

State of knowledge: A historical study on arithmetic problems

Estado del conocimiento: Un estudio histórico sobre problemas aritméticos

État des Connaissances : une étude historique sur les problèmes arithmétiques

Estado do conhecimento: Um estudo histórico sobre os problemas aritméticos

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Abstract

It is currently known that problems in mathematics teaching are present in teachers' school teaching processes, official documents, and textbooks. However, is it possible to say that the problems have always been present in educational practices and official documents linked to teaching? Based on these initial considerations, this production is affiliated with the research field of the history of mathematics education. It aims to scrutinize the permanence and changes, at different historical moments, of the meanings and purposes of *problems* for mathematics teaching from a literature review based on the state of knowledge methodology. From the survey of works such as dissertations, theses, and scientific articles in different digital repositories, it is clear that there have been changes in the conception of problems over time. Between the transition from traditional teaching periods to the intuitive method, the term *problems* is not mentioned. Subsequently, the term is presented as a synonym for *exercises*, for teaching arithmetic. In New School times, however, *problems* became dissociated from *exercises*. They gained both a teaching support character and are also structured as a teaching method to be followed. Finally, problem solving is gaining ground in research in mathematics education field.

Keywords: History of mathematical education, State of the art, Mathematical problems, Arithmetic.

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Resumen

Actualmente se sabe que los problemas para la enseñanza de las matemáticas están presentes en los procesos de enseñanza escolar del docente, tanto en documentos oficiales como en libros de texto. Sin embargo, ¿es posible decir que los problemas siempre han estado presentes en las prácticas educativas y en los documentos oficiales vinculados a la enseñanza? A partir de estas consideraciones iniciales, esta producción se adscribe al campo de investigación de la Historia de la educación matemática y pretende escudriñar permanencias y transformaciones, en diferentes momentos históricos, de los significados y propósitos de los “problemas” para la enseñanza de las matemáticas desde una revisión de la literatura basada en la metodología del Estado del Conocimiento. Del relevamiento de trabajos como tesis, disertaciones y artículos científicos, en diferentes repositorios digitales, se desprende que ha habido cambios en la concepción de los problemas a lo largo del tiempo. Entre la transición de los períodos de enseñanza tradicional al Método Intuitivo, no se menciona el término problemas. Posteriormente, el término problemas se presenta como sinónimo de ejercicios, para la enseñanza de la aritmética. En los tiempos de la Escuela Nueva, en cambio, los problemas se disociaron de los ejercicios. Adquirieron a la vez un carácter de apoyo a la enseñanza y también se estructuran como un método de enseñanza a seguir. Finalmente, la resolución de problemas está ganando terreno en la investigación en Educación Matemática.

Palabras clave: Historia de la educación matemática, Estado del arte, Problemas matemáticos, Aritmética.

Résumé

On sait actuellement que les problèmes d'enseignement des mathématiques sont présents dans plusieurs instances du processus éducatif, comme dans les documents officiels, ou encore dans les livres et le matériel pédagogique, par exemple. Pour autant, peut-on affirmer que les problèmes ont toujours été présents dans les pratiques éducatives et dans les documents officiels liés à l'enseignement ? Partant de ces premières considérations, cette production s'inscrit dans le champ de recherche de l'Histoire de l'enseignement des mathématiques et vise à scruter les permanences et les transformations, à différents moments historiques, des significations et des finalités des « problèmes » pour l'enseignement des mathématiques à travers une revue des littératures, basée sur la méthodologie de l'état des connaissances. En examinant des ouvrages tels que des thèses, des mémoires et des articles scientifiques, dans différents référentiels numériques, il apparaît clairement qu'il y a eu des changements dans la conception des

problèmes au fil du temps. Entre le passage des périodes d'enseignement traditionnel à la Méthode Intuitive, le terme de problèmes n'est pas évoqué. Par la suite, ce mot est présenté comme synonyme d'exercices, pour l'enseignement de l'arithmétique. En revanche, à l'époque de la Nouvelle École, les problèmes étaient dissociés des exercices. Ils acquièrent à la fois un caractère de support pédagogique et sont également structurés comme une méthode pédagogique à suivre. Enfin, la résolution de problèmes gagne du terrain dans la recherche en enseignement des mathématiques.

Mots-clés : Histoire de l'enseignement mathématique, Etat de l'art, Problèmes mathématiques, Arithmétique.

Resumo

Sabe-se, atualmente, que os problemas para o ensino de matemática se encontram presentes em diversas instâncias do processo educacional, como em documentos oficiais, ou também, em livros e materiais didáticos, por exemplo. Entretanto, é possível afirmar que os problemas sempre estiveram presentes nas práticas educacionais e em documentos oficiais ligados ao ensino? A partir destas ponderações iniciais, esta produção se filia ao campo de pesquisa da História da educação matemática e tem por objetivo escrutinar permanências e transformações, em diferentes momentos históricos, dos sentidos e finalidades dos “problemas” para o ensino de matemática mediante uma revisão de literatura, embasada na metodologia do Estado do Conhecimento. Através do levantamento de trabalhos como teses, dissertações e artigos científicos, em diferentes repositórios digitais, constata-se a existência de modificações da concepção dos problemas ao longo dos tempos. Entre a transição de períodos de ensino tradicional para o Método Intuitivo, não há menção ao termo problemas. Posteriormente, este vocábulo se apresenta como sinônimo de exercícios, para o ensino de aritmética. Em tempos escolanovistas, por outro lado, os problemas foram se dissociando dos exercícios. Ganharam tanto um caráter de apoio ao ensino, como também se estruturam como um método de ensino a ser seguido. Contemporaneamente, por fim, a resolução de problemas ganha espaço nas pesquisas em Educação Matemática.

Palavras-chave: História da educação matemática, Estado da arte, Problemas matemáticos, Aritmética.

State of knowledge: A historical study on arithmetic problems

What does the term problems mean? According to the dictionary of the Portuguese language (Ramos, 2011, p. 672), the word problem can designate “a question one attempts to clarify or resolve in any area of human knowledge, and which generally serves as a basis for studies or academic or scientific discussions,” a “complex situation or difficult solution,” or even an “obstacle or difficulty that makes it hard to reach or achieve an objective.” In this initial panorama, the problem encompasses challenging issues or situations that are difficult to resolve, i.e., that have some complexity.

Attributing the relationship between the challenge and complexity of a problem to the teaching of school mathematics, nowadays, some textbooks treat problem solving as an auxiliary method to teach content, enabling students to reflect, and developing students' critical-reflective mathematical reasoning to overcome a difficulty or gap that is presented to them.

The textbook *A Conquista da Matemática - Manual do professor* [The Conquest of Mathematics - Teacher's Manual], written by Giovanni and Castrucci (2018) for the 6th grade of elementary school – final years, brings to the teachers some thoughts about mathematics teaching through methods such as mathematical modeling, problem solving, digital technologies, and communication in mathematics classes as means of support and assistance for teaching this curricular component. The authors highlight that:

The possibility of analyzing various ways of solving specific problems and confronting and validating hypotheses also provides learning that goes beyond mathematics teaching, culminating in the formation of a more active individual in society, a person who relates to groups and faces problem situations, seeking solutions and feeling confident if challenged by complex issues. (Giovanni & Castrucci, 2018, pp. VII-VIII)

A second manual, also for the 6th grade, *Matemática Bianchini* [Bianchini's Mathematics], by Edwaldo Bianchini (2022), proposes problem solving as a didactic proposal for teaching. The purposes of solving problems are:

[...] one of the highlights of contemporary mathematical teaching. To meet the assumptions of a globally formative education, the mathematical problem must, whenever possible, be presented in a challenging context that makes sense to the student. This enables the mobilization of studied content in search of solutions and, above all, opens some space for creating personal strategies and producing new knowledge. (Bianchini, 2022, p. XVI)

According to the author, the objective when using problems is to develop resolution procedures, compare results, and validate the technique or procedure. Therefore, we can see that the problems for teaching mathematics in basic education are integrated with the most current textbooks and are aligned with the National Book and Teaching Material Program (Programa Nacional do Livro e do Material Didático – PNLD), which aims to disseminate such materials suitable for each educational level in Brazil.

