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Knowledge associated with affine functions manifested by high school students

Conocimientos asociados a la función afín expresados por estudiantes de secundaria

Connaissances associées à la fonction connexe exprimées par les lycéens

Conhecimentos associados à função afim manifestados por estudantes do Ensino Médio

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Abstract

The investigation presented in this article is part of a master's research project conducted by the first author, which examined the complexities of subclasses of mixed situations within the class of multiplicative comparison and measure transformation associated with affine functions. For this purpose, a research instrument comprising three situations within this category was designed and applied to a group of 34 students enrolled in the third year of high school at a public school in northwestern Paraná, Brazil. Data were collected through students' written solutions and audio recordings of their dialogues while working in pairs or trios. Specifically, this article presents analyses of the strategies and theorems-in-action mobilized by high school students in solving a mixed situation associated with an affine function, related to the subclasses of multiplicative comparison (unknown referent) and measure transformation (positive transformation with an unknown final state). The results revealed difficulties in interpreting the rate of change of the affine function and in calculating percentages, as

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well as misunderstandings regarding the constant term of the affine function. Based on the analysis of students' schemes and group dialogues, four true and five false theorems-in-action were identified.

Keywords: Mathematics education, Mixed situations, Linear function, Theorem-in-action, Complexity.

Resumen

La investigación presentada en este artículo forma parte de un estudio de maestría realizado por la primera autora, que investigó las complejidades de subclases de situaciones mixtas de la clase de comparación multiplicativa y transformación de medidas, asociadas a la función afín. Para ello, se elaboró un instrumento de investigación compuesto por tres situaciones de esta clase, aplicado en un grupo de 34 estudiantes de 3.º año de la Educación Media de una escuela pública en la región noroeste de Paraná, Brasil. Los datos fueron recolectados mediante las resoluciones escritas y grabaciones de audio de los diálogos de los estudiantes, organizados en parejas o tríos. Específicamente, para este artículo, se presentan los análisis de las estrategias y de los teoremas en acción movilizados por estudiantes de Educación Media en la resolución de una situación mixta asociada a la función afín, relacionada con las subclases de comparación multiplicativa (referente desconocido) y transformación de medidas (transformación positiva con estado final desconocido). Los resultados revelaron dificultades en la interpretación de la tasa de la función afín y en el cálculo de porcentajes, además de incomprensiones relativas al término independiente de la función afín. A partir del análisis de los esquemas y de los diálogos de los grupos, se identificaron cuatro teoremas-en-acción verdaderos y cinco falsos.

Palabras-clave: Educación matemática, Situaciones mixtas, Función afín, Teorema en acción, Complejidad.

Résumé

La recherche présentée dans cet article fait partie d'une étude de master menée par la première auteure, qui a examiné les complexités de sous-classes de situations mixtes de la classe de comparaison multiplicative et de transformation de mesures, associées à la fonction affine. À cette fin, un instrument de recherche composé de trois situations de cette classe a été élaboré et mis en œuvre auprès d'un groupe de 34 élèves de troisième année de l'enseignement secondaire d'un établissement public situé dans la région nord-ouest de l'État du Paraná, au Brésil. Les données ont été recueillies à partir des productions écrites ainsi que des enregistrements audio des dialogues des élèves, organisés en binômes ou en trinômes. Plus précisément, pour cet article, sont présentées les analyses des stratégies et des théorèmes-en-action mobilisés par des élèves de l'enseignement secondaire lors de la résolution d'une situation mixte associée à la fonction affine, liée aux sous-classes de comparaison multiplicative (référent inconnu) et de transformation de mesures (transformation positive avec état final inconnu). Les résultats ont révélé des difficultés dans l'interprétation du taux de variation de la fonction affine et dans le calcul de pourcentages, ainsi que des incompréhensions relatives au terme constant de la fonction affine. À partir de l'analyse des schémas et des dialogues des groupes, quatre théorèmes-en-action vrais et cinq faux ont été identifiés.

Mots-clés: Enseignement des mathématiques, Situations mixtes, Fonction affine, Théorème en action, Complexité.

Resumo

A investigação apresentada neste artigo é parte de uma pesquisa de mestrado realizada pela primeira autora, que investigou as complexidades de subclasses de situações mistas da classe comparação multiplicativa e transformação de medidas, associadas à função afim. Para tanto, foi elaborado um instrumento de pesquisa composto por três situações dessa classe, implementado em uma turma de 34 estudantes do 3º ano do Ensino Médio de um colégio público na região noroeste do Paraná, Brasil. Os dados foram coletados por meio das resoluções escritas e de gravações em áudio dos diálogos dos estudantes, organizados em duplas ou trios. Especificamente, para este artigo, são apresentadas as análises das estratégias e dos teoremas em ação mobilizados por estudantes do Ensino Médio na resolução de uma situação mista associada à função afim, relacionada às subclasses de comparação multiplicativa (referido desconhecido) e transformação de medidas (transformação positiva com estado final desconhecido). Os resultados revelaram dificuldades na interpretação da taxa da função afim e no cálculo de porcentagem, além de incompreensões relativas ao termo independente da função afim. A partir da análise dos esquemas e dos diálogos dos grupos, foram identificados quatro teoremas-em-ação verdadeiros e cinco falsos.

Palavras-chave: Educação matemática, Situações mistas, Função afim, Teorema-em-ação, Complexidade.

Knowledge Associated to Affine Functions Manifested by High School Students

Introduction

The affine function is one of the first functions studied in Basic Education and involves different mathematical notions, such as the graph of a straight line, collinearity of points, constant rate of change, linear coefficient, growth, decrease, constant function, affine function, identity function, and graph translations, among others. According to the Brazilian National Common Core Curriculum (BNCC), the study of functions is formalized in the 9th grade of middle school, at which point students are expected to develop the skills necessary to understand functions as relations of univocal dependence between two variables and to use this concept to analyze functional situations (Brazil, 2018). In high school, this knowledge should be consolidated and expanded so that students are able to solve more complex problems that require higher levels of reflection and abstraction (Brazil, 2018).

However, the teaching of this concept presents challenges due to the diversity of ideas and situations related to the affine function, which may lead to errors and difficulties on the part of students. Studies conducted by Fonseca (2011), Rezende, Nogueira, and Calado (2020), Bernardino (2022), and Calado (2020) have highlighted the presence of incorrect strategies in the resolution of situations involving affine functions, manifested by students completing the ninth grade of middle school and the 12th grade of high school. In this regard, it is necessary to develop further research that investigates the teaching and learning of the affine function in Basic Education.

According to Vergnaud (1996), the understanding of a concept develops throughout the educational process through engagement with different situations that involve other concepts, symbols, representations, properties, and theorems, interconnected within what the author refers to as a Conceptual Field. In order to understand situations involving additive and multiplicative structures, Vergnaud and collaborators defined two specific Conceptual Fields: additive structures and multiplicative structures.

Within the Conceptual Field of Additive Structures, situations involving one or more additions or subtractions are considered. This field comprises a set of concepts and theorems that make it possible to analyze and solve mathematical tasks related to such situations (Vergnaud, 1993). Thus, for students to understand this field, they must be able to solve different types of situations belonging to this structure. Vergnaud (1993) establishes six classes of situations within this field: the composition of two measures into a third, the transformation of an initial measure into a final measure, the comparison relationship between two measures, the composition of two transformations, the transformation of a relationship, and the composition of two relationships.

Within the Conceptual Field of Multiplicative Structures, situations involving one or more multiplications or divisions are considered. This field also includes a set of concepts and theorems for analyzing and solving mathematical tasks related to such situations. Vergnaud (1993) establishes five classes of situations within this field, encompassing measure isomorphism or simple proportion, multiplicative comparison within a single measurement space, the product of measures or Cartesian product, the bilinear function, and multiple proportion.

