

ChatGPT and Mathematics Education

O ChatGPT e Educação Matemática

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Abstract

The present article aims to discuss and address pertinent issues related to the new possibilities and challenges imposed on mathematical education due to the popularization of artificial intelligence. However, what exactly is AI? Let's reflect on whether today's ChatGPT is simply an updated version of the calculator from forty years ago, the desktop computer known as the "PC," and the internet from earlier phases of digital technology in mathematical education. The text seeks to consider recent news on the subject, as well as a series of simple tests conducted by the authors using this resource, in addition to exploring ethical issues and the generation of problems for ever-changing collectives of humans with digital technologies. The study suggests that we will still face immense ethical concerns, but it will be of paramount importance to consider problems to be solved for this type of collective.

Keywords: Mathematical Education, Artificial Intelligence, ChatGPT.

Resumen

El objetivo de este artículo es debatir y abordar cuestiones relacionadas con las nuevas posibilidades y retos que plantea a la enseñanza de las matemáticas la popularización de la inteligencia artificial. Pero ¿qué es exactamente la IA? Consideremos si el ChatGPT actual no es simplemente una versión actualizada de la calculadora de hace cuarenta años, el ordenador

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de sobremesa conocido como "PC" e Internet de las fases anteriores de las tecnologías digitales en la educación matemática. El texto pretende considerar noticias recientes sobre el tema, así como una serie de pruebas sencillas realizadas por los autores con este recurso, además de explorar cuestiones relacionadas con la ética y la generación de problemas para colectivos de seres humanos en constante cambio con las tecnologías digitales. El estudio sugiere que aún nos enfrentaremos a inmensas preocupaciones éticas, pero será de suma importancia que pensemos en problemas a resolver para este tipo de colectivos.

Palabras clave: Educación Matemática, Inteligencia Artificial, ChatGPT.

Résumé

L'objectif de cet article est de discuter et d'aborder les questions relatives aux nouvelles possibilités et aux nouveaux défis posés à l'enseignement des mathématiques par la popularisation de l'intelligence artificielle. Mais qu'est-ce que l'IA exactement ? Demandons-nous si le ChatGPT d'aujourd'hui n'est pas simplement une version actualisée de la calculatrice d'il y a quarante ans, de l'ordinateur de bureau connu sous le nom de " PC " et de l'Internet des phases précédentes des technologies numériques dans l'enseignement des mathématiques. Le texte cherche à prendre en compte les nouvelles récentes sur le sujet, ainsi qu'une série de tests simples menés par les auteurs avec cette ressource, en plus d'explorer les questions liées à l'éthique et à la génération de problèmes pour les collectifs d'êtres humains en constante évolution avec les technologies numériques. L'étude suggère que nous serons toujours confrontés à d'immenses préoccupations éthiques, mais qu'il sera primordial de réfléchir aux problèmes à résoudre pour ce type de collectif.

Mots-clés : enseignement des mathématiques, intelligence artificielle, ChatGPT.

Resumo

O presente artigo tem como objetivo discutir e abordar questões pertinentes às novas possibilidades e desafios impostos à educação matemática devido à popularização da inteligência artificial. Contudo, o que exatamente é a IA? Vamos refletir se o ChatGPT de hoje não é simplesmente uma versão atualizada da calculadora de quarenta anos atrás, do computador de mesa conhecido como "PC" e da internet das fases anteriores das tecnologias digitais em educação matemática. O texto busca considerar as recentes notícias sobre o tema, bem como uma série de testes simples conduzidos pelos autores com esse recurso, além de explorar questões relacionadas à ética e à geração de problemas para coletivos de seres humanos em constante mudança com as tecnologias digitais. O estudo sugere que enfrentaremos ainda

preocupações éticas imensas, mas será de suma importância que pensemos em problemas a serem resolvidos para esse tipo de coletivo.

Palavras-chave: Educação Matemática, Inteligência Artificial, ChatGPT

ChatGPT and mathematics education

The year 2023 was marked by the astonishing arrival of ChatGPT, developed by the OpenAI artificial intelligence research laboratory based in San Francisco, United States. This new technology, equipped with artificial intelligence (AI), initially promised to solve mathematical problems and even draft scientific articles. Furthermore, it could develop computer program codes through interactions in natural language, in a process akin to a conversation between humans (OPENAI, 2023). However, important experts and technology industry leaders, who were initially enthusiastic about the project, are now calling for a halt in the development of this chat, as it could jeopardize democracies or even dominate the human race (Future of Life Institute, 2023).

