the generation of tasks in the classroom, and it can be one of the starting points for access to the mathematical knowledge studied.

According to Nogueira and Soares (2019), in an inclusive school context, for the deaf student who is still in the literacy process, it is necessary to reformulate the presentation of statements in the Portuguese language in the written form – a reformulation that is influenced by their first language, Libras.

Nogueira and Soares (2019) highlight the importance of recognizing the deaf student's linguistic difference, but they also highlight the importance of contemplating visual aspects (such as schemes and illustrations) in task statements. The researchers consider the contemplation of different variables in the task statements as a way of legitimizing the differences of deaf students and contributing to their access to the mathematical knowledge studied.

When studying the two modelings, both the one carried out in the booklet and the one carried out based on Vergnaud's Conceptual Field Theory (2014), we identified that there are barriers for all students to have access to the mathematical knowledge studied. If we consider, for example, an inclusive school context, where the goal is to legitimize differences (an assumption of Inclusive Mathematics Education), access to mathematical knowledge is impaired if the differences of each student are not considered in teaching and learning situations, constituting what Farias (2010) calls "didactic void"[23]. In other words, access to mathematical knowledge for students supported or not by Special Education can be hindered by not considering variables that legitimize their differences in task presentations.

As the focus of the research is access to the mathematical knowledge studied for deaf and hearing students, we also modeled the task types and variables identified in studies in the field of Inclusive Mathematics Education, as shown in Figure 3:

[23]According to Farias (2010), the "didactic void" (*vazio didático*, in Portuguese) is a phenomenon that refers to possible gaps in terms of theoretical support that contribute to didactic practice. Farias (2020) is based on the doctoral thesis entitled "Étude didactique des nombres réels: idécimalité et racine carrée", defended by Alain Bronner, in 1997.

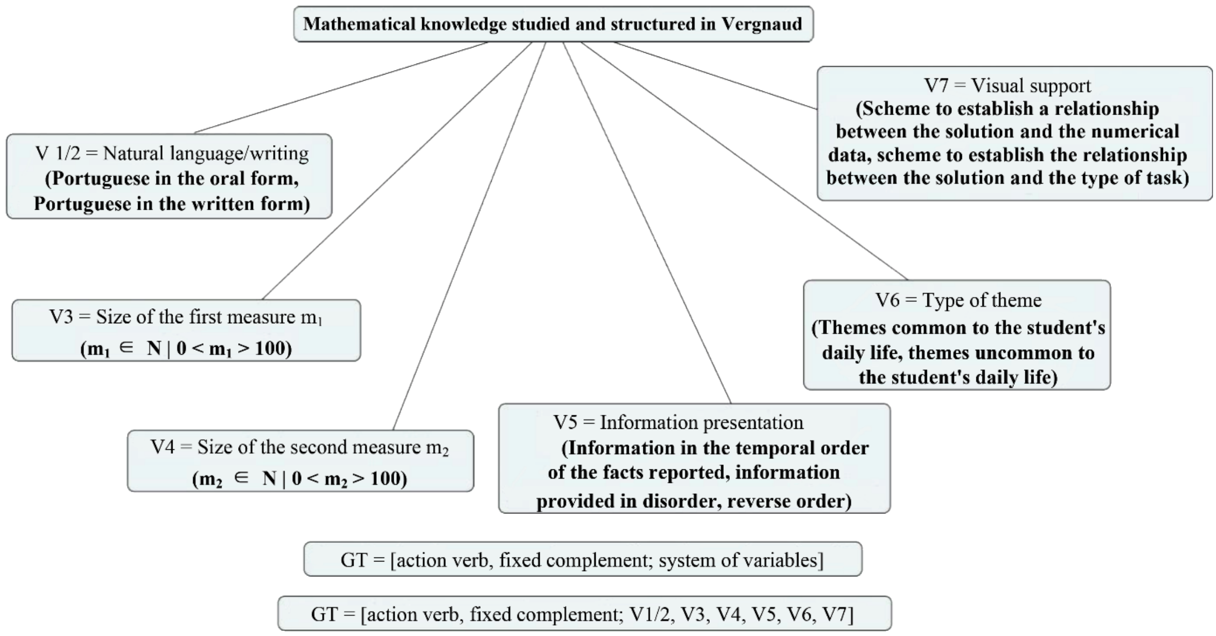

Figure 3.