Situations that simultaneously involve addition (or subtraction) and multiplication (or division) are referred to by Vergnaud (2009) as mixed problems. These situations are associated with the affine function, since its algebraic expression, $f(x) = ax + b$, simultaneously articulates a multiplicative component and an additive component. Miranda (2019) presents a classification consisting of nine classes of mixed situations related to the affine function. In this context, the importance of investigating the complexity of these situations, as well as the strategies and theorems mobilized by high school students in their resolution, becomes evident.

The research presented here is the result of the first author's master's dissertation. In this study, a research instrument was developed consisting of three mixed situations associated with the concept of affine function, belonging to the class of multiplicative comparison and measure transformation, which were solved by thirty-four students enrolled in the 12th grade of high school. The study aimed to identify the complexity among subclasses of mixed problems of the multiplicative comparison and measure transformation type, analyzing the strategies used by students and the theorems related to the affine function that they mobilized.

To this end, this article presents the strategies and theorems-in-action mobilized by students in the situation and subclass of lowest complexity, namely: multiplicative comparison (unknown referent) combined with measure transformation (positive transformation with the search for the final state). This subclass was selected for the present article because it yielded the greatest number of responses and strategies manifested by students. Subsequently, the theoretical and methodological aspects of the research are presented, followed by the analysis and discussion of the results obtained.

Theoretical and Methodological Aspects

The Theory of Conceptual Fields (TCF), developed by the French psychologist Gérard Vergnaud, is a cognitivist theory that seeks to understand the filiations and ruptures among acquired knowledge, providing an analytical framework for the study of the process of concept learning in children and adolescents (Vergnaud, 1993).

For the study of a concept, Vergnaud (1990) argues that it is necessary to consider not only its formal definition, but also other related concepts, situations, symbols, representations, properties, and theorems that are interconnected. These elements constitute what the author calls a Conceptual Field – a complex structure formed by different interconnected elements whose articulation enables a comprehensive understanding of the concept.

Within TCF, concepts are constituted by a set of situations that give them meaning; that is, by a set of operative invariants (properties of the concept) that underlie reasoning, and by a set of symbols used in their representation (Vergnaud, 2017). The development of competencies and conceptions occurs through individuals' experiences with diverse situations, both in and outside the school context. According to Magina e*t al*. (2008), when confronted with a new situation, students mobilize knowledge constructed in previous experiences and seek to adapt it to the demands of the new situation.

To understand the invariant organization of students' actions or operations in relation to a class of situations, it is necessary to understand the concept of scheme. In Vergnaud's view (1993), a scheme is composed of goals and anticipations, rules of action, operative invariants (concepts-in-action and theorems-in-action), and inferences. These components give the scheme a cognitive character that guides and sustains its use. Goals and anticipations correspond to the objectives that orient the subject's action; rules of action are the procedures or guidelines that direct the subject's behavior when dealing with a situation; operative invariants refer to the conceptual and theoretical elements mobilized in action; and inferences correspond to the reasoning and conclusions produced from these elements.

According to Vergnaud (1993), it is within schemes that a subject's knowledge-in-action is found, including concepts-in-action and theorems-in-action, which may be regarded as "operative invariants." A concept-in-action refers to a concept judged relevant in the course of action, whereas a theorem-in-action is a proposition considered true by the subject during action (Vergnaud, 2009). These cognitive elements, present in schemes, enable the subject's action to be operative and contribute to the understanding and resolution of problem situations.

To analyze the classes of situations that give meaning to mathematical concepts in additive and multiplicative structures, Vergnaud established two Conceptual Fields. The Conceptual Field of Additive Structures encompasses situations involving addition and subtraction and includes, according to Vergnaud (1993, p. 13), six classes: "[...] the composition of two measures into a third; the transformation of an initial measure into a final measure; the comparison relationship between two measures; the composition of two transformations; the transformation of a relationship; and the composition of two relationships."[3].

The class of situations referred to as measure transformation is the second described within the Conceptual Field of Additive Structures and is the one that underpins the data production instrument used in this research. For this reason, this class is presented in greater detail. In this class, a dynamic transformation occurs from an initial measure to a final measure. According to Magina *et al*. (2008), transformation situations are related to temporal ideas, in which an initial quantity undergoes transformations – such as loss, gain, increase, or decrease – resulting in a final quantity that differs from the initial one. Such situations involve the notion of variation and require an understanding of the transformations that occur over time. For this class, there are six distinct subclasses, which differ according to the relationships established among the initial state, the transformation, and the final state, namely:

1. When the initial state a and the positive transformation $+b$ are known, the final state c is determined: in this situation, the initial measure a and the increase $+b$ are given. The objective is to determine the final state c after the application of this transformation.
2. When the initial state a and the final state c are known, the positive transformation $+b$ is determined: in this case, both the initial state a and the final state c are known, and it is necessary to identify the positive transformation $+b$ that occurred in order to move from the initial state to the final state.

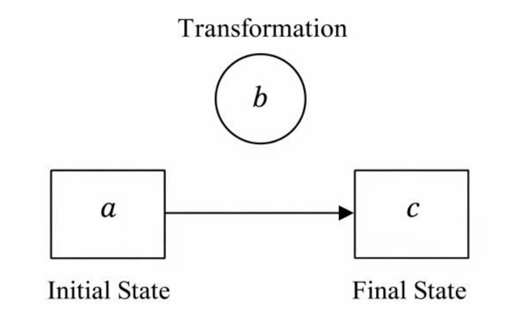
[3] Free translation from the original Portuguese: "[...] composição de duas medidas em uma terceira; transformação de uma medida inicial em uma medida final; relação de comparação entre duas medidas; composição de duas transformações; transformação de uma relação; e composição de duas relações." (Vergnaud, 1993, p. 13).

3. When the positive transformation $+b$ and the final state c are known, the initial state a is determined: here, the positive transformation $+b$ and the final state c are given. The objective is to determine the initial state a from which the positive transformation leads to the final state.
4. When the initial state a and the negative transformation $-b$ are known, the final state c is determined: in this situation, the initial measure *a* is given together with the negative transformation $-b$. The objective is to determine the final state c after the application of this negative transformation.
5. When the initial state a and the final state c are known, the negative transformation $-b$ is determined: here, the initial state a and the final state c are known, making it possible to determine the negative transformation $-b$ that occurred between them.
6. When the negative transformation $-b$ and the final state c are known, the initial state a is determined: in this case, the negative transformation $-b$ and the final state c are given. The objective is to determine the initial state a from which the negative transformation leads to the final state.

Vergnaud proposes relational schemes (sagittal schemes) that assist in the analysis of the structure and classification of classes of situations. These schemes provide a conceptual framework for understanding the relationships among measures, transformations, and the initial and final states involved. Therefore, the relational scheme proposed by Vergnaud organizes the subclasses of the measure transformation class based on the relationship among three quantities, as illustrated in Figure 1.

Figure 1

Relational scheme of the measure transformation class.



Note. Adapted from Vergnaud (2009).

Thus, situations belonging to the measure transformation class can be solved according to the following equation: $x = a + (\pm b)$.

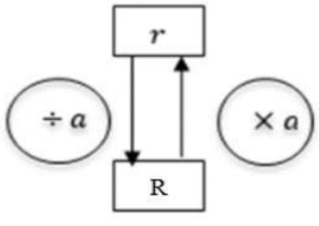
The Conceptual Field of Multiplicative Structures encompasses situations involving multiplication and division. Within this conceptual field, Vergnaud (2009, p. 13) establishes five classes of situations that are relevant to multiplicative structures: "[...] measure isomorphism or simple proportion; multiplicative comparison, in the case of a single measurement space of the same nature; the product of measures or Cartesian product; the bilinear function; and multiple proportion."[4].

