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The math class and the challenges of inclusion: Teaching statistical variables and frequency distribution to a blind student

La clase de matemáticas y los desafíos de la inclusión: Enseñanza de variables estadísticas y distribución de frecuencias a un estudiante ciego

Le cours de mathématiques et les défis de l'inclusion : enseigner les variables statistiques et la distribution de fréquence à un élève aveugle

A aula de matemática e os desafios da inclusão: O ensino de variáveis estatísticas e distribuição de frequências para um estudante cego

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Abstract

Didactic transposition is the phenomenon that describes the trajectory followed by knowledge from its scientific creation, until it becomes an object of teaching in schools. This work is an excerpt from the first author's master's research. The main objective of this article was to analyze the mathematical objects taught to a blind student enrolled in a regular classroom. Thus, we sought evidence of internal didactic transposition (IDT) while teaching the concepts of statistical variables and frequency distribution in a 9th-grade classroom, with the inclusion of a blind student. This research included the class teacher –with a degree in mathematics–, the sighted students, and a blind student. We found that gaps in the mathematical knowledge taught to the blind student cause difficulties in his learning compared to the mathematical objects taught to sighted students. Mathematics teacher education must discuss pedagogical procedures that facilitate the teaching-learning process with blind students.

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Keywords: Student with visual impairment, Distance between knowledge, Internal didactic transposition.

Resumen

La Transposición Didáctica es un fenómeno que describe la trayectoria recorrida por um conocimiento o saber, desde su creación científica hasta su devenir en un objeto de enseñanza en las escuelas. Este trabajo es un extracto de la investigación de maestría del primer autor. el objetivo principal del presente artículo es el de analizar los objetos matemáticos enseñados para un estudiante ciego, matriculado en un salón de clases de condiciones regulares. De esta manera, buscamos evidencias de la Transposición Didáctica en un salón de clases de 9º año con la inclusión de un estudiante ciego. Participaron en esta investigación, el profesor de la clase, con licenciatura en Matemática, el estudiante ciego y los estudiantes videntes. Constatamos que existen, lagunas en el conocimiento matemático enseñado para el alumno ciego que acarrean dificultades en su aprendizaje, en relación a los objetos matemáticos enseñados para los alumnos videntes. Las formaciones de docentes de matemáticas necesitan discutir procedimientos pedagógicos que faciliten el proceso de enseñanza-aprendizaje con estudiantes ciegos.

Palabras clave: Alumno con Discapacidad Visual, Distancia de Saberes, Transposición Didáctica Interna.

Résumé

La transposition didactique est le phénomène qui décrit la trajectoire suivie par la connaissance, depuis sa création scientifique jusqu'à ce qu'elle devienne un objet d'enseignement dans les écoles. Cet ouvrage est un extrait des recherches de maîtrise du premier auteur. L'objectif principal de cet article était d'analyser les objets mathématiques enseignés à un élève aveugle, inscrit dans une classe ordinaire. Ainsi, nous avons recherché des preuves de transposition didactique interne (TDI) lors de l'enseignement des concepts de variables statistiques et de distribution de fréquence, dans une classe de 9e année, avec l'inclusion d'un élève aveugle. Le professeur titulaire, diplômé en mathématiques, l'élève aveugle et les élèves voyants ont participé à cette recherche. Nous avons constaté qu'il existe des lacunes dans les connaissances mathématiques enseignées aux élèves aveugles qui entraînent des difficultés dans leur apprentissage, par rapport aux objets mathématiques enseignés aux élèves voyants. La formation des enseignants de mathématiques doit aborder les procédures pédagogiques qui facilitent le processus d'enseignement-apprentissage avec les étudiants aveugles.

Mots-clés : Étudiant déficient visuel, Distance des savoirs, Transposition didactique interne.

Resumo

A transposição didática é o fenômeno que descreve a trajetória percorrida por um saber, desde sua criação científica, até se transformar em objeto de ensino nas escolas. O presente trabalho é um recorte da pesquisa de mestrado do primeiro autor. O objetivo principal do presente artigo foi analisar os objetos matemáticos ensinados para um estudante cego, matriculado em uma sala de aula regular. Assim, buscamos evidências da transposição didática interna (TDI) durante o ensino dos conceitos de variáveis estatísticas e distribuição de frequências, em uma sala de aula do 9º ano, com a inclusão de um estudante cego. Participaram desta pesquisa, o professor da turma, com licenciatura em matemática, o estudante cego e os estudantes videntes. Constatamos que existe, lacunas no saber matemático ensinado para o aluno cego que acarretam dificuldades em sua aprendizagem, em relação aos objetos matemáticos ensinados para os alunos videntes. As formações dos professores de matemática precisam discutir os procedimentos pedagógicos que facilitam o processo de ensino-aprendizagem com estudantes cegos.

Palavras-chave: Aluno com deficiência visual, Distância entre saberes, Transposição didática interna.

People with disabilities have been stigmatized, segregated, or excluded from social life throughout human history. However, the United Nations (UN), with the participation of several countries, drafted the Salamanca Declaration (1994), which ensures education for people with sensory, intellectual, or physical disabilities must be integrated into the educational system.

The right to education is fundamental for all people, regardless of ethnic-racial issues, religion, social status, and whether they have a disability. The Federal Constitution of 1988, Article 205, guarantees this right. It says, "(...) It is the duty of the State and the family; it will be promoted and encouraged with the collaboration of society, aiming at the full development of the person and their preparation for the exercise of citizenship and qualifications for work" (Brasil, 1988). In line with this, Article 206, Section I of the Constitution establishes that the education provided must be based on the principles of equal conditions for access and permanence at school. Article 208 points out that specialized educational services (SES) should preferably be provided in the regular education network (Brasil, 1988).

