EMP Educação Matemática Pesquisa
ISSN 1983-3156

Qualis A1

http://dx.doi.org/10.23925/1983-3156.2024v26i1p114-150

The academic-professional trajectory of a mathematics teacher educator

La trayectoria académico-profesional de un formador de profesores de matemáticas

Le parcours académique-professionnel d'un formateur d'enseignants de mathématiques

A trajetória acadêmico-profissional de um formador de professores de matemática

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Abstract

This research involves a educator of future Mathematics teachers and the teaching of Linear Algebra, and it is part of an ongoing study. The study has the following objectives: (i) to reflect on and analyze the narratives of a teacher educator regarding the characteristics of the Linear Algebra discipline in a Mathematics Education course, and (ii) Understand how the academic/professional trajectory of a teacher educator shapes and/or interferes in the planning and development of the Linear Algebra discipline and the teaching of systems of linear equations. The data were generated through documentary research and the observation of an episodic interview conducted individually with a Mathematics teacher educator. Interpretative qualitative research was the methodological approach used for constructing the field texts, which involved documentary research and the selection of excerpts from a remote episodic interview. These excerpts were analyzed narratively for insightsand reflections from the data. The results indicated that the personal experience of the

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Mathematics teacher educator influenced the development of their teaching identity in higher education. Regarding the organization of the discipline through the teaching plan, it was observed that the teacher educator aimed to emphasize formalization and argumentation, with the goal of covering the contents outlined in the course syllabus. There is a need for further studies on the roles and identities of Mathematics teacher educators, the weaknesses of the Pedagogical Political Project related to teaching practice, and the involvement of teacher educators in formative contexts.
Keywords: Mathematics teacher educator, Teaching of linear systems, Teaching identity, Narrative analysis.

Resumen

Esta investigación involucra a un formador de futuros profesores de Matemáticas y la enseñanza de Álgebra Lineal, y forma parte de un estudio en curso. El estudio tiene los siguientes objetivos: (i) reflexionar y analizar las narrativas de un formador de profesores sobre las características de la disciplina de Álgebra Lineal en un curso de Licenciatura en Matemáticas, y (ii) comprender cómo la trayectoria académica/profesional de un formador de de profesores moldea y/o interfiere en la planificación de la disciplina de Álgebra Lineal para la enseñanza de Sistemas Lineales. Los datos fueron producidos a partir de una investigación documental y la observación de una entrevista episódica realizada individualmente con un formador de profesores de Matemáticas. El enfoque metodológico utilizado para construir los textos de campo fue la investigación cualitativa interpretativa, que involucró la investigación documental y la selección de fragmentos de una entrevista episódica realizada de manera remota, relacionada con las reflexiones a través de un análisis narrativo de los datos. Los resultados señalaron que la experiencia personal del formador de profesores de Matemáticas influyó en la construcción de su identidad docente en la Educación Superior. En relación con la organización de la disciplina a través del plan de enseñanza, se observó que el formador de profesores buscó enfatizar la formalización y la argumentación, con el objetivo de cubrir los contenidos delineados en el programa del curso. Se identificó la necesidad de más estudios sobre los roles e identidades de los formadores de profesores de Matemáticas, las debilidades del Proyecto Político Pedagógico relacionadas con la práctica docente y la participación de los formadores de profesores en contextos formativos.

Palabras clave: Formador de profesores de matemáticas, Enseñanza de sistemas lineales, Identidad docente, Análisis narrativo.

Résumé

Cette recherche implique un formateur de futurs professeurs de mathématiques et l'enseignement de l'algèbre linéaire, et fait partie d'une étude en cours. L'étude a pour objectifs : (i) réfléchir et analyser les récits d'un formateur de professeurs sur les caractéristiques de la discipline de l'algèbre linéaire dans un cours de licence en mathématiques, et (ii) comprendre comment le parcours académique/professionnel d'un formateur de professeurs modèle et/ou interfère dans la planification de la discipline de l'algèbre linéaire pour l'enseignement des systèmes linéaires. Les données ont été produites à partir d'une recherche documentaire et de l'observation d'un entretien épisodique réalisé individuellement avec un formateur de professeurs de mathématiques. La recherche qualitative interprétative a été la méthodologie utilisée pour la construction des textes de terrain, qui a impliqué la recherche documentaire et la sélection d'extraits d'un entretien épisodique, réalisé à distance, concernant les réflexions à travers une analyse narrative des données. Les résultats ont indiqué que l'expérience personnelle du formateur de professeurs de mathématiques a influencé la construction de son identité d'enseignant dans l'enseignement supérieur. En ce qui concerne l'organisation de la discipline à travers le plan d'enseignement, il a été observé que le formateur de professeurs cherchait à mettre l'accent sur la formalisation et l'argumentation, dans le but de couvrir les contenus du programme de cours. Il est apparu nécessaire de mener davantage d'études sur les rôles et les identités des formateurs de professeurs de mathématiques, les faiblesses du Projet Politique Pédagogique liées à la pratique enseignante et l'implication des formateurs de professeurs dans les contextes formatifs.

Mots-clés: Formateur de professeurs de mathématiques, Enseignement de systèmes linéaires, Identité enseignante, Analyse narrative.

Resumo

Esta pesquisa envolve um formador de futuros professores de Matemática e o ensino de Álgebra Linear e é parte de um estudo em andamento. O estudo traz como objetivos: (i) refletir e analisar as narrativas de um formador de professores sobre as características da disciplina de Álgebra Linear em um curso de Licenciatura em Matemática; e (ii) compreender como a trajetória acadêmico/profissional de um formador de professores modela e/ou interfere no planejamento da disciplina de Álgebra Linear para o ensino de Sistemas de Equações Lineares. Os dados foram produzidos a partir de uma pesquisa documental e da observação de uma entrevista episódica realizada individualmente com um formador de professores de Matemática. A pesquisa qualitativa interpretativa foi o caminho metodológico utilizado para a construção dos

textos de campo, que envolveu a pesquisa documental e a seleção de trechos de uma entrevista episódica, realizada de forma remota, referente às reflexões por meio de uma análise narrativa dos dados. Os resultados apontaram que a experiência pessoal do formador de professores de Matemática influenciou a construção de sua identidade docente no Ensino Superior; sobre a organização da disciplina, por meio do plano de ensino, observa-se que o formador de professores procurou trabalhar a formalização e a argumentação, tendo em vista atender os conteúdos da ementa da disciplina. Verificou-se a necessidade de mais estudos sobre os papéis e identidades dos formadores de professores de Matemática, as fragilidades do Projeto Político Pedagógico relacionadas à prática docente e o envolvimento de formadores de professores em contextos formativos.
Palavras-chave: Formador de professores de matemática, Ensino de sistemas de equações lineares, Identidade docente, Análise narrativa.

The academic-professional trajectory of a mathematics teacher educator

Research involving teacher educators is still little explored, presenting a small number of investigations (Coura & Passos, 2021, 2022), as emphasized by Fiorentini et al. (2016). Few studies present narratives of the life stories and/or professional development of teacher educators who teach mathematics, related to the formation of the professional identity of the teacher educator, in the context of the teaching trajectory, considering that the teacher is the author of the educational process in Higher Education.

Concerning teacher educators, some studies mainly address their role in higher education, that is, in the initial training of future teachers and this training and the teaching practiced by these subjects are capable of interfering in the quality of training of future teachers (Gonçalves, 2000). In this sense, the study by Zeichner (2010) advocates that teacher educators seek a variety of approaches to develop connections between initial training and future teaching practice. Oliveira and Fiorentini (2018) indicate the need for the involvement of teacher educators in investigative and collaborative practices that integrate Mathematics Degree courses with school mathematics, necessary for future mathematics teachers.

From this perspective, Coura and Passos (2017) reinforce the role of the educator in the process of initial training of mathematics teachers, considering the need for research that involves the knowledge that the educator needs for the professional exercise of training Mathematics teachers, both in relation to specific knowledge and pedagogical knowledge. The authors point out the need to prioritize teaching as support for research so that it is possible to reflect on one's own practice and one's role as a teacher educator (Coura & Passos, 2021, 2022). This is because many individuals training to teach Mathematics do not have and/or have not experienced training to be Mathematics teacher educators (Fiorentini, 2004; Silva & Jardim, 2022; Silva et al., 2023).