In this research, the *multiplicative comparison class* (Figure 2) is considered as an analytical instrument. This class addresses situations in which only two quantities of the same nature are compared multiplicatively by means of a scalar, such as a ratio or relationship. Within the multiplicative comparison class, three subclasses can be identified:

1. Multiplicative comparison with an unknown referent: in this situation, the referent (R) is unknown, while the referred quantity (r) and the relationship (ratio) are known. The objective is to determine the value of the referent based on the established relationship.
2. Multiplicative comparison with an unknown referred quantity: in this case, the referent (R) and the relationship (ratio) are known, but the referred quantity (r) is unknown. The objective is to determine the value of the referred quantity based on the established relationship.
3. Multiplicative comparison with an unknown relationship: in this situation, the referent (R) and the referred quantity (r) are known, but the relationship (ratio) is unknown. The objective is to determine the relationship between the quantities through multiplicative comparison.

Figure 2

Relational scheme of the multiplicative comparison class



Note. (Miranda, 2009, p.64).

[4] Free translation from the original Portuguese: "[...] isomorfismo de medidas ou proporção simples; comparação multiplicativa, caso de um único espaço de medidas de mesma natureza; produto de medidas ou produto cartesiano; função bilinear; e proporção múltipla." (Vergnaud, 2009, p. 13).

Accordingly, the equation that represents this class is given by $r = R \times a$.

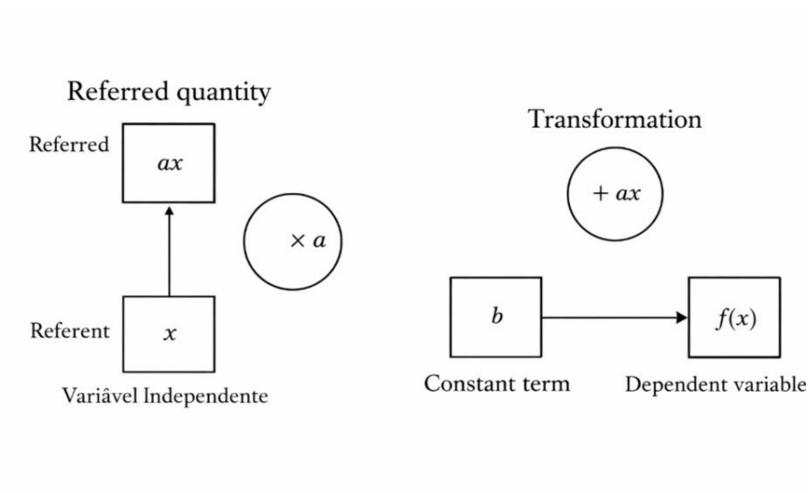
Thus, six (6) subclasses of situations related to comparison – *times more* $(\times a)$ and comparison – *times less* $(\div a)$ are obtained. In this research, only the *times more* comparison variations are used.

Situations that involve, simultaneously in their resolution, at least one operation from the additive field (addition/subtraction) and at least one operation from the multiplicative field (multiplication/division) are referred to by Vergnaud (2009) as *mixed problems*.

For this research, the class of mixed problems of the multiplicative comparison and measure transformation type was considered. This class is composed of a ternary relation from the multiplicative field and a ternary relation from the additive field. According to Miranda (2019), these situations take the analytical form of the affine function $y = f(x) = b + a \cdot x$, with $a, b \in R \; e \; a > 0$. Since Vergnaud does not present a classification for mixed situations, Miranda (2019) identified and classified these problems associated with the affine function. Based on this proposal, the sagittal scheme presented in Figure 3 is introduced to organize the measures, indicating the relationships established among them.

Figure 3.

Sagittal scheme of the multiplicative comparison and measure transformation class.



Note. Adapted from Miranda (2019).

In situations of the multiplicative comparison and measure transformation type, the transformation is composed of a fixed part that undergoes change due to another variable, the latter resulting from a comparison of measures. In this context, the transformation occupies the position of the referred quantity in the multiplicative relation; the independent variable *x* occupies the position of the referent; and the dependent variable *y* corresponds to the final state in the additive transformation relation. Moreover, the comparison ratio between the measures, obtained in the multiplicative comparison, coincides with the rate *a* of the affine function. The equation that represents this situation is given by: $f(x) = b + ax$.

The subclasses of this mixed problem class consider a combination of the subclasses of multiplicative comparison and measure transformation. That is, considering the three (3) variations of the multiplicative comparison class combined with the six (6) variations of the measure transformation class, it is possible to form eighteen (18) subclasses. These variations are presented in Table 1.

Table 1

Subclasses of mixed problems of the multiplicative comparison and measure transformation type

<table>
<tr><th>Multiplicative comparison</th><th>Measure transformation</th></tr>
<tr><td rowspan="6">Multiplicative comparison – unknown referred quantity
Multiplicative comparison – unknown referent
Multiplicative comparison – unknown relationship (ratio)</td><td>Measure transformation (positive transformation with unknown final state)</td></tr>
<tr><td>Measure transformation (positive transformation with unknown initial state)</td></tr>
<tr><td>Measure transformation (positive transformation with unknown transformation)</td></tr>
<tr><td>Measure transformation (negative transformation with unknown final state)</td></tr>
<tr><td>Measure transformation (negative transformation with unknown initial state)</td></tr>
<tr><td>Measure transformation (negative transformation with unknown transformation)</td></tr>
</table>

As an example of this class, consider a situation in which a salesperson receives a monthly salary composed of two parts: a fixed portion, in the amount of R$1,000.00, and a variable portion, which corresponds to a 1% commission on the total sales made during the month. In this situation, the amount received from sales corresponds to 1% of the total sales value, that is, a portion of the total sales amount. Thus, the percentage represents the relationship of a multiplicative comparison, that is, a scalar that relates two measures of the same nature: the referred quantity and the referent. In this case, the referent is the total value of sales made during the period, and the referred quantity is 1% of that value.

Following the presentation of the theoretical and methodological framework, the next section describes the methodological procedures.

Methodological Procedures

As mentioned in the Introduction, this research was conducted in the second semester of 2022 with students enrolled in the 12th grade of high school at a public school in the central-western region of Paraná.

Over a two-month period, from early October to late November, the researcher participated in the class's regular lessons on Monday and Tuesday mornings. In this way, in order to become familiar with the students, the researcher took part in and assisted with classroom activities. Through participant observation, the researcher had the opportunity to establish closer contact with the students, support the classroom teacher in the pedagogical activities developed, and identify the main characteristics and difficulties of the group.

Considering the characteristics of the group, two pilot studies were conducted with the aim of identifying possible difficulties and making the necessary adjustments to the research instrument prior to the implementation of the main study.

In the first pilot study, eight volunteer students participated. They belonged to the same school context previously described and were organized into pairs. Each pair received one of the three situations that composed the research instrument. The researcher explained to the students that their dialogues during problem solving would be audio recorded and clarified that she would not respond to questions regarding the correctness of their answers. The students' problem-solving activity lasted approximately one class period (about one hour).

After the initial analysis of students' responses, difficulties in understanding the wording of items (b) and (c) were identified across all situations. In response, modifications were made to the situations in the research instrument, including the reorganization of the order of the items and changes to numerical values related to salaries and payments, with the aim of avoiding unnecessary complexity in the calculations. These changes were intended to focus students' attention on the central aspects of the investigation and to facilitate comprehension of the problem statements.

Subsequently, a second pilot study was conducted with two participants – one undergraduate student in Business Administration and one pedagogue, who at the time was a doctoral student in Education – both belonging to the researcher's social circle. This second pilot study aimed to verify whether the modifications made to the research instrument were sufficient to address the difficulties identified in the first pilot study.

Following this stage, the research instrument was considered adequate and was finalized for use in the main study.

The implementation of the research instrument with the collaborating class took place on November 8, 2022, over two class periods. In the classroom, a total of thirty-eight students agreed to participate in the research by submitting the Informed Consent Form and the Informed Assent Form. For the development of the activity, the researcher asked students to organize themselves into pairs; however, two groups chose to work in trios, and four students preferred to complete the activity individually. This organization was respected so as not to interfere with the classroom dynamics. However, for data analysis purposes, the records produced by students who worked individually were not considered, since the focus was on analyzing group dialogues and records.