The 1988 Federal Constitution represented a significant milestone in Brazilian education. From this document, the legislation established for education began to be regularized with legal instruments that guarantee the effectiveness of the right to schooling. The established laws referred to all education segments but, specifically in this work, we bring this discussion to the field of special education from an inclusive perspective.

We emphasize that the history of education regarding the schooling of people with disabilities began to be addressed in Brazil in the 16th century. However, at that time and for many years, the "education model" offered to people with disabilities was restricted to custodial care or clinical care, which were impossible to compare with a schooling model (Mendes, 2006).

Fortunately, the right to education for people with disabilities in regular teaching spaces was recognized and made positive after struggles by social movements involving family members, teachers, and other education professionals. These movements focused mainly on the process of school inclusion of people with some type of disability, autism spectrum disorder (ASD), and high abilities. Thus, since the 1990s, this process has been gaining ground in society. As Fernandes and Healy (2010) say, the process of school inclusion is, for educators, a revisiting of their conceptions and beliefs regarding the very notion of diversity since coexistence is present in the school environment, at work, and in life in society.

Regarding Brazilian legislation for those rights, we will highlight the Education Guidelines and Bases Law (Lei de Diretrizes e Bases da Educação - LDB) n. 9.394 of December 20, 1996 and the Brazilian Inclusion Law (Lei Brasileira de Inclusão - LBI) n. 13.146 of July 6, 2015, both guaranteeing the permanence, equality, and accessibility of people with disabilities in regular schools, as well as social inclusion and the exercise of citizenship in all societal spaces. However, even in the face of all legal support for the right to education to be effective, we know that legislation alone cannot guarantee the principles of equality and equity in the teaching of all students enrolled in Brazilian public institutions. Other factors can influence, such as the paths taken by all educators' professional education and the commitment to educating.

Under these conditions, since the right to education is mandatory, it is important to discuss the assurance of equality and equity in the teaching-learning process for blind students in regular schools. In this way, we seek to identify whether there is a gap between the knowledge taught to blind students and the knowledge taught to sighted students.

In this context, Fernandes and Healy (2010) discuss actions aimed at the teaching practices of teachers who teach students with disabilities and identify that the greatest uncertainties of these educators are precisely the pedagogical practices for them. According to the authors, "(...) our conceptions do not always find support in everyday practices and institutional devices. Based on inclusion policies, the educational community must be prepared to receive these students" (Fernandes & Healy, 2010, p. 1113).

Therefore, it has become urgent and necessary to search for new teaching methodologies and strategies that enable the understanding of objects of knowledge by students served by special education from an inclusive perspective and have become participating subjects in regular schools. This search for new teaching practices arises due to the differences and specificities of students, who need adequate conditions for their learning. For Vygotsky (1997), the lack of one of the senses in people with sensory disabilities is not an obstacle to the school learning process, but the use of inappropriate forms of teaching is always a barrier to this process.

Due to Brazilian educational policies focusing on including people with disabilities in regular schools, we sought to determine whether those principles of equality and equity were being assured. In view of this, we highlight Fernandes (2004), Fernandes and Healy (2010), Souza (2014), and Marcelly (2015), who discuss, in their studies, pedagogical practices that enable the development of skills for students with visual impairment at regular schools. The teaching-learning process for students with visual impairment, among other factors, depends on activities that enable teaching through touch and hearing.

Intending to understand the teaching work during the presentation of a specific mathematical topic in an inclusive classroom with sighted and blind students, we delve deeper into the following authors' works: Yves Chevallard (1991), a pioneer in studies of the phenomenon of didactic transposition, and researchers Brito de Menezes (2006) and Araujo (2009), who point out in their investigations discussions about the distance between the knowledge to teach present in the texts of textbooks and the knowledge actually taught by the teacher in the classroom. In their research, those authors focus on the interactions built and developed between the teacher-student-knowledge. This triad is called a didactic system (Brousseau, 1986) and allows studies on the didactic relationships established between these three elements that make up that system. The didactic phenomena related to mathematics teaching emerge from this relationship built between the triad (Brito de Menezes, 2006).

We will now deal with the phenomenon of didactic transposition and the structuring of possible didactic systems (subsystems) based on the inclusion of a blind student in a regular classroom, and the use of ostensive objects chosen by the teacher to transpose knowledge, such as "statistical variables and frequency distribution." We will also address teacher education from the perspective of inclusive education and teaching materials recommended for teaching people with visual impairment in the mathematics curriculum component. Next, we will present the methodology, analysis of the data collected, and our final considerations.

Didactic triangulation: The blind student and the new scenario in the classroom

According to Chevallard (1991), knowledge follows a path until it reaches its destination: the classroom. In this trajectory, knowledge undergoes "transformations, which the author called didactic transposition. This phenomenon makes it possible to analyze the path that knowledge takes, from its production in academia to becoming knowledge to teach in class. For Chevallard (1991), didactic transposition has two phases: external didactic transposition (noosphere) and internal didactic transposition.

The first phase consists of transforming scientific knowledge into teaching knowledge, called external didactic transposition, also identified as noosphere. It comprises people and institutions that define and organize programs and curricula for education systems, i.e., those responsible for dictating the knowledge that should be taught at school. The work of the noosphere can be observed in the National Common Curriculum Base [Base Nacional Comum Curricular] (Brasil, 2018) and the guiding document for the new secondary education (Brasil, 2023). The second phase, internal didactic transposition, refers to the teaching work carried out in the classroom, which results from the didactic relationships that develop between teacher, student, and knowledge, understanding their reflections, questions, and practice. In the phase of internal didactic transposition, during the presentation of the objects of knowledge to be studied, the teacher develops conscious and unconscious actions, and what the teacher prepares in their lesson plan is not always what they actually impart (Brito de Menezes, 2006).

