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Contribuições da teoria da objetivação para a análise do planejamento de tarefas de um professor de matemática envolvendo alunos com transtorno do espectro autista

Contributions of the theory of objectification to the analysis of task planning by a mathematics teacher involving students with autism spectrum disorder

Aportes de la teoría de la objetivación al análisis de la planificación de tareas de un profesor de matemáticas con estudiantes con el trastorno del espectro autista

Apports de la Théorie de l'Objectivation à l'Analyse de la Planification des Tâches d'un Enseignant de Mathématiques impliquant des Élèves atteints de Troubles du Spectre Autistique

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Resumo

O presente artigo tem por objetivo compreender aspectos do planejamento do professor que potencializam a possibilidade do encontro do aluno com transtorno do espectro autista (TEA) com o saber matemático. Trata-se de uma pesquisa de cunho qualitativo, tendo a entrevista semiestruturada e o planejamento de tarefas documentado por um professor como principais instrumentos de coleta de dados. A partir da análise do planejamento do professor, sujeito da pesquisa, discute-se a contribuição do referencial teórico, a teoria da objetivação, idealizada por Luis Radford, que fornece fundamentos que norteiam o componente *delineamento da tarefa*, o qual se traduz como o *projeto didático do professor*. São três as categorias: *considerações gerais, considerações sobre os problemas matemáticos e considerações sobre as formas de colaboração humana*. Como conclusão, as categoriais de análise emergentes dos elementos do componente *delineamento da tarefa* permitiram identificar aspectos do planejamento do professor que potencializam a possibilidade do encontro do aluno com TEA com o saber

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matemático, tais como: considerar conhecimentos prévios que emergem de situações vivenciadas em seu cotidiano; explorar contextos que permitam o encontro do aluno com o saber matemático de forma utilitarista; e explorar diferentes ambientes físicos, propiciando a convivência com regras e comportamentos adequados e a generalização de saberes escolares.

Palavras-chave: Ensino, Matemática, Planejamento, Teoria da Objetivação, Transtorno do Espectro Autista.

Abstract

This article aims to understand aspects of teacher planning that enhance the possibility of meeting students with Autism Spectrum Disorder (ASD) with mathematical knowledge. It is a qualitative research, having as main data collection instruments a semi-structured interview and task planning documented by a teacher. Based on the analysis of the teacher's planning, the subject of the research, the contribution of the theoretical framework is discussed, Theory of Objectification, idealized by Luis Radford, which provides fundamentals that guide the task design component, a component that translates as the project teacher's teaching. There are three categories: general considerations, considerations on mathematical problems and considerations on forms of human collaboration. In conclusion, the categories of analysis emerging from the elements of the task design component allowed identifying aspects of the teacher's planning that enhance the possibility of the student with ASD encountering mathematical knowledge, such as: considering prior knowledge that emerges from situations experienced in their daily; explore contexts that allow the student to encounter mathematical knowledge in a utilitarian way; and explore different physical environments, providing coexistence with appropriate rules and behaviors and the generalization of school knowledge.

Keywords: Teaching, Mathematics, Planning, Objectification Theory, Autism Spectrum Disorder.

Resumem

Este artículo tiene como objetivo comprender aspectos de la planificación docente que potencian la posibilidad de atender estudiantes con Trastorno del Espectro Autista (TEA) con conocimientos matemáticos. Se trata de una investigación cualitativa, siendo los principales instrumentos de recolección de datos la entrevista semiestructurada y la planificación de tareas documentadas por un docente. A partir del análisis de la planificación del docente, objeto de la investigación, se discute el aporte del marco teórico, Teoría de la Objetivación, idealizada por Luis Radford, que brinda fundamentos que orientan el componente de diseño de tareas,

componente que se traduce como el proyecto. enseñanza del maestro. Hay tres categorías: consideraciones generales, consideraciones sobre problemas matemáticos y consideraciones sobre formas de colaboración humana. En conclusión, las categorías de análisis emergentes de los elementos del componente diseño de tareas permitieron identificar aspectos de la planificación docente que potencian la posibilidad de que el estudiante con TEA se encuentre con el conocimiento matemático, tales como: considerar conocimientos previos que emergen de situaciones vividas en su cotidiano ; explorar contextos que le permitan al estudiante encontrarse con el conocimiento matemático de manera utilitaria; y explorar diferentes ambientes físicos, propiciando la convivencia con normas y comportamientos adecuados y la generalización de los saberes escolares.

Palabras clave: Enseñanza, Matemáticas, Planificación, Teoría de la Objetivación, Trastorno del Espectro Autista.