However, how was this conception of the term “problem solving” structured for the teaching that has been consolidated in schools and in the specific educational materials for the mathematics we currently have? Can we consider changes in understanding and purposes of problems and mathematics teaching at different historical educational moments?

Therefore, to address the issue above, we structured this article into four sections. This introductory section addresses the justifications for its elaboration and highlights the theoretical-methodological framework used. The second section discusses the teaching of problems in the pedagogical vacancies of the intuitive method and the New School. The third section explains the teaching of problems in times of the modern mathematics movement and mathematics education. Finally, the last section approaches the conclusions of this paper.

Thus, this work aims to scrutinize the permanence and transformations, in different historical moments, of the meanings and purposes of the “problems” for mathematics teaching based on a literature review based on the state of knowledge methodology. Therefore, the work developed is theoretically guided by a bibliographic review study supported by the state of knowledge methodology.

According to Vosgerau and Romanowski (2014), such studies aim to:

[...] organize, clarify, and summarize the main existing works, as well as provide complete citations covering the spectrum of relevant literature in a field. Literature reviews can present an analysis that provides a historical overview of a topic by considering publications in a field. An analysis of publications can often contribute to the historical reformulation of academic dialogue by presenting a new direction, configuration, and referrals. (Vosgerau & Romanowski, 2014, p. 167)

Thus, the bibliographical review allows us to revisit completed works and present new perspectives and discourses based on the dialogue between the productions. This article formulates a historical overview of changes in conceptions about problems throughout different educational moments.

Using the two expressions “state of knowledge” and “state of the art” as a methodological tool, Ferreira (2002) points out that:

[...] the challenge of mapping and discussing some academic production in different fields of knowledge, trying to answer which aspects and dimensions have been highlighted and privileged in different times and places, in what ways and under what conditions specific master's and doctoral theses, publications in journals, and communications in conference and seminar proceedings have been produced. (Ferreira, 2002, p. 258)

We must note that the literature in Brazil uses “state of knowledge” and “state of the art” as synonyms. However, some authors differentiate those methods. Romanowski and Ens (2006) argue that a state-of-the-art work encompasses the systematized study of a given topic in different spheres of knowledge production, such as master's and doctoral theses, articles, periodicals, and conference proceedings, among other means of dissemination of scientific work, not limited to just one aspect, for a more comprehensive dialogue on the proposed topic.

Studies based on data systematization, called “state of the art”, receive this name when they cover an entire area of knowledge in the different aspects that generated productions. [...] The study that addresses only one sector of publications on the topic studied has been called “state of knowledge.” (Romanowski & Ens, 2006, pp. 39 – 40)

In this sense, the specification of the work aimed at a state of knowledge intends to fill gaps in a given area and theme based on productions already carried out in a given time frame, as well as in just one sphere of publications and, therefore, the inventory of productions of this method remains in a more restrictive instance compared to state-of-the-art research.

Under this assumption, the “state of knowledge” methodology is used as a more restricted bibliographical review, covering productions on problems linked to the history of education and the history of mathematics education in theses and dissertations between 2010 and 2022 and scientific articles between 2017 and 2022.

Below, we present the works that compose the literature review. By selecting works that are close to the theme, we aim to understand the productions in the fields of history of education and the history of mathematics education in Brazil.

Looking for productions from postgraduate programs, master's and doctoral theses, we used three search portals as online databases: the Catalog of Theses and Dissertations of the Coordination for the Improvement of Higher Education Personnel (Catálogo de Teses e Dissertações da Coordenação de Aperfeiçoamento de Pessoal de Nível Superior³ - CAPES), the Brazilian Digital Library of Theses and Dissertations (Digital Brasileira de Teses e

³ Available at: <https://catalogodeteses.capes.gov.br/catalogo-teses/#/>

Dissertações⁴ - BDTD) and the Digital Content Repository (Repositório de Conteúdo Digital⁵ - RCD) of the Federal University of Santa Catarina.

Looking for works on mathematical problems from a historical-educational perspective, the CAPES Catalog of Theses and Dissertations was initially searched for the term [*problemas*], where 53,737 search results mentioned the term at some level in the identification descriptors. As this query involved too many searches, we had to refine it. For a new survey of searches, in a second moment, the terms [*problemas AND história*] were used as keywords, returning 3,671 results.

To bring research closer to mathematical problems and identify productions that point to [*problemas*] as a method, subject, or exercise related to mathematics, we decided to replace [*história*] with [*história da educação*], having as new search words or terms [*problemas AND história da educação*] between 2010 and 2022.

In this last consultation, 70 theses and dissertations returned. Therefore, we chose those productions that deal directly with mathematical problems in a historical conception.

Such productions are identified in the table below:

Table 1.

CAPES Catalog of Theses and Dissertations (Research authors, 2022)

TÍTULO	AUTORIA	MESTRADO/ DOUTORADO	ANO
A RESOLUÇÃO DE PROBLEMAS DE MATEMÁTICA NAS SÉRIES INICIAIS DO ENSINO DE PRIMEIRO GRAU NA REDE ESTADUAL DE ENSINO DO ESTADO DO PARANÁ NA DÉCADA DE 1970: UM ESTUDO HISTÓRICO-CULTURAL	RITA DE CASSIA GOMES WALDRIGUES	Mestrado	2010
O ENSINO DE MATEMÁTICA DA ACADEMIA DE COMÉRCIO DE SANTA CATARINA NAS DÉCADAS DE 1930 E 1940	JULIANO ESPEZIM SOARES FARIA	Mestrado	2011
RETRATOS DE UMA SALA DE AULA PROJETOS E RESOLUÇÃO DE PROBLEMAS NA MATEMÁTICA DOS ANOS INICIAIS	MARIA ANGELA DIAS DOS SANTOS MINATEL	Mestrado	2014
O PROCESSO CONSTITUTIVO DA RESOLUÇÃO DE PROBLEMAS COMO UMA TEMÁTICA DA PESQUISA EM EDUCAÇÃO MATEMÁTICA - UM INVENTÁRIO A PARTIR DE DOCUMENTOS DOS ICMES	ROSILDA DOS SANTOS MORAIS	Doutorado	2015
UM OLHAR PARA A RESOLUÇÃO DE PROBLEMAS NOS ENCONTROS NACIONAIS DE EDUCAÇÃO MATEMÁTICA (ENEMs): DELINEAMENTO DE UMA TENDÊNCIA	TIEGO DOS SANTOS FREITAS	Doutorado	2019
UMA HISTÓRIA DOS PROBLEMAS ARITMÉTICOS: MUDANÇAS NO SABER PROFISSIONAL DO PROFESSOR QUE ENSINA MATEMÁTICA (1870-1960)	ANDREIA FERNANDES DE SOUZA	Doutorado	2021

For the plurality of productions in this perspective, another data collection was carried out at BDTD. The advanced search between 2010 and 2022, with the filters [*problemas*] in the

4 Available in: <https://bdttd.ibict.br>

5 Available at: <https://repositorio.ufsc.br/handle/123456789/1769>

title and [*história da educação*] and [*matemática*] in all descriptor fields, revealed 42 works. We also chose productions that deal with problems historically, involving the history of education and the history of mathematics education. In this survey, the previously explained research was removed, so there would be no duality presented here.

Table 2.

Theses and Dissertations at BDTD (Research authors, 2022)

TÍTULO	AUTORIA	MESTRADO/ DOUTORADO	ANO
A CONCEPÇÃO DA HABILIDADE DE RESOLUÇÃO DE PROBLEMAS ARITMÉTICOS EM LIVROS DIDÁTICOS DO ENSINO FUNDAMENTAL: UM OLHAR SOB A CONCEPÇÃO HISTÓRICO-CULTURALISTA	ANDERSON FELIX SILVA	Mestrado	2021
ÁLGEBRA PARA RESOLVER PROBLEMAS: AS PROPOSTAS DE OTELO DE SOUZA REIS E TITO CARDOSO DE OLIVEIRA, DÉCADA DE 1910	IVONE LEMONS DA ROCHA	Mestrado	2019
OS PROBLEMAS ARITMÉTICOS E OS MÉTODOS PEDAGÓGICOS: PONTOS PARA UM DIÁLOGO SOBRE A HISTÓRIA DA EDUCAÇÃO MATEMÁTICA NO ENSINO PRIMÁRIO ALAGOANO (1924 - 1952)	ELISABETE PEREIRA FERNANDE S	Mestrado	2017
RESOLUÇÃO DE PROBLEMAS: UM ESTUDO SOBRE SEU PROCESSO EVOLUTIVO NOS ESTADOS UNIDOS, NA CHINA E NO BRASIL	YUK WAH HSIA	Doutorado	2013

Furthermore, the RCD of the Federal University of Santa Catarina was consulted, as it houses a community on the history of mathematics education, which prioritizes sources, documents, and investigations on the historical theme proposed in this research. Following in the same direction as previous bibliographical surveys, the table below presents significant research close to the topic. We must note again that the works already mentioned are not shown.