Menezes and Santos (2018) discuss the knowledge effectively taught at school, raising questions about a possible gap between the knowledge taught to hearing people and the knowledge taught to deaf students in an inclusive classroom, in which we have the collaboration of a Libras interpreter translator. For the authors above, including a Libras interpreter translator in school activities directly changes the knowledge to teach to deaf students.

In this same perspective of modifications of knowledge for a particular audience, based on their specific characteristics, and thinking about the specificities of students with visual impairment included in regular classes, we ask: Is there a distance between the knowledge taught to blind students and the knowledge taught to sighted students? How is the didactic relationship between teachers and students developing in mathematics classes with the inclusion of a blind student? Does the inclusion of the blind student change the didactic scenario of the regular classroom?

To understand the phenomenon of internal didactic transposition in the classroom, we went deeper into the formation of didactic systems. According to Brousseau (1996), the formation of the didactic system consists of three elements: the teacher, the student, and knowledge. This triad establishes a didactic relationship that considers the interactions developed between those elements. In these systems, the knowledge to teach transforms into knowledge taught.

The didactic system can be represented by a triangular scheme, in which each vertex is represented by one of the elements. The sides of the triangle represent the relationships built between the elements. Therefore, the interactions in this system are related to several external factors, which is why they are considered dynamic and complex relationships. This triangular scheme is not equilateral, and even if our representations arose from this, a didactic situation does not occur in an equilateral way since it is not possible due to the relationships established between the elements. For example, the relationship established between teacher x knowledge differs from that between student x knowledge.

Next, based on our research, we present the formation of three possible teaching systems (subsystems) observed in a regular class with a blind student. The first subsystem, SD1, is the didactic system formed by the teacher, all the students in the room (sighted and blind), and specific mathematical knowledge. The second subsystem, SD2, is the didactic system formed by the teacher, mathematical knowledge, and the blind student. The third didactic system, SD3, is standard in class and is formed by the teacher, the mathematical knowledge, and the sighted students. Below, we have representations of the possible didactic systems involved in an inclusive class:

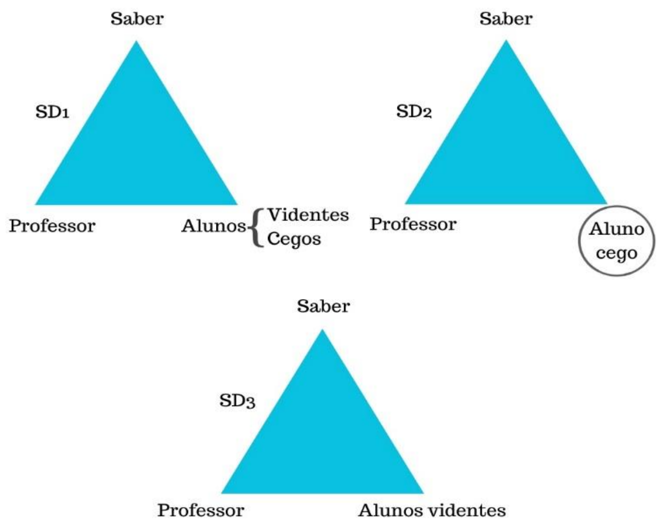

Figure 1.

Regular class (Research data)

According to Araújo (2009, p.30), "This didactic system is an open system whose survival depends on its compatibility with the environment in which it operates; that is, it must respond to the demands that accompany and justify the social project in force." As it is an open system, the environment implies the relationships built in it. The dynamics present in teaching systems are reflections of the conflicts built in the development of those relationships. This occurs due to the presence of human elements, teachers, and students, who add their subjectivities to those relationships, which end up interfering in the changes made to the mathematical knowledge present in this didactic game, i.e., in the dynamics of didactic systems.

Brito de Menezes (2006) states that teachers do not faithfully translate the text of the textbook for their students. When seeking strategies and methodologies so that this knowledge becomes understandable by students, the teacher transforms and rewrites this knowledge to teach, adding their points of view, their relationship with this object of knowledge, and their conceptions and subjectivities. To Chevallard (1991), the teacher builds a metatext from the didactic text to teach, and changes in knowledge are made spontaneously without the teacher noticing them, demonstrating that during the internal didactic transposition phenomenon, the teacher performs conscious and unconscious actions.

Given the interactions in the didactic system, we can find signs of a gap between the knowledge taught to blind students and the knowledge taught to sighted students. Due to the context of this research, we used ostensive and non-ostensive objects to classify the teacher's teaching methodology. We intend to verify whether the methodology adopted by the teacher in the observed classroom makes teaching-learning possible for blind students, which could make the analysis of their teaching practice viable.

Non-ostensive and ostensive objects used during teaching practice

According to Bosch and Chevallard (1999), ostensive objects are perceptible and manipulable by human subjects, have material characteristics, and can be felt or heard. For example, voice, writing, gestures, graphics, and notations. Non-ostensive objects do not have these characteristics, they cannot be heard, touched, or manipulated. We can only represent or invoke ideas or mathematical concepts by manipulating specific ostensive objects. In this way, in the mathematical activity, the non-ostensive objects are associated with ostensive objects. Thus, "(...) the study of ostensive and non-ostensive is essential for understanding mathematical activity" (Bittar, 2017, p. 368).

