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Probability for high school in NTBP 2021 mathematics textbooks

Probabilidade para o ensino médio nos livros de conhecimento do PNLD 2021

Probabilidad para la escuela secundaria en los libros de texto de matemáticas del PNLD 2021

Probabilités pour le lycée dans les manuels de mathématiques du PNLD 2021

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Abstract

Probability is an area of mathematics focused on predicting chances, making decisions and analyzing risks, making it one of the main areas of knowledge developed at school. Knowing that the textbook is an essential tool in the teacher's work, this article aims to analyse the perspective of teaching probability in the Probability and Statistics textbooks approved by the NTBP 2021 for High School. Based on documental research, we analyzed the activities considering the meanings of probability, sample spaces and the structuring of events. We found an asymmetry in relation to the meanings, with a great predominance of the classical meaning, the expressive use of discrete sample spaces and limitations in the conceptual approach to important theorems, such as conditional probability for the composition of events. As a result, teachers' skills are fundamental for complementing and correcting the teaching and learning process of probability.

Keywords: Probability, Textbook, High school.

Resumo

A probabilidade é uma área da Matemática voltada para a predição de chances, tomadas de decisão e análise de riscos, tornando-se assim um dos principais conhecimentos desenvolvidos

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na escola. Sabendo que o livro didático é um instrumento essencial no trabalho do professor, este artigo objetiva analisar a perspectiva do ensino da probabilidade nos livros didáticos do Ensino Médio aprovados pelo PNLD 2021. A partir de uma pesquisa documental, analisamos as atividades considerando os significados da probabilidade, espaços amostrais e a estruturação dos eventos. Encontramos uma assimetria em relação aos significados com grande predominância para o significado clássico, a utilização expressiva de espaços amostrais discretos e limitações na abordagem conceitual de importantes teoremas, como a probabilidade condicional para a composição de eventos. Com isso, as competências dos professores são fundamentais para a complementação e correção de rota no processo de ensino e aprendizagem da probabilidade.

Palavras-chave: Probabilidade, Livro didático, Ensino médio.

Resumen

La probabilidad es un área de las Matemáticas enfocada a predecir oportunidades, tomar decisiones y analizar riesgos, convirtiéndose así en uno de los principales conocimientos que se desarrollan en la escuela. Sabiendo que el libro de texto es un instrumento esencial en el trabajo del docente, este artículo tiene como objetivo analizar la perspectiva de la enseñanza de la probabilidad en los libros de Texto de Conocimientos de Probabilidad y Estadística aprobados por el PNLD 2021 para la Enseñanza Media. A partir de una investigación documental, analizamos las actividades considerando los significados de la probabilidad, los espacios muestrales y la estructuración de eventos. Encontramos una asimetría en relación a los significados con gran predominio del significado clásico, el uso expresivo de espacios muestrales discretos y limitaciones en el abordaje conceptual de teoremas importantes, como la probabilidad condicional para la composición de eventos. Por lo tanto, las habilidades docentes son fundamentales para complementar y corregir el recorrido en el proceso de enseñanza y aprendizaje de la probabilidad.

Palabras clave: Probabilidad, Libro texto, Educación secundaria.

Résumé

Les probabilités sont un domaine des mathématiques axé sur la prédiction des chances, la prise de décisions et l'analyse des risques, ce qui en fait l'un des principaux domaines de connaissances développés à l'école. Sachant que le manuel est un outil essentiel dans le travail de l'enseignant, cet article vise à analyser la perspective de l'enseignement des probabilités dans les manuels de probabilités et de statistiques approuvés par le PNLD 2021 pour

l'enseignement secondaire. Sur la base d'une recherche documentaire, nous avons analysé les activités en tenant compte des significations de la probabilité, des espaces d'échantillonnage et de la structuration des événements. Nous avons constaté une asymétrie par rapport aux significations, avec une grande prédominance de la signification classique, l'utilisation expressive d'espaces d'échantillons discrets et des limitations dans l'approche conceptuelle de théorèmes importants, tels que la probabilité conditionnelle pour la composition d'événements. Par conséquent, les compétences des enseignants sont fondamentales pour compléter et corriger le processus d'enseignement et d'apprentissage des probabilités.
Mots-clés : Probabilité, Manuel scolaire, École secondaire.

Probability for high School in NTBP 2021 mathematics textbooks

We know that one of mankind's greatest desires is to predict the future. Ever since the Greeks played with arrows and used the astragalus bone, when it was believed that the participation of the gods was absolute, probability has been a subject of study in academia (Launay, 2016). However, over the years, this concept has been extrapolated, as today this knowledge is in the classroom and contributes to the development of students' citizenship.