The RCD comprises a virtual platform on which academic works and documents of different natures can be found, thus encompassing a diversity of communities and collections. Uniquely, we justify choosing this platform because:

The repository under discussion [...] aligns with the history of mathematics education sub-community, nested within the UFSC Sciences of Education Center. It is a virtual, open, and institutionalized repository specifically for storing diverse sources, essays, and research focused on the history of mathematics education. (Costa & Valente, 2015, p. 101)

Present in the “TESES E DISSERTAÇÕES em História da Educação Matemática” [THESES AND DISSERTATIONS in history of mathematics education] subcommunity, the search for [*problemas*] returned 37 works. From the dissertations and theses consulted, we chose to delve into texts that cited the history of mathematics education in their title or abstract—or that alluded to the problems and their history linked to the topic.

Table 3.

Theses and Dissertations at RCD (Research authors, 2022)

TÍTULO	AUTORIA	MESTRADO/ DOUTORADO	ANO
PROBLEMAS DE ARITMÉTICA EM TEMPOS DE ARITMÉTICA INTUITIVA: UMA ANÁLISE EM LIVROS DIDÁTICOS (1890-1930)	KARINA CRISTINA DOS SANTOS PAVARIN	Mestrado	2020
DISCURSOS SOBRE PROBLEMAS ARITMÉTICOS (SÃO PAULO, 1890-1930)	ANDREIA FERNANDES DE SOUZA	Mestrado	2017
A RESOLUÇÃO DE PROBLEMAS DE ARITMÉTICA NO ENSINO PRIMÁRIO: UM ESTUDO DAS MUDANÇAS NO IDEÁRIO PEDAGÓGICO (1920-1940)	WELLINGTON PEREIRA DAS VIRGENS	Mestrado	2014

As a result, 13 works were chosen to contribute to the theme of problems for teaching mathematics from a historical-cultural conception, comprising four doctoral dissertations and 9 master's theses. We analyzed the abstract, the research problem, and the results covering the historical constitution to characterize the mathematical problems.

Once the process is complete, the CAPES periodicals portal is also used to access other productions and publications in scientific journals, adopting the years 2017 to 2022 as a time frame, to identify what was produced and what is being produced on mathematical problems in Brazil recently.

Using the search filters [*problemas*] as an exact term for the title and [*história da educação matemática*] in any field of the articles, specifically from the last five years, we found 27 results, 15 of which were selected for this discussion.

Table 4.

Articles on the CAPES periodicals portal (Research authors, 2022)

TÍTULO	AUTORIA	ANO
ESCRITOS DE THEREZA PEREIRA ROCHA: DE PESTALOZZI A THORNDIKE, SABERES PARA ENSINAR PROBLEMAS ARITMÉTICOS	BRUNA LIMA RAMOS GIUSTI; ANDREIA FERNANDES DE SOUZA	2022
OS PROBLEMAS DE ARITMÉTICA NOS LIVROS DIDÁTICOS: FINALIDADES DE USO (1890-1920)	KARINA PAVARIN	2022
OS PROBLEMAS MATEMÁTICOS COMO UM ELEMENTO DA GRADUAÇÃO DOS CONTEÚDOS NO ENSINO PRIMÁRIO: MINAS GERAIS, ALDA LODI, 1941	ANA CRISTINA S. M. ROCHA	2021
PROBLEMAS ARITMÉTICOS COMO ELEMENTOS DA MATEMÁTICA DO ENSINO	LUCIANE DE FATIMA BERTINI; WAGNER RODRIGUES VALENTE	2021
DO CÁLCULO AO PROBLEMA: UM CAMINHO A TRILHAR A PARTIR DE RESULTADOS CIENTÍFICOS	NARA VILMA LIMA PINHEIRO	2021
MAS AFINAL O QUE SÃO PROBLEMAS? UMA ANÁLISE HISTÓRICA SOBRE MUDANÇAS EM DEFINIÇÕES, FINALIDADES E TIPOLOGIAS	LUCIANE FATIMA BERTINI; ANDREIA FERNANDES SOUZA	2021
AS ORIENTAÇÕES PROPOSTAS NOS DOCUMENTOS ALAGOANOS PARA O ENSINO DE PROBLEMAS ARITMÉTICOS A PARTIR DA DÉCADA DE 50	ELISABETE PEREIRA FERNANDES MERCEDES CARVALHO	2021
INTELIGÊNCIA, RACIOCÍNIO E PROBLEMAS MATEMÁTICOS: NOTAS SOBRE UM DEBATE A PARTIR DAS ANOTAÇÕES DE ALDA LODI	ANA CRISTINA S. M. ROCHA; WAGNER RODRIGUES VALENTE	2020
UM TANGO QUE VIROU SAMBA: VÍCTOR MERCANTE E AS APROPRIAÇÕES PARA ENSINAR PROBLEMAS ARITMÉTICOS (SÃO PAULO, 1929 - 1952)	ANDREIA FERNANDES DE SOUZA	2020
OS PROBLEMAS NA 'MATEMÁTICA PARA ENSINAR ENSINADA' EM CURSOS DE FORMAÇÃO DE PROFESSORES PARA O ENSINO PRIMÁRIO (DÉCADAS DE 1920 E 1930)	LUCIANE DE FATIMA BERTINI	2019
ÁLGEBRA NO ENSINO PRIMÁRIO BRASILEIRO: SUA RELAÇÃO COM OS PROBLEMAS DE ARITMÉTICA NO INÍCIO DO SÉCULO XX	IVONE LEMOS DA ROCHA; LUCIANE DE FATIMA BERTINI	2019
RESOLUÇÃO DE PROBLEMAS ARITMÉTICOS: MÉTODO ANALÍTICO OU SINTÉTICO? O QUE REVELAM AS REVISTAS E DOCUMENTOS OFICIAIS ALAGOANOS (1924-1930)	ELISABETE PEREIRA FERNANDES MERCEDES CARVALHO	2019
"RESOLUÇÃO DE PROBLEMAS PELAS EQUAÇÕES ALGÉBRICAS": A PROPOSTA DE TITO CARDOSO DE OLIVEIRA PARA O ENSINO DAS OPERAÇÕES	LUCIANE DE FATIMA BERTINI, IVONE LEMOS DA ROCHA	2018
CADERNOS ESCOLARES: PROBLEMAS ARITMÉTICOS NO ENSINO DE MATEMÁTICA EM UM DIÁRIO ESCOLAR DOS ANOS 1950	NICOLAS GIOVANI DA ROSA; MAYARA BECKER OLIVEIRA DA SILVA; ELISABETE ZARDO BÜRIGO	2017
SINTÉTICO OU ANALÍTICO: COMO ENSINAR PROBLEMAS DE ARITMÉTICA? (SÃO PAULO, 1920)	ANDREIA FERNANDES DE SOUZA	2017

Such productions will contribute to understanding how the topic related to mathematical problems is being produced and interpreted in Brazilian research from a historical perspective.

Exercises and problems in times of the intuitive method and the New School

Firstly, it is important to highlight what problems in mathematics teaching mean. The immediate question is: "But what are problems anyway?" This is the first analyzed work, authored by Bertini and Souza (2021), published in the *Ensino de Ciências e Matemática* [Science and Mathematics Teaching] journal. The authors use already completed work to discuss the problems, whether there are changes over time and their relationship or differentiation regarding the term *exercícios*. Different sources support their arguments.