Résumé

Cet article vise à comprendre les aspects de la planification des enseignants qui améliorent la possibilité de rencontrer des élèves atteints de troubles du spectre autistique (TSA) avec des connaissances mathématiques. Il s'agit d'une recherche qualitative, les principaux instruments de collecte de données étant un entretien semi-structuré et une planification des tâches documentée par un enseignant. Sur la base de l'analyse de la planification de l'enseignant, objet de la recherche, la contribution du cadre théorique est discutée, la théorie de l'objectivation, idéalisée par Luis Radford, qui fournit les principes fondamentaux qui guident la composante de conception de la tâche, une composante qui se traduit par le projet l'enseignement du professeur. Il y a trois catégories: les considérations générales, les considérations sur les problèmes mathématiques et les considérations sur les formes de collaboration humaine. En conclusion, les catégories d'analyse émergeant des éléments du volet conception de la tâche ont permis d'identifier les aspects de la planification de l'enseignant qui augmentent la possibilité pour l'élève avec TSA de rencontrer des connaissances mathématiques, telles que : considérer les connaissances antérieures qui émergent de situations vécues dans son quotidien ; explorer des contextes qui permettent à l'élève d'aborder les connaissances mathématiques de manière utilitaire ; et explorer différents environnements physiques, permettant la coexistence avec des règles et des comportements appropriés et la généralisation des savoirs scolaires.

Mots-clés: Enseignement, Mathématiques, Planification, Théorie de l'Objectivation, Trouble du Spectre Autistique.

Contributions of the theory of objectification to the analysis of task planning by a mathematics teacher involving students with autism spectrum disorder

With the introduction of the Special Education from an Inclusive Perspective National Policy (Política Nacional de Educação Especial na Perspectiva Inclusiva) in Brazil (2008), regular schools now take on increased responsibility for educating students with special educational needs, including students with autism spectrum disorder (ASD), who demand pedagogical and didactic proposals that guide professionals accountable for developing the school skills of students with these characteristics and who were included in the school environment.

This article presents part of the data collected in the scope of a doctoral thesis (Takinaga, 2023), which sought to understand aspects of teacher planning that enhance the possibility of the encounter between students with ASD and mathematical knowledge. It also aims to discuss the contribution of the theoretical framework, the theory of objectification, idealized by Luis Radford, by providing as a foundation the elements that guide the component *task design*, which allowed us to list the categories adopted in the analysis of the task plans prepared by the teachers.

In this sense, our theoretical lens provided a structuring model for analyzing categories listed from the component *task design*: *general considerations, considerations on mathematical problems, and considerations on forms of human collaboration*. The analyses were based on the organization of the tasks in diagrams and, subsequently, described grounded on their structuring guidelines, defined through the identification of object, objective, and task planning.

This research is justified because of the few studies investigating elements of the teaching process that can enhance the mathematics learning of students with ASD.

Regular schools now take on increased responsibility for educating students with special educational needs, including students with autism spectrum disorder (ASD) (Brasil, 2008), who demand pedagogical and didactic proposals capable of guiding professionals responsible for developing school skills of students with these characteristics and who were included in the school environment. Barnett and Cleary (2015, p. 172, our translation) corroborate this statement by arguing that:

> Students with autism spectrum disorders (ASD) are being included more frequently in the general educational environment and, therefore, are expected to increasingly access and master core curriculum content, including mathematics. However, math is often challenging for students with ASD. Interventions to improve math skills in students with ASD have been recommended.

For data collection, we relied on the following resources: semi-structured interviews, audio recordings, and physical and/or digital documents containing the planning and description of the tasks prepared by a teacher from the Specialized Educational Service (Atendimento Educacional Especializado - AEE) in a private school located in the state of São Paulo, and photographic registers of materials used in the tasks.

In the next topics, we present the role of planning tasks from the point of view of enhancing the possibility of encountering knowledge, the theoretical framework used, and the analyses carried out from a task developed by the teacher, the subject of the research.

The role of task planning and the encounter with mathematical knowledge

Based on the discussions about the role played by the planning of teaching activities found in recent works aimed at teaching mathematics to students with ASD, we identified different aspects that emerge from the action of planning, which encourage us to turn our gaze to the moment in which considerations aimed mainly at the target audience and the encounter with mathematical knowledge are intertwined.

Fleira (2016) planned the activities used in her research, aiming to teach her student mathematical concepts that are typically covered in a regular classroom. In doing so, she took into account both the theoretical content and the student's observable learning characteristics. Starting from the mathematical content, the researcher defined the learning objectives and chose materials that would be familiar to the learner. The apprentice's lack of prior knowledge about fundamental operations would make it impossible for him to solve calculations with powers and roots. In this sense, the use of the calculator and multiplication tables, according to the researcher, were alternatives for the student to follow the content worked on in the regular classroom classes.

Delabona (2016, p. 66), on the one hand, considers the importance of planning: "We understand that the satisfactory development of a class and the discussions promoted there are better associated with good class planning". For his research, he prepared curriculum-based tests (assessment of prerequisites) and offered workshops in the environment of the mathematics teaching laboratory (Laboratório de Ensino da Matemática – LEM). The planning was based on the mathematical content to be taught and the characterization of the research subject, a student with Asperger's syndrome. The subject's portrayal included interviews with parents, physicians, and teachers; access to the report; the researcher's view as a teacher; and constitution of social and cultural perspectives. This resulted in a characterization both in relation to the axes of mathematical knowledge (numerical and algebraic thinking, space and

form, magnitudes and measurement, information processing) and personal (cognitive, behavioral, among others).

