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Exploratory-investigative approach in mathematics to include children and young people in situations of school failure

Abordagem exploratório-investigativa em matemática para incluir crianças e jovens em situação de fracasso escolar

Enfoque exploratorio-investigativo en matemáticas para incluir a niños y jóvenes en situaciones de fracaso escolar

Approche exploratoire-investigation en mathématiques pour inclure les enfants et les jeunes en situation d'échec scolaire

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Abstract

This article aims to highlight, in light of discussions about equity and inclusion in mathematics education, the relevance of the results of a survey completed in 2007. This qualitative research aimed to explore the possibilities and contributions of the exploratory-investigative approach to the mathematical learning of students participating in a remedial project, primarily to comprehend and address the issue of academic underachievement. This remedial project was integrated into the Continued Progression system of the public education network in the state of São Paulo, currently adopted by other states, which underscores the ongoing significance of this discussion. The analyzed data were collected in a class equivalent to the 9th grade in the High School (14-15). The research's analytical framework draws from the fields of education, mathematical education, and anthropology. Revisiting this study has brought to light that the exploratory-investigative approach can favor the empowerment and educational inclusion of students, establishing itself as a way to redefine academic failure and to confront a system that perpetuates a state of ignorance and cruelty.

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Keywords: Mathematical education, Exploratory-investigative approach, School inclusion.

Resumen

Este artículo pretende resaltar, a la luz de los debates sobre equidad e inclusión en la educación matemática, la relevancia de los resultados de una encuesta realizada en 2007. Esta investigación cualitativa tuvo como objetivo explorar las posibilidades y contribuciones del enfoque exploratorio-investigativo en el aprendizaje matemático de estudiantes que participan en un proyecto de recuperación, principalmente para comprender y abordar el problema del fracaso académico. Este proyecto de recuperación se integró en el sistema de Progresión Continua de la red de educación pública en el estado de São Paulo, actualmente adoptado por otros estados, lo que subraya la importancia continua de esta discusión. Los datos analizados se recopilaron en una clase equivalente al 3.º de la educación secundaria obligatoria (14-15). El marco analítico de la investigación proviene de los campos de la educación, la educación matemática y la antropología. La revisión de este estudio ha puesto de manifiesto que el enfoque exploratorio-investigativo puede favorecer el empoderamiento y la inclusión educativa de los estudiantes, estableciéndose como una forma de redefinir el fracaso académico y enfrentar un sistema que perpetúa un estado de ignorancia y crueldad.

Palabras clave: Educación Matemática, Enfoque exploratorio-investigativo, Inclusión escolar.

Résumé

Cet article vise à souligner, à la lumière des discussions sur l'équité et l'inclusion dans l'enseignement des mathématiques, la pertinence des résultats d'une enquête réalisée en 2007. Cette recherche qualitative visait à explorer les possibilités et les contributions de l'approche exploratoire-investigatrice à l'apprentissage mathématique des élèves participant à un projet de récupération, principalement pour comprendre et aborder le problème de l'échec scolaire. Ce projet de récupération faisait partie du système de Progression Continue du réseau d'enseignement public de l'État de São Paulo, actuellement adopté par d'autres États, ce qui souligne la pertinence continue de cette discussion. Les données analysées ont été collectées dans une classe équivalente la 6ème du collège en France (14-15). Le cadre analytique de la recherche puise dans les domaines de l'éducation, de l'éducation mathématique et de l'anthropologie. La révision de cette étude a mis en lumière que l'approche exploratoire-investigatrice peut favoriser l'autonomisation et l'inclusion scolaire des élèves, se présentant

comme un moyen de redéfinir l'échec scolaire et de confronter un système qui perpétue un état d'ignorance et de cruauté.

Mots-clés : Éducation Mathématique, Approche Exploratoire-Investigatrice, Inclusion Scolaire.

Resumo

Este artigo pretende ressaltar, à luz das discussões sobre equidade e inclusão em educação matemática, a atualidade dos resultados de uma pesquisa concluída em 2007. Esta pesquisa, de abordagem qualitativa, buscou investigar possibilidades e contribuições da abordagem exploratório-investigativa para a aprendizagem matemática de alunos participantes de um projeto de recuperação, sobretudo para compreender e enfrentar o problema do fracasso escolar. Este projeto de recuperação integrava o regime de Progressão Continuada da rede pública de ensino do estado de São Paulo, adotado atualmente por outros estados, o que torna esta discussão ainda relevante. Os dados analisados foram coletados em uma classe que equivale ao atual 9º ano do ensino fundamental (14-15). Os referenciais analíticos da pesquisa são oriundos dos campos da educação, da educação matemática e da antropologia. A revisitação deste estudo permitiu evidenciar que a abordagem exploratório-investigativa pode favorecer a emancipação e a inclusão escolar de alunos, constituindo-se como uma forma de ressignificar o fracasso escolar e de enfrentar um sistema que visa manter a barbárie.

Palavras-chave: Educação matemática, Abordagem exploratório-investigativa, Inclusão escolar.

Exploratory-investigative approach in mathematics to include children and young people in situations of school failure

Recent research "reveals a close relationship between the vulnerability of children and young people and a greater likelihood of school failure, stigma and/or social and school exclusion." (Martins et al., 2023). For the authors, what characterizes vulnerability is the fact that children are or have been involved in violence in the family or at school. In their article, they present various actions aimed at including students in this situation, such as: working in pairs, using "structured teaching methodologies and strategies, and developing personal and social autonomy skills" (Martins et al., 2023, p. 22. Our translation).

In contexts like this, our responsibility as teachers goes far beyond teaching, especially if teaching is still rooted in the idea of "transmitting content", as this type of teaching tends to increase school failure. When it comes to teaching mathematics, which is considered a difficult subject for students, care needs to be taken with the teaching approach.

Faced with this problem, in this article we revisit a master's research project carried out over a decade ago (Cristovão, 2007), with the aim of highlighting the relevance of its results to the discussion on school inclusion. This is a proposal for school inclusion without domination or subordination of the subject, involving ordinary students who attend regular classes and who carry the stigma of school failure, who were submitted to actions for recovery in basic education, aiming, on the one hand, to include these students in math classes through an exploratory-investigative approach and, on the other, to understand and rethink what is still understood by school failure.

At the time the research was carried out, elementary school in the state of São Paulo was divided into two cycles (1st to 4th grade and 5th to 8th grade), and the Continuous Progression[3] system was adopted, so there was a "Cycle Recovery Project" at the end of each cycle. The class in which the research was carried out was a Cycle Recovery II (RC II) class in the 8th grade (now the 9th grade). Although, on the surface, the project seemed interesting in terms of meeting the needs of students with greater learning difficulties, in practice, these students were isolated from the rest of their classmates to form classes with 25 students, characterized by their different difficulties and needs, which further reinforced the stigma of failure.

[3] "[...] its main mechanism of operation is recovery in its various forms, as stated in SE-SP Resolution No. 6, of January 24, 2008: continuous (in the daily classroom), parallel (at a time other than regular classes), intensive (aimed, in previously established periods, at students who present learning difficulties that have not been overcome) and cycle (a school year of studies to assist students who do not demonstrate conditions to proceed to the next stage)" (Melo & Roggero, 2023, p. 8. Our translation).