Inclusive Mathematics Education

In this figure, we can observe the variables and the values attributed to them, where: the underlined values were identified in the studies of the didactic booklet; the values in bold were identified in Vergnaud's Conceptual Field Theory (2014); the italicized values were identified in studies in the field of Inclusive Mathematics Education, carried out by Nogueira and Soares (2019) and Nogueira and Borges (2019).

Nogueira and Soares (2019) and Nogueira and Borges (2019) conducted investigations into the influence of different forms of statement presentation on deaf students' performance in solving tasks involving additive structures. The results of these investigations corroborate other studies, such as those of Coutinho (2011), and indicate that visual aspects are crucial for deaf students' understanding of mathematical task statements and that any task that concerns the differences of a group of students will possibly successfully meet the other students present in the classroom.

By modeling in studies in the field of Inclusive Mathematics Education, we identified possible variables that legitimize the differences of deaf students, namely: interlanguages, Portuguese in the written form (presenting one sentence per line), Libras, scheme, and illustration. These variables legitimize deaf students because they recognize, consider, and value their way of understanding and interacting with the world through visual experiences and their linguistic difference.

Concerned with access to mathematical knowledge for deaf and hearing students and aiming to eliminate the identified didactic gap, we consider it necessary, in generating task types, to contemplate, in addition to the variables identified in Vergnaud's Theory of Conceptual Fields (2014), the variables identified in studies in the field of Inclusive Mathematics Education, in the specific case, for the deaf. We present, in Figure 4, an example of a task that contemplates, in the presentation of the statement, variables that legitimize deaf students, which are in italics:

T = In teacher Marisa's classroom there are 4 boys and 3 girls.

How many children are there in teacher Marisa's classroom?

Values assigned to the variables that were included in the generation of this task:

- size of the first measure less than 100;

- size of the second measure less than 100;

- information presentation in temporal order;

- common theme of the student's daily life;

- *Portuguese in the written form (presenting one sentence on each line)*;

- *illustration to establish the relationship between the solution and the type of task*

Figure 4.

Task that contemplates variables legitimizing deaf students in the statement presentation

In this task, we can identify the variables that meet the conditions imposed by the investigated institutions regarding the mathematical knowledge studied: the size of the first and second measures; the presentation of information in temporal order; and the common theme of the student's daily life. We also identified the variables legitimizing deaf students: Portuguese in the written form (presenting one sentence per line) and the illustration to establish the relationship between the solution and the task type (relational calculation).

Considering the studies conducted, a didactic device was generated with 70 tasks produced by T4TEL. Initially, it was implemented in the 3rd year of elementary school in a bilingual school for the deaf, both in the regular and adult education modalities, with the intention of identifying whether the variables considered as legitimizing the differences of deaf students were appropriate, and the result was positive. Subsequently, the implementation was carried out in a 3rd year classroom in an inclusive school, aiming to identify whether the device favored access to knowledge for other students in the classroom. As hearing students are still in the literacy phase, the effectiveness of the visual support variable and the value “one sentence per line” in the writing variable of the statements for the understanding of the statements by hearing students was identified.

The variables considered in this study aim to legitimize the differences of deaf students, but, in other contexts, other variables can be modeled, contributing to the legitimization of the differences of other students supported by Special Education. This occurred, as previously explained, during the implementation for the validation of the didactic device developed from the T4TEL task generator. Considering the specificities of the students in the class in question: a deaf student not using Libras and a student with low vision, the values of the didactic variables were altered.

Thus, this unforeseen circumstance was addressed by contemplating, in the T4TEL model, other variables that allowed the inclusion of these students. For the deaf student not using Libras, the writing in the interlanguage was fundamental for his understanding of the statements. The consideration of the value “Portuguese in the written form (presented in enlarged font)” in the “writing” variable allowed meeting a specificity of the hearing student with low vision. The possibility of assigning new values to the variables reveals the efficiency of this model, provided that the specificities of the students are considered, which is essential for success in accessing knowledge.

Some considerations

We consider that adding variables that legitimize deaf students to the variables that meet the conditions of the institutions related to the mathematical knowledge studied does not compromise access to knowledge for other students in the classroom. On the contrary, it provides more tools for them to have such access.