As a result, the school has acquired both the role of academically educating students to deal with these situations and develop their cognitive aspect of probability, as well as the task of situating students in their daily role, using this knowledge to make daily decisions. For this purpose, the role of the teacher is fundamental, and the textbook is one of the most important tools to support the teacher's work (Brown, 2009; Santos, 2013; Vieira, 2013; Barbosa & Oliveira, 2018).

In Brazil, the National Textbook Program (NTBP) is responsible for establishing guidelines for the preparation and distribution of these materials, defining their prerogatives and validity periods in specific public notices. In doing so, it adapts to the current concepts for each educational segment, which in turn, in the context of secondary education, had a reform established by Law 13.415/2017, which defined new ways for the structure and functioning of this stage of Brazilian basic education.

Thus, the NTBP Notice 2021 (Brasil, 2020) has already complied with the changes established by the law, such as the use of the National Common Curriculum Base (NCCB) (Brasil, 2018) to parameterize how and what content should be present in secondary education. Therefore, textbook objects have emerged in this new format, namely: object 1 - books on integrative and life projects; object 2 - works on specific knowledge; object 3 - works on continuing education; and objects 4 and 5, which include digital resources and literary works.

When it comes to probability, Silveira (2021) and Silva (2023) show how this knowledge is being developed in textbooks, and Araújo and Guimarães (2022) look at project books and the use of the normal curve concept. However, despite recent data pointing to deficiencies in the learning of Brazilian students, such as PISA, for example, there are few studies that detail the details that these books consider developing probabilistic knowledge. Therefore, this study aims to analyze the perspective on the concept of probability in the NTBP 2021 high school textbooks, using documentary research.

Probability in high school

The development of the concept of Probability in Basic Education has been a guideline since the early years, in terms of the Common National Curriculum Base (NCCB). With this, the document guides the development of the concept from vocabulary to sophisticated quantification methods, such as the Total Probability Theorem or even Conditional Probability. For the NCCB,

> Regarding Probability, elementary school students have the opportunity, from the earliest years, to construct the sample space of equiprobable events, using the tree of possibilities, the multiplicative principle, or simulations, to estimate the probability of success of one of the events. (Brazil, 2018, p. 518)

This highlights the need for high school to promote the deepening and contextualization of the knowledge acquired in elementary school. According to different authors (Batanero, 2005; Gal, 2004, 2019; Ross, 2015), the teaching of probability and statistics is essential to develop not only the cognitive but also the critical sense of students, thus making the school a promoter of citizenship.

To this end, we have used the concept of probabilistic literacy structured by Gal (2004), which is composed of two main components: the elements of knowledge, which are part of the concept of probability and its applications (the subject of this article), and the elements of disposition, which constitute the aspects that are intrinsic to each citizen and are related to this concept. For the author, a probabilistically literate adult can mobilize these two elements when exercising his role as a critic and citizen in society.

PROBABILISTIC LITERACY (GAL, 2004)	
ELEMENTS OF KNOWLEDGE	1. Big Ideas
	2. Calculating Probabilities
	3. Probabilistic language
	4. Context
	5. Critical Issues
DISPOSITIONAL ELEMENTS	1. Critical Posture
	2. Beliefs and attitudes
	3. Sense of Risk

Figure 1.

Probabilistic literacy (Gal, 2004)

The *elements of knowledge* are the variants that give meaning to the concept of probability, in which the school fulfills its pedagogical role by developing these skills in students. Five ideas are developed: (i) big ideas, which work on situations related to the randomness and variability of probabilistic experiments; (ii) probability calculations, which are those developed to quantify different events in a random experiment, such as the total probability theorem, Bayes' theorems, among others; (iii) probabilistic language, which is concerned with the ways in which events are verbalized, whether in oral or written language; (iv) context, which is concerned with the situations in which probability is used to make statements; and (v) critical questions, which are those developed to understand probabilistic messages, such as the expression *P(A|B)* in conditional probability.

The main characteristic of dispositional elements, on the other hand, is to deal with issues inherent in the subjectivity of those who read or state probabilities. This notion includes the critical stance, which reflects how a subject positions itself in the face of probabilistic data, whether it has the vocation to question it when necessary, how it allows its beliefs and attitudes to guide its judgment or whether it remains independent of the reality of the facts, and finally *the sense of risk*, which examines how the individual reacts to risk and weighs its possibilities in the face of the reality that pervades it.