Does ChatGPT represent the end of mathematics teachers? Does it signify the beginning of the end for education? In this editorial, we will address these questions and attempt to reflect on whether today's ChatGPT is merely an updated version of the calculator from four decades ago, the desktop computer known as the "PC," and the internet from earlier phases of digital technologies in mathematics education, as highlighted by Borba, Silva, and Gadanidis (2020). As members of the research group GPIMEM, focusing on digital technologies and mathematics education, we take advantage of the occasion of the 25th anniversary of the journal *Educação Matemática Pesquisa*, which has already published several articles on this topic. However, are we facing a different moment now, with researchers calling for a halt to AI research?

In this essay, we propose to attempt to answer these inquiries and raise other relevant questions. Nevertheless, we acknowledge that before proceeding, it is crucial to provide the reader with a comprehensive understanding of what Artificial Intelligence is.

But what is AI?

There are diverse types and definitions of AI, as there are various approaches and tools with different degrees of sophistication and applications. Artificial intelligence can relate to different processes of thinking, reasoning, and behavior, as well as to rationality, as pointed out by Russell and Norvig (2022). It encompasses a wide range of areas, from search algorithms and content recommendation to the control of autonomous cars and industrial robots. Therefore, AI is an extremely comprehensive and diverse field. Some enthusiasts believe that these technologies have the potential to eradicate hunger and war, while pessimists fear that they might drive us to extinction.

Indeed, as researchers and educators, we have heard similar stories over the years. The introduction of new technologies, such as the calculator and computers, has always raised concerns and questions about their impact on students' reasoning abilities and the future of the teaching profession. Additionally, the popularity of digital videos and video lectures has also sparked debates about the possible end of schools and teachers.

These questions have been recurrently asked over the past five decades. However, it is important to note that although technologies have brought significant changes to the educational field, they do not entirely replace the role of the teacher. In fact, these tools can be leveraged as supplementary resources and provide support to the teaching and learning process.

For instance, the calculator allowed students to perform calculations more quickly, freeing up time for the development of advanced mathematical skills and promoting new ways of learning (Selva; Borba, 2010). Computers expanded access to information and enabled the creation of interactive learning environments. Digital videos and video lectures offered new ways of presenting content, stimulating visualization and understanding among students, even though there is a discussion about whether the use of videos is creative or merely reproduces the dynamics of traditional lectures (Borba; Souto; Canedo Jr, 2022).

Therefore, despite the significant transformations brought about by technologies, the figure of the teacher remains essential in the educational process. The role of the teacher goes beyond knowledge transmission, involving mediation, encouragement of critical thinking, guidance, and individualized student support. Thus, instead of being replaced by technologies, teachers can utilize these tools as allies to enrich teaching and promote better learning experiences.

But will it be different this time?

Indeed, we are witnessing significant advancements in the field of generative artificial intelligence, which can be considered a disruptive technology. According to Bill Gates, one of the founders of Microsoft, this technological progress is the most important in decades (BBC News Brasil, 2023a). The ability of generative AI systems to create entirely new content, be it in the form of text, images, audio, or videos, is something that can have a profound impact across various domains.

A notable example is the capabilities of ChatGPT, a language model that utilizes generative artificial intelligence to produce original responses based on prompts or questions. It is capable of generating coherent and contextually relevant text, even without direct access to specific information. The inputs to interact with this tool are referred to as "prompts", which

essentially consist of commands that provide instructions or examples of how to successfully complete a task (OPENAI, 2023). This ability to autonomously generate content could have significant implications in areas such as virtual assistance, customer service, content creation, and education.

An interesting feature is the integration of these chatbots with other platforms through APIs. This means that they can be incorporated into applications like browsers, text editors, instant messengers, customer service chatbots, and others, expanding their potential uses in conjunction with other digital tools (Openai, 2023). With this integration, there is no longer a need to access a specific website to ask questions, making interaction with generative AI more seamless and convenient.

This capacity for creative and personalized responses, within a familiar and everyday context, makes interaction with generative AI more appealing and relevant to various audiences and situations.