Studying the end of the 19th century, the authors, in contact with different manuals during the intuitive method pedagogical period, identify in *Compêndio de Pedagogia*

[Pedagogy Compendium], by Bráulio Cordeiro, 1874, recommendations for the use of problems to teach operations. However, the manual does not define those problems.

Bertini and Souza (2021) verify that the first strict definition of problems was found in 1888 in the manual *Arithmetica Pratica* [Practical Arithmetic] by Felipe Nery Collaço. In this manual, the arithmetic problem was delimited as a question involving unknown quantities to be determined.

This definition would improve what had been learned, i.e., understanding unknown elements through given numbers and quantities. This understanding does not separate problems and exercises since the problems were intended to exercise. Antonio Trajano's work, *Arithmetica Primária* [Primary Arithmetic], published in 1895, differentiates and introduces problems for teaching as a method, not just for exercise. Therefore, for Trajano, “the difference between a problem and an exercise would reside in adding a statement with a question” (Bertini & Souza, 2021, p. 10).

At the end of the 19th century, the problems began to be acclaimed and differentiated from the exercises. This movement did not just happen nationally, and research on the importance of teaching problems began to emerge.

Hsia (2013) indicates different understandings of problems, such as John Dewey's, George Polya's, George Stanic and Jeremy Kilpatrick's, and John Van de Walle's, among others. Regarding the emergence of problems for teaching, Dewey identified five stages: “1. Felt need; 2. Analysis of the difficulty; 3. Alternative solutions; 4. Experimentation with several options, until the mental test approves one of them; and 5. Action as final proof for the proposed solution, which must be scientifically verified” (Hsia, 2013, pp. 49–50).

Thus, Dewey's studies provided the first understanding of how problems can be used in educational processes. In his book *How We Think*, published in 1910, Dewey advocated training in thinking from the perspective of problems, the elaboration of reflective, meaningful thinking that emerges from experimentation (Hsia, 2013).

Over the years, the problems gained space in international research, and George Polya became known as the “father of problem solving.” Polya produced important references, such as the book “*How to Solve It*.” According to Hsia (2013), Polya defended that students should construct and create knowledge instead of being presented with a non-reflective science closed in itself. Solving a problem means finding the solution and the dynamics to be adopted, despite all the obstacles and difficulties. Obtaining this resolution is the achievement of human intelligence; our thoughts and actions are always aimed at some purpose.

Problems were further classified into two types: 1) Routine, those mechanical problems that would be solved by following a rule, and 2) Non-routine problems, problems with a “purpose for the student”, thus linked to their interests and daily life.

The first mention of problems by the “father of the New School,” John Dewey, at the end of the 19th century, brings some theoretical considerations for teaching problems, justifying it as a means of developing “reflective thinking” defended from experimentation. Corroborating the idea of reflective education, Polya develops methods and guidelines for teaching problem solving based on his concern with education, recognizing the potential of the problem as an aid in the educational process.

Interest in research on problems and problem solving spread on Brazilian soil mainly during the New School era. Therefore, Dewey's New School ideas about teaching at the time permeated new national educational proposals. Bertini and Souza (2021), in contact with São Paulo pedagogical journals from the early 20th century, and with the change in educational ideas following the New School, problems became high-profile and differentiated from the exercises.

The authors defend the appearance and detailing of problems in São Paulo's teaching programs. Different views on the treatment of problems are highlighted. In one of the articles published in *Revista Educação* in 1929 by Anna Nogueira Ferraz and analyzed by Bertini and Souza (2021), problems are understood as content to teach. Other periodical productions deal with problems, not just as a means of introducing the content but presenting data that relate to students' practical lives. According to the authors, “Exercise was increasingly related to the use of memory, repetition, and exercise of the mind, while the problems were more associated with refining reasoning” (Bertini & Souza, 2021, p. 16).

It is worth mentioning that this production focused mainly on pedagogical manuals circulated in Brazil and educational journals from the 1920s and 1930s in São Paulo. In this regard, these representations of problems for teaching mathematics mainly encompass a perception of education in São Paulo. The analysis of the other bibliographies highlighted the representation of problems from different spatial and temporal perspectives, sources, and understanding of problems for teaching.

Likewise, starting from the conception of the problem related to students' lives, Pinheiro (2021) brings international discussions and references about the scientific investigations of the American educator Carleton Washburne and his team from the Commission of Seven in their work in the schools of Winnetka, Chicago (USA). Based on students' difficulties in learning arithmetic, Washburne proposed using tests to understand the reasons for such difficulties and

look for appropriate solutions to improve teaching. Students at the Chicago school were administered different types of tests. One specified the order of teaching processes using the problem method or the mechanical method⁶.

Due to this and other tests performed, Washburne and his team found an answer to their inquests about the difficulties of teaching problems. The problem statement was “the problem.” Statements that integrated the student's life as examples of family relationships and children's activities instead of dealing with problems in the practical life of an adult favored a better understanding of the problem and, consequently, better results for solving them (Pinheiro, 2021).

Pinheiro (2021) says that those representations were disclosed in Brazil in the 1930s: in the capital, Rio de Janeiro, by Alfredina de Paiva Souza, in São Paulo by teachers from the Normal Schools who added problem methods to primary school programs, and in Minas Gerais by Alda Lodi from her pedagogical mission in the Teachers College.

Regarding the teaching of problems in Brazil, from the constitution of the republican education, Pavarin (2020) analyzed arithmetic problems in textbooks between 1890 and 1930, during the pedagogical waves of the intuitive method and the New School. In short, based on references to intuitive arithmetic linked to the ideals of the intuitive method, the author establishes four purposes for the problems: exploring, instructing, applying, and verifying.

In the same time frame, Souza (2017a) sought conceptualizations of the problems and how they integrated teaching in São Paulo, supporting his analysis of pedagogical journals between 1890 and 1930. We realize that the time frame is close to what was previously seen, readily weaving a dialogue between the intuitive and new-school methods. The author indicates continuities and changes in the teaching of arithmetic problems in the articles. She also defines five different classes, or trends, as the author herself calls, on those problems and their relationships in historical sources: 1) Absence as an indication; 2) Problems as synonyms of exercises 3) Problems as a symbol of pedagogical modernity; 4) Arithmetic to teach problems; 5) Problems from the centers of interest.

The articles in the first category do not discuss the use of problems between 1890 and 1896. Going through the heritage marks of traditional teaching and the emergence of the intuitive method, the second categorization (1896 – 1906) presented by Souza (2017a) clarifies

⁶ “The problem method was based on the idea that students would effectively learn a mathematical process if faced with a problem requiring such a process. For this to happen, the problem must be real, involve a family situation, and arouse interest in solving it. [...] In turn, with the mechanical method, teachers taught mathematical processes and operations independently, without using problems, which would only be used when students mastered the operations. In both methods, the same types of problems were used” (Pinheiro, 2021, p. 8).

the association between problems and exercises as synonyms, without specific differences for teaching as an act of reinforcement of content at the end of each exhibition.

According to Souza (2017a), the third category, “Problems as a symbol of pedagogical modernity” (1897 – 1922), the problems were seen as a method for teaching arithmetic that should connect to the students’ reality. The fourth categorization, “Arithmetic to teach problems,” came from experiences with the analytical method. The problems went from supporting teaching arithmetic to becoming a teaching method. Problems surrounded students’ lives and interests, and a method for solving them was presented. Therefore, the teaching method through problems provided the ability for analysis, observation, and reasoning instead of only helping to solve elementary operations. The last category links the problems to the teaching of the interest centers. Based on reading and dialogue about the sources, “students’ curiosity and interest were used to organize the content that should be taught” (Souza, 2017a, p. 84).

In dialogue with the categories referenced above, Souza's (2020) article, intending to apprehend appropriations made for teaching problems proposed by Argentine professor Victor Mercante in his work *Cultivo y Desarrollo de la Aptitud Matemática del Niño* [Cultivating and developing children's mathematical skills], released in 1905, emphasizes that problems should be a means of teaching arithmetic, and not a reinforcement of learning, thus marking the difference between problems and exercises, each with its purpose.