In this article, we are particularly interested in analyzing one of the knowledge elements of probabilistic literacy, the calculation of probabilities. In this sense, we understand that the concept of probability for secondary education includes: the sample space, the configuration of the event, the counting model and the meaning (Figure 2). Therefore, for each situation, it is necessary to evaluate these structuring elements.

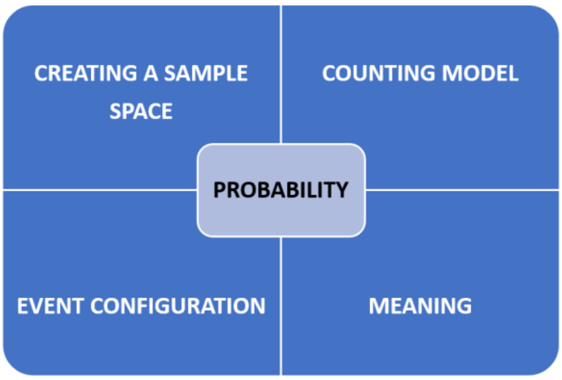

Figure 2.

Structuring Probability (The Authors)

The Common National Curriculum Base - NCCB (Brazil, 2018) determines the following skills to be developed in secondary education.

- (EM13MAT311) Solve and prepare problems that involve calculating the probability of random events, identifying and describing the sample space and counting the possibilities.

- (EM13MAT312) Solve and work out problems involving calculating the probability of events in successive random experiments.
- (EM13MAT511) Recognize the existence of different types of sample spaces, discrete or not, of equiprobable events or not, and investigate the implications for calculating probabilities.

The assessment of the structuring elements of probability is based on the recommendations in the NCCB, on problems in textbooks and on research already carried out.

Sample spaces and counting methods

In terms of the basic structure of the elements that make up probability as a mathematical concept, the sample space is central to solving and modeling problems. For Hoel et al. (1978), a probability space consists of an algebra of sets with a non-empty set of elements. This set is the sample space, which can be characterized by different properties.

Thus, corroborating with skill EM13MAT511, we bring in the configuration of sample spaces through their nature, which can be characterized by the variable of the problem (discrete or continuous) or by the formatting of the sample space itself (equiprobable or non-equiprobable).

The discrete sample space is characterized as one in which the variable involved is discrete in nature, i.e., it comes from a count of elements (Hoel et al., 1978). For this type of space, elements are counted in two ways: simple counting and indirect counting.

In simple counting, no special technique is used. For example, suppose we analyze the probability that the result of throwing a dice face-up will be a prime number. In this case, we use simple counting in the sample space because we have 6 possible outcomes (1, 2, 3, 4, 5, 6) and only 3 of them are favorable (2, 3, and 5), giving us a probability of 3/6. Note that these are honest dice, i.e., each face has an equal chance of coming up. In this case, we are dealing with an equiprobable sample space, in which all its elements have the same chance of being drawn (Ross, 2015).

However, when counting involves a process in which listing is inadequate, we use a technique to obtain the quantity of the results without necessarily listing all of them, thus resulting in an indirect counting process. Example: if we want to choose a committee of 2 people from a group of 4 men and 6 women, what is the chance that the committee will be made up of 2 women? For this problem, we don't have the number of possible committees. For the sample space, we calculate: $(4\times3+6\times5+4\times6)/2=(12+30+24)=66/2=33$. For the chance of there being two women: (6×5)/2=15. With this, the probability is 15/33.

As for variables that come from measurements, such as area, perimeter, height, mass, among others, called continuous variables, the way of counting depends on the type of problem (Ross, 2015). For example, if a rectangle with dimensions of 4 × 5 c.u. has a second rectangle inside, with dimensions of 2×3 c.u., what is the chance that when you throw a dart, it will hit the region of the inner rectangle? In this case, the calculation would be of area, with the larger square having 20 a.u. and the smaller 6 a.u., giving a probability of 6/20.

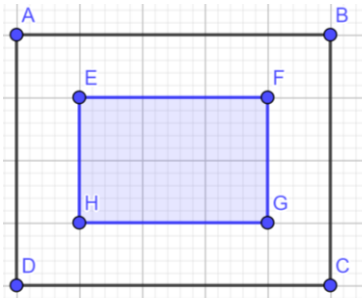

Figure 3.