For data production, students' written records for the three situations included in the research instrument were considered, as well as the dialogues that occurred during problem solving. Some groups allowed the recording of their discussions, while others preferred to submit audio recordings explaining their solution strategies. The researcher agreed to this request, since the main objective was to understand students' strategies, and verbal explanations of the strategies used contributed to the research.

The situations from the research instrument were delivered simultaneously to the groups, with each student receiving three colored A4 sheets of paper, each corresponding to one situation. The different colors were used to distinguish this activity from other regular classroom activities. Although each group member recorded their calculations and answers individually, data analysis was conducted at the group level, since students, when collectively discussing the situations, displayed similar problem-solving strategies.

Considering the number of groups and the availability of technological resources, a hybrid recording strategy was adopted: some groups used audio recorders provided by the researcher, while others used their own mobile devices to record their discussions. To facilitate the submission of audio files and any additional explanations, the researcher wrote her email address and WhatsApp number on the board. In addition, the researcher explained that she would be available to clarify doubts, but would not interfere with the groups' solution strategies. Upon completion, students handed in their answer sheets to the researcher and took photographs of their solutions so that they could be discussed in the following class, with the researcher's mediation.

In the class following the implementation, the researcher presented the situations to the students and encouraged them to present their solutions on the board or to orally share the procedures they had adopted. After each presentation, the researcher asked whether other students had solved the problem differently or wished to make comments or add explanations. This process allowed for the identification of misconceptions and the visualization of different strategies for solving the same situation. At this point, the researcher was also able to question students about their solutions and their impressions of the situations. Although this was not the main objective of the research, at the request of the classroom teacher, the researcher formally introduced the concept of the affine function, using the situations from the instrument to present its definition and general aspects, such as coefficients, variables, domain, graph, and zeros of the function.

The research instrument consisted of three situations of the multiplicative comparison and measure transformation type. Initially, several attempts were made to design one situation for each variation of the multiplicative comparison and measure transformation class. However, after discussions held within the Mathematics Education Teaching and Research Group (GEPEDIMA), it was decided to vary only the multiplicative comparison class, encompassing its three possibilities, while the measure transformation class would always remain positive, with the final state unknown. This decision was made due to the considerable number of possible subclasses related to the multiplicative comparison and measure transformation class, totaling 36 variations, which would be unfeasible to include in a single research instrument.

The research instrument was therefore composed of three situations considering the following variations:

1. multiplicative comparison (unknown referent) – transformation of measures (positive transformation with unknown final state);
2. multiplicative comparison (unknown reference) – transformation of measures (positive transformation with unknown final state);
3. multiplicative comparison (unknown relationship) – transformation of measures (positive transformation with unknown final state).

The macro-context adopted for the situations was that of salary/payment, involving a fixed amount and a variable amount resulting from a commission. These situations were selected taking into account the students' reality, as they live in a small town in the interior of the state of Paraná, where job opportunities are limited, especially for those who have just completed basic education. Therefore, three professions were chosen within the macro-context of each situation: gas station attendant, truck driver, and store salesperson.

The statement of each situation consisted of four items, following the same pattern: in item (a), the use of natural language for generalization, with the intention that students would reflect on the situation without yet mobilizing algebraic symbols; item (b) consisted of a comparison involving the referent, the reference, or the unknown relationship; in item (c), students were asked to determine $f(x)$ for a specific value of x, that is, to find the final state of the transformation; and in item (d), the analytical expression was required, that is, the generalization in its algebraic representation.

For this article, Situation 1 of the research instrument was selected for analysis. This situation was chosen based on the analysis of the degree of complexity attributed to it, as it was considered the least complex, and because it presented a larger number of student solutions.

Next, the situation and the data analyses are presented.

Data Analysis and Discussion

For the analyses, the following aspects were considered: whether the situation was solved by the students or not; the adequacy of the strategies expressed by the students; and the identification of theorems-in-action, both false and true, in the students' responses.

The 34 students were distributed into 16 groups, two of which were composed of three students and fourteen formed by pairs. In order to preserve the participants' identities, in this analysis each student is identified by the letter S, followed by a number from 1 to 34, and each group is identified by the letter G, followed by a number from 1 to 16.

The theorems-in-action are identified by the acronyms TAV (true theorem-in-action) and TAF (false theorem-in-action), followed by a number indicating their order. Next, a discussion is presented of the solution strategies adopted by the groups for the two situations, the least and the most complex, including excerpts from students' dialogues and written records.

Situation 1 belongs to the subclass multiplicative comparison (unknown referent) – transformation of measures (positive transformation with unknown final state).

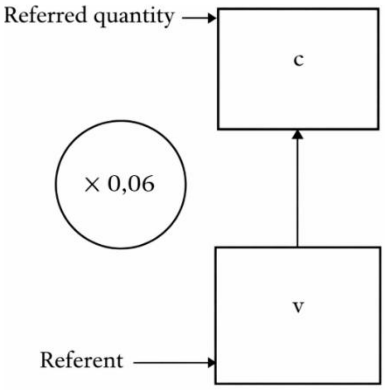
Situation 1: *Gerônimo owns a truck and hired a driver to transport cargo. The driver receives R$ 2,500.00 plus 6% of the truck's monthly gross revenue.*

a) *In your own words, explain how the driver's payment is calculated.*
b) *In December, the truck grossed R$ 36,200.00. How much did the driver receive that month?*
c) *Knowing that in February, the truck grossed R$ 54,000.00, how much was paid to the driver?*
d) *Write an expression that allows you to calculate, for any month, the driver's payment (p) as a function of the gross revenue (v).*

For this situation, the reference is the amount (v) corresponding to the gross monthly revenue generated by the truck. The referent, in turn, is six parts per hundred, and the relationship is given by 6%. Thus, the amount received by the driver as commission on the gross monthly revenue (v) is given by $c = 0.06 \times v$. In other words, the amount received by the driver is equal to 0.06 times the gross revenue generated. Therefore, this constitutes a multiplicative comparison, represented by the following sagittal diagram:

Figure 4

Sagittal diagram of the multiplicative comparison in the situation

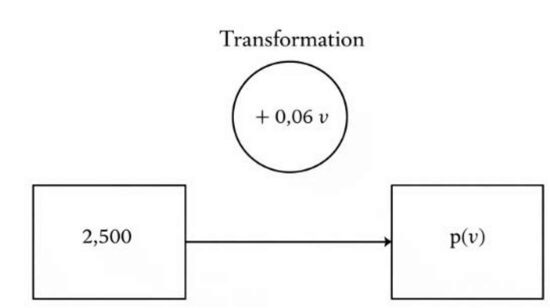


Note. Prepared by the authors.

Thus, the driver's payment situation p(v) consists of a fixed amount of R$ 2,500.00 (initial state) added to a commission of $0.06 \times v$ (positive transformation), resulting in a final amount (final state) to be paid. Accordingly, there is a positive transformation with the search for the final state, as shown in the diagram presented in Figure 5.

Figure 5

Sagittal diagram of the transformation of measures in the situation

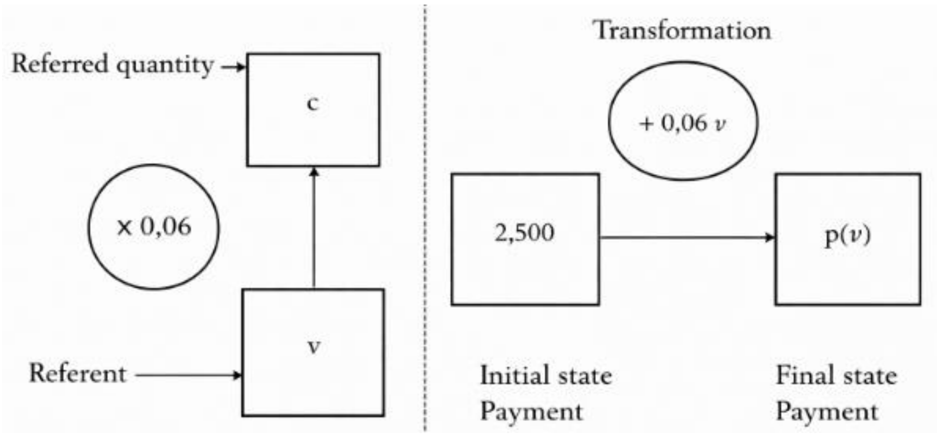


Note. Prepared by the authors.