On the other hand, Viana (2017, p. 90) says, "The characteristic and motivating elements that permeate the path of a didactic situation of mathematics teaching and learning" enables the direction of didactic planning. To the researcher, it is necessary to know the student and the "characteristic and motivating elements" that direct their dealing with the didactic situation. According to the author:

> [...] we also observed that sometimes the characteristic and motivating elements that the student with autism presents in a didactic situation leave the school conventionality zone, that is, the territory of doing and communicating mostly accepted by the school community. It is not plausible that the school community requires students with autism a doing or communicating that set them apart from the characteristic and motivating elements that govern their dealing with the didactic situation, such as calculating using algorithms written on paper, verbalizing a response, or even copying a transcribed definition on the classroom board. (Viana, 2017, p. 90)

For planning the activities, or sessions, focused on his research, Flôres (2018, p. 91) considered the content and "the condition of the student with ASD". The author was based on scientifically based recommendations for this public to determine this condition, such as a center of interest.

In Francisco's work (2018), the teaching activities were carried out with his research subject, a student with ASD, attending regular classes with classmates. To meet the learning specificities of his subject, the author describes, "in the planning of activities, [...] the making of short and objective statements was made available, the use of figural representations [...] and the incorporation of tools in the virtual format to arouse students' interest" (p. 36).

When mentioning the article by Chequetto and Gonçalves (2015), Nascimento (2020, p. 80) defines planning as a collaborative work, highlighting the characteristic the authors attributed to it as the one that "into account the specificities of the autistic student and that it is elaborated together with the teachers of the specific areas and the Specialized Educational Service".

Takinaga (2015) points out that the planning and structuring of teaching activities must take into account the characteristics that impair this group from learning mathematics – generalizing knowledge; building abstract thinking; poor concentration; among others – and the proper use of materials and resources, the organization of the environment where learning takes place and the way of systematizing the content.

In this sense, investigations point to the fundamental role of task planning for the encounter of the student with ASD with mathematical knowledge, the former being strongly influenced by the choices made by teachers and their understanding of mathematical knowledge. To Viana and Manrique (2020), the way the teacher understands mathematical knowledge influences the articulation between the planning and the selection of resources for didactic work with students with ASD.

In the next topic, we define task planning from our theoretical lens, *task design*, and its relationship with the encounter with historical and cultural knowledge.

Theory of objectification and task design

Our theoretical lens was based on assumptions of the theory of objectification (TO), idealized by Luis Radford. Such assumptions refer to one of the methodological aspects of the theory associated with *activity task design,* or *the teacher's didactic project.*

TO fits into the contemporary sociocultural theories that are based on sociocultural theories developed in sociology and anthropology. Its role, on the one hand, is to move away from individualistic and transmissive theories of knowledge and, on the other hand, to conceive the classroom as a space for social transformation and not as a reproduction of society. It is inspired by Freire's concept of education (1987) and Vygotsky's cultural-historical theory, having dialectical materialism as a common point.

TO's contribution to the research was provided by providing a guiding model based on its assumptions of analysis categories – *general considerations, considerations related to mathematical problems, and specific considerations of human collaboration* – which allowed identifying aspects of teachers' task planning – *task design of the activity* – that enhances the encounter of the student with ASD with mathematical knowledge during the *activity*.

Based on the dialectical-materialist philosophy, the key concept of learning and all of TO is the *activity*. In TO, the concept of *activity* is distinct from "doing things", referring, in the educational context, to the joint work of teachers and students to achieve a common object. This joint work is called in the theory of objectification *joint labor*. The encounter with cultural-historical knowledge – in this case, mathematical knowledge – takes place in the *activity*.

In this sense, teaching and learning are conceived as a single process, *teaching-learning*. This does not imply that teachers and students do not play different roles, but in the context of the *activity,* conceived in TO as *joint labor*, both "engage together, intellectually and emotionally, for the production of what we call a common work" (Radford, 2017, p. 252).

From the TO perspective, a *teaching-learning activity* has an *object*. For the *activity* to develop towards its *object*, one or more *goals* may be identified. To reach the *goals* of the *teaching-learning activity*, one specific *task* may be conceived. (Radford, 2021, p. 123).

We emphasize that the *task*, from the perspective of our theoretical lens, although it may affect the *teaching-learning activity*, is not deterministic. According to Radford (2021, p. 126):

> The dialectical nature of the activity can be better understood if we keep in mind that an activity is a process located in space and time that, although affected by the didactic project, cannot be previously determined. Teachers and researchers may have an idea, but the process is neither mechanical nor deterministic. How the activity takes place will depend on how students and teachers engage in it, how they respond to each other, and their dynamic relationships with knowledge in general and with institutions, etc.

To understand the contributions to our research proposal, we must observe the position occupied by the task in *the structure of the teaching-learning activity*. The structure is represented by the Greek letter Φ (*phi*).

Figure 1 presents the *activity* structure.