The problems had the potential to promote the development of student reasoning. Victor Mercante then proposed five steps for developing simple and compound problems: objectification, induction, decomposition, analysis, and response (Souza, 2020).

Regarding the specifications between the intuitive method and the New School and their transitions for teaching problems, Souza (2021) analyzes sources such as pedagogical manuals, schoolbooks, pedagogical journals, and São Paulo programs. According to the author, intuitive problems, linked to understanding of the intuitive method,

[...] had different purposes (introducing, assessing, exercising, and verifying what was taught), used empirical materials, graduated the difficulty levels, starting from the simple to the complex and from the concrete to the abstract, and considered everyday situations in a miniature adult context. (Souza, 2021, p. 132)

The intuitive problems in this categorization were often confused with the term exercises due to its purpose of exercising content and its purpose of verification. Another critical point is the contexts of the problems in this excerpt, where the problem situations did not follow the child's cognitive development.

For Souza (2021), the transition from intuitive problems to tailored problems, or rather, the problems linked to the intuitive method to the New School movement, can be substantiated by three trends: 1) Problems exercising the contents, 2) Problems such as synthesis of content grading, 3) Problems defining the sequence of content. As a result, tailored arithmetic problems prioritized the distinction between problems and exercises. The teaching of problems began to be systematized from a teaching degree, propositions close to situations in students' daily lives, and specific methods for solving them.

Mentions to problems in São Paulo were not only seen in manuals and pedagogical journals. Giusti and Souza (2022) delved into the improvement notebook of Thereza Pereira Rocha, a prospective primary school teacher (*normalista*) in São Paulo, analyzing the document she wrote for the course practicum. That notebook integrates Thorndike's references in the preparation of his material, corroborating the use of problems linked to realities and, to solve them, his interpretation and interest in what was offered was valid, as Pestalozzi's use of graphic elements and three phases for the initiation to mathematics: "First, check the child's knowledge, as soon as they enter school; then, give them the notion of number; and finally, make her recognize a group of things and objects" (Giusti & Souza, 2022, p. 16). In this teaching pace, problems appear as a component to teach.

However, a new representation for teaching problems can be found in the work of Rocha (2019), which aimed to characterize algebra to teach arithmetic problems in two didactic manuals, one by Reis (1919), *Álgebra - primeiros passos* [Algebra - first steps] and another by Oliveira (n.d.)⁷ *Arithmetica complementar para os cursos primário complementar, normal e comercial* [Complementary Arithmetic for Complementary Primary and Normal and Commercial Courses], for the 1910s. The delimited spatial section referred to the place of publication of the manuals, Rio de Janeiro and Pará, respectively.

Rocha (2019) conceives that the teaching pace in the two works would lead to arithmetic teaching and arithmetic problems through an "algebraic method." This presents the method intended to be disseminated, mainly in the mentioned states, to promote economic advancement. Consequently, this advance would come with new educational prescriptions, mainly for mathematics teaching. In this design, algebra was not established as a component or as a subject to be taught in primary school but rather as a support for arithmetic teaching. The

⁷ The author used the 8th edition as a reference, but the 4th edition of this same manual was dated 1919, the same date as Reis' manual.

author understands that this method facilitated the resolution of arithmetic problems, allowing data to be converted into simpler operations that could be solved by arithmetic.

Otelo Reis (1919) and Tito de Oliveira (n.d.) provided pedagogical guidelines for teaching arithmetic and riddle problems through this algebraic base. However, teaching methods for solving problems through algebra were not the same. One author defended a path that introduced the notion of algebraic reasoning, while the second pointed out the opposite: beginning through arithmetic and leading toward interpretation through algebra. However, the authors operate through algebra as a facilitating element for solving arithmetic problems, riddles, or enigmas (Rocha, 2019).

For teacher education and mathematics to teach problems, Bertini (2019) investigated the place allocated to them, considering the school notebooks from teacher education courses by José Burlamaqui Freire, 1923, and Imene Guimarães, 1932. The author indicates two movements aimed at teaching graduation. The first movement, found in José Burlamaqui Freire's notebook, introduces problems for teaching arithmetic, sometimes as a method and sometimes as support. The second resulted from learning the four operations supported by the teaching of arithmetic problems presented for teaching purposes in this scenario. However, interest-oriented problems appear as a characteristic in both aspects.

On the other hand, supported by the contributions from professor Imene Guimarães' notebook, alluding to the teachings of Alda Lodi's teacher education course, it is clear that the interest was not only linked to the students' universe but also integrated the activities that provided her with curiosities and enthusiasms.

In Minas Gerais, based on the study notes of Minas Gerais teacher Alda Lodi in the teacher education course at Teachers College from Columbia University, and through the dialogue between her role as a student in this course and as a primary teacher in Minas Gerais, Rocha and Valente (2020) address the debate about knowledge for teaching along with international travel to acquire knowledge and its consequences for arithmetic teaching.

According to Rocha and Valente (2020), the teacher's notes and exchange confirmed that the teaching of problems was linked to “reasoning tests” that measured the resolution of the four fundamental operations. Consequently, students who master the operations would have the competence to use the “arithmetic machinery.” The problems would only be taught after they were mastered.

Alda Lodi pointed out some characteristics of problem-teaching from her studies at the Teachers College. Mastering the machinery was important, but it was not everything.

Understanding the problem, identifying the data, and using it to solve it were essential to achieving the final answer (Rocha & Valente, 2020).

Reasoning was important for achievement, but the main point was knowing how to make associations, and consequently, Lodi understood that choosing the statement was important for its promotion. The influence of psychological tests on these circumstances is predominant in defining the differences between operations calculation and reasoning and their components for solving problems. In this article, we note the marks left by the improvement course at the Teachers College and the understanding of problem solving supported by tests and psychology.

Alda Lodi's guidelines were also found in the Minas Gerais 1941 primary education program. As reported above, Lodi's ideas brought from the United States penetrated and fed the prescriptions for arithmetic and geometry teaching, as well as problem teaching for Minas Gerais education. In the Minas Gerais program that Aldo Lodi endorsed, Rocha (2021) identifies two graduation levels for problem teaching. The first concerns children's experience divided into "current problems" linked to the child's scenario in the classroom and "practical problems" related to adult life. Other problems were related to the ability to interpret and associate previous knowledge.

Different guidelines and systematizations based on Alda Lodi's contributions are presented in Minas Gerais. Problems abandoned the purpose of being an instrument for teaching basic operations and became a method to be followed, the knowledge to teach.

Following the topic of the New School, Virgens (2014) analyzes the objectives and purposes of the problems in reference manuals for teaching arithmetic for this pedagogical wave. The author uses as a source the manuals by Thorndike, *A Nova metodologia da Aritmética* [The New Methods in Arithmetic], of 1936; Backheuser's *Como se ensina a aritmética: Fundamentos Psicopedagógicos* [How arithmetic is taught: Psychopedagogical Fundamentals], of 1946; Miguel Aguayo's *Didática da Nova Escola* [New School Didactics], of 1959; and Faria de Vasconcelos', *Como se ensina a raciocinar em aritmética: psicológica aplicada e didáctica* [How to teach reasoning in arithmetic: applied psychology and didactic], of 1934, and articles in *Revista do Professor*, published in São Paulo and *Revista do Ensino*, published in Minas Gerais.

According to Virgens (2014), supporting the discussions proposed by Thorndike in *A Nova Metodologia da Aritmética* [The New Methods in Arithmetic], the author prescribes a representation for teaching "good problems" defined as those that would be most likely to happen in the students' real and practical lives.

Adopting precepts close to Thorndike's but with reservations, in dialogue with questions of the intuitive method, Virgens (2014, p. 73) indicates that Backheuser “would have defended as characteristics of a good problem the need to correspond with reality and the importance of the child's interest, but questioned aspects defended by the New Schools, such as the need for applicability.”

In the same vein, Aguayo and Vasconcelos agree with the use of these “*good problems*”, defined by Virgens (2014), and complement by indicating that they would aim to verify the skills acquired in mathematics, also seen as a method for teaching school arithmetic. Along this path, the author points to different appropriations and purposes for problem teaching, gaining notoriety during the New School movement to measure learning, using it as a teaching method, and bringing problems closer to the student's reality.