This fact was observed by Ping et al. (2018), who indicated that there are two common routes for teacher training: one related to teaching practice initiated in a school environment, later moving on to teaching in higher education, and another involving evolution as a student in initial training in which he remains in teaching higher education after obtaining a postgraduate doctorate. When discussing professional activity, the authors Ping et al. (2018) indicate that there are different roles and responsibilities, depending on the context associated with the teacher educator's work. In this sense, they reveal that there is a set of learning involving what (the professional learning content), the how (the professional learning activity) and the why (the reason/motivation for professional learning) the teacher educator learns, and

this is constituted throughout a professional trajectory. Associated with the subject who trains teachers, the research by Belo et al. (2021) suggests that developing a teacher identity is an important process in becoming a teacher educator. In this case, they refer to the repercussion of this identity on the training of future teachers and the teacher educator's own learning.

Based on the reflections presented, this study will present the narratives of a teacher educator, listening to him analyze the initial training of the teacher who will teach mathematics, mainly through his perspective as a teacher educator, participating in spaces that reflect his teaching identity, his process training and the contribution to thinking about teaching in the Higher Education space. In this context, the questions that guide this study emerge: how is a Linear Algebra discipline structured to teach the content of systems of linear equations and what are their interfaces in High School? From the point of view of a teacher educator, how can the content of systems of linear equations from the Linear Algebra discipline contribute to the teaching of mathematics in high school? How does the academic/professional profile of a teacher educator contribute and/or interfere in the planning of the Linear Algebra discipline and the teaching of the content of systems of linear equations?

To answer these questions, the following specific objectives were defined: (i) Reflect and analyze the narratives of a teacher educator about the characteristics of the organization of teaching the discipline of Linear Algebra in the Mathematics Degree course at a public educational institution Higher education and (ii) Understand how the academic/professional trajectory of a teacher educator shapes and/or interferes in the planning and development of the Linear Algebra discipline and the teaching of systems of linear equations.

In order to meet the objectives, this article has theoretical assumptions focusing on the professional identity of the teacher educator and the teaching of Linear Algebra and Systems of Linear Equations. The methodological procedures adopted consisted of two stages. The first, related to the context of the Linear Algebra discipline in initial training, encompassed documentary research; and the second consisted of the selection of excerpts from narrative episodes referring to the reflections of a Mathematics teacher educator. The two stages made it possible to construct a narrative analysis, in the context of the teacher educator's work in his teaching practice in the bachelor's degree in mathematics.

Theoretical framework

Professional identity of the teacher educator

Some researches states that the professional identity of Mathematics teacher educators in Higher Education is seen as constructions and/or development, in social situations, in

practical experiences (Dalby, 2017; Losano & Fiorentini, 2018; Belo, 2018; Belo et al., 2021). It involves the construction of the subject as a professional (Marcelo, 2009), fundamental for the development of teaching professionalism (Morgado, 2011) and for the development of teaching professional learning (Avalos, 2011; Ping et al., 2018).

To begin reflections on the professional identity of the teacher educator, it will be necessary to discuss the terms educator, teaching professionality and professional identity. Regarding the educator, Mizukami (2005, p. 3) explains that this term is understood as all professionals involved in the training processes of "teaching learning", suggesting that these are the ones who train future teachers (initial training) and/or in-service teachers (continuing training), whether in higher education or in schools. For this study, the teacher educator who works in Higher Education will be considered, just as Coura and Passos (2017) conceptualize that the teacher educator is a teacher who trains in the exercise of his profession,

> as he needs to mobilize his knowledge to undertake practices that meet the demands of his professional context. In this way, he maintains a double relationship with teacher training: as an agent in his own training and in the training of his students, future teachers. (Coura & Passos, 2017, p. 9, our translation).

Regarding teaching professionalism, Morgado (2011) understands that it is linked to professional identity. The author expands and clarifies by structuring the terms "professional identity", "professional competence" and "teaching professionality", considering that there is a relationship between the concepts and their interpretation. Thus, professionalization focuses on the process of transforming a subject into a qualified professional by "assuming complex and varied professional functions". And professional skills, "professional culture and professional identity are, therefore, three essential pillars of professionalization for the development of teaching professionalism" (Morgado, 2011, p. 797).

In the field of Mathematics Education, professional identity, related to the subjects who teach, is discussed in the research by Dalby (2017), who states the need for this to be established based on the meaning of the term "identity". This is because this term as a concept has emerged in educational research and offers a useful bridge to explain how social interactions shape people and how their voices, with their individual speeches, relate to the voice of a community. Losano and Fiorentini (2021) base their conceptualization of professional identity on the work of Holland et al. (1998), which has its roots in sociocultural theories. These theories provide

> three theoretical notions to capture these mutually constitutive relationships between the teacher and the social worlds in which they participate: identity, agency and figurative worlds, in which they recognize that they act purposefully and reflexively to reiterate,

> reconstruct or transform their teaching practices and ways of understanding each other as professionals (Losano & Fiorentini, 2021, p. 1221, our translation).

Teacher educators, by engaging in reflection and criticism about their own practices, contribute to teacher improvement in their professional identity, teaching and learning (Avalos, 2011; Lunenberg et al., 2011; Loughran, 2014; Ping et al. al., 2018). In this regard, Belo (2018) reflects on the personal dimension of Mathematics teacher educator s, with mathematical training as an important factor in the constitution of professional teaching identity. In this sense, social interactions and the personal dimension are seen as forming identity, constructed and socially situated, prolonged throughout professional life, always changing, and this is promoted by a process that involves professional skills, professionalization (Morgado, 2011; Dalby, 2017; Losano & Fiorentini, 2021).

Considering the importance of listening to mathematics teacher educators in their professional contexts, the research by Belo et al. (2021) concluded that, by provoking the mobilization of these individuals' experiences, it was possible to lead them to reflect on the gaps in their training and the practice developed. In this way, they accessed new assessment strategies, experienced interactions, participated in reflective movements and transformed their practices, contemplating the development of new training models (Silva et al., 2023).

Ping et al. (2018), when researching teacher educators' learning, they found that this is also constituted by their teaching identity, as they observed, when presenting the category entitled professional identity, two subcategories involving identity as a teacher educator and as a researcher. Thus, they indicated that there is academic involvement in teaching and research. In formal education, in the social and professional interactions that support teaching practice, participation in collaborative investigative communities provides opportunities for professional learning. This learning is rooted in the professional identity of a teacher educator (Ping et al., 2018).

The research generally addressed the complex, dynamic and changing nature of the teacher educator's professional roles based on their professional identity. This approach allows for reflection on the trajectories of what it means to become and be a teacher educator. The professional identity of Mathematics teacher educators is related to their professional development as a teacher, in their work context. It aligns with professional learning needs, throughout their careers, and presents itself as an important element when examining the professional agenda of this subject.

Teaching Linear Algebra and Systems of Linear Equations

Regarding the discipline of Linear Algebra[4], some authors consider the teaching and learning of this discipline to be complex and difficult (Rodriguez, 2011; Barros, 2018; Ramírez-Montes, 2020). Other studies are concerned with the involvement of the teacher and teacher educator so that they can use the concepts of Linear Algebra connected with the contents of basic education when teaching in the final years of Elementary School and High School. These studies also explore the duality between mathematics learned in initial training and mathematics for teaching in Basic Education[5] (Dias et al., 2017; Barros, 2018; Bianchini et al., 2018; Stewart et al., 2019). Moreover, there is a recognized need for research on teaching and learning Linear Algebra in the field of Mathematics Education (Bianchini et al., 2018; Bianchini et al., 2019).