Melo and Roggero (2023) take a stance on the system of continuous progression, which has currently been adopted in several states across the country. The authors recognize several of its benefits, but denounce the economic side of its adoption in Brazil and problematize the way it is practiced in schools. For the authors, this system offers pseudo-training that has served to keep children and young people in a state of alienation and submission to the authorities. This perpetuates the logic of social domination under contemporary capitalism and denies "the constitution of subjectivity to these students, making it impossible for them to develop autonomy and emancipation; in this way, they remain regressed and incapable of overcoming barbarism" (Melo & Roggero, 2023, p. 18. Our translation).

It is not our intention in this article to discuss the viability or otherwise of the system of continuous progression, but given this scenario of expansion of the system and the current discussions on inclusion and equity, the results obtained in the research become relevant to help us understand and reflect on the system itself and our role as educators in the face of it. When we refer to school failure, whose failure are we talking about? Is it the students, the teachers, the school, or the system?

If we are interested in promoting a truly inclusive education, if we want to act with equity, meeting everyone's unique needs, whether special or learning, we need to change the way we look at the practices of our students who are considered "failures", we need to reframe what we mean by school failure. Sun (2014) distinguishes between the concepts of equity and equality by stating that to act with equity is to offer everyone the necessary support to succeed, while equality presupposes treating everyone the same, regardless of their preconditions.

In the field of Mathematics Education, Gutiérrez (2012) carried out a study that addresses four dimensions of equity: access, achievement (related to the results obtained), identity (cultural, racial, ethnic, sexual, special needs, etc.) and power (referring to the levels of social transformation achieved). Although the research predates this and other studies that have discussed inclusion and equity, the axes of analysis adopted in the research (Cristovão, 2007, p. 63-64. Our translation) dialog very closely with these dimensions:

- the mathematical production of the students, mainly highlighting their ways of thinking and communicating mathematically their reasoning, procedures, conjectures, and attempts at justification and argumentation to validate them;
- the mobilization and (re)signification of the knowledge acquired, both during the previous period of schooling and in their daily practices;
- changes in students' attitudes and attitudes towards knowledge, production, and mathematics itself, as well as confidence in themselves and their ability to produce mathematical knowledge;

- students' protagonism and active participation when communicating mathematically;
- the resistance or "negativeness[4]" shown by students during exploratory-investigative practices, i.e. an ability to question proposals, to respond in a totally and unpredictably different way to what was expected. A kind of mathematical and social empowerment.

It is not possible to think of teaching mathematics for equity if all students start from the same point. For everyone to learn mathematics in a meaningful, effective and empowering way, it is necessary to consider the needs of each individual or group. The results obtained from the research show that an exploratory-investigative approach to teaching can contribute to this process. This is not just a theoretically based suggestion, but one based on empirical findings.

To this end, in this article, we first present the conceptions of some authors who can help us understand the nature of the exploratory-investigative approach to teaching. In the next section, we present the cycle recovery project and some discussions on the concept of school failure. In the same section, intertwined with the theoretical discussions, we present the two investigative work hypotheses that helped establish the methodological paths of the research.

In the section "Diving into everyday classroom practices", we present the analysis and qualitative interpretation of the data. To make up this analysis, episodes that occurred during the research are recalled, which, intertwined with the axes adopted by the researcher, point to the contributions of the exploratory-investigative approach to school inclusion of students considered "failures". We conclude the article by highlighting how teaching mathematics, from a more exploratory-investigative perspective, makes it possible to treat everyone equally and helps us reframe what we mean by school failure.

On the exploratory-investigative approach

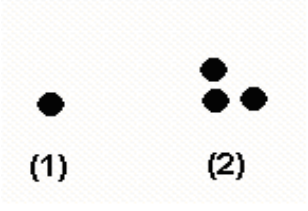
Investigative tasks differ from other teaching proposals in that they focus on challenging and open-ended problem situations that allow students to engage in an activity with multiple possibilities for exploration and investigation. For Ponte et al. (2003, p. 9. Our translation),

> In teaching and learning contexts, investigating does not necessarily mean dealing with very sophisticated problems at the frontier of knowledge [as mathematicians do]. It simply means that we formulate questions that interest us, for which we have no ready answer, and we search for that answer in as reasoned and rigorous a way as possible.

[4] Negativity is understood as "the capacity that the other always possesses to be able to dismantle with their own counter-strategies those of which they feel they are the object" (Ardoino, Barbier & Giust-Desprairies, 1998, p. 68).

Below are two examples of tasks that can be considered investigative, due to their degree of openness and possibilities for exploration. The first is taken from Fernandes et al. (2006) and the second from Ponte et al. (2003).

Example 1: Investigating and discovering sequences (Fernandes et al., 2006. Our translation)

> Today, we're going to work with sequences of balls and their shapes. How about discovering relationships between the way the sequence is constructed, the number of balls in a certain position, and their position in the sequence? *I challenge you to investigate and discover the next positions in the sequence!*
> Take a look at the first two positions in the sequence of balls below:
>
> 
>
>
> ...
>
> Did the group find it complicated? Below are some questions to guide the study.
> **1.** Continue the sequence, drawing up to the 10th position.
> **2.** Could the group find other ways to continue this sequence? What would they be?
> **3.** If the group has thought of more than one type of sequence, choose the one you like best to find a way of saying in writing what your 100th position would look like. Furthermore, would you be able to say how many balls the 100th position will have?
> **4.** Can you now write down a rule that could represent the number of balls or the shape of any (undefined) position in the sequence?

Example 2: Explorations with numbers (Ponte et al., 2003. Our translation)

> Try to find relationships between the numbers:
>
> | | | | |
> |---|---|---|---|
> | 0 | 1 | 2 | 3 |
> | 4 | 5 | 6 | 7 |
> | 8 | 9 | 10 | 11 |
> | 12 | 13 | 14 | 15 |
> | 16 | 17 | 18 | 19 |
> | ... | ... | ... | ... |
>
> Record the conclusions you reach.

In a more recent publication, Ponte et al. (2015) broaden the notion of tasks to include not only exercises and problems, but also explorations and their formative contributions:

> The exploratory approach values the development of reasoning through tasks that are somewhat open-ended or challenging. This approach represents a significant change from teaching in which the teacher first demonstrates the solving method and then

> presents exercises for the student to solve (Ponte et al., 2015, p. 114. Our translation and emphasis added).

These changes are essential in order to re-signify school failure and understand who really fails in this process. Complementing this idea, Fiorentini (2012) conceptualizes exploratory-investigative classes as those in which

> in which tasks and activities are mobilized or triggered that are open, exploratory and non-directive of the student's thinking and that present multiple possibilities for alternative treatment and meaning. These lessons are generally used to introduce a new topic of study or to problematize and produce meanings for a mathematical concept.
> Depending on how these exploratory-investigative lessons are developed, the activity may be restricted to the exploration and problematization phase. However, if, during the activity, questions or conjectures are formulated which trigger a process of testing and attempting to demonstrate or prove these conjectures, then we have a typical mathematical investigation situation. (Fiorentini, 2012, p. 72. Our translation).