Other notable examples include Midjourney and DALL-E, which represent significant advancements in the field of artificial intelligence. While previously robots were used for text analysis and facial recognition, they are now capable of generating new content and producing original images (O Globo, 2023). They can generate original illustrations based on provided descriptions or concepts. These technologies have the potential to impact areas such as graphic design, advertising, entertainment, and much more.

In light of these advancements, it is important to be mindful of the potential impacts and ethical challenges that generative AI may bring. Considerations such as the authorship of machine-generated content, the spread of misinformation, and the potential for manipulation need to be taken into account. Responsible development and the establishment of ethical guidelines are crucial to ensure that generative AI is used in a beneficial and conscientious manner.

However, it is important to highlight that even with these advancements, generative AI is still evolving and has limitations. Artificial intelligence requires extensive training with large volumes of data to make accurate predictions. The development and implementation of machine learning models involve various stages, such as training and inference (Oracle, 2023).

It relies on vast amounts of data to learn and generate content and does not always produce flawless or bias-free results. Therefore, while it is a promising technology, careful and critical oversight of its development and application is still necessary.

Nevertheless, it is worth noting that the generative nature of these systems can raise concerns about their reliability. It is worth remembering that intelligent systems integrated into

society are potentially dangerous, even without consciousness, as harm is more due to the competence and the extent of the impact radius of actions.

These systems have the potential to be highly detrimental to society, and one of the reasons for this destructive possibility is the lack of reliability in the information they provide. Despite their apparent intelligence, these systems have no idea what they are talking about and sometimes "hallucinate" a term in AI that refers to responses that are out of touch with reality (BBC News Brasil, 2023b).

These programs have the ability to create invented information and present it to users as facts. Furthermore, it is important to consider that these tools are not neutral and can be used by individuals with malicious intent to create and spread misinformation, fake news, or attack reputations, driven by personal or corporate interests.

Currently, researchers are pursuing general intelligence, which is a form of artificial intelligence capable of handling a wide variety of tasks and learning and adapting similarly to human intelligence.

How Can ChatGPT Assist in Mathematics Education?

The use of ChatGPT in Mathematics Education, despite some limitations and issues, presents challenges and opportunities. The tool has the potential to create personalized exercises tailored to the students' level and ability. However, the ease with which AI handles natural language can pose a challenge for plagiarism control. Despite limitations, Gonzalez, after conducting experiments with this technology, believes that, with the intentionality of the teacher, the tool can be a great opportunity in teaching mathematics (Gonzalez, 2023).

To assess the capabilities and accuracy of ChatGPT, we also conducted a series of simple tests. One of the mathematical problems addressed was in financial arithmetic, specifically a problem involving the installment payment of a purchase with an initial down payment and fixed installments. The problem involves calculating the value of each installment, taking into account the total purchase amount of 1200 reais, the provided down payment of 300 reais, and the agreed upon number of installments, which is 12. This type of problem is commonly encountered in situations involving installment purchases or financing.