Barros (2018) relates the difficulties in learning Linear Algebra to the complexity of understanding it and the specific use of formalism, that is, the use of algebraic structures without meanings. In this sense, formalism, symbolism, language and abstraction are present and play distinct roles that influence the pedagogical projects of the courses and the practices of educators in the Degree in Mathematics (Barros, 2018). Fact observed by Dias et al. (2017) when recording that such aspects may be directly related to the low skills of future teachers in teaching some Linear Algebra content in Basic Education

From this perspective, the discipline of Linear Algebra, integrated into the curriculum of several higher education courses, also generates one of the obstacles related to the lack of understanding of future Mathematics teachers regarding formal definitions and resolutions of conceptual and procedural problems (Dias et al., 2017; Ramírez-Montes, 2020). This means that these future Mathematics teachers are unable to relate them to meanings and reality so that they can be taught in basic education (Barros, 2018).

When we deal with systems of linear equations, there are not many studies on possible approaches and/or methods that students and undergraduates can use (Bianchini et al., 2018; Ramírez-Montes, 2020). However, research reflects on decontextualized approaches, in which students do not master basic skills and have difficulties in learning concepts and skills related to solving equations, as well as the significance of this curricular topic (Bertolazi & Savioli, 2018; Bianchini et al., 2018; Diniz & Ferreira, 2019). This decontextualized approach,

[4] As a mathematical discipline taught in undergraduate Mathematics courses (Degree and Bachelor's Degree), in exact sciences and service courses, covering in its contents such as systems of linear equations (algebraic or differential), the conceptualization and foundation of mathematical structures. In other words, it involves the study of lines and planes (vectors), vector spaces, mappings for linear transformations and systems of linear equations and matrices (Barros, 2018).

[5] Regarding the National Common Curricular Base (In Portuguese, Base Nacional Comum Curricular – BNCC), it states that high school (in this case the 2nd year) teaches matrices, determinants and systems of linear equations.

according to Bertolazi and Savioli (2018), is found in several textbooks (Basic Education), because, by not covering aspects considered essential to the concept of systems of linear equations, teaching is traditionalist and learning is focused on a technical and decorative process of the concept.

In the algebra curriculum for Basic Education, solving linear equations is one of the foundations on which students transition from reasoning with numbers to reasoning with unknowns. In such a scenario, problem-solving skills are important, and it is emphasized that

> (...) this content is related to others, in multiple ways, both within the scope of mathematics itself and in other areas of scientific knowledge. (...) present in the mathematical curriculum of Elementary, Secondary and Higher Education. It is configured as a prerequisite and directs the study of various topics related to Linear Algebra, assists and equips the systematic study of Analytical Geometry, collaborates with the resolution of algebraic, geometric and trigonometric problems, among other potentialities (Bertolazi & Savioli, 2018, p. 34, our translation).

The difficulties encountered in teaching and learning Linear Algebra constitute a dilemma, becoming guidelines for investigations that enable the implementation of teaching strategies that can lead to significant changes in student learning. Thus, it can be seen that a large part of the theory of Linear Algebra revolves around finding solutions for a Linear System, which is a content that is present in both Higher Education and Basic Education. This aspect results in and enables its countless applications, from engineering to natural and social sciences.

Methodology

The study is part of a thesis in development[6], made possible by qualitative-interpretative research (Creswell, 2014; Scheiner, 2019). Focusing on meeting the objectives of this study, we intend to understand the nature of the teaching work of a teacher educator and the tensions experienced by those who work in teaching degrees, between the world of academic mathematics and the world of school mathematics (Moreira & David, 2003) to understand his professional identity and the relationship between this and his teaching practice to teach systems of linear equations, in a Linear Algebra discipline, in the Mathematics Degree.

Context and participant

The research in question was carried out within the scope of a Mathematics degree course, with a Mathematics teacher educator working in the Linear Algebra discipline at a Brazilian public higher education institution (Silva et al., 2023). The data collected regarding the documentary research and the teacher educator were synthesized and inspired by the model tabulated by Losano and Fiorentini (2018), as shown in Table 1. To this end, an episodic interview (Flick, 2015) was transcribed with this teacher educator[7], with a view to organizing the analyzes (Table 2).

Data collection instruments

The data from this study are documentary (Bauer & Aarts, 2017) and observational (Lüdke & André, 2015) in nature. Therefore, data collection instruments were documents relating to the teaching of the discipline of Linear Algebra in the Mathematics Degree and documents specific to the course in which the participant works as a teacher. So, the instruments of a documentary nature are National Curricular Guidelines for the Degree in Mathematics (In Portuguese, Diretrizes Curriculares Nacionais da Licenciatura em Matemática – DCNLM), Information on the topic of systems of linear equations in the National Common Curricular Base (In Portuguese, Base Nacional Comum Curricular – BNCC), Pedagogical Political Project for the Degree Course in Mathematics (PPC) and the Teaching Plan (TP).

As an observation, an episodic interview (Flick, 2015) was carried out with the teacher educator to understand his impressions and academic-professional trajectory, in addition to better understanding the aspects related to his teaching practice. The choice for the episodic

[6] First stage of a collaborative training process that aims to explore the stages of teaching practice between a teacher educator and a researcher (first author of this article) (Silva et al., 2023).
[7] Meeting held via videoconference, using the Zoom Meetings tool.

interview occurred because it combines the possibility of narrating concrete events relevant to this article, considering the experience of the interviewee (Mathematics teacher educator), based on an interview guide. This guide directed the field study, as it can be created from different sources regarding experience, results, theoretical dimensions, analyzes and preliminary understanding of the area under study. With these data, it was possible to construct field texts (documentary research and episodic interviews), which made it possible to transform them into research texts through a narrative analysis of the data.

Data analysis procedures

The analysis process took place in two stages. The first occurred through description based on reading documents. The second consisted of expanding and deepening this description of documents through the inclusion of narrative excerpts from the episodic interview carried out with the educator.

To organize the data in the first stage, it was necessary to create a field of the analyzed perspectives of each data and the method used to obtain these analyzes (Table 1). In this sense, documentary data was used as reference and the transcription of the episodic interview with the Mathematics teacher educator.

Table 1.

Data chosen to construct the narrative analysis

Data	**Perspectives analyzed**	**Method of obtaining the Study**
(DCNLM)	The entire document, focusing on the Degree in Mathematics course	Documentary description
Information on the topic of Systems of Linear Equations in the National Common Curricular Base (BNCC)	Presentation; Introduction; The Elementary School stage; The Mathematics area; Mathematics in Elementary School – Final Years – High School; Competency #3, and Ability (EM13MAT301)	Documentary description
Political Pedagogical Project for the Degree Course in Mathematics (PPC)[8]	Presentation; Course objectives; Curricular Organization and Syllabus Disciplina de Álgebra Linear I	Documentary description

[8] The choice for the analyzed PPC is justified by the fact that the Mathematics Degree course is from the HEI co-participating in this doctoral study, which is the one in force at the time of the research and, for ethical reasons, will not be included in the references of this article. Approvals from the Research Ethics Committees (CEP) of

Teaching Plan (TP)[9]	Objective; Program Content, and Methodology	Documentary description
Interview with a Mathematics teacher educator	Professional Training and Performance – training process; Profile as a educator; Teaching at undergraduate level; Professional Training and Performance – Teaching the curricular topic of Systems of Linear Equations	Episodic interview (transcribed video-audio)

The second stage, the interview with the teacher educator, aimed to organize the criteria that involved the structuring of data collection, the interviewee's point of view and the construction of perspectives for performative narrative analysis (Riessman, 2005) as observed in the organization of Table 2. After transcription, some episodes from the interview were selected. In this context, it was possible to carry out a narrative analysis of the data, supported by Riessman's proposal (2005, p. 5), called "performative analysis" which, according to the author, is an interactional approach, "appropriate for studies of communication practices and for detailed studies of identity construction". Based on Riessman (2002, 2005), the authors Losano and Fiorentini (2018, p. 10) reflect that this type of analysis presents "the positions of the participants in the episode, the context of the episode, the dialogues between them, the paralinguistic signs and the interlocutor's audience", which goes beyond the word itself.

Table 2.