How to solve a mathematical activity is the teacher's choice. Those chosen forms for activity resolutions are not independent. Those actions are guided and justified by mathematical concepts that maintain an interdependent relationship with the chosen forms of resolution.

We can then understand that those choices to solve a given mathematical activity are guided by non-ostensive objects, but all representation and symbology that are perceived to solve the activity are ostensive objects, "(...) the non-ostensive objects regulate all manipulation of the ostensive ones" (Bosch & Chevallard, 1999, p.11, our translation). As an example, we can cite the expression: $2 + 6 = 8$. In this case, we have a manipulation of ostensive objects that represent quantities; the resolution of this operation is guided by a non-ostensive object specific to this activity, which is the concept of addition.

From this, we understand the relationship between ostensive and non-ostensive objects. While the ostensive ones are perceptible and manipulable, the non-ostensive ones regulate the orientation of this manipulation, justifying and explaining all the actions so that it is possible to solve a mathematical activity. However, even given the dimension of non-ostensive objects,

“(...) we should not ignore the importance and omnipresence of ostensive objects in the study of mathematics” (Kaspary & Bittar, 2013, p.1425).

Teacher education and teaching materials for teaching visually impaired students

When we refer to the inclusion of students with disabilities in regular schools, we ask ourselves about the factors that classify a school as inclusive. We believe that an inclusive school allows everyone access to education that recognizes the differences and abilities of the students involved, offering inclusive activities that enable school development, thus providing education with real opportunities for everyone.

When reflecting on the learning of blind students in regular classes, it is pertinent to reflect on the teaching strategies and methodologies adopted by teachers that favor those students’ learning, because their blind condition does not compromise their cognitive development.

According to the authors Fernandes and Healy (2007, 2010), Silva (2010), and Souza (2014), the academic development of students with visual impairment depends on practices that enable teaching through touch and hearing. These researchers emphasize the importance of using tactile teaching materials in high relief or computer programs that use voice synthesizers, which can collaborate with this public’s teaching and learning.

According to Souza (2014, p.19), “(...) blind people can and should participate in the educational program. For this to actually happen, it is necessary to adapt or build materials that facilitate and significantly enable the teaching and learning process for people with visual impairment”. To discuss the potential of activities and teaching materials adapted for teaching students with visual impairment, we will describe below the materials constructed by the authors mentioned during their investigations. Fernandes and Healy (2010), faced with the challenge of teaching the concepts of area and perimeter, alternatively created low-relief geometric figures for figure recognition through touch.

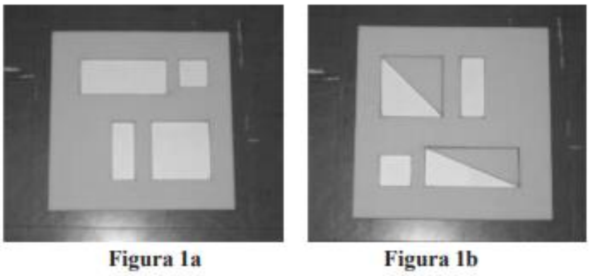

Figure 2.

Board for area and perimeter studies (Fernandes & Healy, 2010, p. 1119)

Silva (2010) created a comic book to present the concept of Thales's theorem. The comic book was developed in Braille, with the illustration of the description of the story settings and the characters' gestural reactions.

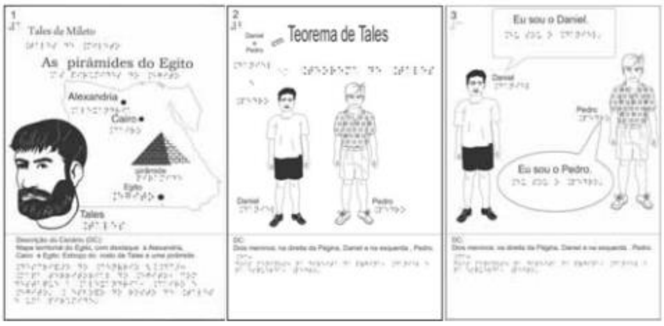

Figura 31 Figura 32 Figura 33

Figure 3.

Adapted comics (Silva, 2010, p. 58)

Souza (2014) used a digital game to study ratio and proportion, the speaking tool mediated the game with students, giving instructions for them to find solutions to the questions. The game presented a positive result; the students constructed strategies that led them to solve the problems involved in the activity.

Figure 4.

Player using the tool (Souza, 2014, p. 69)

The research by Fernandes and Healy (2010), Marcelly (2010), and Souza (2014) is in line with the studies by Santos (2020, p. 60), which state that "the handling of teaching materials during the teaching of mathematical concepts for blind students or those with low vision helps build these mathematical objects in the students' minds."

From reading those studies, we understand that students who interact with the proposed activities have better performance in class, confirming what Fernandes and Healy (2010) say about practical activities for people with visual impairment: that these activities must be developed in a pleasurable and motivating way, so that students are provoked to change their behavior in the classroom, becoming active and participatory subjects in their schooling. In this line of thought, we understand that visually impaired students must be seen and noticed. However, for this to occur, everybody in the classroom must collaborate, i.e., the teacher is not the sole but the main responsible for this didactic interaction.

> The teacher must make them discover that the classroom offers alternative ways of learning, that it is a pleasant space, and that they can feel welcomed and safe to take a participatory stance during classes. However, for this to happen, the classroom must be the space we describe. (Santos, 2020, pp. 60-61).