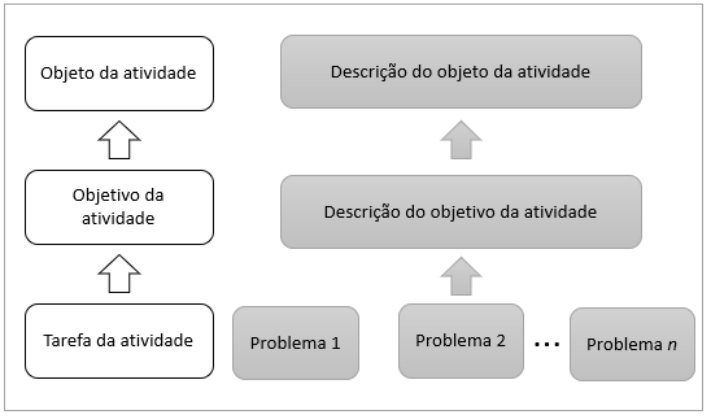

Figure 1.

Activity structure (Radford, 2021, p.25)

The arrows represented in the *activity* structure indicate alignment and not direction; the *task* is aligned with the *goals*, which, in turn, are in line with the *object*. In practice, usually, the object of the activity is defined, then the goal (objective), and finally, the task with its various problems (Radford, 2021, p. 125).

The position the *task* occupies in the structure of the *teaching-learning activity* Φ aligns it with the *activity*'s objectives. This point of view guided us in the analysis of the *tasks* elaborated by our research subject. In this sense, investigating what the teacher considers in planning *tasks* for them to reach the objectives he considered and related to the mathematical object in question, we sought to identify aspects that would enhance the possibility of the

student with ASD encountering mathematical knowledge. For this purpose, we analyzed the *tasks*, contextualized as *the teacher's didactic project*, based on the TO guidelines to design the *teaching-learning activity* considering its structure – *task design*[3].

According to Radford (2021, p. 173), all *teaching-learning activity* is associated with a *task*, which consists of a series of problems that students are asked to deal with. The author says that, in this context, three important questions arise:

> 1. What guidelines should be considered when choosing the mathematical problems students will be encouraged to solve?
> 2. In what order should the problems be presented to students?
> However, as we cannot think about the mathematical dimension without also thinking about the social dimension, we need to ask another question:
> 3. What are the forms of human collaboration that can help students deal with these problems? (Radford, 2021, pp. 173-174).

Based on these determinations, we prepared Table 1, containing a summary of the TO guidelines that answer the questions raised; therefore, they are related to the *task design*. The TO elements for *task design* were considered as categories of analysis for our investigation.

Table 1.

TO elements for task design (Radford, 2021, pp. 174-177)

ELEMENT	OBJECTIVE	GUIDELINES
(1) General Considerations	a. Take into account what students already know; b. Involve, as far as possible, the use of artifacts (concrete, technological, etc.).	▪ Consider prior knowledge (familiar mathematical knowledge) so that students can "immerse" themselves in the activity.
(2) Considerations about mathematical problems	c. Be interesting from students' point of view; d. Offer students opportunities to engage with mathematical knowledge at deep levels of conceptualization; e. Be organized according to a conceptual and contextual unit;	▪ Assignments should, as much as possible, arouse students' interest; ▪ Seek to conceive mathematical problems in a playful or narrative context in which individual actions motivate mathematical reflection;

[3]The term *task configuration* has been used, but it was replaced by *task design* to adapt to a terminology widely used in mathematics education and to separate a term that expresses the idea of a technical problem. (Radford, 2021).

	f. Having an increasing conceptual complexity.	▪ To promote a progressive encounter with cultural-historical knowledge, task planning should include three different *conceptualization levels;* ▪ Problems are organized according to increasing conceptual complexity; ▪ Resort to a conceptual and contextual unit.
(3) Considerations about specific forms of human collaboration	g. Encourage reflections and criticism; h. Provide strong interaction between students and between teacher and students.	▪ Foster forms and means of interaction that promote critical positions, solidarity, responsibility and care for others; ▪ Contribute to the creation of a space for reflection to promote *joint labor*.

We highlight below the *conceptualization levels* included in planning tasks with the proposal to promote the progressive encounter with cultural-historical knowledge. "It should be noted that these levels of conceptualization do not imply a strict order. On the contrary, they can overlap or even appear simultaneously during the teaching-learning activity of students and teachers" (Radford, 2021, p. 176).

- The first level is associated with a *concrete sensory experience*, that is, experimentation and reflection through the use of concrete materials;
- The second level of conceptualization involves a *theoretical reflection* based on the use of concrete objects that could enhance possible emergent links that give meaning to mathematical objects;
- The third level of conceptualization appears with the *manipulation of mathematical symbols* with which students elevate previous experience (sensory, concrete experience) to *another level of consciousness*.

Organization of research data

We designed a *task* named "Introduction to the addition operation", taking as reference the data provided by the teacher of the Specialized Educational Service (AEE, in the Portuguese

acronym), to introduce the addition operation and as object the addition operation with natural numbers.