Data collected – performative narrative analysis

Data	**Criteria**	
Episodic interview (transcribed video-audio)	Structuring the collection	Interview guide, with open questions for the narrative
	Interviewee's point of view	Organization of questions giving space for the narrative

Higher Education Institutions (HEIs): proposing institution Universidade Federal do ABC (UFABC) (CAAE 56236822.4.0000.5594, Report 5.400.645/2022) and co-participating institution (CAAE 56236822.4. 3001.5083, Report 5,478,447/2022).

[9] Only Teaching Plan provided by the teacher educator, collaborator of this study. For ethical reasons, it will not be included in the references of this article.

Perspectives analyzed	● The discipline's profile: academic, professional and practical trajectory of the Mathematics teacher educator ● The discipline of Linear Algebra presented in a PPC – Degree in Mathematics ● Contribution of the Linear Algebra discipline to Basic Education and to the training of future Mathematics teachers. Expertise in teaching the discipline of Linear Algebra. ● Teaching Systems of Linear Equations

Therefore, the educator's performative narrative analysis was guided by the documents analyzed, organized according to the criteria presented for analytical focuses. Narrative analysis is categorized based on narrative content or structure. To this end, each episode was described in an attempt to give a voice to the teacher educator, highlighting the "narrative's ability to explain experiential phenomena" (Losano & Fiorentini, 2018, p. 11). Throughout the process, different sources of data and references were used for complementation and preliminary versions were shared, with a view to providing opportunities for changes and corrections by the research collaborator (Silva et al., 2023).

The participant in the research: teacher educator João (FPJ)

João[10] is a teacher educator with 19 years of experience in Higher Education. Because he liked and had an affinity with the discipline of Mathematics in Basic Education, he chose to pursue a degree in Mathematics. Studying and experiencing the universe of academic Mathematics at the Universidade Federal de Viçosa, in the interior of Minas Gerais/MG, he was able to observe the different career possibilities related to the area, in addition to teaching.

During that period, he began to dedicate himself to teaching, as well as giving private classes to his university colleagues. These facts influenced him to simultaneously study his degree and his bachelor's degree. He continued his studies at master's level in postgraduate studies in the area of Algebra at the University of Brasília. After finishing, he continued to have concerns about continuing his studies in Mathematics (field of Algebra) or pursuing the teaching career that he so desired. So, even though he was in doubt, he decided to take a break for a year, at which point he dedicated himself to his teaching career in Higher Education, as he states:

[10] The name of the Mathematics teacher educator, collaborator in this research, was changed to a fictitious name.

> (...) I had an inner scream, so I allowed myself, I started working. But where to work? I had a master's degree in Mathematics, I went looking for a job at colleges that had Mathematics courses, but, in general, what happens in private colleges, the courses are Mathematics Degrees. So, he was a pure mathematician, despite having a degree. (...) I had taken a course on didactics there, one on educational psychology and one on teaching structure, I didn't have such a solid pedagogical training, I found myself in the classroom, full of Mathematics in my head, but working on degree courses (FPJ).

The teacher educator decided to pursue a doctorate, but, simultaneously with his studies, he continued his work as a teacher in the Mathematics degree and other higher education courses. He has always been involved in Higher Education, highlighting that he has never worked outside of this environment, although he has participated through guidance or projects that provided some experience in basic education (FPJ). As for teaching experiences, the FPJ considers that these were transformed throughout the relationships it had the opportunity to experience. These experiences allowed this teacher educator to guide several future Mathematics teachers at various levels, including guidance for teaching practice, internships, course completion work, Pibid[11], scientific initiations and production of teaching materials, such as books aimed at teaching practice (FPJ). He reports:

> My trajectory was completely guided by my first professional experience in 2004, at a private college in the Federal District, where I met some people, who were experiencing the reality of Mathematics Education. Contact with two educators and a teacher educator, with initial training in Mathematics and a postgraduate degree in the area of Mathematics Education, who worked in teaching subjects, who had a different perspective on teaching, something I had never even heard of to speak. By relating, it was possible to establish partnerships, and this dictated the direction I took from this experience, I never stopped experiencing things along this path (FPJ).

This shared account of the educator João is based on his initial training, the construction of his identity and the experiences of teaching practices, mainly having his experiences in initial training and as a master and doctor in Mathematics in the field of Algebra (Morgado, 2011; Dalby, 2017; Losano & Fiorentini, 2021).

Data analysis

National Curriculum Guidelines for the Mathematics Course – Bachelor's degree

For this study, it will first be necessary to report to CNE/CES Report nº 1.302/2001, approved on November 6, 2001 – National Curricular Guidelines for Mathematics Courses (Brazil, 2001), which presents aspects related to bachelor's and degree in Mathematics. Its

[11] Institutional Teaching Initiation Scholarship Program

objective is to define the organization and pedagogical functioning of these courses in a Higher Education Institution (HEIs).

These guidelines establish the basic standards for the development of undergraduate courses in Mathematics (bachelor's and licentiate's degree), such as control and recording of academic activities, indicating what is necessary to compose a Pedagogical Project of Course (PPC), so that this consider: the profile of the graduates; their skills and abilities; general and specific training curricular contents; the characteristics of complementary activities; the format of the internships; the course load and the course load, and the assessment methods. Beltrão and Mandarino (2014) observed that, in the introduction, it is stated that Bachelor's degrees in Mathematics prepare professionals for a career in Higher Education and research, while Degree courses in Mathematics aim to train teachers for Basic Education. In this regard, the authors reflect that

> (...) the misconception that bachelors will work in research, which stricto sensu would only occur with the continuity of postgraduate studies. On the other hand, in a more current view of the field of Education, any teacher should have training to exercise an investigative stance, being a researcher of their own practice. At the same time, **if the Bachelor can work as a teacher, moving on to a career in Higher Education, it would be necessary to incorporate skills linked to teacher training into his/her training**. (Beltrão & Mandarino, 2014, p. 749, emphasis added)

In this research, we will focus our attention on the Degree in Mathematics, which has as its main objective the training of teachers for basic education (Brazil, 2001). From the perspective of improving the quality of teaching and strengthening training profiles in the various specialties, the Curricular References for the DCNLM were developed throughout 2009. These references prioritize historically consolidated nomenclatures, supported by professional regulations and curricular guidelines for courses in graduation (Zaidan et al., 2021).

The document, in its initial text, outlines the profile and qualifications of the graduates as possibilities. Thus, it presents that graduates must be aware and have a vision of their social role as a future Mathematics teacher, of the contribution that mathematical learning can offer in or to the student's training for the exercise of citizenship, mathematical knowledge accessible to all "and awareness of their role in overcoming prejudices, translated by anguish, inertia or rejection, which are often still present in the teaching-learning of the discipline" (Brasil, 2001, p. 3).

As a educator, Professor João shared that, at the university, he works on other courses in initial training, in addition to Mathematics. Regarding the training of future Mathematics

teachers, he is concerned with their social role, considering it a personal priority not to neglect the training of this public, "the experience so that they can get to know and learn about the profession that also aims to build citizenship, of a country, because it can transform lives" (FPJ).

The process of building a degree course in Mathematics, according to the aforementioned document, needs to consider the skills and abilities to be acquired by undergraduates throughout their initial training, which are indicated in the DCNLM. These standards add, for undergraduate courses, that the curricular contents include representations that students have of mathematical concepts and their school processes, with the following guidelines: "a) based on the representations that students have of mathematical concepts and school processes to organize the development of approaches during the course b) build a global view of the contents in a theoretically meaningful way for the student" (Brasil, 2001, p. 4, our translation).