With these modelings, we generated important data for research in the field of Inclusive Mathematics Education. The conducted investigation allowed us to verify that, based on T4TEL and structured around legitimizing variables, we can generate a set of tasks that contribute to deaf and hearing students simultaneously accessing the mathematical knowledge studied. As a collateral result, although not initially foreseen but conjectured, this set of tasks allows, through the choice of legitimizing variables, differentiating teaching to facilitate access to mathematical knowledge “problems of additive structures” for students with diverse specificities. T4TEL enables the generation of similar didactic devices for different objects of mathematical knowledge, considering relevant the "action verbs and complements", and for different contexts and subjects, with the choice of suitable didactic variables. Thus, task modeling is possible while preserving diversity in the classroom, i.e., it is possible to differentiate teaching so that "inequalities in the face of school diminish, and simultaneously, teaching improves", as advocated by Perrenoud (2000, p. 9).

We proposed a possible evolution towards the sense of Inclusion, emphasizing that the path is not limited to adapting task statements; on the contrary, it includes generating task statements designed for students supported by Special Education and extending contributions to other students. One possibility is that teachers and textbook editors, in parallel, discuss and consider more inclusive forms of presenting task statements that meet the diversity of classrooms. Furthermore, it is hoped that Portuguese in the written form, which has been shown to be an elitist and homogenizing variable according to Bourdieu and Passeron (2014), is not the only possible variable.

References

Assude, T.; Perez, J., Suau, G., Tambone, J.; Vérillon, A. (2014). Acessibilidade didática e dinâmica topogenética: um estudo de caso. *Pesquisa em Didática da Matemática*, 34(1), 33-57.

Bosh, M.; Chevallard, Y. (1999). La sensibilité de l'activité mathématique aux ostensifs. Objet d'estude et problematique. *Recherches em Didactique des Mathématiques*. Grenoble: La Pensé Sauvage-Éditions, 19(1), 77-124.

Brasil. *Declaração Mundial sobre Educação para Todos*: plano de ação para satisfazer as necessidades básicas de aprendizagem. (1990). Jomtiem/Tailândia: UNESCO. https://unesdoc.unesco.org/ark:/48223/pf0000086291_por.

Brasil. (1994). *Declaração de Salamanca e linha de ação sobre necessidades educativas especiais*. Brasília: UNESCO. http://portal.mec.gov.br/seesp/arquivos/pdf/salamanca.pdf.

Brasil. (2005). *Decreto n° 5.626 de 2005.* Regulamenta a Lei nº 10.436 de 2002, que dispõe sobre a Língua Brasileira de Sinais – Libras e o art. 18 da Lei n° 10.098, de 19 de dezembro de 2000. Brasília, Distrito Federal. https://www2.camara.leg.br/legin/fed/decret/2005/decreto-5626-22-dezembro-2005-539842-publicacaooriginal-39399-pe.html.

Brasil. (2008). *Política Nacional de educação especial na perspectiva da Educação Inclusiva*. Brasília: MEC/SEESP, Secretaria de Educação Especial. http://portal.mec.gov.br/arquivos/pdf/politicaeducespecial.pdf.

Brasil. (2009). *Decreto nº 6.949 de 2009*. Promulga a Convenção Internacional sobre os Direitos das Pessoas com Deficiência e seu Protocolo Facultativo, assinados em Nova York, em 30 de março de 2007. https://www.planalto.gov.br/ccivil_03/_ato2007-2010/2009/decreto/d6949.htm.

Brasil. (2015). *Lei n° 13.146 de 2015*. Institui a Lei Brasileira de Inclusão da Pessoa com Deficiência (Estatuto da Pessoa com Deficiência). Brasília, Distrito Federal, 2015. https://www.planalto.gov.br/ccivil_03/_ato2015-2018/2015/lei/l13146.htm.

Brasil. (2023). Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira (Inep). *Censo Escolar da Educação Básica 2022:* Resumo Técnico. Brasília. https://download.inep.gov.br/publicacoes/institucionais/estatisticas_e_indicadores/resumo_tecnico_censo_escolar_2022.pdf.

Brousseau, G. (1997). *Theory of Didactical Situation in Mathematics*. Springer Editions.

Bourdieu, P. Passeron, J. (2014). *A reprodução*: elementos para uma teoria do sistema de ensino. Petrópolis, RJ: Vozes.