In an exploratory-investigative approach, the proposed tasks do not necessarily require a large base of prior knowledge, so they can facilitate the mobilization of the knowledge and meanings that students already have or bring from their schooling process or everyday practices, in other words, they can mobilize their senses and their potential ability to think and establish relationships. It was on the basis of these senses and meanings mobilized in the students' activity that we began to systematize the concepts covered in each task proposed during the field research. In this systematization, we tried to highlight the students' productions and discoveries, and what this represents for their mathematical learning and for their school inclusion process.

The research, its contexts and its protagonists

Cortesão (2004), with his concept of action research as a transgressive practice, helped the researcher to understand the true meaning of educational research. Based on this author's ideas, she realized how much the methodology we choose to conduct our research has to do with our worldview and the historical and cultural context in which we are situated. For a teacher who was just beginning to become a researcher, it made perfect sense to study the RC II classes, in partnership with other teachers at the school, from the perspective of action research as advocated by Cortesão. It made sense to study the RC II classes from the inside to

understand and transform - or at least to show what needed to be transformed - the logic of exclusion that classifies individuals as "failures" or "successes[5]".

The research therefore took place in a collaborative group, in a context of partnership between the researcher who led the group and one of the participating teachers. Both believed that an exploratory-investigative approach could contribute to students' mathematical learning by providing them with a legitimate involvement based on their interest in the social and mental activity they were undertaking, as it resulted from open-ended tasks that allowed them to create their own relationships with the mathematical knowledge to be constructed and/or (re)signified. Thus, what justified the use of this approach at the time was the possibility for students to become the protagonists of their learning. This was the teacher's and researcher's idea of inclusion.

During the preparation of the tasks, the researcher, and the teacher were careful not to deviate too much from the program planned for these classes, based on the use of their notebooks (São Paulo, n.d.) to guide the work and the need to compile a portfolio of the activities developed by the students. This portfolio was to be submitted to the school board, which regulated the work of the city's schools, as a record of the work done during the year and as a way to assess whether teachers were using the materials intended for the project. But, as De Certeau (1994) warns, they could use "loopholes" to produce something else. Therefore, although the tasks were based on the material, they tried to go beyond what the material suggested.

In reviewing the path taken, we realized that by initially proposing the formation of a group of teachers to intervene with them in classroom practice and to analyze the actions and results obtained, the researcher developed a set of practices typical of action research:

> In an action research project, a set of research practices is developed with the aim of producing knowledge that is admittedly necessary because it will make it possible to intervene better in a social and/or educational problem that is thought to be important to tackle. And this set of practices always uses the results obtained from analyzing what happens as a result of the intervention to produce new knowledge (and so on) (Cortesão, 2004, p. 1. Our translation).

For the author, action research corresponds to

> (...) a different form of traditional scientific work (a new methodology for some, a new paradigm for others) that is extremely transgressive. Transgressive because of

[5] At the school where the researcher worked, during the field research several teachers were indignant at the choice to carry out research "precisely" with RC II students. It was as if the research should only be interested in investigating classes or students without learning problems.

> everything that has already been said, and because it dares to introduce action into the sphere of competence of theoreticians and because, in responding to the needs of practitioners (who are faced, in the field, with the urgency of solving the problem), it dares to propose that they stop to reflect and produce knowledge, in the midst of the drudgery of action (Cortesão, 2004, p. 3. Our translation).

The exploratory-investigative approach allows students and teachers to have entirely unique experiences from those that take place in a traditional classroom. It allows a polysemy (Alro & Skovsmose, 2006) to emerge in the classroom, which will be seen in the analysis, where we will recover, through the episodes experienced by the students, their voices, their way of interpreting and understanding the mathematics they produce and the relationships they establish with knowledge.

Considering the objectives of this article, we will do better to explain the problem of the study, i.e., the cycle recovery classes and the stigma of failure carried by their students, as well as the concepts that underpin the analyses carried out. It is not enough to take an exploratory-investigative approach into the classroom in projects that reinforce the stigma of failure. You have to prepare yourself to understand and give new meaning to failure!

Tactics (and strategies?) for reframing "school" failure

Authors such as Omuro (2006), De Certeau (1985, 1994), Dubet (2003) and Cortesão (2000) help to give a new meaning to failure from a theoretical perspective. The dialogue with these authors allowed us to understand the magnitude of the challenge of promoting the inclusion of students in a class that carries the stigma of failure. Working with students in remedial classes, or classes that separate "weaker" students from "stronger" students, is an arduous task for teachers, who find themselves alone in solving the problems caused by a school system that automatically admits students to save resources and not to improve the quality of teaching. In the case of RC II, when the students saw themselves in the project, which already differentiated them by grouping them in a class with a reduced number of students (maximum 25), all repeaters of the current 9th grade, or with a history of difficulties in previous years, their initial reaction was revolt, since they saw Cycle Recovery as a punishment, mostly associated with their unruly behavior. The material used was also problematic, presenting concepts from earlier grades without an appropriate approach for their age group.

We can't blame the students for this sense of revolt; there is a lot to rethink to understand this situation. Dubet (2003) draws attention to the social role of the school, stating that it cannot be seen only as a reproducer of social inequalities. Like Cortesão (2000), the author criticizes

the most common interpretation of the problem of school exclusion, which sees it because of students' social origin. For a long time, the school has preserved its image by relying on these differences to justify the "educational failure of its students", but it is also responsible for this process, as Dubet (2003, p. 43. Our translation) explains:

> The problem of exclusion teaches us that the relationship between school and society has changed and that the school has lost its "innocence". It is itself the agent of a specific exclusion that transforms the experience of students and opens up a crisis of meaning in studies, sometimes even in the legitimacy of the school institution.

With more inclusive legislation, the school has welcomed students with Special Educational Needs (SENs), but it has not prepared itself or its teachers to meet these needs. Furthermore, although it has welcomed students from all social classes, it has not embraced the culture of these young people. It welcomed them in greater numbers, but began to exclude them more efficiently by affirming "both the equality of individuals and the inequality of their performance. In this sense, the school integrates more and excludes more than before, despite its principles and ideologies" (Dubet, 2003, p. 44. Our translation). By no longer failing, students have internalized their exclusion (Freitas, 2002b), giving up a losing fight and taking the blame for their failure on themselves.

> Unsuccessful students gradually discover that their work "doesn't pay", that they can't achieve honorable results despite their efforts. [...] they decide not to play the game anymore, not to take part in a competition in which they have no chance of winning. [...] This strategy is not without rationality if one admits that it allows students to preserve their dignity, their self-esteem, since they themselves contribute to their exclusion. [...] They lost the match, but their honor is saved since they did nothing to win, instructed by a long history of failures (Dubet, 2003, p. 41. Our translation).

What can be done to change this situation, "given" by the macrostructure we have? Freitas (2002b) points to elements of what he calls an "alternative policy" focused on the school's formative responsibilities, which aim to transform the relationship between people and between people and nature. To this end, he presents a framework which, according to him, summarizes the characteristics of an alternative public policy and which guides a different way of looking at school, cycles and assessment, and is aimed at building a real school for all - as a possibility of access and as quality. But while this alternative doesn't happen (which recently has become even more distant, given the strength of neoliberalism, which is now presenting itself in an even more fascist light), what can we do, as teachers, for our students? What exits or loopholes do we have left that we can take advantage of, thinking from the micro perspective

of the classroom, so that the macro condition, as Freitas puts it, can emerge over time? What tactics[6] can we use, since in this current game of forces - system vs. teachers - it still seems difficult to reach the level of strategies?