With regard to Curricular Contents, the document stipulates that the themes provide professional knowledge, considering both profiles (bachelor and graduate), aligned with the indicated skills and abilities. The organization of these subjects in a Higher Education Institution (HEI) must include "content common to all Mathematics courses, complemented with organized subjects" (Brazil, 2001, p. 5), according to the profiles. However, it is noted that the mathematical contents for undergraduate training are practically the same as those recommended for bachelor's training. In this context, Linear Algebra is a common content that must be present in Mathematics courses (Bachelor's and Bachelor's degrees). The teacher educator highlights the importance of prioritizing mathematical knowledge, stating that "it doesn't matter at what level, from the most basic to the most complex, I think it is their right, if it is proposed in a course's curriculum, I try not to neglect this content" (FPJ). However, in the educator's own narrative, it is not stated how he approaches this knowledge. Thus,

> (...) **there was a time when I thought the content was enough. I had my moment as a content teacher, totally a content teacher**. Today I see that the content is completely important, that you cannot give up on it, but when I think about the classroom, about teacher training, it is necessary to understand that there are tools that help in this communication. **Just having knowledge of the content does not mean teaching, communicating content is not teaching. So, today, I believe that I must combine the technical knowledge of mathematics with strategies that allow the appropriation of this knowledge**. (FPJ, emphasis added)

The training of graduate students still needs to include content for professional training, along with the contents of Basic Education and the National Guidelines for Basic Education and Secondary Education. Another aspect indicated in these documents is that the student

acquires “familiarity with the use of the computer as a work tool that contributes to mathematical teaching” (Brasil, 2001, p. 6, our translation).

According to the document in question, it is expected that Mathematics courses will be references for reflection and constant discussion of the training process of future Mathematics teachers. Regarding this, in Freire’s (1996) critical conception, related to theory and practice, the author attributes that both are inseparable and, through their relationship, enable subjects to reflect on action. When narrating his teaching practice, the teacher educator is concerned with the relationship between theory and practice in the initial training of future teachers in the Mathematics Degree course, socializing that:

> I can’t say that I can make this relationship in any subject taught. When I have the opportunity to teach more introductory subjects, I try to ensure that the student is able to understand how that university content is present in Basic Education, because they often forget this and don’t bother to make that link (FPJ).

Therefore, discussing issues related to initial training and the curriculum of bachelor’s and undergraduate courses requires time and initiatives. It is evident that the educational practice for the future educator and/or teacher needs to be articulated to prepare these professionals. According to the DCNLM, they are prepared for real teaching, learning and research situations, both in Basic Education and Teaching Higher education. Additionally, they should possess a thorough understanding of the context of their training and professional qualifications (Bianchini et al., 2019).

Information about the content of the Linear System in the National Common Curricular Base (BNCC)

The BNCC is a normative document, currently in force in Brazilian schools. It is a state policy, foreseen since the Federal Constitution of 1988, which states, in its Article 205, that all Brazilians have the guarantee of education, aiming at full personal development and preparation for the exercise of citizenship. To meet these purposes, in its Art. 2010, it assumes the need to establish a minimum set of contents to be worked on in Basic Education, with the aim of ensuring basic and common training, aiming to guide learning in Brazilian schools and promote the equity of student learning (Brasil, 2017; Zandonay, 2020).

The BNCC determines the essential learning to be developed with Basic Education students, organized based on ten general competencies understood as the “mobilization of knowledge (concepts and procedures), skills (practical, cognitive and socio-emotional), attitudes and values to resolve complex demands of everyday life, the full exercise of

citizenship and the world of work" (Brasil, 2017, p. 8), which are covered in the different curricular components. In this regard, the teacher educator understands that

> (...) when you look at the documents, there are interesting elements that make it possible to work on the proposal. However, sometimes I think that teachers have not been able to keep up. When we are at university, we deal with students who have just arrived from high school, they have difficulties, for example, in knowing, dealing with an ordered pair or dealing with a polynomial. Maybe you've never heard of them in your life (...) I think the problem isn't in the proposal of the documents, which bring expected skills, which are coherent. But I can't really see if teachers are managing to put this into practice (FPJ).

The teacher educator's report sets a precedent for many discussions, both those related to the professional practice of teachers who work in Basic Education, as well as initial and continuing teacher training.

The document is organized by area of knowledge, namely: Languages and their Technologies; Mathematics and its Technologies; Natural Sciences and their Technologies, and Applied Human and Social Sciences. Each of these areas of knowledge has its specific skills to be developed throughout each teaching stage. In the case of Mathematics and its Technologies for High School, it has five specific competencies, and, for each competency, there are so-called skills. Linked to his speech, when observing the necessary teaching work, educator João highlights that, as a educator of future teachers, he realizes that there is a need for constant training, as "you will look at a simple math problem and perceive the perspective of that future teacher who is completely different from yours" (FPJ).

Considering the five competencies in the area of Mathematics and its Technologies for High School, we will focus our attention on the competency that encompasses the ability related to the content of systems of linear equations, specifically competency number three. This competence is defined as the ability to use strategies, concepts, definitions and mathematical procedures "to interpret, build models and solve problems in different contexts, analyzing the plausibility of results and the adequacy of proposed solutions, in order to build a consistent argument" (Brazil, 2017, p. 531). This competence indicates that the skills necessary for its development involve the interpretation, resolution and formulation of mathematical problems related to the topics of arithmetic, algebra, geometry, probability and statistics (Brasil, 2017). The teacher educator states that this is a component that connects, because

> [...] if there is a component that is important within the training of a mathematics student, it is the contents of Linear Algebra, this is almost unanimous. In fact, I often hear this type of argument, as it allows a lot of interaction with Basic Education (FPJ).

Regarding resolution and formulation, the document highlights that high school students "must develop and mobilize skills that will serve to solve problems throughout their lives – therefore the proposed indications must have **meaning** for them" (Brasil, 2017, p. 535, emphasis added). It also emphasizes that the construction of meanings for mathematical problems must be based on everyday school life, as the BNCC proposal considers it fundamental for learning, as pointed out in research by Moreira and David (2003), Dias et al. (2017) and Diniz and Ferreira (2019).

Competency three also highlights mathematical tasks as problems that are evident, as it considers that they must be solved by students through the mobilization of their knowledge and skills with the purpose of identifying these concepts and "designing a resolution process", as well as the possibility of building a model that can provide the generation of appropriate responses (Brasil, 2017, p. 535). We highlight, within this competence, the skill identified as EM13MA301, which can be addressed from the first year to the third year of high school. This skill focuses on solving problems arising from quantities that vary linearly, as highlighted by Diniz and Ferreira (2019, p. 8), and it

> (...) expands the work developed throughout Elementary School (in particular, in the skill EF08MA08) bringing the possibility of using technological resources to visualize the variation of quantities involved in the situation and, thus, collaborating with the development of General Competence 5 of the BNCC towards understanding and using digital technologies (...) making it easier for students to transpose between algebraic and graphic languages. This expands the repertoire of strategies for solving problems, also diversifying the form of recording that the student has to portray a phenomenon.

In this sense, the student, with mastery of this skill, will have the knowledge to work with systems of 1st degree linear equations with two or three unknowns, use graphs to understand the linear growth of equations, something evidenced in the teacher educator's speech related to BNCC and the teaching of Linear Algebra. It highlights that there is a connection with Basic Education: "(...) because it can formulate simple situations through systems of linear equations, which allows the student to seek to solve problems that involve more than one variable" (FPJ).

Diniz and Ferreira (2019) highlight that contextualized situations mobilize learning and favor problem-solving skills, allowing the construction of arguments and expanding the relationship between mathematical modeling and communication through verbal language. Depending on the didactic proposal, it is possible to use technological resources for teaching and learning systems of linear equations, which provides the opportunity to visualize the variation of magnitudes. Regarding this, the teacher educator's narrative adds that

> (...) these contents are very connected with Basic Education, but not only them. A more advanced part of Linear Algebra, such as vector spaces and linear transformations, may seem like there is no interface with the content of Basic Education, however (...) transformations in areas, you can deal with various geometric problems, which, in fact, they are linear transformations (FPJ).

The lack of these retakes implies the non-recognition of this relationship between Basic Education and what is covered in the undergraduate course, further accentuating the distance between what is taught in initial training and what actually occurs on the classroom floor. classroom (Moreira & David, 2003). In addition to the observations regarding the BNCC, the teacher educator states that it is important that this content is included in a Mathematics course curriculum (degree and bachelor's degree), due to its relevance in training "for the future teacher, because they will understand, algebraic, but also geometric aspects of Mathematics" (FPJ).