Reverberations of problems in the New School era could also be seen in Fernandes (2017), where the author brings the interpretations of problem teaching from articles in *Revista de Ensino*, in Alagoas, which circulated between 1924 and 1952. Fernandes (2017) uses analytical and synthetic methods to analyze the space given to problems in journals and teaching programs. Hence, the author appropriates and interprets such methods as:

The synthetic method comes from Euclidean mathematics and is linked to the idea of element, or elementary, which comes first. Teaching from this perspective means starting from the beginning, from the simple to the complex, from the smallest to the largest, from the easiest to the most difficult, always gradually. In the analytical method, the idea is to start from the child's daily life, from what they experience, without considering whether they have mastered all the content or previous elements to understand what is being addressed. Access to the whole is believed to help the child better understand the parts. (Fernandes, 2017, p. 43)

Based on these methods, we used six educational articles published in different editions of the *Revista de Ensino* de Alagoas –by Escobar (1927), Campos (1927), Laisant (1927), Barreto (1930), and Carneiro (1952a; 1952b)⁸– to understand the role of problems for teaching in Alagoas.

⁸1) Escobar (1927), 2) Campos (1927), 3) Laisant (1927), 4) Barreto (1930), 5) Carneiro (1952a), and 6) Carneiro (1952b) were the articles used as historical sources by Fernandes (2017). Such documents are available respectively at: 1) <https://repositorio.ufsc.br/handle/123456789/135356>;
2) <https://repositorio.ufsc.br/xmlui/handle/123456789/126738>;
3) <https://repositorio.ufsc.br/handle/123456789/126742>;
4) <https://repositorio.ufsc.br/handle/123456789/179874>;
5) <https://repositorio.ufsc.br/handle/123456789/179872>;
6) <https://repositorio.ufsc.br/handle/123456789/179873>.

Following the chronological order of publication of articles in the Alagoas journal, Fernandes (2017) considers that the nature of the articles in relation to problems is diverse. The first four mentioned above considered synthetically teaching problems, starting from the parts to the whole. The author also adds that the purpose of the problems was to teach the content and/or reinforce learning. Escobar (1927) gives guidance on the application of simple problems, student problems, problems without numbers, and illustrated problems; and such problems integrate the reality and interest of the students.

According to Fernandes (2017), only the two texts by Carneiro (1952a; 1952b) show a different approach to the problems that focused on the center of interest, “Man's nutrition.” In this context, the problems would not serve as support to teach arithmetic or reinforce learning, but the arithmetic teaching would take place through problems; therefore, the basis becomes the analytical method.

Thus, in this production, problems are characterized as elements that help teach arithmetic content and reinforce content already taught, except for Carneiro's articles. We infer that the temporal distance between some of these publications and the New School movement may have contributed to this change in attitude toward using arithmetic problems.

In the South of Brazil, Rosa, Silva, and Búrigo (2017) also deal with sources that correspond to school notebooks for the fourth grade of primary education at Grupo Escolar Ramiz Galvão in Rio Pardo – RS, belonging to Juvenal Rosa Nunes, 1954. A range of problems for teaching arithmetic was found in this document. Based on an interview with the owner of the notebook and consultation with the 1939 Programa Mínimo de Matemática [Minimum Mathematics Program] in force at the time, the authors verify the relevance of the problems and the central role of arithmetic teaching for teaching operations.

Another characteristic that is confirmed by evaluating the other productions communicated here, is that the problems brought common components to students' lives. Regarding problem elaboration, Rosa, Silva, and Búrigo (2017), inferring their own productions but connected with the ideas followed in the Minimum Program and Souza Lobo's 1954 book *Primeira Aritmética Para Meninos* [First Arithmetic for Children], identify the proximity between the problems in the book and the class notebook.

Speaking about other aspects, in the New School era in Santa Catarina in the 1930s and 1940s, Faria (2011) presents, based on a socio-historical study, problems in components of the partial exams of the Academy of Commerce⁹.

The sources used were test content, a monthly breakdown of content, textbooks, tests, a list of textbooks, teachers, internal regulations, and minutes of the Academy of Commerce congregation for secondary education. In these sources, a range of problems linked to mathematics content was found: problems on the four operations of fractions and whole numbers; problems about areas, volumes, and weights; recapitulation problems on ordinary fractions, decimals, and the decimal metric system; problems about compound interest; problems with quotient progressions; simple rule of three problems; algebraic problems; problems about 1st degree equations and one unknown; and solving functional problems (Faria, 2011).

Therefore, Faria (2011) says that discussions about using problems in secondary education, specifically at the Academy of Commerce in Santa Catarina, began in the 1940s. However, its predominance in test content and actually in tests intensified from 1946 onwards. However, the author must thoroughly examine the objectives and purposes of using problems in the Academy of Commerce courses.

Problem solving in times of modern mathematics and mathematics education

In the change of ideas, Waldrigues (2010) explains that problem solving in times of modern mathematics in the 1970s in Paraná, using as sources a student's notebooks dated from 1975 to 1977, “[...] ceases to be an element of operationalization of content and becomes recognized as a fundamental methodological element in conducting the subject” (Waldrigues, 2010, p. 94). From this perspective, problem solving becomes a methodology with steps for its production and solution, where mathematical rigor is strictly linked to each resolution step.

However, the author reinforces that the problems were not linked to students' interests, as during New School times, with plots that approached more rural than urban elements. The

⁹“Santa Catarina Academy of Commerce was the responsibility of the State which, through Decree n. 782 of April 5, 1935, received the Polytechnic Institute by public deed; [...] The name 'Academia de Comércio de Santa Catarina', in fact, was established on May 23, 1942” (Faria, 2011, pp. 75-76).

author also emphasizes that the problems encountered brought unrealistic questions and solutions to students' lives, with large amounts to solve that were not consistent with the grade of schooling.

The resolutions had a technical content because “The problem situations analyzed refer to a prescriptive way of solving problems, with an emphasis on solving mathematical models, rules, and procedures, with the teacher playing the role of instructor and transmitter of information” (Waldrigues, 2010, p. 94). Fixation and memorization exercises were highlighted in this section of Paraná. The problems became a method for teaching mathematics, and the exercises became a way of fixing the content.

Few studies in the bibliographical survey dealt with discussions on teaching problems for the modern mathematics movement. However, Morais (2015) produces an inventory of the constitution of research on problem solving based on the proceedings of eleven editions of the *International Congress on Mathematical Education* (ICME), in which the outline of the time frame is shaped between 1969 and 2008. Consequently, two important milestones radiate: the modern mathematics movement and the emergence and consolidation of the field of mathematics education.

In Morais's (2015) research, problem solving was divided into four phases. The first phase (1969), synthesized from ICME-I, was called “Problem Solving is not a topic for discussion in lectures and plenary sessions.” According to the author, despite no in-depth discussion on problem-solving, Henry O. Pollak's research, “*How Can We Teach Applications of Mathematics?*” stands out for its observation of the teaching of problems from the 1960s and their misappropriation. This perception corroborates, to a certain extent, the issue of problems referred to by Waldrigues (2010) in his research in the state of Paraná about mathematics teaching problems aligned with “eccentric problems” and those that “pretend to be applied.”

In Phase 2 (1972), delimited from the analysis of proceedings of ICME-II, three works were presented on the theme of problem solving: The lecture *As I read them* by George Polya had a different format as it brought considerations from mathematicians such as Socrates, Archimedes, Descartes, among others, to address the importance of problem solving. Edith Biggs' work pointed to problem solving as a methodological approach, and Efraim Fischbein's

work pointed to modifying and improving the resolution method, mainly regarding reading, language, and interpretation.

In Phase 3 (1976-1984), “focus on teaching and learning mathematics with problem solving”, Morais (2015) exposes a significant increase in scientific productions that addressed the theme present in ICMEs III, IV, and V. The author considers the consolidation of problem solving as a trend in mathematics education based on the growing number of research and discussions on teaching and learning on the topic in international mathematics education congresses.