Thus, we realize the importance of initial and continuing education from an inclusive perspective so that teachers are not "caught by surprise" by the differences in everyday school life. Teachers must be aware of the importance of inclusive methodological practices such as those mentioned previously since such activities contribute to the teaching and learning process of students with visual impairment.

Teacher improvement is necessary to meet the diversity present in the school. Therefore, we emphasize the importance of this requalification in teacher education since it is a viable alternative for effective changes that will enable learning for all students in regular schools.

From this perspective, Santos (2020) developed the educational product called "Soroban: ferramenta didática no ensino de matemática para alunos cegos" [Soroban: a didactic tool in teaching mathematics for blind students], which is a material recommended for teacher education to assisting teachers when teaching mathematical concepts for students with visual impairments involving calculations with the four fundamental operations: addition, subtraction, multiplication, and division. In this educational product, Santos (2020) demonstrates how to use Soroban, then presents three proposals for inclusive didactic sequences, that is, which must be applied to all students in the common regular classroom.

We understand that inclusive practices will only be present in the daily life of a regular school through several paths, among them continuing teacher education from the perspective of inclusive education and studies and dialogues with the school community to identify the educational needs of students with visual impairment. After completing these steps, the mathematics teacher can present adapted activities and materials for teaching mathematics to visually impaired students.

Methodological path

In this research, we chose to work with a qualitative methodology. Since "(...) in qualitative research, the direct source of data is the natural environment, with the researcher being the main instrument (...) the investigation is descriptive, and researchers are more interested in the process than simply the results or products" (Bogdan & Biklen, 1994, p. 50). These properties of qualitative research meet the interests of this investigation, as they point to the natural investigative environment as the direct source for data collection and the researcher as the main instrument of the study in question.

The choice of the school and the setting for this research should meet the following criteria: having students with visual impairment and having projects that favor those students' inclusion process. Thus, we selected as subjects participating in this research a basic education teacher with a degree in mathematics, a blind student, and sighted students from a 9th-grade class from the public school system in the state of Paraíba. Data was produced through participant observation, class videos, voice recording for the interview with the participating teacher, a field diary for notes, and systematic observations of classes.

The analyses were based on ostensive objects the teacher chose to present mathematical objects during the class on statistical variables and frequency distribution. We classify these objects into the following categories: choices, information, and leaps (or lacks). In the choices category, we identified all the teaching resources and nomenclature used by the teacher during the class to work on the same mathematical concept with the blind and the sighted students.

We then classify the information category. In this category, we observe the conceptual information the teacher added or removed so that the students understood the mathematical object presented. This category highlighted the information the teacher considered relevant and irrelevant for teaching the students in this research.

Adding or removing information from knowledge can make possible or jeopardize the understanding of visually impaired students. Therefore, the teacher must be aware of visually impaired students' skills and possible difficulties in assimilating a particular content. To this end, developing the pedagogical strategies to be used before adding or removing certain conceptual information is essential.

Finally, we classified a third category, skips (or absences), comprising the concepts about central tendency measures presented only to sighted students.

Class description and analysis: statistical variables and frequency distribution

Our analyses were based on the use of ostensive objects chosen by the teacher to verbalize the "statistical variables and frequency distribution" knowledge during the formation of the possible didactic systems (subsystems) SD1, SD2, and SD3, of which we found evidence in our research based on the participation of a blind student inserted in a regular classroom.

The SD1 didactic system refers to the composition: teacher, blind/sighted student(s), and specific mathematical knowledge, SD2 is constituted by the triad: teacher, blind student, and knowledge, and the SD3 didactic system comprises the teacher, sighted students, and knowledge. The ostensive objects studied were classified into categories, as outlined in the methodology. Thus, we use the following categories to classify these objects during the internal didactic transposition phenomenon: choices, information, and skips (or lacks).

The data collected and analyzed includes filming of classes, interview recordings, and notes in the field diary of informal conversations on site. We created a descriptive report that presents the results obtained through each of the points studied, sought to observe the presentation of mathematical objects for blind students and sighted students, and discussed the possibilities of new teaching systems (subsystems) based on the inclusion of visually impaired students in the regular classroom. Research on the inclusion of a blind student in mathematics classes shows that "the teacher needs knowledge about the specificities of the disability, foundations, and elements of special education" (Borges et al. 2022, p. 413).

The class described below involves "statistical variables and frequency distribution" mathematical knowledge. It took place on July 17, 2018, lasting 24'57" of effective knowledge verbalization. During the class, the teacher used a textbook called *Vontade de Saber Matemática* [Willingness to Know Mathematics] (Souza & Pataro, 2015). At the beginning of the class, we observed a possible formation of SD3, and this, throughout the class, was more prevalent in relation to the SD1 and SD2 teaching systems. In excerpts from[4] the dialogues of SD3 below, we observed an expository class.[5] We identified the categories of choices and information, which will be discussed as they appear during the class. Let us see the beginning of the class as follows:

> (T) Do you remember the procedure in which we will get an idea of how to organize the table, what is needed to do it, that I have already done the step-by-step with you, doing each thing, the list, then the construction?

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[4] In the transcription of the dialogues, we will represent the teacher with a T, sighted students with an Ss, and the blind student with a Bs. When the teacher refers to the blind student, we will identify him by the fictitious name Jorge. Also, we will be guided during the formation of didactic systems by $SD1$, $SD2$, and $SD3$.

[5]We relate this class to the baldist conception, discussed by Câmara dos Santos (2002), about teaching-learning in mathematics. According to the author, in this teaching model, knowledge is "poured" into the student's head, they know nothing about this new knowledge, it is like an empty bucket, which fills up as the teacher explains the new knowledge. And the student will have "learned everything" when this bucket is completely full. This verification of learning is given based on the assessment grades.