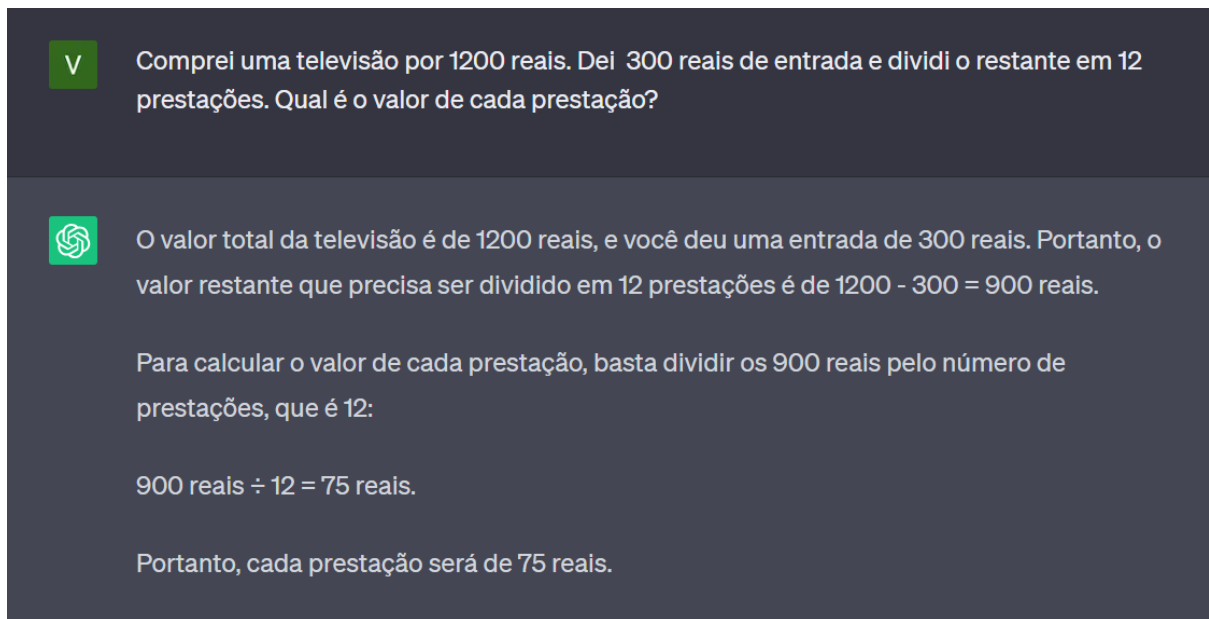


Figure 1.

Response of ChatGPT to a Financial Arithmetic Problem

The performance of ChatGPT in solving the problem was quite satisfactory. The response provided a clear and detailed explanation of how to calculate the value of each installment based on the given information. Firstly, ChatGPT correctly identified that the total cost of the television is 1200 reais and that an initial payment of 300 reais was made. Then, the model subtracted the initial payment from the total cost to obtain the remaining amount to be financed, which is 900 reais.

Afterward, ChatGPT correctly divided this remaining amount by the agreed-upon number of installments, which is 12, to calculate the value of each installment. The obtained result, 75 reais, was presented clearly and accurately. The explanation was well-structured, outlining the necessary steps to arrive at the final answer. Additionally, the model demonstrated an understanding of the concept of installment purchases and how to calculate the value of each installment.

In this problem, ChatGPT provided a thorough and complete response, standing out for the quality of explanations and presented details. This showcases the model's ability to provide clear and useful information in mathematical problems of this nature.

Do you believe that the truth always prevails? Then get ready to face one of the most intriguing paradoxes in the history of logic: the Liar Paradox. Will ChatGPT be able to unravel the enigma and offer a coherent logical answer to the Liar Paradox? After all, how does a machine handle intricate logical contradictions?

Imagine a child who unfailingly always tells the truth. Now, this same child makes a perplexing statement: "I am lying". How do you deal with this situation? Is it possible to determine whether the child is telling the truth or lying?