Phase 4 (1988-2008), entitled “Problem solving is incorporated into the mathematics curriculum in some countries,” included the analysis of proceedings of ICMEs VI, VII, VIII, IX, X, and XI, where they reaffirm the consolidation of problem solving and its relevance for mathematics education. Thus, two more aspects of the analyzed research emerge: the relationship between problem solving and curriculum studies and the maturity of problem-solving research.

The changes in conceptualizations at different times are perceived in Morais (2015). Initially, problem solving was absent in discussions, although there were points about mathematics problems that were close to the ideas of modern mathematics. Throughout the international congresses cited before, the term problem solving became notorious and gained space for research in the educational area, establishing a research trend in the field that had been consolidated over the years, mathematics education, and its incorporation and institutionalization into the mathematics curriculum in different countries.

Internationally, more precisely in the United States, Hsia (2013) shows contributions to this debate from the 1980s and 1990s led by George Stanic and Jeremy Kilpatrick. In 1985, Kilpatrick defined a problem as “(...) a situation in which the goal to be achieved is blocked. So, from this perspective, the problem can be like an activity generated from a theme” (Hsia, 2013, p. 26).

Stanic and Kilpatrick, as cited in Hsia (2013), indicate three themes in problem solving: 1) Problem solving as context; 2) Problem solving as capabilities; and 3) Problem solving as art. The first theme uses resolution and its solutions to achieve a specific purpose. The second

goes toward developing perception, memory, imagination, understanding, and intuition through elaborating and executing problems. The third is linked to the question of interest, of the “art of discovery” through problems, a legacy of Polya’s works.

Bringing American historiography closer to contemporary times, studies on problem solving have become institutionalized as a research topic for mathematics education, gaining a sizeable scientific basis for studies on the curriculum and teaching of problem solving.

According to Van De Walle (2010), teaching via problem solving requires a paradigm shift from the teacher, i.e., they must not only change some aspects of their teaching, but they must also change their philosophy, the way students think, the way they learn and how they can improve their learning. (Hsia, 2013, p. 67)

The idea of learning to solve mathematical problems is reinforced to develop skills such as reasoning construction, meaning construction, decision-making through data collection, and emotional and confidence development.

Furthermore, studies by Schoenfeld (2007), cited in Hsia (2013), contemporary to Walle, show that the American curriculum permeated two aspects of problem solving, one of them as seen before in the development of reasoning and understanding and in the technical methods of how to solve them. We must note that all contextualization of the problems, especially the references above, initially announced as *problems* and later presented as *problem solving*, allows us to understand the changes that have occurred in an international context.

In the Brazilian context, Hsia (2013) limits his approach to teacher education on problem solving in the mathematics education field through the seminars of the Working and Study Group on Problem Solving (Grupo de Trabalho e Estudo em Resolução de Problemas - GTERP), in particular, Seminars I and II in Problem Solving (Seminários em Resolução de Problemas - SERPs), both promoted by Unesp – Rio Claro, and three other publications found by the author. There is no in-depth discussion about paradigm changes in the Brazilian context, the formation, and resolution of problems, or the specific objectives and purposes found in these works; it is just an exposition of the results of each published research paper.

Finally, the contributions of Bertini and Valente (2021) add a diverse aspect to what has been seen so far. The authors consider problem mobilization as a category of analysis linked to the dialogue between mathematics for teaching¹⁰ and teacher education. With the articulation coming from the *school culture* and the production of professional knowledge, the authors

¹⁰ The concepts of school culture, knowledge to teach, knowledge for teaching, and mathematics teaching are discussed by Bertini and Valente (2021), available at: <<https://repositorio.ufsc.br/handle/123456789/234157>>.

present two research papers on problems to understand mathematics *for teaching* and *to teach* in teacher education.

In conclusion, the other productions by Pavarin (2022), Fernandes and Carvalho (2019; 2021), Rocha and Bertini (2019), Bertini and Rocha (2018), and Souza (2017b) are close to productions previously discussed in Pavarin (2020), Fernandes (2017), Rocha (2019), and Souza (2017a), respectively, regarding arithmetic problems, as they are seen as developments of master's and doctoral research, and present similar results as evidenced in this literary review.

Final Considerations

Returning to the objective of this article, which aims to scrutinize the permanence and changes, in different historical moments, of the meanings and purposes of the “problems” for the teaching of mathematics from a literature review based on the state of knowledge methodology, we observed in the range of works found and analyzed on different scientific dissemination platforms, different meanings for problems related to mathematics teaching.

Initially, we note the change in understanding of problems at different times. In the passage from traditional teaching to the intuitive method, problems and exercises were not seen as separate entities. Such discussions about this differentiation between exercises and problems were nonexistent. Definitions and the first considerations on using problems for teaching were found in 1888 in the work *Arithmetica Pratica [Practical Arithmetic]*, by Felipe Nery Collaço. Such a definition would improve what had been learned and reinforce learning. Based on given numbers and quantities, unknown elements would be found. This understanding does not separate problems and exercises since the problems were intended to exercise.

Discussions about the use of problems emerged from the New School movement. In Brazil and internationally, problems became popular with the support of Dewey's first conceptions and problems focused on students' interests, such as George Polya's efforts in the United States. At the national level, some Brazilian states have different appropriations of problems, such as São Paulo, Rio de Janeiro, Minas Gerais, Alagoas, and Pará.

In some productions, problems are understood as an element to finalize the content studied. In others, the problem was seen as a means of introducing content. In both cases, the problems were seen as a support for learning and closely related to students' practical lives. The

relationship between statements and elements of students' daily lives was privileged. Another point that emerged in the literature review was the relationship and use of problems and the Centers of Interest. Here, many understandings of the problems and New School teaching can be seen. Finally, a specific method for problem-teaching began to be established, so they went from being a support for teaching to a teaching method that would promote the development of students' abilities to analyze, observe, and reason.

From the 1960s onwards, during modern mathematics, problems were seen as a teaching method. However, the problems “pretended,” in a way, to be linked to students' lives, as they presented problem situations that should be connected to everyday life. However, the problems brought unrealistic questions and solutions in large amounts to be solved, often not consistent with students' grades of schooling. The resolutions had technical content emphasizing solving mathematical models, rules, and procedures. The differentiation between problems and exercises was made explicit: problems became a method to be followed for teaching, and the exercises became a means of fixing the content.

Finally, we comment on the problems for contemporary mathematics teaching and as a research trend in mathematics education. Notably, the nomenclature used in productions that address the problems in a more recent time frame does not cover the term *problems* but rather the expression *problem solving*. With the establishment of a specific method over the decades, this change in terminology can be seen.

Regarding the direct association between mathematics education and problem solving, it is clear that their discussions at the ICMEs increased interest and willingness to study in this area, becoming a research trend and gaining a large scientific basis for studies on curriculum and teaching. The idea of learning to solve mathematical problems is reinforced to develop skills such as reasoning construction, meaning construction, decision-making through data collection, and emotional and confidence development.

Such research had resonances in the implementation of problem solving in the curricula. The most current textbooks in this production indicate a method capable of formulating hypotheses, planning means of resolution, raising hypotheses, and validating the resolutions.

Solving problems thus contributes to developing logical-deductive reasoning, providing more meaningful, comprehensive, and critical mathematics learning.

Therefore, from this work, supported by the state of knowledge methodology, we could identify changes in the conception of problems. In summary, the problems were not specifically mentioned during the transition from traditional teaching to the intuitive method. Subsequently, the relationship between *problems* and *exercises* was evident, requiring differentiation. Consequently, the problems became dissociated from the exercises, gaining a supporting character for teaching and, later, a method to be followed. The constitution of a method changed the term from *problems* to *problem solving*. Finally, problem solving became relevant in research in mathematics education, and its reverberations began to reach educational curricula to promote reasoning and critical and meaningful learning.