Illustration of the example (The authors)

In this problem, we can see that the chance of hitting the blue area of the rectangle is less than the chance of hitting the white area, so we are using a non-equiprobable continuous space because the elements of this sample space do not have the same chance of being selected. Thus, the NCCB assumes that the probability discussed in the classroom has as its guiding axis the constitution of discrete and continuous sample spaces, whether equiprobable or not.

Event configuration

In a probability problem, events are partitions of a sampled space. At the secondary school level, we can consider events as those events for which we are interested in analyzing their chances of occurrence, considering their nature and their influence on probability calculations (Godino et al., 1996). Events can be analyzed as a single event or in parallel, which leads to different interpretations and procedures for calculating probabilities (Batanero, 2005).

Table 1.

Classification of events (The authors)

EVENT CLASSIFICATION	
Simple	**In Parallel**
Unique event	Sets
	Mutually exclusive
	Conditionals
	Correlation Analysis
	Independence Analysis

At the high school level, *simple events* are those that are analyzed in a single step, without sequence, and with the goal of visualizing a single partition of the sample space. For example, if we are interested in analyzing the probability of drawing a white ball from an urn containing 4 white balls and 6 black balls, the probability of drawing a white ball is 4/10.

When we analyze e*vents in parallel*, they are given different classifications according to the dynamics to which they are related in the quantification method.

Joint events form situations in which they occur simultaneously. Textbooks often instruct us to use the product rule or the "and" conjunction. For example: If Carla throws a dice twice, what is the probability of getting the numbers 3 and 5, in that order, on those throws? We can see that the first roll does not prevent or affect the second roll, so the probability is 1/6 on the first roll and 1/6 on the second, with the probability calculated as $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$.

When we are dealing with *mutually* exclusive events, they cannot occur at the same time. If we say that we will go to the beach by motorcycle or by car, it is understood that one option excludes the other. In the field of probability, it is common to use a Venn diagram to represent this situation, as it provides an important visual representation to illustrate the problem. For example, what is the probability of drawing a multiple of 3 or 5 from an urn of balls numbered from 1 to 20?

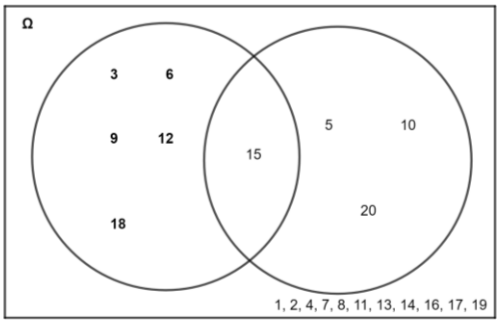

Figure 4.

Illustration of the problem (The authors)

The diagram shows the outline of the sample space of the problem, with multiples of 3, multiples of 5, and the number 15, which is a multiple of both. This shows the relationship between the diagram and the probability rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.. Thus, the problem would be answered as $P(A \cup B) = \frac{6}{20} + \frac{4}{20} - \frac{1}{20} = \frac{9}{20}$.

Another important relationship is the conditional probability, which is an event that depends on the occurrence of another event. This relationship is one of the points for deepening the probabilistic knowledge inherent in secondary education because, in addition to being one of the important probability theorems, it also has the function of updating opinions (an issue that will be discussed in the next subsection) (Ross, 2015). For example, if we analyze the probability of adding the values obtained by the faces up when throwing two dice and getting the number 8, knowing that one of them resulted in the number 3, we realize that one event clearly influenced the other. If we get the number 3 on the first roll, the only number that favors us on the second roll is 5, which is 1/6.

Finally, correlation and independence analysis are factors that involve observing two or more variables and how they behave in terms of visualizing future scenarios (Ross, 2015). When we discuss whether the number of popsicles sold by a street vendor correlates with his income, we are looking at the behavior of two variables and how they can provide a model for predicting this worker's future income.

Meanings of probability

We know that the need to respond to the transformations and the evolution of humanity was one of the basic rules for the development of mathematics. In the case of probability, since its first appearance - still as the *geometry of chance* - in the correspondence between Pascal and Fermat, its development has been linked to the evolution with which the concept has been developed, encompassing the solution of different problems.

In this sense, Godino et al. (1996) propose meanings for probability in a historical and epistemological evolution to offer an organization for understanding the concept. Thus, Batanero (2005) assumes some meanings that can be developed in basic education, where their construction is given from the first years of schooling, already in line with the NCCB (Brazil, 2018).