The tasks were planned to be developed in a multifunctional resource room (sala de recursos multifuncionais - SRM) for the AEE for students with ASD in the ninth grade of elementary school, other environments (library, stockroom) and external environment (bakery).

The participating student has a moderate degree of autism, is verbal, is in the literacy process, reads with stick-type words represented in capital letters, and has difficulty interpreting what he reads and maintaining attention and focus, which prevents him from completing the proposed tasks. He has good social interaction and is willing to perform the suggested tasks. The contents addressed are equivalent to the first year of elementary school (initial years). His main difficulties in mathematics refer to maintaining focus and attention, interpreting and understanding abstract terms and concepts, and memorizing.

As these are tasks developed within the scope of the AEE, we believe it is relevant to present aspects considered by the teacher in his planning before the application of the problems that make up the task since, in this environment, differentiated practices are developed.

The teacher thinks that his initial contact with the student with a disability must be based on interactions that allow identifying means and strategies with the aim of establishing a two-way communication, *teacher-student* and *student-teacher*.

> [...] I worked with deaf students, it was my first contact with inclusion as an educator. I remember that it was very difficult to understand what they meant and vice versa, but that period was also very rich, because we learned how to communicate with them. We learn some mathematical terms that lead us to communicate better with them. And I think it all starts from there. From there, you make a lot of effort to understand what the student is trying to convey and, on the other hand, what you want to convey, what you want to pass on to him, to have this exchange. I think this is the beginning of everything, and then you see what happens to all the others, the other students, regardless of the question. (Teacher).

Regarding the differences presented by people with ASD, when considering their classification along a spectrum, the teacher highlights the need to develop tasks that meet these characteristics, but suggests the possibility of adopting some standards in preparing teaching tasks. In this sense, students' prior knowledge should be the starting point for expanding the their repertoire.

> The autistic spectrum disorder has varying degrees. ASD comprises various degrees of autism, so sometimes an activity that makes sense for student x doesn't make sense for student y. So there is this nuance of adopting some standard characteristics in the activities, for example, using capital letters. Using capital letters facilitates the autistic

> person's reading; some can read with normal letters, others find it easier to read capital letters. So you adopt writing with capital letters, using short, simple, and objective statements. The idea is not to confuse, the idea is for the student to interpret, to be able to read and understand what they are being asked, or what is happening. We are adopting this kind of thing, always bringing more to the visual, always bringing to what they already know, and, from there, expand their knowledge of the world. You can evaluate the things they like, of course, you will start with what they like, but you expand, you bring something new and you try [...]. This issue of bringing the student's knowledge and expanding it, I think it works, that breaks boundaries. I'm not just talking about inclusion students, but any student, any student. So, we assume this type of pattern and, of course, we have to know our student, who is the person we are dealing with, who we are talking about, with whom we are working, so that we can think of a more specific activity [task] for them. Well, this is the pattern I learned and brought here. Anyway, so, I think this is a bit of my education [...]. We learn a lot of new things every day, it's an exchange: at the same time that we are conveying something to them, we also learn a lot from them. (Teacher).

We should clarify that the task design presented was prepared for a student enrolled in the ninth grade, but complies with content from the early years of elementary school. A diagnosis carried out by the institution identified an incompatibility with the school stage and its performance, which revealed the need for curriculum flexibility and re-signification of its purpose from content-driven to utilitarian.

> At the beginning of the year, a diagnostic activity was carried out [...], he did this diagnostic activity to check if he understood sum [addition], quantification, this kind of thing. It turned out that, through the diagnostic activity, we diagnosed that the student still needs some elements related to mathematical literacy. What do you mean by elements in mathematical literacy? The student did not have much notion of quantification or order. He had a very mechanized counting of numbers, 1, 2, 3, 4, 5, but if we asked him which is bigger, 5 or 4, he could guess right because the chance is 50%, but he wouldn't say for sure. So, given that, we made a long-term plan, that plan in which mathematics would enter the student's life, a mathematics for life, a mathematics that accompanies him, for use in life, like going to the market, doing his shopping. Understand, for example, I want two bars of chocolate... think about cost and benefit, I want two bars of chocolate, but this one is on sale, this mathematical logic, like, more of everyday life. So we want him to have that, that he knows how to interpret the world, that's what we want for him, what we think for him. (Teacher).

As this student has a moderate degree of autism, a structured and objective approach is necessary.

> [...] we eventually fall into the idea of a pattern, because the student still needs a pattern, something very mechanized, standardized. His reading is also very mechanized, I think it is because of the issue of the degree of autism being more pronounced, but he is very mechanized. His reading... he reads, if you give him this big text [extensive text] that you see here, he'll read it, but when you ask him something about that text, he doesn't know how to answer because it's too mechanized [...]. (Teacher).

After presenting the factors that precede the tasks but influence their planning, we presented, in Table 2, the sequential order of the problems that make up the task.

Table 2.