References

- Bertini, L. de F. (2019). Os problemas na “matemática para ensinar ensinada” em cursos de formação de professores para o ensino primário (décadas de 1920 e 1930). *HISTEMAT*, 5(1), 5–19. <https://repositorio.ufsc.br/handle/123456789/197216>
- Bertini, L de F., & Rocha, I. L. da (2018). “Resolução De Problemas Pelas Equações Algébricas”: a proposta de Tito Cardoso de oliveira para o ensino das operações. *Revista de História da Educação Matemática*, 4(3). <https://histemat.com.br/index.php/HISTEMAT/article/view/234>
- Bertini, L. de F., & Souza, A. F. (2021). Mas afinal o que são problemas? Uma análise histórica sobre mudanças em definições, finalidades e tipologias. *Revista de Ensino de Ciências e Matemática*, 12(5), 1–19. <https://doi.org/10.26843/rencima.v12n5a08>
- Bertini, L. de F., & Valente, W. R. (2021). Problemas Aritméticos como elementos da Matemática do Ensino. *Cadernos CEDES*, 41(115), 230–238. <https://doi.org/10.1590/cc245623>
- Bianchini, E. (2022). *Matemática Bianchini: Manual do Professor - 6º ano*. 10a ed. FTD.
- Costa, D. A. da., & Valente, W. R. (2015). O Repositório de Conteúdo Digital nas pesquisas de história da educação matemática. *RIDPHE_R Revista Iberoamericana do Patrimônio Histórico-Educativo*, 1(1), 96–110. https://doi.org/10.20888/ridphe_r.v1i1.9231
- Faria, J. E. S. (2011). *O Ensino de Matemática da Academia de Comércio de Santa Catarina na década de 1930 e 1940*. [Dissertação de Mestrado em Educação, Universidade Federal de Santa Catarina, Centro de Ciências da Educação].
- Fernandes, E., & Carvalho, M. (2021). As orientações propostas nos documentos alagoanos para o ensino de problemas aritméticos a partir da década de 50. *Tangram*, 4(3), 25–42. <https://doi.org/10.30612/tangram.v4i3.13609>

- Fernandes, E. P. (2017). *Os problemas aritméticos e os métodos pedagógicos: pontos para um diálogo sobre a história da educação matemática no ensino primário alagoano (1924 - 1952)*. [Dissertação de Mestrado em Educação, Universidade Federal de Alagoas].
- Fernandes, E. P., & Carvalho, M. (2019). Resolução de problemas aritméticos: método analítico ou sintético? O que revelam as revistas e documentos oficiais alagoanos (1924-1930). *Revista Cocar*, 6, 73–94. <https://periodicos.uepa.br/index.php/cocar/article/view/2476>
- Ferreira, N. S. de A. (2002). As pesquisas denominadas “estado da arte”. *Educacao & sociedade*, 23(79), 257–272. <https://doi.org/10.1590/s0101-73302002000300013>
- Giovanni, J. R., Jr., & Castrucci, B. (2018). *A Conquista da Matemática: Manual do professor - 6º ano* (4o ed). Editora FTD.
- Giusti, B. L. R., & Souza, A. F. de. (2022). Escritos de Thereza Pereira Rocha: de Pestalozzi à Thorndike, saberes para ensinar problemas aritméticos. *Ensino em re-vista*, 29, e017. <https://doi.org/10.14393/er-v29a2022-17>
- Hsia, Y. W. (2013). *Resolução de problemas: um estudo sobre seu processo evolutivo nos Estados Unidos, na China e no Brasil*. [Tese de Doutorado em Educação, Pontifícia Universidade Católica de São Paulo].
- Morais, R. dos S. M. (2015). *O processo constitutivo da Resolução de Problemas como uma temática da pesquisa em Educação Matemática: um inventário a partir de documentos dos ICMEs*. [Tese de Doutorado em Educação Matemática, Universidade Estadual Paulista]. <https://repositorio.unesp.br/items/695e90f6-ea9a-4110-9be3-afbed34c4548>
- Pavarin, K. C. dos S. (2022). Os Problemas de Aritmética nos livros didáticos: finalidades de uso (1890-1920). *Revista de História da Educação Matemática*, 8, 1–15. <https://histemat.com.br/index.php/HISTEMAT/article/view/502>.
- Pavarin, K. C. dos S. (2020). *Problemas de aritmética em tempos de Aritmética Intuitiva: uma análise em livros didáticos (1890-1930)*. [Dissertação de Mestrado em Ciências, Universidade Federal de São Paulo]. <https://repositorio.ufsc.br/handle/123456789/222267>.
- Pinheiro, N. V. L. (2021). Do cálculo ao problema: um caminho a trilhar a partir de resultados científicos. *Ciência & Educação* (Bauru), 27. <https://doi.org/10.1590/1516-731320210038>.
- Ramos, R. A. (2011). *Dicionário Didático de Língua Portuguesa* (2o ed). Edições SM.
- Rocha, A. C. S. M. (2021). Os Problemas Matemáticos como um Elemento da Graduação dos conteúdos no Ensino Primário: Minas Gerais, Alda Lodi, 1941. *Revista de História da Educação Matemática*, 7, 1–18. <https://histemat.com.br/index.php/HISTEMAT/article/view/402>.
- Rocha, A. C. S. M., & Valente, W. R. (2020). Inteligência, Raciocínio e Problemas Matemáticos: notas sobre um debate a partir das anotações de Alda Lodi. *REMATEC*, 15(34), 173–194. <https://doi.org/10.37084/rematec.1980-3141.2020.n34.p173-194.id269>
- Rocha, I. L. (2019). *Álgebra para resolver problemas: as propostas de Otelo de Souza Reis e Tito Cardoso de Oliveira, década de 1910* [Dissertação de Mestrado em Ciências, Universidade Federal de São Paulo]. <https://repositorio.ufsc.br/handle/123456789/201746>

- Rocha, I. L., & Bertini, L. de F. (2019). Álgebra no Ensino Primário Brasileiro: sua relação com os problemas de aritmética no início do século XX. *Revista de História da Educação Matemática*, 5(3). <https://histemat.com.br/index.php/HISTEMAT/article/view/289>.
- Romanowski, J. P., & Ens, R. T. (2006). As Pesquisas Denominadas do tipo “Estado da Arte” em Educação. *Revista diálogo educacional*, 6(19), 37–50. <https://www.redalyc.org/articulo.oa?id=189116275004>.
- Rosa, N. G., Silva, M. B. O., & Búrigo, E. Z. (2017). Cadernos Escolares: problemas aritméticos no ensino de matemática em um diário escolar dos anos 1950. *Revista de História da Educação Matemática*, 3(1). <https://histemat.com.br/index.php/HISTEMAT/article/view/140>
- Souza, A. F. (2017a). *Discursos para ensinar problemas aritméticos (São Paulo, 1890-1930)*. Dissertação de Mestrado em Ciências, Universidade Federal de São Paulo]. <https://repositorio.ufsc.br/handle/123456789/178612>.
- Souza, A. F. (2017b). Sintético ou analítico: como ensinar problemas de aritmética? (São Paulo, 1920). *HISTEMAT*, 3(1), 196–211. <https://repositorio.ufsc.br/handle/123456789/189564>
- Souza, A. F. (2021). *Uma história dos problemas aritméticos: mudanças no saber profissional do professor que ensina matemática (1870-1960)* [Tese de Doutorado em Ciências, Universidade Federal de São Paulo]. <https://repositorio.ufsc.br/xmlui/handle/123456789/230713>
- Souza, A. F. (2020). Um tango que virou samba: Victor Mercante e as apropriações para ensinar problemas aritméticos (São Paulo, 1929-1952). *Revista de História da Educação Matemática*, 6(3), 73–85. <https://repositorio.ufsc.br/handle/123456789/218972>
- Virgens, W. P. (2014). *A Resolução de Problemas de Aritmética no Ensino Primário: um estudo das mudanças no ideário pedagógico (1920-1940)*. [Dissertação de Mestrado em Ciências, Universidade Federal de São Paulo].
- Vosgerau, D. S. R., & Romanowski, J. P. (2014). Estudos de revisão: implicações conceituais e metodológicas. *Revista diálogo educacional*, 14(41), 165. <https://doi.org/10.7213/dialogo.educ.14.041.ds08>
- Waldrigues, R. C. G. (2010). *A resolução de Problemas de Matemática nas séries iniciais do ensino de primeiro grau na rede estadual de ensino do Estado do Paraná na década de 1970: um estudo histórico cultural*. [Dissertação de Mestrado em Educação, Pontifícia Universidade Católica do Paraná].