At the micro level, for example, we can organize ourselves into groups and try, together, to find loopholes and collaboratively build strategies that can help tackle the problems that arise in school practice. Proposing a teaching alternative that makes it possible to tackle the problem of exclusion is not a task to be carried out in isolation or independently, either by school teachers or researchers. Therefore, the first hypothesis raised in the research was that tackling this problem could be based on collaborative work between the researcher and the teacher, with the support of the group set up for the research.

This hypothesis was reinforced by the findings of Omuro (2006), who points out a flaw in the process of continuing the RC II project, stating that when teachers started working alone, they ended up abandoning not only the material, but also the problem-solving methodology proposed by the project, returning to traditional lectures and the procedural and mechanical approach of repetitive exercises.

The collaborative group realized that to change teaching perspectives and conceptions and really promote change, it wasn't enough to train teachers as technician pedagogies and neoliberal policies prefer. By living experiences of change and reflecting on them, teachers can realize "how enriching it will be for everyone if the 'other-different', instead of being considered strange or ignorant, is considered someone who has a lot to unveil for the other members of the group, who has their contribution to make to resolving the learning issues they face" (Cortesão, 2000, p.5. Our translation). And this was the perspective adopted in the group and also in the classroom.

To preserve the identity of the students and the partner teacher, two or three letters have been used for each student and also for the teacher. With the aim of seeking evidence and understanding of the emancipatory possibilities - in other words, of promoting students' inclusion in school - of the exploratory-investigative approach in the context of RC II classes, the four axes described above were established to analyze and interpret the productions and the

[6] "I call 'strategy' the calculation of the relations of forces that becomes possible from the moment when a subject of will and power is isolable from an 'environment'. It posits a place capable of being circumscribed as one's own and therefore capable of serving as the basis for managing one's relations with an extinct exteriority. Political, economic or scientific nationality was built according to this strategic model [...]. On the other hand, I call 'tactics' a calculation that cannot count on a self, nor, therefore, on a border that distinguishes the other as a visible totality. Tactics only has the other as its place. [...] It has no basis on which to capitalize and ensure independence in the face of circumstances. The 'proper' is a victory of place over time. On the contrary, because of its non-place, tactics depends on time, watching to 'capture in flight' the possibility of gain" (De Certeau, 1994, p. 47. Our translation).

relationships that the students established with mathematical knowledge, with themselves, with others and with the teaching and learning process, and which were evidenced during the development of the proposed tasks. These axes were not defined a priori; on the contrary, they were constructed based on the interpretative dialogue that was established between the evidence from the field research and what the literature said, as we will show in the next section.

Diving analytically into everyday classroom practices

The events were analyzed chronologically, grouped into three different moments of contact with the students. The first, called "Searching for a positive synergy", which lasted two lessons, aimed to allow the researcher to get closer to the students through two dynamics developed in the classroom. The second moment aimed to get to know the students better - their desires, interests, characteristics, and needs - from the perspective proposed by Santos and Martins (2015). This moment, whose task was called "Who is the CR student?", lasted eight lessons spread over four meetings. In the third moment, also held over four meetings (eight lessons), the students were involved in solving the "Triangles" task.

The possibility of interpreting the RC II classes as homogeneous was soon ruled out. It was easy to see, from the first contact, the multicolored "rainbow" (Cortesão, 2000) that constituted that class. A rainbow of attitudes and relationships with school and mathematics, which manifested themselves in entirely different ways from one activity to the next, from one day to the next. Therefore, due to the limited space of this article, we have chosen to give an analytical and interpretative account (Clandinin & Connelly, 2000) of the first two moments and some relevant aspects and episodes relating to the third moment.

First: initial search for positive synergy

In the praise dynamic, each student stuck a piece of paper on their back and the class went around the room to write compliments on their classmates' papers. When discussing what they had written, it became clear that the relationship between the students was far removed from the standards expected by any teacher in their classroom. They all accepted the offensive words they attributed to their classmates as if it were something common, naturalized, showing that these words were already part of their daily lives in a trivial way.

For McDavid and Harari (1980), it is through self-perception that a person forms their self-concept. These authors define the term as an organized structure, which derives from the experience that each person establishes with their self. The evaluative aspect of self-concept, i.e., whether we like who we are (or a part of who we are) and the degree to which this is the

case, is the "measure of our self-esteem". A high level of self-esteem corresponds to a feeling of general approval of what we perceive in ourselves, i.e., our self-concept, and the opposite corresponds to a low level of self-esteem.

The dynamic revealed signs of low self-esteem in several of the students in the class in relation to their self-concept and that of their peers. This situation may have led many students to drop out over the course of the year. The project started with only 15 students out of the 25 enrolled, but not all of them finished the school year attending classes.

The second dynamic, that of the figures, consisted of forming a large circle in which each student chose a figure from among the figures displayed at the front of the room and taken from magazines; then they had to write a short text relating that figure to their impression or mental image of what mathematics represented. After a while, everyone commented on what they had written, trying to justify why they had chosen that picture. Below, we highlight some sentences as they were written on the sheets they handed in, including the spelling mistakes.

> **De:** My relationship with mathematics is more or less living without mathematics, you are nothing. Without Re we wouldn't pass the grade. (based on a photo of indigenous people)

De doesn't make it clear what relationship he has with the indigenous people in the photo, but he values the teacher (Re) as the holder of a power: that of passing them. Thus, as Omuro (2006) observes, it was possible to see that the students don't understand what continuous progression means, in other words, they don't understand that, in CR, the important thing is to recover the learning they should have had and not just a gimmick to pass.

> **Pa**: I like math because I like doing calculations and using my head. And also because when I grow up, I want to be a civil engineer and I know I'm going to use a lot of math. And this picture reminds me how much I have to work to make my intelligence grow like these vegetables (based on a picture of a plantation).

Pa shows that although, on the one hand, RC II students have low self-esteem in relation to their perception of themselves and don't value themselves, their self-esteem in relation to their ability to learn mathematics seems not to have been affected by the condition of being an RC II student. However, they recognize that learning this important knowledge for their future requires effort and greater dedication to their studies. Like Pa, other students associate learning mathematics with an opportunity for social advancement.

> **Me**: I don't like math very much, but I know it's necessary to learn it. I don't like it very much because I find some things difficult, and I'm embarrassed to ask, but I know that

> our lives fit perfectly into math for millions of things. (photo of an animal at the bottom of the sea)

Me tells me that he doesn't like math because he has difficulties and seems to want to escape from the world of math. Perhaps the figure of an animal at the bottom of the sea represents a distancing from this social world that "fits perfectly into mathematics for millions of things". In other words, although she recognizes that mathematics is present in life, by not explaining how this happens, student Me establishes a distancing relationship with mathematics, or perhaps she perceives that school mathematics has no relation to her experiential and cultural world.