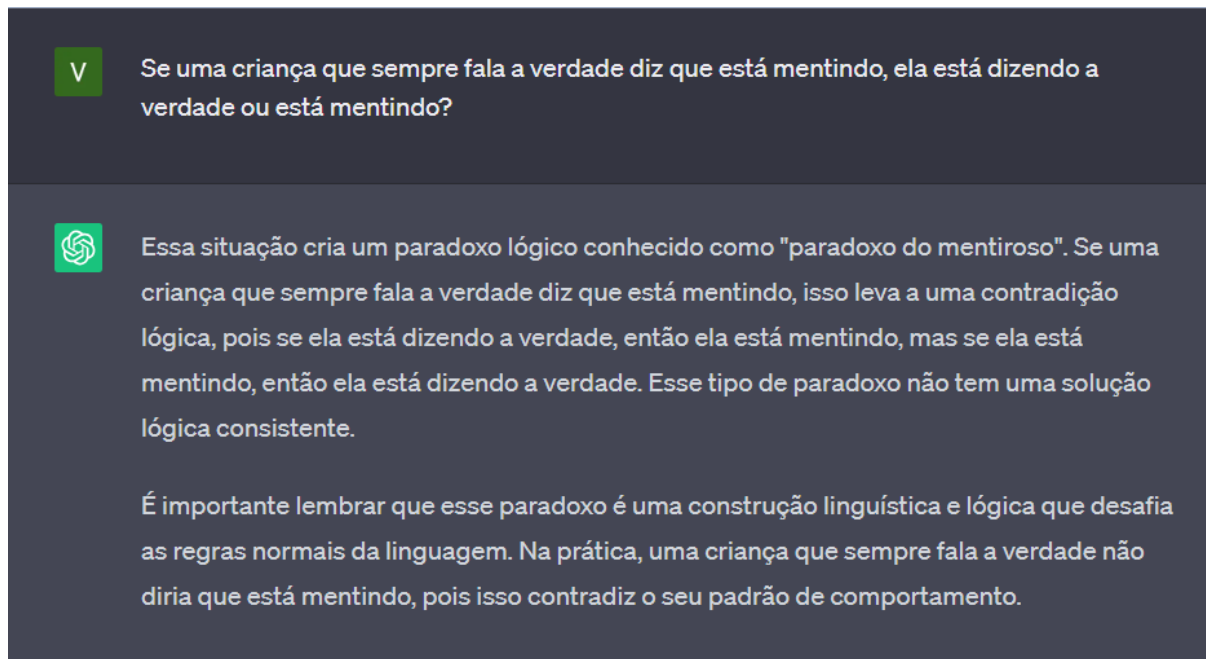


Figure 2.

ChatGPT's Response to a Logic Problem

ChatGPT correctly explained that the paradox arises from a fundamental contradiction. If the child is telling the truth by claiming to be lying, then they are lying. On the other hand, if they are lying, then they are telling the truth. This explanation reveals a solid understanding of the paradox and the logical implications involved.

Furthermore, the model recognized that the paradox does not have a consistent logical solution. This demonstrates that the AI is aware of the logical limitations and the challenging nature of the liar paradox, emphasizing that there is no clear or coherent answer within logic.

However, it's important to note that ChatGPT cannot experience the same sense of bewilderment or emotional ambiguity that humans might encounter when faced with this paradox. The model provides a logical response based on its programming and training, but the emotional understanding and subjective experience associated with the paradox may be unique to humans.

AI doesn't always get it right. Let's present another problem, involving a geometry question related to the area of a regular hexagon with one side measuring 12 cm.

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Um hexágono regular é formado por seis triângulos equiláteros com os lados medindo 12 cm. Determine a área do hexágono.



Para determinar a área de um hexágono regular, precisamos conhecer o comprimento de seus lados. No caso, você já forneceu essa informação: os lados do hexágono são triângulos equiláteros com 12 cm de comprimento cada.

A fórmula para calcular a área de um hexágono regular é:

$$\text{Área} = (3\sqrt{3} * \text{lado}^2) / 2$$

Onde "lado" representa o comprimento de um lado do hexágono.

Substituindo o valor de lado na fórmula, temos:

$$\text{Área} = (3\sqrt{3} * 12^2) / 2$$

$$= (3\sqrt{3} * 144) / 2$$

$$= (3 * 12 * \sqrt{3}) / 2$$

$$= 36\sqrt{3}$$

Portanto, a área do hexágono regular é igual a $36\sqrt{3}$ cm².

Figure 3.

ChatGPT's Response to an Area Problem

During the problem-solving process, the AI started well by detailing the steps and presenting the correct formula to calculate the area of a regular hexagon. However, as it progressed in the calculations, the AI got lost and made errors, resulting in an incorrect answer. The correct solution to the given problem is an area with a measurement of $216\sqrt{3}$ cm². However, the AI provided the value of $36\sqrt{3}$ cm², demonstrating an inaccuracy in its calculation process.

Mathematics Education and ChatGPT

Will ChatGPT be able to answer all mathematical questions in the future? There are reports that ChatGPT performs well in the Brazilian National High School Exam (ENEM), but not as well in the mathematics section of our exam according to recent news (Folha De São Paulo, 2023). Students of the first author of this editorial have used ChatGPT to help them

create a video that was presented as one of the assessments in a university course. Was the professor right to accept it?

Again, the answer is not easy. Nevertheless, we would like to share a perspective on knowledge production that emphasizes the importance of technologies. Villarreal and Borba (2010) illustrate how compasses, rulers, and various other technologies have been important in the history of mathematics. They also suggest that the abundance of paper is a condition that has allowed for lengthy pencil-and-paper demonstrations to be developed over nearly two centuries.