In this way, the driver's payment (p) as a function of the gross monthly revenue (v) is expressed by the algebraic expression $p(v) = 2{,}500 + 0.06v$, or alternatively, $p(v) = 0.06v + 2{,}500$. In this situation, neither the value of $0.06v$ (the referent) nor the amount to be paid to the driver is known; therefore, this corresponds to the subclass of multiplicative comparison (unknown referent) – transformation of measures (positive transformation with unknown final state). Figure 6 presents the complete sagittal diagram of this situation.

Figure 6

Sagittal diagram of the subclass multiplicative comparison (unknown referent) and transformation of measures (positive transformation with unknown final state)



Note. Based on Miranda (2019).

Item a) of this situation aims for the student to interpret the proposed statement and present a generalization of the situation using natural language, explaining how the calculation of the amount to be paid to the driver is performed. Only eight (8) of the sixteen (16) groups presented the correct strategy. These groups were: G1, G2, G5, G7, G9, G13, and G14.

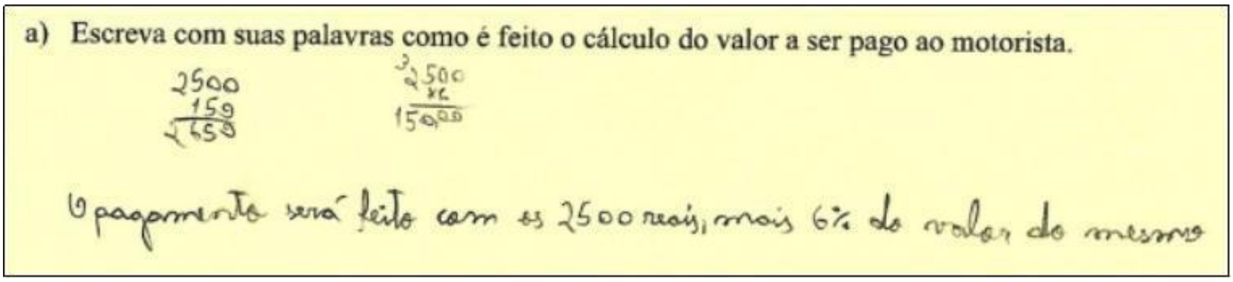
Based on the written records and dialogues, the students demonstrated that the calculation of the amount to be paid to the driver is obtained by adding a fixed amount of R$ 2,500.00 to 6% of the truck's gross monthly revenue.

These groups implicitly used a true theorem-in-action when generalizing the driver's payment (p) as a function of the truck's monthly revenue (v), that is, $p(v) = 2{,}500$ reais $+6\%$ of v. This theorem-in-action was identified as TAV1, implicitly expressed in the students' actions. Based on the students' dialogues and strategies, it is possible to infer that they implicitly considered the coefficients $a = \frac{6}{100} = 0{,}06$ and $b = 2.500$, thus establishing a functional relationship. In this way, the following theorem-in-action can be modeled: *TAV1: If a is the rate and b is the linear coefficient, then the functional relationship f is given by $f(x) = ax + b$.*

Of the sixteen (16) participating groups, seven (7) groups – G3, G8, G10, G11, G12, G15, and G16 – presented an incorrect strategy for item (a). In this strategy, the students considered the data provided in the statement and calculated 6% of the fixed payment amount and then added this value to the initial salary. Therefore, the groups did not correctly identify the requirement of the item, which was simply to describe how the driver's salary would be calculated for any given month. Instead, they presented a specific calculation, indicating that they used a fixed value to represent a quantity that could vary. Figure 7[5] illustrates this strategy.

Figure 7

Resolution by Group G12 for item (a) of Situation 1



Note. Research archive.

This incorrect strategy corroborates the findings of Calado (2020), who highlights that students experience difficulties in understanding that a variable may assume any possible value and tend to generalize facts by verifying their validity only in a particular

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[5] Free translation of item a) and the response provided by G12 (Figure 7): a) Write, in your own words, how the calculation of the amount paid to the driver is conducted. "*The payment will be made with 2,500 reais, plus 6% of that amount*".

situation. Therefore, the groups that adopted this strategy show evidence of mobilizing a false theorem-in-action, labeled TAF1: Any variable x is associated with a fixed value.

Within this same strategy, the groups also mobilized another false theorem-in-action, defined as TAF2: *If* a *is the rate and* b *is the linear coefficient, then* $f(x) = a \cdot x = a \cdot b$*, with* $a, x, b \in R$*.*

The groups display an incorrect interpretation of the linear coefficient b. In this case, the linear coefficient b in the function should not be interpreted as the truck's fixed gross revenue, but rather as the constant term of the function. It represents a constant value added to the product of the rate a and the variable x. By interpreting b as a fixed gross revenue, the students incorrectly assumed this quantity to be constant, without considering the variability of the revenue over time. However, the truck's revenue is a variable quantity.

Group G12 considered the sum of the fixed payment plus 6%, but did not specify what this percentage referred to. By considering only $y\%$ instead of $y\%$ of x, the group made an interpretative error. This misunderstanding can be modeled as a false theorem-in-action, referred to as TAF3: the percentage of y in relation to x is represented by $y\%$.

In addition to the strategies presented above, it is important to note that a specific group, G4, did not provide a solution for this item of the situation. It is possible that the group encountered difficulties either in understanding the problem or in formulating a solution, which resulted in the absence of a response for the item in question.

For item (b), three adequate strategies and three inadequate strategies were identified among the students. The first adequate strategy was adopted by seven groups: G2, G5, G6, G7, G11, G14, and G15. These groups demonstrated understanding by calculating the commission amount the driver received based on a gross revenue of R$ 36,200.00. They multiplied this amount by the centesimal rate of 6%, that is, $6\% = \frac{6}{100} = 0{,}06$, obtaining the result of R$ 2,172.00. They then added this value to the driver's fixed payment (R$ 2,500.00), arriving at a total of R$ 4,672.00, which corresponds to the amount the driver received in December. This strategy indicates that the students appropriately associated the need to multiply the percentage by the gross revenue to calculate the commission and then add the fixed amount.

In this strategy, when calculating the amount received as sales commission, the students mobilized a true theorem-in-action. This theorem can be represented mathematically as follows: $f(36.200)$ = 0,06 (commission rate) × 36.200 (gross monthly revenue of the truck) = 2.172 (reais).

Thus, TAV2 is modeled by considering $f(x)$ as a proportional relationship between two quantities, in which $f(x)$ represents the driver's commission-based payment, a is the commission rate, and x is the truck's gross monthly revenue. Therefore, the true theorem-in-action can be represented as follows: *TAV2: let f be a proportional relationship and a the rate; then $f(x) = a \cdot x$, with a and $x \in R$.*

We therefore observe the presence of TAV2, which represents the standard property of the coefficient of proportionality across various levels of schooling and in different mathematical situations. Studies by Vergnaud (1996), Pavan (2010), and Rodrigues (2023) show that TAV2 is identified in students' solutions to multiplicative and mixed situations, including those produced by students in elementary education. In the study presented in this article, its presence is also observed among high school students when solving situations involving affine functions. This highlights the importance of this theorem-in-action for understanding proportionality and for generalizing functional relationships. By identifying the manifestation of TAV2 in high school students' solutions to affine function situations, it is possible to recognize the continuity of conceptual development and students' ability to establish proportional relationships in more abstract contexts.