The teacher educator also pointed out that all the content, as well as the topic of systems of linear equations and linear transformations, make it possible to study different problem models. According to the educator, "(...) here at the university, we study vector spaces with inner product, it may seem something outside the reality of Basic Education, but this type of theory allows us to describe others, such as a basic internal product, which uses the trace of a matrix" (FPJ)

He considers it necessary to understand the interaction between mathematics learned in initial training and school mathematics (Moreira; David, 2003; Dias et al., 2017). Furthermore, he reports that there is no guarantee that other educators will establish this connection: "I confess that I did not observe this in practice. It is not common to see teachers who are working in higher education courses paying attention to making these interconnections, but they do exist, they exist..." (FPJ).

The Pedagogical Project of Course

The Pedagogical Project of Course (PPC)[12] for Graduation in Mathematics – Degree underwent adjustments in 2017 for implementation in 2019 (Matrix 2019), using as a guide: (a) CNE/CP Resolution No. 02, dated 01 July 2015 - National Curriculum Guidelines for initial higher education training and continuing training (Brazil, 2015); (b) two resolutions from the institution, one that presents general regulations for undergraduate courses and the second referring to the basic education teacher training policy; (c) Maintain a close relationship with

[12] The confidentiality of this PPC will be maintained.

the Bachelor's degree in certain common training subjects; (d) restructuring and reorganization of some disciplines, and (e) review of the need for prerequisites and corequisites. According to the teacher educator, in relation to this reformulation, the course went through a process of unifying the subjects.

This PPC for the Degree in Mathematics course – Matrix 2019 – aims to train a professional capable of working in Basic Education, so that they have mastery of the mathematical and didactic-pedagogical knowledge necessary for educational practice. Thus, with mathematical knowledge, logical reasoning, a critical stance and the ability to formulate, interpret and solve learned problems, they enhance the formation of contemporary citizens (Course Pedagogical Project, 2019). In addition to this, five specific objectives are presented related to teacher training for Basic Education and the construction of mathematical and didactic-pedagogical knowledge for this practice, articulation of theory and practice in the domain of scientific and didactic knowledge in the tripod that governs the Education institution Higher education: teaching, research and extension.

Before presenting the list of curricular organization, the PPC presents the guiding principles for professional training, divided into five sections, namely: (1) professional practice; (2) technical training; (3) ethical training and the social role of the professional; (4) interdisciplinarity; (5) the articulation between theory and practice. Regarding the profile of graduates on the course, the document indicates that they are able to follow 13 objectives, in addition to being guided by making considerations about the "Mathematics teacher", for 17 skills and abilities that they consider the teacher to have (Pedagogic Project of Course, 2019).

The curricular matrix of this PPC reserves a total of 2,620 (two thousand six hundred and twenty) hours, divided into eight academic periods (semesters), with 41 mandatory disciplines and 33 optional disciplines, separated by the curricular components: Common Core; Mandatory Specific Core; Optional Specific Core; Free Core and Complementary Activities. Of these mandatory disciplines, 15 disciplines are from the Common Core (bachelor's degree and degree in Mathematics) and 26 disciplines from the Specific Mandatory Core for the Degree in Mathematics course.

Of the disciplines of Common Core, nine must be taken in the first two academic periods, regardless of the academic degree (bachelor's degree or degree in Mathematics). The disciplines of the Common Core (Evening Degree and afternoon ABI), in the first period, are: Differential Calculus (96 h); Plane Geometry (64 h); Analytical Geometry (64 h); Fundamentals of Mathematics (64 h); Language and Mathematics (32 h). In the second period: Linear Algebra (96 h); Integral Calculus (96 h); Spatial Geometry (64 h); Introduction to Computing (64 h).

The PPC ensured the creation of disciplines to meet the provisions of Article 13, paragraph 2, of Resolution No. 2, of June 1, 2015, of the National Education Council. Even with this curricular organization, the teacher educator shares his concern, as this pedagogical project is once again in the process of being reformulated, with the discipline of Linear Algebra being one of them:

> (...) at the current time, it is a 2019 curriculum, the discipline of Linear Algebra, organized, was designed with great care for the mathematics course (...) the thought was to extend its workload, which even then it went from four hours a week to six hours a week, totaling 96 hours per semester. So, it is a 96-hour course designed to rescue Basic Education content, due to the difficulty of dealing with some topics (...) so that the teacher has a little more space to cover some of the deficiencies of these new students and delve into Algebra topics. Linear itself (FPJ).

The teacher educator also adds that the extension curricularization contributed to "reformulating our PPC again and is currently being done. I'm also from NDE, I'm actively participating" (FPJ). This fact had a major impact on the reformulation of the discipline of Linear Algebra, "we had the need to return the discipline of Linear Algebra to 64 hours again, so it will lose that initial momentum to rescue the students' deficiencies, because it needs to dedicate 10% of the course load to the extension" (FPJ).

It is worth noting that university extension will make up the curricular matrix of initial training courses. Regarding this, Curado Silva and Kochhann (2018, p. 710) reflect on the importance of thinking about this process of curricularization in Higher Education Institutions - HEIs, because when discussed "among the peers of the institutions the felt conception and construction of university extension through legalization, to think about how to express it in the curriculum and carry it out in a qualitative way, which benefits the academic training and society". Regarding the deficiencies and "lack of time" pointed out by teacher educator João, these can be rescued within an extensionist proposal.

To discuss this subject, it is necessary to present a plan by a educator from the institution, considering the PPC and the syllabus presented. Therefore, the next section will bring excerpts from a plan for the Linear Algebra I discipline.

Planning of the discipline: Linear Algebra I

In this section, we present a teaching plan (TP) for the discipline of Linear Algebra by the teacher educator João, developed during the second semester of 2015. This plan is based on the syllabus of the discipline, as shown in Figure 1:

Figure 1.

Syllabus and program of the teaching plan of teacher educator João (Program of the discipline Linear Algebra TP, 2015, p. 1)

02: Syllabus:

Linear systems and matrices. Vector spaces. Linear transformations. Eigenvalues and eigenvectors. Space with inner product.

03: Program:

1. Systems of Linear Equations: linear systems and matrices. Operations with matrices and properties. Elementary operations. Solutions of a system of linear equations. Determinant. Adjoint matrix and inverse matrix.
2. Vector spaces: definition and examples; Vector subspaces. Linear combination. Linear dependence and independence. Base e dimension of a vector space. Change of base.
3. Linear Transformations: definition. Linear transformations and their matrices.
4. Eigenvalues and Eigenvectors: definition and example of eigenvalues and eigenvectors. Diagonalization of matrices.
5. Internal Product: standard. Gram-Schmidt orthogonalization process. Orthogonal complement.

The discipline program links systems of linear equations to matrices and describes the contents in item 1. With regard to systems of linear equations, there are discussions about the matrix form, providing two possible approaches: one with debates about matrices before the systems of linear equations and another through the study of matrices together with systems of linear equations. Based on the Teaching Plan presented, in organizing the schedule, the teacher opts for the first approach, starting with a review of the study of matrices (six hours) before covering the content of Systems of Linear Equations (six hours).

Regarding this choice, the teacher educator highlights that the component related to systems of linear equations is one of the parts that pleases him most, as he feels that he can motivate future teachers "(...) to study, as it has the possibility to be contextualized" (FPJ). A specific approach to the study of the matrix theory of systems of linear equations treats the content of matrices in a way that is useful for solving ideal system problems. Regarding this, the educator reports that some colleagues do not propose to develop it with future teachers, opting to introduce matrix theory first and then apply it to solving systems of linear equations (Bertolazi & Savioli, 2018). Thus,

> (...) there are different ways to solve systems of linear equations. The Gaussian elimination path, in which pivoting is applied over rows or columns of the system. But there is also a way to discuss the resolution of the system, using matrix inverses. There is an old rule called Cramer's rule, which many students end up knowing. I particularly like to treat matrix theory well, to show the different paths and compare with students the number of algebraic operations that are performed, (...) allowing the student to choose possibilities later, if they think that one of them is enough, ok, but I like to offer other paths (FPJ).