The sequence of problems for the task "Introduction to the addition operation"

PROBLEM / ELEMENT	**IMAGE / TEXT**	**DESCRIPTION**
Problem 1/ Action 1	Research Project – "Understanding addition" 1) Interview with teachers, staff, and colleagues about how important addition is.	Describe the meaning of the addition operation through its use in everyday life.
Problem 2/ Action 1	Let's investigate how the operation we're dealing with, addition, works. 2) For that, let's look at the objects we have here at school in everyday situations [in] which addition would help. Find the following objects: – books; – basketballs and soccer balls; – chalks.	Associate the addition with the possibility of solving everyday problems involving objects that can be counted.
Problem 2/ Question 1	3) Analyze the following situations and help your friends solve them. – The librarian needs to separate the books that the sixth-grade class will study. She has already sorted out ten books, but there are still five books to go. How many books does she need in total? – The Physical Education teacher will separate the materials for the class. There are still four basketballs and seven soccer balls to go. How many balls are left in total?	Introduce the operation of adding natural numbers by counting.

	– The math teacher needs four white chalks, three green chalks and two orange chalks. How many chalks does he need in total?	
Problem 2/ Action 2	4) Make a record with photos or drawings of the solution to these situations.	Formalize the record of the addition operation and associate it with the meaning of joining/gathring.
Problem 3/ Question 1	5) Is there another way to register these situations?	Enter the symbolic register of the addition operation.
Problem 3/ Action 1	6) Write using this register.	Represent, using symbolic register, the addition operation and its result.
Problem 4/ Action 1	Project – "Paying for products at the bakery". 7) Let's create a price list with the items you like from the bakery. To do this, take a picture of the products that you like, then, we will create a table with these items and their prices.	Organize information through tabular register.
Problem 4/ Action 2	8) Choose some items from the table to buy at the bakery. Add up their values.	Carry out the operation of adding monetary values and performing rounding.
Problem 4/ Action 3	9) Let's go to the bakery to buy the items you chose. Before that, separate the money needed to pay.	Perform the operation of adding and comparing numbers.

With **Problem 1 / Action 1**, the teacher intends to involve the student in the school environment, taking advantage of spaces and relationships. As a first contact with the addition

operation, the learner will conduct an interview to conceptualize it from its application in the daily life of the interviewees.

> [...] I like that he enjoys the school space, this space is his, I want him to occupy the school space, I want him to feel and recognize the school space as his. I think of situations, as I said at the beginning, [A3] is cute, I propose things to him through which he could help teachers, staff, librarians [...].
> [...] this interview activity [...]. He will have to ask teachers, classmates, what is the importance of the sum [addition]. We are already bringing this word sum [addition] and associating it with joining/putting together. "What is this, this sum [addition] thing that is showing up in my life?" "What do other people think of it?" [...]. (Teacher)

In **Problem 2 / Action 1**, the student will separate the objects necessary to carry out the sequence of problems. For students with ASD, breaks are suggested, so that, at these times, they can perform tasks of interest. These intervals help to improve their attention and focus.

> "How many books did you have to bring?", then he draws the books. "Don't you want to improve?", so he drew "Boitatá", he loves reading this book about Brazilian legends. We also have a deal, because here we have the class period of a regular student, I believe that this time for them is excessive, keeping focused for this long period is difficult, so we have the option, in the last 10 minutes of class he can read a book, he can take a break, he can have a little longer break so he can focus a little more on the activity while he is doing it. (Teacher)

In **Problem 2 / Question 1**, the proposal is to associate the operation of addition with the resolution of problems involving counting and the operation of addition with the meaning of joining/putting together. The teacher suggests questioning the learner during the participation in tasks to lead him to the conclusion of the task, directing him to the proposed objectives.

> [...] First, it is separated, then everything is put together, and there was the last one, that was chalk. "Let's help the [math] teacher? "How many white chalks do you have?" He counts everything together, I say: "No, separate it. Count how many whites there are". He answers "Four", but there were three, then he noticed, he noticed by himself and then he took the pencil and drew the fourth. "Cool!", "How many greens are there?", "Did it hit? Yes, it did!" And the orange one had two, then he counted one, two, three, and then he goes back here, back there, always with his finger, he needs to touch, so he has to erase two. "How do you erase it?". I'm always questioning him, so he doesn't get lost [...]. This is the task, already thinking about the sum [addition], but sum [addition] without saying it is sum [addition]. The idea of joining/putting together, this idea of joining, I've been working with him for a long time. (Teacher)

As **Problem 2 / Action 2,** the teacher guides the student to register informally. For the student with ASD, registering can contribute to the focus and interpretation of the situations experienced.

> [...] we are now with the issue of registers. He must register. As I said, he [student] has a very mechanized reading, he interprets little.
> [...] It helps him interpret and helps him think about what's going on, because of the focus issue and all that. His autism is very strong, so, in this very question of the research project, he must register. (Teacher)

With **Problem 3 / Question 1** and **Problem 3 / Action 1,** the teacher suggests that the student considers other ways of registering the additions performed in the previous problem. In this context, mathematical language is presented as an alternative to informal registration.