Within this same strategy, the seven groups used direct addition to determine the final payment amount received by the driver. They added the fixed payment of R$ 2,500.00 to the commission earned from the truck's revenue, which was calculated as R$ 2,172.00. The sum of these amounts results in R$ 4,672.00. In this way, the groups recognized that the fixed amount and the commission amount are two distinct components of the total payment and, therefore, must be added to obtain the final value. This strategy can be modeled, based on Rodrigues (2023), by considering F as the final state, I as the initial state, and T as the transformation, as expressed in *TAV3: if F is the final state, I the initial state, and T the transformation, then $F = I \pm T$.*

For item (b), groups G1, G3, G9, G13, and G16, unlike the previous groups, used a ratio-and-proportion strategy to determine the value of the commission on the truck's monthly revenue. They established a proportion between the gross revenue (R$ 36,200.00) and the commission percentage (6%) in order to find the corresponding commission amount. This strategy is based on the concept of proportion, in which a relationship is established between two quantities: the revenue amount and the commission. By applying the rule of three, they determined the value proportional to the commission percentage relative to the gross revenue. Subsequently, this amount was added to the initial fixed payment to obtain the driver's final payment.

This strategy is aligned with TAV2, previously discussed, in which the functional relationship is represented by $f(x) = a \cdot x$, where a denotes the commission rate and x represents the gross revenue. It can also be observed that these groups mobilized and applied TAV3, which represents the direct additive transformation between the initial state (I), the transformation (T), and the final state (F). In this case, the initial state corresponds to the fixed payment (R$ 2,500.00), the transformation corresponds to the commission based on the truck's revenue (R$ 2,172.00), and the final state corresponds to the driver's total payment (R$ 4,672.00).

The final adequate strategy was exhibited by group G8. This group employed a decomposition of the monthly revenue in order to calculate the commission. The following dialogue, originally in Portuguese, illustrates this line of reasoning.

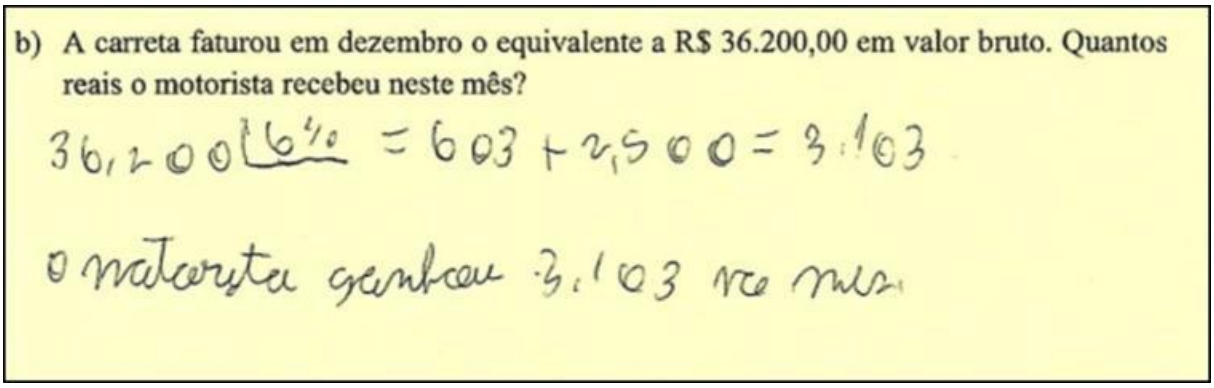
Student 17: *So, now I'm going to explain it by audio for you who will be listening. I don't really do calculations, so I'm going to explain how I'm going to calculate this. Six percent of 36,000. I know that 6% of 10,000 reais is 600. I know that of 20,000 it will be 1,200. I know that of 30,000 it will be 1,800. So I know that 30,000 reais… 6% of 30,000 will be equivalent to 1,800 reais. I know that of 5,000 it will be equivalent to 300. So now I already have the amount of 2,100 reais.*
Student 17: *Is the amount 36,200?*
Student 18: *Yes, it's 36,200.*
Student 17: *Oh, it's wrong—no, it's right. So am I still right? Okay. So, 1,200. Six percent of 1,000—6% of 1,000 is 60. So, 2,160. Six percent of 200. Of 100 I know it's 6… so 12. So I have 2,172 reais. So I know that 6% of 36,200 will be 2,172 reais.*
(After a few moments.)
Student 17: *So, since here at the top you have to add it, you have to add the amount he's going to receive from the load, which is the percentage, to the payment, so you have to add it. So I have there the amount of 2,172, and I have to add another 2,500. So 2,172 plus 2,500 is equal to 4,672.*

Thus, the group considered $6\%\ of\ 36.200 = 6\%\ of\ 10.000 + 6\%\ of\ 10.000 + 6\%\ of\ 10.000 + 6\%\ of\ 5.000 + 6\%\ of\ 1.000 + 6\%\ of\ 100 + 6\%\ of\ 100$, and manifested TAV2 by calculating the percentage of each value. In addition, the group manifested TAV3 by adding the commission amount to the fixed salary. However, unlike the previous groups, G8 also manifested a theorem-in-action that establishes a valid property for direct proportionality relations. This theorem states that, when the same quantity (*x'*) is added to or subtracted from the variable *x* in a direct proportionality relation, the image of the function *f(x)* undergoes the same addition or subtraction. Based on the studies by Rodrigues (2021), Calado (2020), and Vergnaud (2007), and on the modeling proposed by Rodrigues (2023), a true theorem-in-action, referred to as TAV4, was identified. Its formulation is equivalent to *TAV4*: *if f is a direct proportionality relation; then* $f(x \pm x') = f(x) \pm f(x')$*, with x* $\in$ *N.*

The first incorrect strategy for item (b) was manifested by group G12 (Figure 8[6]).

Figure 8

Resolution by group G12 for item (b) of Situation 1



b) A carreta faturou em dezembro o equivalente a R$ 36.200,00 em valor bruto. Quantos reais o motorista recebeu neste mês?

36,200 ⌊6% = 603 + 2,500 = 3.103

o motorista ganhou 3.103 no mês

Note. Research archive.

Based on group G12's solution, it is evident that the students made two errors: they used division instead of multiplication to calculate the percentage, and they replaced the decimal point with a comma in the numerical representation. These errors led to misinterpretations and an incorrect result. Thus, it is possible to identify the false theorem-in-action (TAF4) mobilized by group G12, according to which:

TAF4: if a is the centesimal ratio of a percentage rate, then the value obtained by applying this rate to a value x can be expressed by the functional relation $f(x) = \frac{x}{a}$, with $x\ e\ a\ \in R$, and $a \neq 0$.