> **Ci**: The first picture, of a father and son, is very representative of mathematics. It looks like the father is teaching his son to count with the rims of a bicycle, it's an incentive for the son to want to learn about mathematics (based on a photo of a father and son looking at a bicycle).

It was precisely this student, who spoke of her father's encouragement to study, who was withdrawn from school by her mother when she thought her daughter's performance was insufficient to continue studying, due to her low grades. These grades, the student's previous failure and the lack of clarification to parents about the meaning of the Cycle Recovery project, lead parents to believe that their children are incapable of learning the mathematics that the school teaches. In addition, they do not realize that their children are in a "situation of school failure" (Charlot, 2000) and that this situation may be the result of the school's own prioritized practices and, therefore, they do not demand a change in these pedagogical practices.

Although their self-concept in relation to themselves seems very affected, in most cases, in relation to their ability to learn mathematics, they showed hope that they could overcome their limitations. For these students, whose relationship with their math teacher was good, learning this subject, although challenging, was perceived as necessary and important for their lives. This, in a way, indicates a favorable predisposition towards learning mathematics. However, gaining the attention of the RC II students to establish this dialog was not an easy task. Among the many attempts by the partner teacher to get them to calm down and accept the proposal, the researcher tried to talk to them about the importance of valuing themselves and others, and asked for their collaboration in this regard. She also pointed out that the research she was starting was precisely intending to value the RC II students, bringing them a different teaching proposal where they could express their meanings and be themselves.

Over time, the researcher's (first author's) dialog with them became more fluid and frequent due to collaborative work with the teacher, who shared the strategy she used to use: she never explained orally what they had to do or any of the content. She would write a lesson plan on the blackboard, containing the tasks they had to carry out. The nature of the tasks, however, became more open and exploratory, geared towards the students' culture in a way that could encourage them to develop culturally relevant mathematical activity, giving them a leading role and the opportunity to think mathematically.

Second moment: the exploratory-investigative approach enters the classroom: Who is the CR student?

The two exploratory-investigative tasks, proposed after the initial dynamics period, were developed with the support of a collaborative group of interested teachers and in negotiation with the partner teacher, seeking to address concepts present in the material adopted in the project. The pedagogical objective of the first task, entitled "Who is the CR student?" was to sharpen the students' critical sense so that they could rethink their view of themselves, while at the same time learning to use basic statistical tools. The task consisted of an initial text, as shown in Chart 1.

Box 1: Motivating task proposed to the students

I spent the weekend trying to come up with a script for our investigative task and decided to a very real situation for me. **At the university where I study, when I told my professors and colleagues from the Masters and Doctorate programs that I was going to do a research project with CR students, they all wanted to know: "Who is a CR student?" I tried to explain that CR students are students who were held back in the 8th grade, or students who have been doing poorly since the 5th, 6th and 7th grades, students who are always being reinforced. I think this description I've given of you is very poor and superficial, because I know very little about what and how you really are, not only in appearance, but also what you think, do, like or dislike, not only at school and in math lessons, but also in life.**

Source: Prepared by the researcher (Cristovão, 2007. Our translation)

In addition to the motivating text, the task also presented guiding questions for the work (another contribution from the partner teacher, who knew the students and knew the need for this direction), such as: "How do you think we could produce material that would be able to truly answer "WHO ARE THE CR STUDENTS?"? What idea would you have for us to take this information with more details about you to the University? Is there any mathematical knowledge that we could use to "show" the characteristics of 8th grade students? How could

we organize this information and present it so that everyone who sees it understands who you are?"

There were also guidelines about forming groups of 3 students, taking on different roles (writers and rapporteurs), and about moments for exchanging ideas and organizing the material. These guidelines were not followed to the letter, as the groups were often incomplete.

Although the task was restructured to make it easier to get started and to provide a starting point for statistical processing, not everything worked out as planned. Their concern was to try to characterize the students from their perspective, without developing questions that could support a statistical survey of the data. The teacher and researcher tried to help them realize that what they were writing was conjecture that had to be proved or disproved, as the following dialogue shows.

> **Researcher:** *I haven't seen you discuss anything, have you?*
> **Student[7]:** *Discuss? I'll write down what it is here. Who is the RC student?*
> **Researcher:** *How are you going to write it down? What's your idea for answering this question? Are you just going to answer in your head? Does it matter what the class thinks?*
> **Student:** *The class or the group?*
> **Researcher:** *You can't take the opinion of the class, what does the class like, do?*
> **Student:** *The class just likes to mess around!*
> **Researcher:** *So you think RC students are messy? Is it a characteristic?*
> **Pa:** *Ah... not all of them!*
> **Researcher:** *So! Let's try to formulate a question to see if the class gives this opinion. Couldn't you collect this opinion from the whole class?*
> **Student:** *Ah... I'll put it... some!*
> **Researcher:** *But do you think you can put it in your head? What if the class doesn't agree?*
> **Pa:** *But it's not my group?*
> **Researcher:** *Your group will have the ideas, but the class has to agree...[several discussions] Think of a way to put the information, but have the whole class take part!*
> **Pa:** *No way!*
> **Researcher:** *Oh, I think there is! What if you came up with a way of getting everyone's opinion across? Haven't you learned anything in math that allows you to do that?*
> **Pa:** *In math we only do graphs, drawings...*
> **Researcher:** *So you can't make a graph with people's opinions?[pause] So try to think what questions you could ask to make the graphs. Try to think what information you could ask the class. (Field diary 1, p. 47 – 49. Our translation)*

Students who attend remedial classes come with school baggage. Their history of failure or the fact that they are in a remedial class is one more reason for us to try to recover the

[7] In the recordings, it wasn't always possible to identify the student who was speaking. Therefore, this will be the form used in those cases.

knowledge they have already built up over the course of their schooling. They may not be clear about how and where to use it, but it's our role to build on this knowledge, whether it's in the most appropriate way or not, to allow the student to (re)signify it.

Alro and Skovsmose (2006) use the concept of "privileged views" to explain how the initial start of the lesson takes place, in which the students, even though they don't know exactly where they are going, explore various ideas about the proposed topic and then begin their discoveries. Working with this type of lesson may initially generate certain inconveniences, but it allows students and teachers to break away from the traditional classroom perspective:

> There are different aspects involved in the process of changing the paradigm from exercises to research scenarios. Communication patterns can change and open up new types of cooperation and new forms of learning. [...] Both the teacher and the students can be plagued by doubts when they come to work in a research scenario, without the protection of the well-known operating "rules" of the exercise paradigm. Thus, leaving the exercise paradigm also means leaving a comfort zone and entering a risk zone (p. 58. Our translation).

The teacher and the researcher relied on this teaching approach, on these "scenarios", as a means of **mobilizing and re-signifying the students' previous knowledge** in order to promote their integration in school. By not presenting ready-made concepts for them to apply, the teacher accepted the risk of losing her authority, of leaving her "comfort zone" and entering a "risk zone" which is "closely related to the emergence of possibilities for student involvement, different communication patterns and, consequently, new learning qualities" (Alro & Skovsmose, 2006, p. 58). Teachers and researchers sometimes felt too directive in their interventions, but Alro and Skovsmose (2006) clarify that these interventions act as fuel for a dialogical process to take place in the classroom. It's not just the students' perspectives that need to be considered. An intervention by the teacher can bring new energy and new perspectives to the students' investigation:

> Although students' perspectives are a source for the research process, dialog can reveal something radically new. The teacher can see new things too. In this sense, we see a dialog as a collaborative process of constructing perspectives (Alro & Skovsmose, 2006, p. 127. Our translation).