> With this question, I want him to know that there are different ways to represent the addition operation. In this case, the mathematical form. In the interview, he already came across this representation, now I'm making him learn this particular language of mathematics to register operations. There is a symbol for that, and he needs to know it. (Teacher)

With **Problem 4 / Action 1,** the suggestion is to make a table with products that the student usually consumes at the bakery to register their prices. The actions required in problem 4 aim to immerse the apprentice in the mathematical object defined for the task design and correct behaviors that are not associated with appropriate conduct in a given environment.

> [...] when he went to the bakery, he used to simply grabbed his things and wanted to leave, so he gave his wallet to the girl at the cashier, cut in line... So this issue of him perceiving the environment where he is: there is a queue, so he must stand in line and wait for his turn. What will I take? What do I want to buy? There's a table I made with him with the bakery products he likes. (Teacher)

The proposal of **Problem 4 / Action 2** is to perform the addition of monetary values with decimal places making rounding, because the intention is to manipulate natural numbers.

> [...] Then mathematics comes in and, of course, there are products of 19.50 and the [student] doesn't know decimals, but then I say to him: "[Student], 19.50, let's round it up to 20.00 or 19.00". The important thing is that the number is integer or natural. (Teacher).

With **Problem 4 / Action 3**, the professor suggests, as an initial approach to adding monetary values, associating the addition operation with the idea of putting together, without using the addition algorithm to refer to the amount, based only on the bills/checks.

> The potato [...] cost 20.00, [the soda] cost 4.00. "Cool, how much is that, [Student]? Let's get the bills/checks together." Because we want him to understand that he must pay this price to have this product, so here, in this first moment, I work with these junctions that are more immediate, 20.00 and 2.00; 20.00 and 5.00. 30.00 is no longer

> so immediate, 20.00 with 10.00 does not come out so immediately from our mouths. 50.00 and 2.00, it is the same thing when you add the bills together, putting together a 50.00 bill, joining a 2.00 bill: 52.00. (Teacher).

In the next topic, we move on to the analysis of data referring to the design of the task "Introduction to the addition operation" in the light of the TO, which allowed us to identify aspects of the teacher's planning that enhanced the possibility of the student with ASD encountering mathematical knowledge.

Analysis categories and identification of planning aspects

The mathematical object of the task design "Introduction to the addition operation" is the operation of addition with natural numbers, and its objective is to introduce the addition. The task includes a sequence of four problems planned to be performed in the Multifunctional Resource Room (MRS), in different school environments (library and storeroom) and in an external environment (bakery). The teacher, the student with ASD and the people involved in the dynamics proposed in the different environments will participate in the task.

Regarding the category **(1) general considerations**, in his planning, the teacher bears in mind the student's prior knowledge that emerges from experienced situations in his daily lives: collaboration with teachers and professionals at the school and the habit of going to the bakery. Through experienced situations in these environments, we seek to enhance the possibility of meeting the student with ASD with the mathematical object during the teaching-learning activity.

The suggested artifacts are aligned with the objectives and object of the task. The objects intended for the tasks to be developed in the school environment and in the bakery prove to be adequate to enable, through their manipulation, the student's engagement in carrying out the sequence of problems proposed in the task.

We can consider the teacher's choices as a proposal for curriculum adaptation, since his choices are based on the utilitarian and non-formal presentation of mathematics. About the detailed and interactive adaptations, Nascimento (2020, p. 151) says:

> These changes were made taking into account the student's specificities and the teacher's diagnosis of the contents the student had acquired in the grades before. In the activities, the changes occurred in the elaboration of more specific activities. In general, the adaptations are effective in autistic students' mathematics learning.

Through the recommendations of the category **(2) considerations on mathematical problems**, there is a conceptual and contextual unity in the sequence of problems created by

the teacher, which combines the suggested collaborative work with everyday situations presented by different environments, where it is possible to explore the mathematical object and its initial ideas regarding the addition operation.

The sequence of problems and their elements, actions, and questions suggest an organization according to an increasing conceptual complexity. The use of artifacts chosen by the teacher enhances the possibility of encountering increasingly elaborate concepts about the object.

In this regard, the forms the teacher proposed to enable a growing conceptual complexity about the object and objectives defined for the task are developed from proposals that explore the idea of presenting different registers of the addition operation, including the register in mathematical language.

Figure 2 summarizes the curve of increasing conceptual complexity of the different actions and questions that make up problems 1, 2, 3, and 4 of the task design "Introduction to the addition operation".