(Ss) Teacher, I understood [how] to put together the table, I just didn't understand the one that has four things more, that has more than one.
(T) Guys, quiet, please! Firstly, we have the information presented in a small table, which will not determine an order, it will bring disorganized data. Let's base ourselves on some grades in the class, who could tell me your math grade for the second quarter?
(Ss) Six, seven, eight...
(T) Calm down!
(Av) Eight and a half, six and a half, and nine and a half.
(T) Let's round up!
(T) Let's say that these were the grades that were presented, collected to find out what the mathematics grades of a ninth grader were. Okay? The first step is, in this case, the second step, after the information, is it to call the roll? list
(T) What is a roll?
(Ss) Organizing in ascending and descending order, from the smallest to the biggest.
(T) So that I can advance my work and not be repeating it, I will analyze it. 5, 5, 5, then 6, 6, 6, then, how many 7, 7, 7.
(Ss) Two.
(T) How many eights?
(Av) Three.
(T) What did we do here guys?
(Ss) The roll.
Source: Research data.

When we analyze the previous dialogues, the teacher does not seem to present difficulties in imparting knowledge to sighted students. He uses the terminologies from the textbook, the ostensives: "speech and mathematical symbols," and the teaching resources: "textbook, board, and marker."

The ostensive objects and the materials the teacher chose helped present knowledge and carry out the mathematical activity guided by non-ostensive objects (variables and frequencies). We characterize these teacher actions based on the category of choices. After this verbalization, the following excerpts point us to signs of the formation of SD1:

(T) Now, in the third step, building the table. For the table to be built, it is necessary to have the segment of rows and columns. Is the name vertical or horizontal direction line?
(Ss) Vertical!
(T) Please, Jorge, what does the line give an idea of?
(Bs) Horizontal!
(T) What does the line give an idea of?
(Bs) I don't know!
(T) But didn't you say it now?
(Ss) lol"
Source: Research data.

During the emergence of SD1, the teacher asks the blind student about the position of the lines on a table. He quickly says it is "horizontal," but when the teacher asks the question again, he says he does not know. We were unsure whether the student responded mechanically and would have felt insecure about the answer or embarrassed as the sighted colleagues began to enjoy the situation.

The possible formation of this system is short and apparently does not mean much for the blind student's learning, as it does not enhance the ostensive teaching resources for the student's understanding of the mathematical object. The teacher only used speech, did not use other ostensives indicated to explain the position of the rows and columns, and also did not lead the student to questions. According to Borges et al. (2022), classes with a considerable number of students can make the work of teachers difficult concerning the development of teaching resources for visually impaired students.

Next, the teacher returns to the explanation to the class as a whole and again does not favor the blind student due to how he explains. We understand that it only favors sighted people, as he does not use teaching resources that help present these concepts to people with visual impairment. Thus, we again observed evidence of the formation of SD3. Let us see:

(T) And the relative frequency? How do I find it? I have to remember that's basically it. Make the total divided by how much? How many bills do we have?
(T) 1, 2, 3 ,4, ...15
(Ss) $\frac{3}{15} = 0.2$
(T) How much is this result?
(Ss) 0.2
(T) Sure? Multiply by 100. How much does it give?
(Ss) 20
(T) 20%
(T) So, please, how would you do the next one, divided by the total?
(Ss) $\frac{2}{15} = 0{,}1333$
(T) Multiply by one hundred, equal to 13%.
(T) And the total?
(Ss) 100%
(T) So do it: twenty, plus twenty, plus twenty.
(Ss) Sixty
(T) Sixty plus thirty-nine?
(Ss) Wow, 99%!
(T) What happened that didn't close 100%? You saw that there was this 13.333. If you put it in rounding form, you would have to put 14%, but it generated approximately 99%, one less one more; it is more a question of rounding. Basically, for the error estimate, we have to have 100%.
(T) Oh! Let's add all the variables.
(Ss) Seven, eight, nine, and ten.
(Ss) Too smart people!
Source: Research data

In SD3, the teacher can present the concepts of statistical variables and frequency distribution: absolute, relative, accumulated, and relative accumulated. Furthermore, during the class, there was a need for calculations with decimal results, percentages, and rounding, which, apparently, did not create difficulties in this system.

However, we classify as an information category the absence of presentation of knowledge about class intervals during these studies, since the teacher did not present this knowledge, and, in the textbook, it was one of the topics that completed the knowledge oriented to be presented during the "Information Processing" chapter. SD3 ends, and soon after, the teacher approaches the blind student, so this moment gives us signs of the formation of SD2. In it, we identify the categories of choices, information, and skips (or lacks). Let us see:

> (T) So, oh, Jorge, get up! Come over here!
> (T) Oh! Let's run our hands here on the wall to understand a little, I didn't bring the things, the pieces. When we talk about the relationship between row and column, we have to realize that, in this case, the row will be horizontal, and in the table, there are always elements in that row. When talking about columns, you will place numbers in the columns with reference to the search.
> (T) Does anyone have erasers? I need kind of four erasers!
> Source: Research data.