This historical perspective has been developed from works that view mathematical knowledge being produced by collectives of humans-with-media. In this view, whether through orality, writing, or digital technologies, knowledge is transformed. Thus, while the results may remain the same, mathematics expressed is different when shaped by different media. In his classic book "Intelligence technologies", Lévy (1993) refers to orality, writing, and informatics as technologies of intelligence. In the research of GPIMEM, several members use this notion to say that mathematics is generated differently. With an emphasis on software such as Cabri, or the software used in graphing calculators, it is possible to emphasize an experimental approach, similar to that practiced in the natural sciences (Borba; Villarreal, 2005).

Orality shapes different ethnomathematics of peoples who do not have writing like us; writing shapes what is considered academic mathematics, and ever-changing technologies have shaped areas of mathematics and mathematics education as a whole. In our classes, the combination of various technologies also shapes the type of knowledge developed. In this sense, knowledge is seen as a product of humans but also of non-humans, of things. Different humans, different technologies develop mathematics, and socially, what is considered valid is agreed upon. During the pandemic, digital technologies that were seen by many educators as potentially dulling students' thinking were adopted, supported or not by research like that already published in this journal. Borba (2021) discusses how different trends changed during the pandemic, particularly digital technologies. He illustrates how technologies, in this delicate moment, altered what it means to be human, and how different humans shaped and used these digital technologies.

In this sense, it is possible to think of ChatGPT as another digital technology, and we will have to consider how a collective of humans-with-ChatGPT produces knowledge. In GPIMEM, it has been argued that technologies have agency, the power of action (Cunha, 2023). While this assertion was often contested in the case of technologies like the famous Cabri software, Derive, among others, it is less contested when the technology in question is

ChatGPT, artificial intelligence. Researchers who did not adopt this notion propose that now is the time to consider the power of media. With the public's knowledge of ChatGPT, the role of media has become more evident, whether it be orality, writing, compasses, Geogebra, in collectives of humans-with-media that produce knowledge.

Is it possible for ChatGPT to answer everything?

One way to think about this is to refer to the notion of a problem. What is a problem? Polya addresses this question by explaining how to solve mathematical problems. Much later, Borba (1987) discusses this issue to underpin matters concerning ethnomathematics. He asserts that a problem has an objective aspect - "I don't know something" - and a subjective aspect - "I am interested in knowing". This interest is linked to culture and is the root of a given ethnomathematics, which is mathematics enveloped in a culture, stemming from a cultural interest.

This view has been used over the course of twenty-five years in the development of the construct "humans-with-media" (Borba et al, 2023, forthcoming). The idea was developed to say that plotting the graph of $y=x^2+5x+6$ can be a problem, if there is interest, for collectives involving student-pencil-and-paper. However, it won't be a problem for a collective involving software like Winplot or Geogebra because the answer is automatic, meaning there's no non-objective aspect. Much of the research conducted in GPIMEM resulted from or was supported by generating problems for collectives of humans - constantly changing with digital technologies - using digital technologies that are created at all times by humans-with-technologies. Borba (2009) asks: what will become of the classroom if the internet is embraced? There's no way around it... we will have to, as we partly did, generate problems for collectives of humans-with-internet. Today, no one dismisses the use of the internet in mathematics education, although the discussion about the role it should play is extensive.

Do we mean that ChatGPT should not raise concerns and that we only need to think about problems to be solved by collectives of humans-with-ChatGPT? No! It's important to research ChatGPT and think about problems for this type of collective, just as Borba, Souto, and Canedo Jr (2022) did the same to think about problems for collectives of humans-with-videos.

On the other hand, ChatGPT and artificial intelligence bring immense ethical concerns, as they have been used to generate bubbles of fake news, with strong support from conservative sectors that want to insist that the world was made only for an elite and not for all. The recent

resurgence of flat Earth beliefs shows how algorithms, combined with actions from extreme right sectors of society, can revert to pre-Galileo theories. Furthermore, as Minister Flávio Dino has consistently alerted us, the algorithms behind artificial intelligence are not neutral (UOL Notícias, 2023). And if they influence global politics, what happens to education? How will we know how teachers and students are being influenced by undisclosed hidden algorithms?

The debate about ChatGPT and artificial intelligence in education in general, and in mathematics education in particular, is certainly going to intensify, and we hope this editorial will help guide the discussion.

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