By asking the RC II students about the need to collect the opinions of their classmates in order to draw up a profile of the RC II student, the teacher, and researcher pointed them in the direction of a new working perspective that would lead them to more consistent data, but at the same time they encouraged the students to **mobilize and re-signify their previous**

knowledge. They expected the proposed task to lead them to the need for a statistical treatment, but by encouraging them to arrive at this treatment, they gave credibility to the results obtained in their questions and provided them with a moment of reflection that was not part of their plans. This phase of solving the task-developing questions and alternatives with a view to collecting data-allowed the students to do something that is not part of everyday classroom life: **be the protagonists of the learning process.** This became even clearer when discussing the results of their data collection. The students were able to assert their will, their interpretations, to become the authors of the questions.

The initial topics chosen were: age, occupation, and hobbies. They all agreed that these questions would be interesting to characterize them, and the topic of hobbies was suggested by **Pa,** who argued that he thought it would be better than asking about types of music, sports, etc.. He said that *this way all would be covered!* This intervention shows an ability to think statistically, which is a first sign of **mathematical production.** What opportunities do we offer them to expose their thoughts, with the **protagonism of mathematical production**, when we work with questions that are already formulated?

Based on the questions they formulated and the data they collected, the use of tables and graphs also had to be negotiated, as they didn't feel the need. We could interpret this as a lack of will, interest, or involvement for the students, but if we look at Charlot (2000), we can see that another view is needed. This author argues that there are two ways of "translating" school failure in order to think about it. The first interprets it as a difference and is based on theories of reproduction; the second requires interpreting school failure not only as a difference, but also as "an experience that the student lives and interprets and that can become an object of research" (p. 17. Our translation). Therefore, in this study it was necessary to look at the students' school history, their activities, and behaviors, without forgetting that

> the school experience of the student in a situation of school failure bears the mark of difference and lack: he encounters difficulties in certain situations, or guidelines that are imposed on him, he constructs a devalued image of himself or, on the contrary, manages to calm this narcissistic suffering that is failure, etc. School failure is then studied "from the inside", as an experience of school failure (Charlot, 2000, p. 17. Our translation and emphasis added).

So this positive outlook was an important step towards changing **attitudes and behavior**. Despite this search, the researcher was worried about the pace at which the investigations were progressing, so the teacher reassured her by saying that, in RC II, *our spirit has to be different*. For **Re**, it was already a victory to be able to work with them with a

beginning, middle and end. The teacher seemed to see, in this activity, a possibility of school inclusion for these students who, at other times, didn't even finish what they started. It was the first sign of **changes in the attitudes and behaviors** of the students, but also of the teacher and the researcher, who were building together what Charlot (2000) calls a "positive reading" for the learning process of these students:

> Practicing positive reading also means paying attention to what people do, achieve, have and are, and not just what they fail at and what they lack. It means, for example, asking what students in situations of failure know (despite everything) - what they know about life, but also what they have acquired from the knowledge that the school tries to provide them with. In this sense, it is an "optimistic" reading, for those who want to use that word [...] practicing a positive reading is not only, nor fundamentally, perceiving knowledge acquired alongside deficiencies, it is reading in another way what *is read as lacking* by the negative reading (p. 30. Emphasis added and our translation).

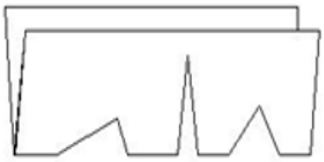
In this task, they were involved in: designing questions and answering alternatives, collecting and organizing data, constructing tables and graphs, calculating percentages. And they produced posters with the results of this first task, to show everyone their **mathematical production**. This first task also influenced the results of the triangles task because in order to produce the reports, they even wrote entire pages trying to argue about the classifications they had found.

Third moment: entering an exploratory-investigative mathematical practice

Although it won't be described and analyzed in detail, it's worth noting that the second task proposed to the students was more focused on teaching geometry, especially the study of triangles, which was also provided for in the project material, but in a fairly traditional way, based on definitions. The task had two parts. The first focused on exploring pre-constructed triangles, with the aim of constructing lines with a ruler on these triangles to produce other triangles, in order to explore classifications by angle and side. No pre-established definitions were given, but the students could consult their notebooks and books and thus draw on previous knowledge. In the second part, they were challenged to produce triangles with different classifications from cut-outs on a folded sheet of paper, as shown in Figure 1 below.

1) On a sheet of paper folded in half, cut out equilateral, isosceles and scalene triangles.

Take the pieces of paper you have obtained, unfold them and say which geometric shapes they contain. Write everything down in the group report.



2) Now we want to get the equilateral, isosceles and scalene triangles by making two cuts. Which cuts should we make?

Draw pictures of the cuts you have made and comment on your findings.

Figure 1

Exploratory task with different types of triangles (Ponte et al., (2003, p. 72. Our Translation)

In the explorations carried out by the collaborative group, and also in the book from which the task was taken, the conclusion was that it would be impossible to construct a scalene triangle with these conditions, but one pair of students questioned this certainty. Da and Em, realizing that it was impossible to get a scalene triangle by making two cuts on the folded sheet, changed the rules of the game. They made a cut in the bottom right-hand corner of the folded sheet and then unfolded the cut-out part, obtaining an isosceles triangle. They then made a second cut, obtaining a scalene triangle. Teacher Re and the researcher went to check what they had done:

> **Researcher:** How did you manage to form it?
> **Da:** Two cuts, right? Now the one on different sides, look... one cut, okay? [Da folds the paper in half, makes a cut in the corner, as Em had done before, and starts to open the paper].
> **Researcher and teacher** Re: But... it's all with it folded!
> **Da:** No, no! Here it doesn't say that it's only folded... [Points to item 2 of the task]
> **Researcher:** [Reads the task sheet and agrees]... yeah... it was missing the words: on a sheet folded in half! (Lots of laughter) (Brackets by the authors of this article)

By mobilizing their thoughts and finding a loophole in the rules of the proposed task, the students managed to surprise the teacher and the researcher, producing a possible interpretation that was accepted by them. These students' resolution brings us back to the concept of **negatricity** (Ardoino et al., 1998), understood as an incredible ability to de-play, that is, to respond in a completely unpredictable way and in a way that is different from the objectives set by the educational action. This shows the educational potential of the exploratory-

investigative approach, as it provides students with the opportunity to take the lead and be authors, and enables a form of teaching that offers everyone the necessary support to succeed, i.e., teaching with equity (Sun, 2014). This approach also highlighted the **mathematical and social empowerment** of these students who, in another teaching context, seemed incapable of reasoning and producing ideas and resolutions with convincing arguments.