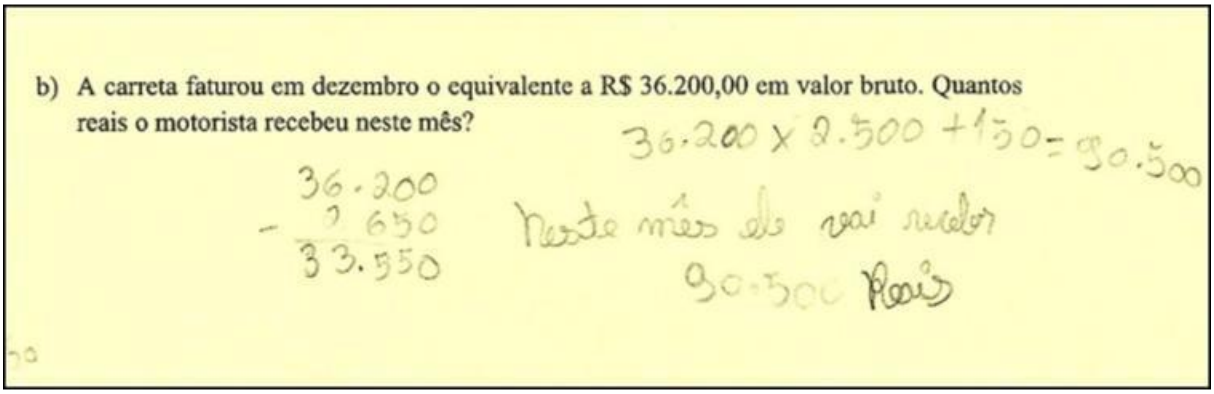
However, this interpretation is incorrect, since the correct relation should be $f(x) = a.x$, where a represents the percentage rate. Another inadequate solution for item (b) was presented by group G10, which calculated the 6% commission based on the fixed payment rather than on the monthly revenue, thereby mobilizing TAF2. In the last inadequate strategy, group G4 calculated 6% of R$ 2,500.00, also manifesting TAF2, and then added this value to the product of the monthly gross revenue and the initial payment (Figure 9[7]).

[6] Free translation of item b) and the response provided by G12: *b)* The truck generated gross revenue of R$36,200.00 in December. How much did the driver receive that month? *"The driver earned R$3,103 during the month"*.

[7] Free translation of item b) and the response provided by G4: *b)* The truck generated gross revenue of R$36,200.00 in December. How much did the driver receive that month? "*In that month, he will receive 90,500.00 reais*".

Figure 9

Resolution by group G4 for item (b) of Situation 1



Note. Research archive.

This inadequate strategy indicates that the members of the group did not correctly understand the proportional relationship between the monthly gross revenue and the commission to be received by the driver. They treated the 6% as a fixed portion of the initial payment, rather than as a percentage of the gross revenue.

In item (c), the students were asked to determine the amount paid to the driver in a month in which the truck generated a gross revenue of R$ 54,000.00. That is, the relational calculation is identical to that of the previous item; therefore, the adequate strategies did not vary.

Groups G2, G6, G7, G14, and G15 converted the 6% percentage into decimal form and multiplied this value by the monthly gross revenue of the truck, that is, *0.06 × 54,000 = 3,240*, thus manifesting TAV2. They then added the commission amount to the driver's fixed payment (R$ 2,500.00), obtaining a total of R$ 5,740.00. In this strategy, they also manifested TAV3, which involves adding the transformation to the initial state.

On the other hand, groups G9, G3, G10, and G16 used a ratio-and-proportion strategy to calculate the commission based on the truck's monthly gross revenue. They established a proportion between the monthly gross revenue (R$ 54,000.00) and the commission, considering the 6% rate. After determining the commission amount, they added it to the driver's fixed payment (R$ 2,500.00), mobilizing both TAV2 and TAV3.

Group G8, in turn, chose to decompose the monthly gross revenue value (R$ 54,000.00) in order to calculate the commission. They decomposed the number, calculated 6% of each part, and summed the resulting values. After that, they added the obtained amount to the driver's fixed payment. This strategy also involved the application of TAV2, TAV3, and TAV4.

Therefore, the students employed different adequate approaches – such as ratio and proportion, numerical decomposition, and multiplication of the rate by the corresponding quantity – to determine the amount paid to the driver based on the truck's monthly gross revenue.

In item c), group G12 once again mobilized TAF4. The students in this group calculated the value of the truck's monthly gross revenue divided by 6%, rather than multiplying it by the 6% rate. It is worth emphasizing that the TAF4 mobilized by group G12 is false, since the correct functional relationship for calculating the commission requires multiplying the gross revenue by the percentage rate, not dividing it.
Although this item was similar to the previous one, a new inadequate strategy emerged. By calculating only the amount the driver received from the truck's monthly revenue, groups G1, G5, G11, and G13 adopted an inadequate strategy in item c). In this case, the students ignored the fixed payment and considered only the commission related to the gross revenue. This approach fails to account for the fixed amount the driver receives in addition to the commission. Therefore, by calculating only the commission on the monthly revenue, these groups did not consider the direct additive transformation that involves the fixed payment together with the commission. This inadequacy can be associated with the false theorem-in-action, TAF5: if a is the rate and b is the linear coefficient, then the functional relationship f is given by $f(x) = ax + b = ax$, in which the students disregarded the constant term b, which represents the *fixed* payment.

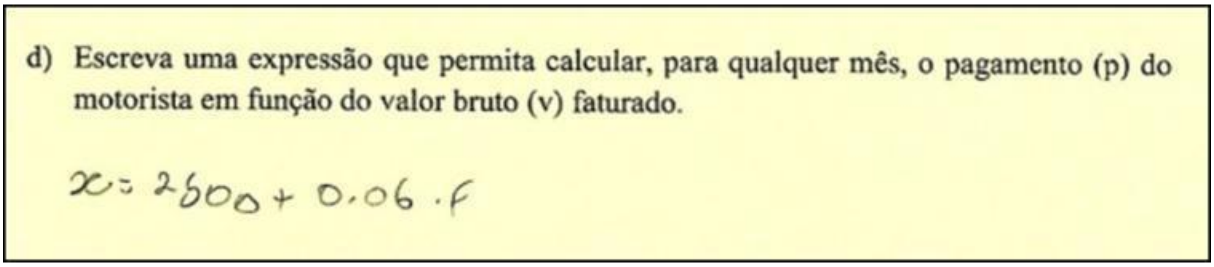
It is also worth noting that groups G1 and G13 presented an adequate solution for item b), but an inadequate one for item c). In addition, group G4 did not present a solution for item c) of the proposed situation, even though it had presented an adequate strategy for the previous item.

In item d), students were challenged to generalize the function that represents the driver's payment (p) as a function of the monthly gross revenue (v). Among the solutions presented by the groups, only group G2 provided an adequate response. Four groups (G6, G8, G9, and G14) presented partially adequate responses. On the other hand, nine groups (G1, G3, G5, G7, G10, G11, G12, G13, and G15) presented inadequate solutions. Groups G4 and G16 did not provide a response to this item. Figure 10[8] shows the solution provided by group G2.

[8] Free translation of item d): d) Write an expression that allows the calculation, for any month, of the driver's payment (p) as a function of the gross revenue (v).

Figure 10

Solution by group G2 for item d) of situation 1



d) Escreva uma expressão que permita calcular, para qualquer mês, o pagamento (p) do motorista em função do valor bruto (v) faturado.

x = 2500 + 0.06 . f

Note. Research archive.

Although they did not use the proposed variables, their generalization was adequate and aligned with the true theorem-in-action TAV1, which describes the functional relationship $f(x) = ax + b$, where a is the rate and b is the linear coefficient. Group G14 generalized the situation in item a), expressing it as: $Sm = 2.500 + 0{,}06x$. However, when asked in item d) to use the variables p and v, the group responded with the expression $p = v + 0{,}06x$. Thus, they replaced the fixed value of 2,500 with a variable v. Groups G8 and G9 presented a partially adequate strategy by establishing an expression that allows the calculation of the driver's payment for any month, considering both the fixed part and the variable commission. However, the students made the mistake of treating coefficients as variables when representing the expression $F = P + 0{,}06V$. Despite this, they correctly manifested TAV1.

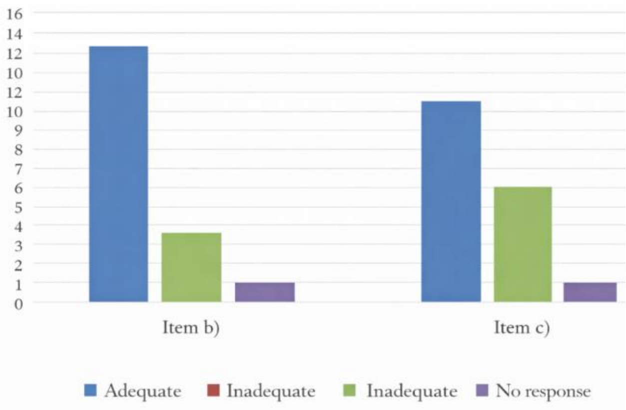
Group G6 also presented a partially adequate strategy. They identified that the situation corresponded to a first-degree function, whose expression is $y = ax + b$, and used the expression $p = av + b$. However, they did not define the coefficients a and b, considering that it was not necessary for this item.