Chaachoua, H; Bessot, A. (2008). A noção de variável no modelo Praxeológico. In D: Almouloud, S. A.; Farias, L. M. S.; Henriques, A (orgs.), *A teoria antropológica do didático: princípios e fundamentos*. (pp. 119-134). CRV.

Chevallard, Y. (1992). Concepts fondamentaux de la didactique: perspectives apportees par une approche anthropologique. *Recherches en Didactique des Mothématiques*. IREM d'Aix-Marseille, Université Aix-Marseille II, 12(1), 73-112.

Coutinho, M. D. M. C.(2011). *Resolução de problemas por meio de esquemas*. *In*: XIII Conferência Interamericana de Educação Matemática**.** *Anais***...**Recife.

Farias, L. M. S. (2010). *Étude des interrelations entre les domaines numérique, algébrique et géométrique dans l'enseignement des mathématiques au secondaire: Une analyse des pratiques enseignantes en classes de troisième et de seconde*. [Thèse de Doctorat, Université de Montpellier II, France, France]. https://theses.hal.science/tel-00588484/preview/ThA_se_Complete_Luiz_Marcio_SANTOS_FARIAS.pdf.

Filgueiras, L. M.; Pereira, L. H. L.; Melca, F. M. A. (2008). *Processo ensino-aprendizagem dos alunos com necessidades educativas especiais*: deficiente visual. Rio de Janeiro: Unirio.

Gascón. Josep. (2011). Las três dimensiones fundamentales de um problema didáctico. El caso del álgebra elemental. *Relime*, 14(2), 203-231.

Gomes, M. C. (2010). *Lugares e representações do outro*: a surdez como diferença. Porto: CIIE/Livpsic.

Henriques, A.; Nagamine, A.; Nagamine, C. M. L. (2012). Reflexões Sobre Análise Institucional: o caso do ensino e aprendizagem de integrais múltiplas. *Boletim de Educação Matemática*, 26(44), 1261-1288.

Magina, S.; Campos, T.; Nunes, T.; Gitirana, V. (2001). *Repensando Adição e Subtração*: Contribuições da Teoria dos Campos Conceituais. PROEM.

Morás, N. A. B. (2013). *Um dispositivo didático com potencialidades inclusivas: um estudo a respeito de problemas de estruturas aditivas com números naturais*. [Tese de Doutorado em Educação Matemática, Programa de Pós-Graduação em Educação em Ciências e Educação Matemática, Universidade Estadual do Oeste do Paraná, Cascavel]. https://tede.unioeste.br/handle/tede/6618.

nogueira, c. m. i.; borges, F. A. (2019). Formação docente para a inclusão nos Anos Iniciais do Ensino Fundamental: uma análise a partir da formulação e adaptação de enunciados de problemas matemáticos. *Educação Matemática em Revista*. 24(65), 04-28.

Nogueira, C. M. I.; Soares, B. I. N. (2019). A influência da forma de apresentação dos enunciados no desempenho de alunos surdos na resolução de problemas de estruturas aditivas. *Educação Matemática Pesquisa*. 21(5), 110-120.

Nogueira, C. M. I., (2020). Educação Matemática Inclusiva: do que, de quem e para quem fala? In D: Martensen, A. M.; Kallef, R.; Pereira, P. C. (orgs.) *Educação Matemática*: diferentes olhares e práticas. (pp. 109-132). Appris.

Perrenoud, P. (2020). *Pedagogia diferenciada*: das intenções à ação. Porto Alegre: Artes Médicas.

Skliar, C. B. (1998). Um olhar sobre nosso olhar acerca da surdez e as diferências. *In*: SKLIAR. C. B. (orgs). *A Surdez*: um olhar sobre as diferenças. Porto Alegre: Mediação.

Skovsmose, O. (2019). Inclusões, encontros e cenários. *Educação Matemática em Revista*, 24(64), 16-32.

Soares, B. I. N.; Nogueira, C. M. I.; Borges, F. A. (2018). Diferentes formas de apresentação de enunciados de problemas matemáticos: subsídios para inclusão de estudantes surdos. In: VII SIPEM. *Anais*... Foz do Iguaçu.

Vergnaud, G. (2014). *A criança, a matemática e a realidade*: problemas do ensino da matemática na escola elementar. Curitiba: Ed. da UFPR.