Without a positive outlook, and in a context that favors only the traditional classroom, they could be considered students with intellectual disabilities, as it is common to hear teachers claiming that students do not have a report, but have serious learning problems. In an integrative review on school inclusion, Silva and Carvalho (2017, p. 293. Our translation) analyzed "the content of national publications from January 2011 to April 2016 to understand the facilitators and limitations of the school inclusion process in Brazil from the teachers' point of view". In their results, the researchers point out the factors that interfere in the process of school inclusion, highlighting the teachers' lack of knowledge about the inclusion policy and about the student's abilities and limitations in terms of disability and personal interests, the lack of resources offered by public authorities to make their actions effective, and the lack of specialized professionals in the area of special education to support teachers. In short, the limitations were highlighted, but few results were presented in relation to the facilitators. This was another motivation for writing this article!

Further reflections and discussions on the results of the two tasks

Returning to the analysis of the first task (statistics), it is worth noting that the same students had already found innovative strategies for presenting their results. **Da** organized a flowchart[8] constructed with paper and glue by connecting the graphs and tables with small strips of paper. This strategy made the presentation of the results very efficient and creative. **Erk** wanted to present the statistical data collected by his group in the form of a television news report, and spent much of the class rehearsing his speech as if he were a reporter. In the end, he offered to start the presentations and encouraged his shy classmates. **Erk's** enthusiasm for presenting and his ease in expressing himself during rehearsals infected the class, prompting the researcher to suggest that the teacher begin the presentations by talking about the project. The students liked the idea and agreed on the order of the presentations from one group to the next. **Erk** eventually realized the idea of turning the presentation into a TV news format! At the

[8] A flowchart is a type of diagram and can be understood as a schematic representation of a process, often using graphics that illustrate in an uncomplicated way the transition of information between the elements that make it up (Source: <http://pt.wikipedia.org/wiki/Fluxograma>, consulted on 22/03/07).

end of the presentations, everyone applauded, happy to see the work successfully completed. The teacher and the researcher felt as if they were at one of those gymkhanas or parties that the school usually organizes, for which a play or a dance is rehearsed, and at that time, despite the certainty that it won't work, everything goes wonderfully, as if the actors or dancers were professionals. Everyone stares in amazement, not believing that the show was a success, that it exceeded all expectations... But the effects of the first task didn't stop there. There was more to do because that first task was to submit a report to the university explaining who the CR student was. Faced with the students' refusal to do any more writing (they thought the posters were more than enough), the teacher and researcher decided to record an oral report. During the recording, when they were asked about the need for the CR II classroom, based on the answer to a question about the importance of the material used by the students to help them catch up on what they hadn't learned, a significant dialogue began:

> **Researcher**: *Isn't this CR initiative good? Doesn't it help you?*
> [several students say it does, but a "no" resounds in the room]
> **Researcher:** *Who doesn't think so?*
> **Da:** *Me.*
> **Researcher:** *Why?*
> **Da:** *We don't learn things from the eighth, only from the seventh.*
> **Researcher:** *Oh, yes! So you think it could be this material, but you'd have to learn something about the eighth?*
> **Da:** *Of course!*
> **Me:** *But when we had the chance to learn stuff from the eighth notes, we didn't really appreciate it, did we?*
> **Researcher:** *Yes, but there are people who didn't repeat the eighth grade, right?*
> [After some demonstrations, it seems that three students in the class are in this situation: **Ri, Erk and Er**, who was absent that day, but was reminded by the teacher].
> **Re** [addressing **Da**]: *Do you feel disadvantaged by this?*
> **Da:** *Of course, how am I going to get to the first year of high school without the content of the eighth grade?* [Brackets by the authors of this article].

The dialog leads to a discussion that encourages the students to listen carefully to the teacher's suggestion to work in a compact way during the past two months on the eighth grade content that is most relevant to this continuity. There, they seemed to experience a kind of collective reflection, triggered by the teacher, on the need to **change their attitudes and approach to** knowledge, even in the face of knowledge presented in a more transmissive and typically school-based way.

After some time, it was possible to see that this reflection had awakened in them an awareness of the importance of the active participation of all students in the learning process. This had already been reflected in the second task, but the teacher reported that after the project

they began to really commit to carrying out the activities she had suggested, even though they were not of an investigative nature. For her, the impact of the first task had already exceeded our expectations.

Skovsmose (2000[9]) discusses social issues surrounding the teaching of mathematics and points to what he calls "scenarios for inquiry" as an environment conducive to more meaningful teaching of the subject for students. After an example where the author presents dialogues resulting from a lesson in a "scenario for investigation", he clarifies:

> A scenario for investigation is one that invites students to ask questions and seek explanations. The teacher's symbolizes the invitation "What happens if...?". The students' acceptance of the invitation is symbolized by their "Yes, what happens if...?". In this way, the students get involved in the exploration process. The teacher's "Why is this...?" represents a challenge, and the students' "Yes, why is this...?" indicates that they are taking up the challenge and are looking for explanations. When students take on the process of exploration and explanation, the research scenario becomes a new learning environment. In the inquiry setting, the students are responsible for the process (Skovsmose, 2000, p. 5. Our translation).

This assignment provided a real "investigative scenario" in which the students went far beyond the scope of the assignment. It was not expected that the very questions they raised would, at the end of the presentation of the results, generate a discussion that would make them rethink their role as students. The intention was to allow them to understand the statistical method as a process of obtaining reliable data, but they used these data to reflect on their condition as students carrying the stigma of school failure, and to move forward, **to produce mathematically,** to take the **lead in the learning process, in short, to empower themselves mathematically and socially.**

This reflection may have been the beginning of a process of inclusion that ended up making the students feel more interested or capable of solving problems, something they had not felt motivated or confident to do before. However, this study has shown that care must be taken to ensure that teaching activities are not limited to the exercise paradigm and focus more on the syntax than the semantics of doing mathematics, an aspect that is still common in schools. Matos (2004) argues that "economic, social and political transformations have determined different ways of looking at the phenomenon of education, but situations of inequality and social exclusion worldwide suggest an even more critical perspective" (p.1. Our translation).

[9] For the quotations from the dissertation, Cristovão (2007) used an online version of the text, found on the website: <http://www.spce.org.pt/sem/01Ole.pdf>. Accessed on 05/10/06.

We believe that the proposed tasks and their solutions have made it possible to come closer to this critical perspective.

Combining the ideas of Matos (2004) with the learning environments proposed by Skovsmose (2000), we believe that it is necessary to rethink what we mean by democracy in school. If we understand it as effective participation in school activities, we need to give students the space and opportunity to play a **leading role.** This means promoting education "for citizenship" - where the student is involved in processes that are in an "inquiry scenario" where problem formulation, problem-solving and decision-making are privileged, rather than promoting education "for citizenship" as an imaginary action that will take place only in adult life.

Although only two exploratory-investigative tasks were developed with the RC II students, the concern to focus on a topic related to the reality of the students was present, especially in the development of the first task. With this task, the teacher and researcher moved closer to the four dimensions of equity presented by Gutiérrez (2012): access - everyone was able to do the activity; achievement - related to the results obtained and the joy of seeing a piece of work successfully completed; identity - by controlling the course of the presentation and, at the same time, learning important lessons about themselves and their ability to learn; and power - related to the levels of social transformation achieved, albeit to a small extent, related to the change in students' attitudes.