The objectives are presented in Figure 2:

Figure 2.

General objectives and specific objectives of the teacher educator João's teaching plan

(Linear Algebra TP discipline program, 2015, p. 1)

05: General Objectives:

Study topics in Linear Algebra; introduce mathematical formalization; develop in the individual the ability to understand the fundamental concepts of algebra and their abilities to apply them to problems.

06: Specific Objectives:

1. Allow the understanding and elaboration of mathematical arguments through symbolic language. 2. Introduce basic operations in matrix space and extend the ideas to other vectors. 3. Provide the student with an integrated view of Linear Algebra concepts and their applications. 4. Make the student capable of recognizing and solving problems in the area.

07: Methodology:

The program will be developed, essentially, using exposure on the board and reflections on approaches made through solving exercises, discussing problems or demonstrations. Lists of exercises and problems will be presented to students to create the habit of frequent study and analysis of the content covered, in addition to promoting the development of skills and encouraging creativity in solving problems. The use of other bibliographies in addition to textbooks will be encouraged for theoretical complementation and additional examples. Furthermore, whenever possible, comments on computational approaches will be presented. The teacher will, when necessary, change the order of the syllabus units and redistribute the hours allocated to each topic.

Upon reviewing the objectives, it is evident that the educator organized, for that year, to introduce formalization, as well as work on argumentation through symbolic language, with a view to geometric motivations in the second dimension; in the system classification part, he develops nullity (specific objective number 2) and works on contextualized problems, as it is a case of restriction on the set of solutions (specific objectives numbers 3 and 4).

Regarding planning, the teacher educator states that he organizes it so that two moments occur, one for collective discussion with future Mathematics teachers and the other for approaching concepts and properties, as well as the possibility of using technological resources. In relation to technological resources, he considers it to be a relevant moment to work on the topic with future teachers through examples that can be solved with technological tools. He exemplifies this in the narrative "(...) he can do a Gaussian elimination, using a spreadsheet, there is no need to have specific Mathematics software, I even separate some classes to show these possibilities" (FPJ).

Regarding the methodology, it is described that the teacher will use the presentation on the board, as well as reflections on approaches taken in solving exercises, discussing problems or demonstrations, lists of exercises. It is intended to promote the development of skills and encourage a variety of strategies in solving problems. In this part, there is also an indication of the use of other bibliographies, in addition to those proposed in the course syllabus, as well as the possibility of presenting comments on computational approaches, possible readjustment in the order of the program content units and the redistribution of the workload.

Regarding the use of technological resources, it adds meaning to solving problems with questions and discussions:

> (...) this can often have the opposite effect, just with the technological resource, so I face a situation where the machine fails. I set the correct parameters, but the response that the software presents is not compatible with the problem we have (FPJ).

The educator is concerned about the criticality of the future teacher, as the mathematical answer to a problem does not always solve a physical problem: "the solution is mathematical, acceptable, but it does not solve the physical problem. You have to criticize, you have to restrict the mathematical solution, so as I said, I like to provoke, to make this type of movement when possible" (FPJ).

Regarding the evaluation, three individual writings are proposed, with the dates of their occurrences established. In this regard, the teacher educator shares that regarding evaluation processes, it was an area that he considers having made the least progress in his teaching performance "not just due to lack of will, the institutional or behavioral issues themselves, which cause a little hardening" (FPJ). As Luckesi (2013) rightly relates, this distance and hardening between theory and assessment practice is a characteristic of almost every educational system, historically marked by traditionalist, classificatory and exclusionary means of assessment.

The educator states that, during the pandemic, teachers were forced to have a new look at assessments, which went through different experiences: "the results were not so positive because I began to observe that this left room for students not to dedicate themselves to the way I expected" (FPJ). When using the available resources, the teacher educator observed many repeated records, in which he was unable to see "the subject within his assessment, it was as if I saw the same assessment for many different subjects" (FPJ). At this moment, the educator socializes his experience related to post-pandemic teaching, of the great pedagogical and

cognitive loss of students in initial training, without realizing that, when talking about an event, he focuses only on the aspects of the graduate:

> I had a situation, which illustrates this, gives a shout out about evaluation. Last semester we had a remote part of the semester and transitioned to in-person. There was a student who scored 8 and a half in two remote assessments, and there was a single in-person assessment, in which this same student scored 0.3. **I try to understand how the teacher evaluates this student, in which, without supervision, he gives you back the material that you can evaluate out of eight and a half, when he has supervision, he gets a 0.3... I don't have an answer for that** (...) so the evaluation is something that, in fact, I still encounter some obstacles (FPJ, emphasis added).

In addition to this shared experience, the teacher educator detailed how he generally organizes assessment processes in the subjects he teaches, such as: tests; seminars and, depending on the type of discipline, project development. He points out that: "when I teach numerical calculation, this subject allows many actions to be applied to students as a team, with a project that starts from theory to the application of concepts, with critical discussion of the results" (FPJ).

Teacher educator João complements the narrative by stating that there are disciplines that do not allow them to be developed in this way. Thus, assessments are direct applications, using the test format. This obstacle to evaluation allows us to talk about the importance of evaluative moments that can explore and evaluate errors and how they provide opportunities for rethinking and teaching. In them, the primary interest will be in examining them regarding their pedagogical effectiveness in the learning process, as observing the perceptions of students' errors "as a center of reflection on pedagogical practice, aiming at the possibility of, through its logic, that the same be used in a positive and constructive way as a guiding resource for the teacher and his teaching [re]-doing" (Félix Correia, 2010, p. 183). In view of the narrative analyses, the next section aims to present the discussions and final considerations of this study.

Discussion of results

The results show how the personal experience of the teacher educator João shaped and influenced the construction of his identity as a higher education instructor (Dalby, 2017; Ping et al., 2018; Belo et al., 2021). This identity, intertwined with his narratives and professionalism, plays complex and varied roles, potentially linked to his critical personal reflection, giving meaning to past personal experiences, and facilitating transformations in his role as a teacher educator and researcher (Morgado, 2011; Dalby, 2017).

The narrative analysis conducted reveals the complexity involved in the development of the professional identity of a Mathematics teacher educator (Marcelo, 2009; Losano & Fiorentini, 2018; Ping et al., 2018). It also highlights the human aspect of teacher educator João as he perceives himself in this role, even if it is belatedly recognized in his practice as an educator (graduate). The narrative underscores the considerable effort required in this context (Morgado, 2011; Dalby, 2017; Losano & Fiorentini, 2018; Cunha Neto & Costa, 2018; Belo et al., 2021).

In relation to this, the reflections of teacher educator João on his initially content-focused approach are noticeable, aligning with the observations of Losano and Fiorentini (2018, p. 22), who, in the results of their study, assert that the training of mathematics teachers "is conceived purely in terms of academic mathematics, and participants experience strong tensions in the development of their professional identity," which directly influences the future practice of the teacher or teacher educator.

Observing the documents, it is apparent that the organization of the curriculum structure of the course was reflected and thought out to relate it to the pedagogical practices in the training of future Mathematics teachers. As an institutionalized text, the course curriculum provides brief indications in formulating this curriculum, presenting the mathematics disciplines as well as didactic-pedagogical disciplines, even though the syllabi continue to be traditionally organized (Bianchini et al., 2019; Zaidan et al., 2021).

Regarding the preparation of a Teaching Plan (TP), it is a process for decision-making during the course of a discipline. In the organization of the plan, it is observed that the teacher educator sought to work on formalization and argumentation. It is worth reflecting that there is a concern to address the contents of the discipline syllabus, with reflections in the narrative of teacher educator João about his concern in teaching Linear Algebra content (Rodriguez, 2011; Barros; 2018; Ramírez-Montes, 2020).