Nine groups (G1, G3, G5, G7, G10, G11, G12, G13, and G15) presented incorrect strategies for generalizing the function that represents the situation. Three groups – G1, G5, and G13 – incorrectly used the expression $T = P + V$. Groups G11 and G12 used the expression $\frac{6}{100} = \frac{x}{1}$, which bears no relation to the correct expression nor to the answers presented previously. Groups G7 and G15 used the expression $p \times 0{,}06 = v$, failing to consider the fixed payment and accounting only for the commission. These groups manifested TAF5, as they did not consider the constant term. The last group, G10, used the expression $f(x) = v \times p\,(2500)$, identifying that it would be a function of x but failing to present the variable and considering only a multiplication. Groups G3, G4, and G16 did not present a solution for item d) of the situation.

Regarding the results for Situation 1, related to the subclass of multiplicative comparison and transformation of measures, it was observed that most groups presented adequate solutions in items b) and c), which involved the same relational calculation but had different objectives. In item b), thirteen groups presented correct solutions, while three groups responded inadequately. In item c), there were ten correct solutions, five incorrect ones, and one blank response. This indicates that some groups did not perceive the connection between the two items or had doubts regarding the resolution of the first item. The graph in Figure 11 presents a synthesis of the groups' solutions.

Figure 11

Group performance on items b) and c)



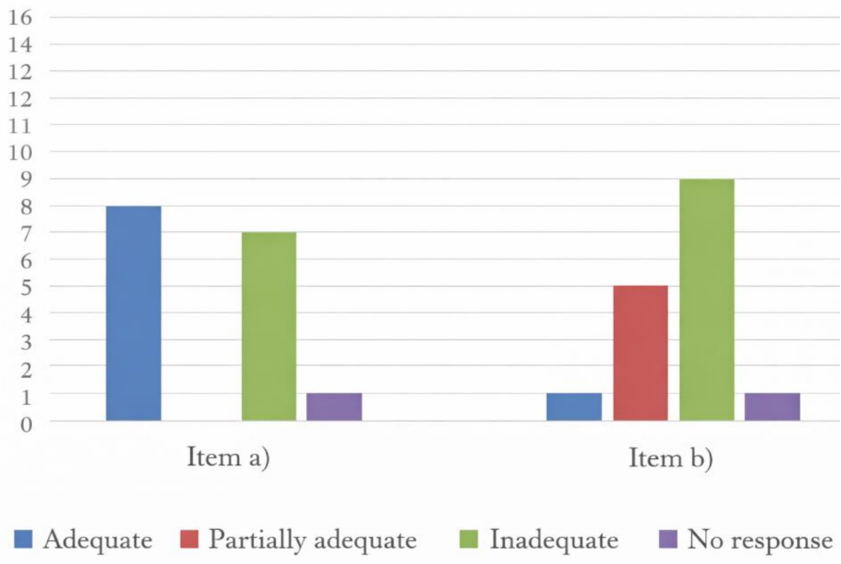
Note. Prepared by the authors.

From the graph, a decrease in students' performance from item b) to item c) can be observed, indicating that these students lacked confidence in their own solutions. Thus, even after succeeding in the first situation, they did not recognize that the items involved the same strategy.

In items a) and d), which involved generalizing the situation, the difficulties manifested by the students were more pronounced. In the first item, only eight groups were able to express the generalization adequately in natural language, while the others considered specific cases to represent the situation. Regarding the algebraic expression, only one group responded correctly, while four groups had difficulties by treating coefficients as variables, and the others presented inadequate strategies or did not respond (Figure 12).

Figure 12

Group performance on items a) and d)



Note. Prepared by the authors.

There was a significant increase in the number of inadequate strategies in the transition from item a) to item b). This result suggests that students are still in the process of developing the skills required to effectively convert a representation in natural language into an algebraic language. It also indicates that this situation constituted a new type of problem for the students, who did not yet have ready-made schemas to solve it.

The students' solutions indicate the manifestation of both true and false theorems-in-action. In item a), the mobilization of one true theorem-in-action (TAV1) and three false ones (TAF1, TAF2, and TAF3) can be observed. In items b) and c), three true theorems-in-action (TAV2, TAV3, and TAV4) and two false ones (TAF4 and TAF5) were mobilized. The false theorems-in-action are associated with errors in percentage calculation, the substitution of coefficients with variables, and the failure to consider the independent term of the affine function.

Vergnaud (1996) and Bittencourt (1998) emphasize the importance of recognizing the false theorems manifested by students, as this enables teachers to propose situations that challenge these misconceptions, leading students to reflect on their errors and reconstruct their mathematical understanding. Understanding the difficulties presented by students is therefore essential for comprehending the organization of their knowledge and, consequently, for designing more effective teaching strategies.

Final Considerations

The research presented herein aimed to analyze the solutions produced by groups of students when faced with a mixed situation of the multiplicative comparison–measure transformation type, associated with the affine function. The strategies used by the groups were identified, as well as the presence of true and false theorems-in-action in their solutions. The situation selected for this article was chosen because it presented a lower level of complexity compared to the other situations included in the first author's master's dissertation, which allowed for a greater amount of data to be collected.

In the analysis of this situation, it was possible to identify that, among the relevant forms of knowledge mobilized by the students, the following stand out: an understanding of the relationship between the truck's revenue and the driver's payment; the identification of the proportionality coefficient as the commission rate applied to the gross revenue; and the recognition that the driver's final payment is composed of a fixed part and a variable part. However, misconceptions were also identified. Some groups of students showed confusion in interpreting the commission rate and, as a result, performed incorrect percentage calculations. In addition, some students failed to include the driver's fixed payment in the general expression, which indicates a lack of understanding of the constant term of the affine function. These errors can be attributed to distinct factors. Overall, they indicate that students show limited familiarity when solving contextualized situations associated with the affine function, especially those involving salary contexts derived from commissions, as addressed in this investigation. They also reveal that students do not possess previously constructed and sufficiently organized schemes to solve affine function situations belonging to the multiplicative comparison–measure transformation class.

According to Vergnaud (2009), a concept is understood by students throughout the schooling process and as a result of the situations they encounter. Considering that the class of multiplicative comparison–measure transformation situations addressed in this study is one of the least explored in middle school and high school textbooks, as shown by Miranda (2019), and considering the analysis of the solutions produced by the participant students, it can be concluded that the situations proposed in the research instrument were novel for the students.

Furthermore, the results of this study indicate that, within the groups' schemes, four (4) true theorems-in-action and five (5) false theorems-in-action were identified. The latter are associated with confusion between variables and constants; a lack of understanding of the rate of change and the linear coefficient; the failure to indicate the quantity to which the percentage refers; the incorrect interpretation of the centesimal ratio of the percentage rate; and the omission of the constant term of the affine function.

Thus, by identifying the false theorems-in-action mobilized by students and their difficulties, future research is encouraged to design mixed situations related to the affine function that allow for the destabilization of these TAFs. In addition, the results may contribute directly to teaching practice by helping teachers understand the types of knowledge mobilized by students when faced with problem situations involving the affine function. By having access to students' solutions and the conceptual errors they exhibit, educators can adapt instructional content to students' levels of knowledge and skills, providing a gradual progression in the complexity of the situations presented. Therefore, it is expected that the results of this research may contribute both to academic inquiry and to educational practice, fostering a more effective approach to the teaching of the affine function and to the development of students' mathematical skills related to this concept.

Acknowledgments

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