In the second task, although this focus was not again privileged, the problematizing aspect - which Skovsmose (2000) considers necessary to characterize a "scenario for investigation" - was strongly present in the discussions of the group that helped design the task, which was reworked several times until it reached its final version.

One question that may have remained is: why do students want to do exercises after having experienced an exploratory-investigative approach? We are aware that there is a long way to go to change this perspective in the classroom, and we know that many of the students' behaviors are reactions to the teaching they receive, teaching governed by "mechanisms through which subjective exclusion is constructed" (Freitas 2002a), which prevails at school; for this reason, we believe that an exploratory-investigative practice can help in this process.

By developing an investigative attitude together, teachers and students will be able to question these mechanisms. Another way of interpreting this attitude would be to see it as a sign of inclusion for these students, a way of breaking the cycle of barbarism (Melo & Roggero, 2023). In fact, after going through an exploratory-investigative experience and feeling like

protagonists, these students may have wanted to show that they are also capable of learning *the mathematics at which they had failed.*

By going through this experience, the students, and the teacher were able to try out a methodological approach that made it possible to develop the "personal and social autonomy" skills recommended by Martins et al. (2023). Alro and Skovsmose (2006), when discussing the need to vary the type of class, point out the main limitation of the traditional classroom paradigm: it doesn't allow for a change in communication, no matter how good the teacher-student relationship or how innovative the teacher's intention. Thus, when they point to research scenarios as an alternative, they warn that "taking this step presupposes a change in the educational situation and a change in perspective" (Alro & Skovsmose, 2006, p. 28. Our translation).

Overcoming "bureaucratic absolutism" (Alro & Skovsmose, 2006) requires more than just a change in the teacher's attitude towards the students because the school logic is not just made up of attitudes. This logic places the teacher as the sole holder of knowledge and the student as the recipient of this knowledge. In order for school logic to be challenged, we first need an openness that allows for a change in perspective. Seeking to point the way, Alro and Skovsmose (2006, p. 31. Emphasis added and our translation) state that "One way of doing this is to create a situation in which, on the one hand, certain structures and premises are well-defined and established and, on the other, there is relative openness for the students themselves to create concepts".

Reflecting on the attitudes of these students has allowed us to raise some questions: Do students who are failing at school not produce knowledge or are they just unable to adapt to a rigid, closed school system, where everything has to be done as prescribed and within a set time? If we give them more freedom to show their creativity, won't they surprise us? In RC II, or in any other recovery proposal, wouldn't this be a way to break the vicious cycle of continuous progression, to recover these students' self-esteem and make them believe again that they are capable of learning mathematics and overcoming barbarism (Melo & Roggero, 2023)? By interpreting students' attitudes as evidence of their ability to question and deny (Ardoino et al., 1998), finding loopholes and other possibilities for interpreting a task or problem situation. It also provides an opportunity for the students to argue and justify an answer that the teachers did not expect. This result suggests that we value even more the use of an exploratory and problematizing approach that allows students to be the protagonists of their learning, in the sense that they produce multiple meanings, resolutions, and conjectures on the one hand, and arguments and justifications that validate them on the other.

Some conclusions and final considerations

There are students who submit to an imposed school culture of transmitting knowledge and doing a battery of exercises because they already have a cultural capital that has prepared them for this. However, not all students are like this. There are those who don't want to be mere repeaters of procedures or ideas formed by others. They want to be considered as epistemic agents who bring with them their experiences and resources to produce their own ideas and resolution processes. It is precisely these students who are forcing education to rethink its role. It is the school's duty to make room for these students, valuing their culture, their way of thinking which, if properly interpreted, can be seen not as a difficulty, but as a different way of appropriating knowledge. This cultural diversity in today's classrooms can be seen, according to Cortesão (2000), not as a problem, but as a richness which it is up to the school and the teacher to take advantage of.

Throughout this study, we have worked with students who carry the stigma of school failure. In other words, a class already characterized a priori as incapable, with low self-esteem, who face prejudice from teachers, other students and even the students themselves who constitute the RC II classes. How can we study the output of these students? How can we study their practices, based on assumptions that expect the results standardized by school culture? De Certeau gave us another perspective by pointing out the need to study everyday practices, the practices of ordinary people, the excluded, the dominated, and he drew our attention to the fact that we can't think that these "dominated" are really dominated. They may not benefit from the strategies of the dominant, but they are capable of striking a "blow", not in the sense of evil, but in the sense of finding ways out of their problems through tactics that are possible in their environment.

We are not advocating that the exploratory-investigative approach is the only resource that teachers can use to get students involved in activities that allow them to re-signify and mobilize concepts, argue mathematically or feel that they are the protagonists of the learning process and authors of their own ideas. However, it must be acknowledged that the exploratory-investigative approach to teaching mathematics, such as the one we have investigated here, seems to be opportune and necessary to break this vicious cycle of students not learning because they are not interested and not being interested because they don't learn.

The concern to make a presentable poster, to rehearse the speech and agree on the rules, to present themselves as if they were reporters, seems to be an indication of the pleasure the

students felt in presenting not only their discoveries, but also themselves as subjects of the action. In the exploratory-investigative classes, during the many moments in which the students became involved in the activity and began to raise their own questions, sometimes the teacher and researcher together were unable to cope with the small number of students in the room. They became the protagonists of the action and not mere spectators: it was their questions that drove the lesson, not a script of definitions and exercises pre-established by the teacher or the researcher. It was the paths they chose that generated these doubts.

Looking at the students' attitudes, based on a different conception, allows us to see that, in the words of De Certeau (apud Giard, 1994), "everyday life is seeded with wonders, scum as brilliant [...] as that of writers or artists. Without a name of their own, all kinds of language give rise to these ephemeral festivals that appear, disappear and reappear" (p. 18. Our translation).

By proposing an exploratory-investigative approach to the students, we were able to change their relationship with mathematics, the teacher's relationship with the teaching process and also the relationship between teacher and students. De Certeau allows us to think about new interpretations of the relationship of domination that exists in society and draws attention to the need to value the practices of ordinary people, to consider these practices not just as mechanical actions, but as intelligent tactics for surviving domination.

He warns that they deserve to be considered because they are not just the uniform practices of a dominated mass; on the contrary, within these practices there are tactics, used by ordinary people, which prove that we cannot believe that by researching what is consumed, we can generalize its consumers. We have to research the uses they make of these products. Students in situations of school failure (Charlot, 2000) are not mass consumers who accept everything that is handed to them. Working with them requires a change in attitude for teachers and managers, who will need to see them not just as rebels who don't produce, but as critical consumers of the knowledge they are offered.

In short, the exploratory-investigative approach, based on more open-ended tasks, helped bring out these practices, making the students see themselves as producers of knowledge, including mathematical knowledge. Despite all this, we have to remember that it's difficult to adopt this approach at all times. Often, as teachers, we leave a lot to be desired in order to be what, as researchers, we think is necessary to promote the inclusion of students in situations of school failure. Being a teacher-researcher does not exempt us from failings because

we are, as Freire (1997) says, unfinished beings. But we certainly become more questioning of our practices.

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