The discipline of Linear Algebra, observed at the HEI in this study, makes up a basic area for those entering undergraduate and bachelor's degrees and, until the end of 2022, had a course load of 96 hours. The disciplines that had their workload expanded, with a view to training future Mathematics teachers, currently cause concern for teacher educator João. This

is because, with the curricularization of the extension, Linear Algebra will now have a minimum load mandatory. Depending on the teacher educator responsible for its instruction, this may still imply that it lacks the possibility of establishing connections with Basic Education (Dias et al., 2017; Barros et al., 2019; Stewart et al., 2019; Ramírez-Montes, 2020).

From the point of view of teacher educator João, it is evident that he presents the importance of mathematical knowledge, as well as didactic knowledge for the future teacher, as he considers it important that there is connection, indicating that they do not just occur in isolation (Moreira & David, 2003; Dias et al., 2017; Ramírez-Montes, 2020).

Final considerations

To meet the objectives and questions of this study, we made some considerations based on the narrative as a data source. Processes of reflection on the construction of the professional identity and pedagogical practices of teacher educator João are observed. This understanding was possible from the analysis of documents BNCC, DCNLM, PPC and the TP (which was prepared by the teacher educator João). This process made it possible to outline his trajectory by portraying some processes of construction of his professional identity as a teacher educator.

In their speeches, some conceptions related to the knowledge of the undergraduate student when entering university are observed, especially when the teacher educator, when questioned about the BNCC's skills and abilities, presents concerns when understanding that many students enter the initial training course. without minimal mathematical knowledge, which is considered worrying, considering that many of these students have already entered straight into graduation after high school. In this context, one can see, in his speech, a view considering that the problem does not lie in the proposed competencies indicated in the BNCC. In this study, the teacher educator focuses only on the teacher who teaches in Basic Education when indicating that he is not able to follow the skills or put them into practice.

In view of some speeches by teacher educator João, the lack of teaching experience in Basic Education is evident (at least in this study), as he did not present relationships of the different natures and learning that are associated with pedagogical practices and there was no reflection on the difficulties faced in everyday school life and not the ways or strategies to

overcome them. Regarding the content of the system of linear equations, there is recognition by teacher educator João that this content is included in a Mathematics course curriculum, as it covers knowledge that goes beyond algebraic.

Regarding the TP, it appears that it meets the discipline's syllabus, but does not present information aimed at training alternatives that involve future teaching practice in teaching this content. In the speech of teacher educator João, there is a strong tendency in the explanations of the contributions that involve mathematical knowledge of this content, considering that there are no notes with significant references nor for teaching the system of linear equations in High School. This is something already evidenced in the research by Rodriguez (2011) and Ramírez-Montes (2020), in which the authors present the formalization in the teaching of Linear Algebra as a discipline in Higher Education. Its teaching in the classroom follows a pattern, valuing theory and little practice (Rodriguez, 2011; Dias et al., 2017; Barros, 2018; Ramírez-Montes, 2020), which we infer is strongly under the exercise paradigm (Skovsmose, 2000).

From this, it is worth reflecting that, even though we have a PPC that meets the DCNLM and presents as its objective professional training to work in Basic Education, through the syllabus of the Linear Algebra I discipline, it is evident that it is structured in the contents that include it There are also some high school contents (such as a system of linear equations); however, through the teacher educator's EP and even when narrating his teaching practice, he informs that, when possible, he "tries" to make the teacher understand that certain content is present in Basic Education; Therefore, it is neither evident nor guaranteed that other HEI teacher educators establish opportunities for undergraduate students to have: (1) mastery of pedagogical didactic knowledge for future teaching practice (as recommended by the DCNLM and the PPC itself) and, (2) that these academic mathematics contents establish interfaces with the school mathematics necessary in high school.

Regarding the approaches to the "floor" of the Basic Education classroom, due to the organization and the syllabuses themselves, it was not possible to say whether and how this theme is developed in the classroom, the relationships of school mathematics or a perspective approach. This is because it is not made clear that knowledge is, in fact, a construction and not an imposition, as the PPC itself recommends in its document related to the initial training of the

future teacher in the Degree in Mathematics, something evidenced, including, in the speech of the teacher educator João.

The teacher educator João himself, in the narratives, presents personal accounts that show the construction of his professional identity. Beginning with a degree and bachelor's degree in Mathematics, followed by a master's and doctorate in the same field, João reflects on his own initial training challenges, particularly the limited disciplines he studied. he reflects on his actions, shares obstacles in his own initial training related to the few teaching subjects studied, as well as being an initially content teacher in Mathematics. Despite initially focusing solely on content in Mathematics, he now critically examines his role as an educator, engaging in introspective analysis and observing the context he operates in. Since 2004, he has also been researching the challenges faced by students in Basic Education regarding Algebra. Additionally, he actively participates in informal training processes aimed at professional development (Silva et al., 2023), providing continuous opportunities for self-reflection on his teaching practice.

It is worth reflecting that the training of some Mathematics teachers in Higher Education presents itself as a current concern. These educators often transition directly postgraduate programs in Mathematics to be responsible for the training of future teachers in Mathematics Degrees. This transition demands a high level of responsibility for individuals who may not have received specific training in teacher education (Gonçalves, 2000; Ping et al., 2018; Belo et al., 2021). There are also some teacher educators who do not recognize themselves as teachers in their degree, resulting in their non-participation in continuing education spaces that involve reflecting on teaching practice or on the teaching and learning processes in higher education. This is often due to their perceived identity as mathematics researchers, which has impacts, as they do not always, as subjects they train, realize the links with Basic Education.

In this case, the teacher educator has a fundamental role in the organization and planning of disciplines and the future teaching practice of their students in initial training. Therefore, it is necessary for Mathematics teacher educators not only to identify themselves as researchers, but to establish links with teaching and Basic Education, as educators of future Mathematics teachers. In this study it is considered that the academic and professional trajectory of a teacher educator can contribute and interfere in the planning of an algebraic discipline, which, as seen

both by the TP presented and in the narratives of the teacher educator João, impact the teaching of systems of linear equations.

It is considered, then, that teacher educators must be prepared to teach the disciplines in a Mathematics Degree course, teach at levels and adapt their knowledge and teaching, that their teaching practice requires qualities and skills that go beyond knowledge specific mathematical content (Coura & Passos, 2017; Lozano & Fiorentini, 2018, 2021). Regarding the discipline of Linear Algebra, there should be opportunities to reflect on the technicist vision of its teaching (Bertolazi & Savioli, 2018; Bianchini et al., 2018; Ramírez-Montes, 2020).

Regarding the work of the teacher educator, the preparation of a TP is important for organizing choices and decision-making during the teaching of a discipline, so that there are considerations when planning a particular discipline aimed at the Mathematics Degree course, which can consider a means for reflection that involves academic mathematics and school mathematics and not just a formalization. That the act of planning is seen by Mathematics teacher educators as a reflective part of their teaching practice (Serrazina, 2017; Pina Neves; Fiorentini; Silva, 2022).

This research highlights the need to study the roles and identities of Mathematics teacher educators, the weaknesses of PPC and TP related to the teaching practice of teacher educators and the mathematics taught in undergraduate courses and the lack of articulation with mathematics school. With this, we hope to better understand who teacher educators are, what they do, how they understand their roles and professional development, but also how they see themselves as educators of future teachers (Santos, 2016; Cunha Neto & Costa, 2018).

To continue this study, we consider possibilities for involving teacher educators in training contexts that allow the improvement of their training and teaching practice (Silva et al., 2023). Additionally, we observe that these teacher educators are conscious of their social role, continuously engaging in professional development, as pointed out by teacher educator João at various moments narrated in this study.

Acknowledgements

To the Study and Research Group on Science, Mathematics and Sexuality Education (In Portuguese, Grupo de Estudos e Pesquisa em Educação em Ciências, Matemática e Sexualidade – GECIMAS/UFABC); the Mathematics Teaching Research Group (In Portuguese, Grupo de Investigação em Ensino de Matemática GIEM/UnB); to the Postgraduate Program in Teaching and History of Science and Mathematics (In Portuguese, Pós-graduação em Ensino e História das Ciências e da Matemática – PEHCM/UFABC) and to the Coordination for the Improvement of Higher Education Personnel – Brazil (In Portuguese, Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – CAPES) – Financing Code 001.

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