Initially, we identified the category of choices since the teacher uses the voice and objects, ceramic pieces on the classroom wall, and erasers as ostensives to present knowledge to the blind student. In SD2, the book is not used, as it is not adapted, and the nomenclature (words) used during this verbalization is confusing, making the teacher and student feel insecure. Furthermore, once again, the lack of teaching material led the teacher to manipulate objects, which, in our opinion, may not have contributed to the student's learning. Let's look at the following excerpt from the class:

> (T) Then, when it's a class, I'm going to demonstrate how perception makes you organize the lines.
> (T) From a single line, [show] how many columns?
> Source: Research data

In the excerpt above, we cannot understand what the teacher means by "how perception makes." Perhaps he wants to say how to make the "distinction between rows and columns" in a table. The student does not question the teacher's explanation, and the teacher continues:

> (T) Oh friend, come here! Quiet, please! Your colleague must listen [to it]. I want you to place the erasers in a single, very organized line.
> (T) From a single line, [show] how many columns?

(Bs) Four.
(T) Now, I want you to assemble a column with these same pieces, one column, just one column.
(Bs) Ready.
(T) Are you confident? Is that a column? The vertical one.
(T) Now, make a table with two rows and two columns.
(Bs) Like this?
(T) Not yet. You need to make two rows and two columns.
(T) See there!
(Bs) Like this?
(T) See there! Okay! What characterized this? You built two lines and two columns, the graphics part; before this part, we worked on making this information in these lines and columns, which generate tables in which, in each place, there is a single element.
(T) Can you initially form what a table is with rows and columns?
(Bs) Yes!
(T) High five! Great!
Source: Research data

In this system, the teacher intended to present the construction of a table and the distinction between rows and columns by touching the tiles on the classroom wall and manipulating erasers. However, he could not explain the non-ostensive objects, rows and columns, and the mathematical objects proposed to be presented during the class. Of all the content planned to be shown during this class, the concepts of lines and columns were the ones that the teacher tried to explain to the blind student, yet he could not expose them clearly, nor was he able to construct a table using erasers.

We identified the category of “skips (or lacks)” in SD2, as the concepts of variables, frequency distribution (absolute, relative, accumulated, relative accumulated) and the development of calculations with decimal numbers and percentages were conveyed during the class without favoring the learning of the blind student; and when the teacher dedicated his time only to explanations to this student, these concepts were not addressed.

We also identified, in SD2, the information category since knowledge of class intervals for visually impaired students was also not presented. This teacher practice can lead to gaps in students’ future studies. Furthermore, the teacher makes some mistakes in his speeches; unconsciously, he says the expression: “See there!” which occurs frequently during the class. These automatic speeches repeat ways of speaking aimed at the sighted people. These actions are unconscious actions that can cause discomfort for the blind student.

At the end of the class, we observed that teaching work can be more favorable to teaching sighted students than to blind students. Furthermore, the teacher explains the concepts to the blind student in a “separate” way, with unconventional teaching resources, which he believes will favor this student's learning. We are aware that even though we point out that the teacher presents mathematical objects in a "separate" way, we know that this does not entirely exclude the blind student, as he is present in the classroom and hears what the teacher explains, but does not participate, does not question. Therefore, we do not know whether he can understand the concepts covered during the class. We observed that the teacher's explanations during the class almost entirely favored sighted students' learning over blind students' learning.

Final considerations

In our research, we sought evidence of the phenomenon of internal didactic transposition during the class on "statistical variables and frequency distribution" for a group of one blind student and sighted students in a regular classroom.

The formation of possible teaching systems SD1, SD2, and SD3 allowed us to observe the internal dynamics of a regular class, with the participation of a blind student, which leads us to understand that n-teaching systems can be formed in other cases situations, such as the participation of deaf, autistic students, and others.

Using ostensive objects as a category helped us in a more detailed analysis of the teacher's actions during his practice, indicating a distance between the knowledge effectively taught to blind students and the knowledge effectively taught to sighted students.

In this system, the teacher intended to show the construction of a table and the distinction between rows and columns by touching the tiles on the classroom wall and manipulating erasers. It is essential to be aware that developing an inclusive activity does not imply using any objects. They must choose objects based on their potential to mediate the relationship between the mathematical knowledge addressed and the student involved in the teaching situation, making the learning process viable.

Even though those are initial data, they point to gaps in the mathematical knowledge taught to blind students that cause difficulties in their learning. However, we do not want to hold the teacher responsible for all the challenges in the student's learning, as we understand that improving the quality of this teaching depends on several factors, ranging from specific education for work with people with visual impairment to teaching resources that contribute to the teaching and learning process of the mentioned public.

Even in the face of the difficulties observed during the class, perhaps a closer relationship between the teacher and the blind student would strengthen this didactic relationship and enable the discovery of ways of teaching to make the learning process viable. Furthermore, if the teacher allowed a constructive interaction between all the students in the room when knowledge was introduced, perhaps this could awaken questions that would lead to students' good intellectual development regardless of their differences since we learn from coexistence and with the relationships established between the groups.

The way the teacher chose the "separation" for conveying knowledge did not allow for an exchange of knowledge between the students attending class. We understand that this was how he imagined it most appropriate for the student to learn. However, this model implies a lack of interaction between students, so we cannot say there is true inclusion in this analyzed school space.

In this way, we emphasize the importance of continuing teacher education from an inclusive perspective. Through initial and ongoing education, the school community will be prepared to develop appropriate methodologies that respond to the special educational needs of all students enrolled in the regular classroom, regardless of their differences.

We understand that the teacher is primarily responsible for making inclusion a reality in the classroom through public policies that will allow this reality to come to fruition. However, the teacher cannot be the sole person responsible for including people with disabilities in regular schools; the entire education system must be accountable.

References

Araujo, A. J. de. (2009). *O ensino de álgebra no Brasil e na França: estudo sobre o ensino de equações do 1º grau à luz da teoria antropológica do didático*. [Tese de Doutorado em Educação Matemática, Universidade Federal de Pernambuco].