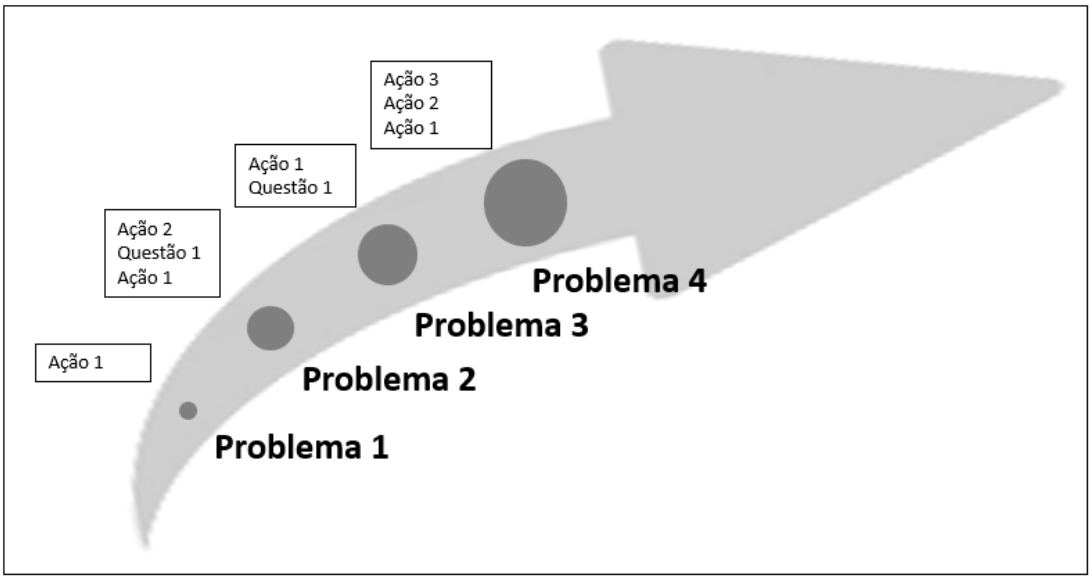

Figure 2.

Task design, based on conceptual and contextual unity in increasing conceptual complexity

As for the levels of conceptualization, we understand that the privileged level in the sequences of problems was the *concrete sensory experience*, since the use of artifacts aimed to bring out the idea of counting and joining/putting together embedded into the addition operation without having been explored through algorithms.

About the **(3) Considerations about specific forms of human collaboration**, the strategy planned to explore different environments, school and bakery, promotes interactions and the creation of situations that facilitate reflection and responsibility, in addition to favouring the student's encounter with mathematical knowledge.

The teacher considers the characteristic of the students with ASD in planning, both through the available artifacts and their practice and organization of the classroom environment.

Table 3 presents a summary of the aspects of the teacher's planning related to the student with ASD that can enhance the encounter with mathematical knowledge in *teaching-learning activities*, identified through the categories of analysis.

Table 3.

Aspects of planning related to the student with ASD

CATEGORY	PLANNING ASPECTS
(1) General Considerations	- Consider previous knowledge that emerges from situations experienced in their daily lives; - Use of artifacts that are aligned with the object and objective(s) of the task.
(2) Considerations on mathematical problems	- Explore contexts that allow the student to encounter mathematical knowledge in a utilitarian way[4]; - Plan structured and objectively presented tasks; - Alternate tasks with specific objectives with tasks of interest to the student; - Encourage different forms of registers, even if informal.
(3) Considerations about specific forms of human collaboration	- Interact with the student to develop strategies that make it possible to establish two-way communication: teacher-student, student-teacher; - Explore different physical environments, providing coexistence with appropriate rules and behaviors and the generalization of school knowledge.

Based on the analyses carried out, we understand that the task design "Introduction to the addition operation" brings aspects that enhance the possibility, at the time of the activity of *teaching-learning*, of the encounter with the mathematical object, albeit in a concrete way, defined for this task in a collaborative and critical way.

[4] With practical application.

Final considerations

The theoretical lens that enabled us to dialogue with the data took place through assumptions of the theory of objectification (TO), idealized by Luis Radford. Such assumptions refer to one of the methodological aspects of the theory associated with *task design* of the teaching-learning activity the teacher presents to the students.

TO's contribution was to deliver a guiding model of analysis categories based on its assumptions – *general considerations, considerations relating to mathematical problems, and specific considerations of human collaboration* – which allowed us to identify aspects of the teacher's task planning – *activity task design*. Such an approach enhances the possibility of the student with ASD encountering mathematical knowledge during the teaching-learning activity.

Remember that these aspects are translated as possibilities, as they are actions thought from the teacher's planning. Only in the teaching-learning activity will these actions be tested and validated.

To present the data, we adopted the following standard to organize the task design:

I. Presentation of the descriptive elements of the task [mathematical object, objective(s), description and organization of the environment, prerequisite(s), main characteristics of the student(s) with ASD and resource(s)], organized in chart format;

II. Sequential presentation of problems;

III. Teacher considerations;

IV. Analyses guided by fundamental categories in our theoretical lens: *general considerations, considerations on mathematical problems and specific forms of human collaboration*, and based on our literature review.

The presented structure allowed the dialogue between the data collected and the theoretical assumptions guiding our analyses.

In conclusion, the methodological path adopted, and its guidelines, allowed identifying elements that contribute to the development of mathematical skills in students with autistic spectrum disorder, since its categories of analysis make it possible to consider conceptual and contextual aspects that promote the development of such skills.

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