Bosch, M., & Chevallard, Y. (1999). A sensibilidade da atividade matemática aos ostensivos. Objeto de estudo e problemática. *Pesquisa em Didática da Matemática* , *19* (1), 77–124. https://revue-rdm.com/1999/la-sensibilite-de-l-activite/ Acessado em 10 de junho de 2023.

Bogdan, R., & Biklen, S. **Investigação qualitativa em educação: uma introdução à teoria e aos métodos**. Tradução Maria J. Alvarez, Sara B. Santos e Telmo M. Baptista. Porto (Portugal): Porto Editora, 1994.

Borges et al. (2022). Processo de inclusão escolar de uma aluna cega em aulas de Matemática Disponível em https://revistas.pucsp.br/index.php/emp/article/view/54260 Acessado em 28 de julho de 2023.

Brousseau, Guy. (1986). Fondements et méthodes de la didactique des mathématiques. In: Recherches en Didactique des mathématiques. Vol. 7, n° 2, p. 33-115. Grenoble.

Brasil. (1988). *Constituição da República Federativa do Brasil*. Senado Federal. disponível em: https://www2.senado.leg.br/bdsf/bitstream/handle/id/518231/CF88_Livro_EC91_2016.pdf

Chevallard, Y. (1991) *La transposition didactique*. Grenoble, La pensée Sauvage.

Conferência Mundial sobre necessidades educativas especiais. (1994). Declaração de Salamanca. http://portal.mec.gov.br/seesp/arquivos/pdf/salamanca.pdf Acessado em 14 de maio de 2023.

Fernandes, S. H. A. A. & Healy, L. (2010) A Inclusão de Alunos Cegos nas Salas de Aulas de Matemática: Explorando área, Perímetro e Volume Através do Tato. *Bolema.* v.23, nº 37, p.1111-1135. https://www.periodicos.rc.biblioteca.unesp.br/index.php/bolema/article/view/4306

Kaspary, D. & Bittar, M. (2013). Ostensivos como instrumento no estudo das operações de adição e de subtração dos números naturais. Anais do VII CIBEM (pp. 1424-1434). http://funes.uniandes.edu.co/19206/1/Kaspary2013Ostensivos.pdf

Menezes, A. P. A. B. (2006). *Contrato didático e transposição didática: Inter relações entre fenômenos didáticos, na iniciação à álgebra na 6ª série do ensino fundamental*. [Tese de doutorado em Educação Matemática, Universidade Federal de Pernambuco].

Mendes, E. G. A. (2006). radicalização do debate sobre inclusão escolar no Brasil. Revista Brasileira de Educação.v.11, n.º33, p.387-405 https://www.scielo.br/j/rbedu/a/KgF8xDrQfyy5GwyLzGhJ67m/

Menezes, M. B. de. & Santos, W.F. dos. (2018). As modificações no saber efetivamente ensinado em uma sala inclusiva para alunos surdos: o caso do conjunto dos números naturais. In: Revista do Programa de Pós- Graduação em Educação Matemática (UFMS). V. 11. nº 27. https://periodicos.ufms.br/index.php/pedmat/article/view/7217

Mello, H. de. & Machado, S. (2017). A formação histórica da educação para cegos no Brasil: uma análise contextualizada das leis do Império à República – 1º Seminário Luso Brasileiro de Educação Inclusiva: o ensino e aprendizagem em discussão (p. 26-40). https://ebooks.pucrs.br/edipucrs/anais/i-seminario-luso-brasileiro-de-educacao-inclusiva/assets/artigos/eixo-1/completo-2.pdf

Menezes, M. B. de. (2020). Praxeologia do professor e do aluno: uma análise das diferenças no ensino de equações do segundo grau. [Tese de Doutorado em Educação Matemática, Universidade Federal de Pernambuco].

Ministério Público do trabalho. (2016). *Lei Brasileira de Inclusão da Pessoa com Deficiência*. Procuradoria Regional do Trabalho. 60 p.

Ministério da Educação. (2001). *Diretrizes nacionais para educação especial na educação básica.* Secretária de Educação Especial. MEC, SEESP.79 p.

Santos, V. L. O. dos. (2020). Análise sobre o fenômeno da transposição didática interna no ensino de estatística: um estudo com a inclusão de um aluno cego em uma sala de aula regular. [Dissertação de Mestrado em Educação Matemática, Universidade Estadual da Paraíba].

Santos, V. L. O. dos. & Menezes, M. B. de. (2020). Soroban: ferramenta didática para alunos cegos no ensino de matemática. [Produto Educacional do Mestrado em Educação Matemática, Universidade Estadual da Paraíba]. http://tede.bc.uepb.edu.br/jspui/handle/tede/3922

Silva, L. M. S. (2010). As histórias em quadrinhos adaptadas como recurso para ensino da matemática para alunos cegos e videntes. [Dissertação de Mestrado em Educação Matemática, Universidade Estadual Paulista].

Silva, L. M. S. (2015). Do improviso às possibilidades de ensino: estudo de caso de uma professora de matemática no contexto da inclusão de estudantes cegos. [Tese de Doutorado, Universidade Estadual Paulista, Instituto de Geociências e Ciências Exatas]

Souza, N. T. (2014). Conversando sobre razão e proporção: uma interação entre deficientes visuais, videntes e uma ferramenta falante. [Dissertação de Mestrado em Educação Matemática, Universidade de Anhanguera de são Paulo].

Vygotski, L. (1997). Obras escogidas – V: Fundamentos da defectología. Tradução: Blank, J, G. Madrid. Visor.

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