There are five meanings: intuitive, classical, frequentist, subjective and axiomatic. The historical evolution of the concept presents different situations. With the first appearances in game situations, the development of the probabilistic vocabulary with expressions such as most likely, certain, impossible, among others, was fundamental. For this meaning, called *intuitive*, the sources of quantification of probability are inherent to the subjects who declare it, and the communication of probability takes place through these expressions.

However, it was necessary for these same quantifications to be expressed numerically. This gave rise to the first ideas of understanding probability as the ratio of the number of favorable cases to the number of cases in the sample space (classical meaning). Laplace, in his philosophical essay on probability, already stated this idea in his first principle. However, it should be noted that equiprobability was already present in the First Principle when it defined probability as *"the ratio between the number of favorable cases and that of all possible outcomes"* (Laplace, 2010, p.49) and illustrated it with the toss of a coin, in which the outcomes head and tails have the same chance of occurring (50%), i.e. there was still the understanding that the chances of occurrence of the elements of the sample space were equal.

In practice, however, the idea of equiprobability was problematic in all cases, since randomized experiments did not always have this configuration. This gave rise to the concepts of theoretical and experimental probability, confirming the frequentist sense to which Jacob Bernoulli contributed greatly when he presented the law of large numbers, showing that the more an experiment is repeated in practice, the closer the experimental probability comes to the theoretical probability. One of the assumptions of this theorem is that the experiments are repeated a sufficient number of times under the same conditions. For example, if we simulate flipping a coin and observe the frequency with which the "tails" side is obtained, we can easily see how close it is to 50% (Figure 5).

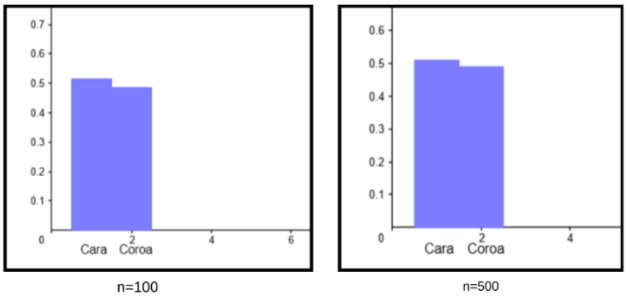

Figure 5.

Repetition of the coin toss experiment 100, 500 times[3] *(The authors)*

As we increase the number of simulations, the graph approaches 0.5 (50%), confirming the convergence of the result. Nevertheless, there are some valid questions about the frequentist meaning, such as how many repetitions are necessary to make a probability statement, or what can be done in experiments where repetition under similar conditions is not possible.

To answer these problems, the *subjective* meaning is mainly anchored in the work of Thomas Bayes, who proposed a model in which successive events could update the a priori probabilities. To do this, the *a priori* probability is measured numerically based on the experience and intuition of an expert, and the facts that are "learned" are incorporated, thus updating the a priori probability based on other situations.

Thus, in light of the above, different authors (Batanero, 2005; Ross, 2015) propose that the study of probability should also take place through experiencing the different meanings that give body to the object probability. We agree with these authors and argue that the view of the concept should be broad and respond to as many problems as possible.

It should be emphasized that the meanings are not watertight, i.e., they can interact within the same situation. For example, let's imagine that a doctor wants to assess the probability of success of an operation, and to do so, he examines *a priori* that of the last 10 operations, 9 were satisfactory. From this, he concludes that there is a success rate of about 90% (we can see the use of frequentist probability in the foreground). However, this new patient has a different characteristic, so the doctor must analyze the chances of success in this case based on this new information, and for this he will use subjective probability, with the a posteriori generation of a new degree of belief in the success of the procedure.

Thus, in this article we are interested in analyzing the perspective of probability teaching proposed in the textbooks approved in the NTBP 2021 for Brazilian High School.

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[3]Graph drawn up in Geogebra software, using an applet available at the following link: <https://www.geogebra.org/m/v5fh6sqq>

Method

This article uses documentary research as its methodology, since we analyze the documents (textbooks) as they are presented (Sá-Silva et al., 2009).

We analyzed the 10 (ten) books approved by the National Textbook Program - NTBP 2021, more specifically the Object 2 books, which are knowledge books.

To conduct the analysis, we *specified* criteria based on the definitions already presented in the theoretical framework, with the meaning criterion linked to the sub criteria: intuitive, classical, frequentist, subjective and axiomatic; *sample space and counting method with equiprobable or not*, discrete or continuous and direct or indirect counting, and finally, event, linked to the simple or compound subcriteria, which involves conditional events, correlation, mutually exclusive, independent, and sets (Figure 6). The data was processed using SPSS software.

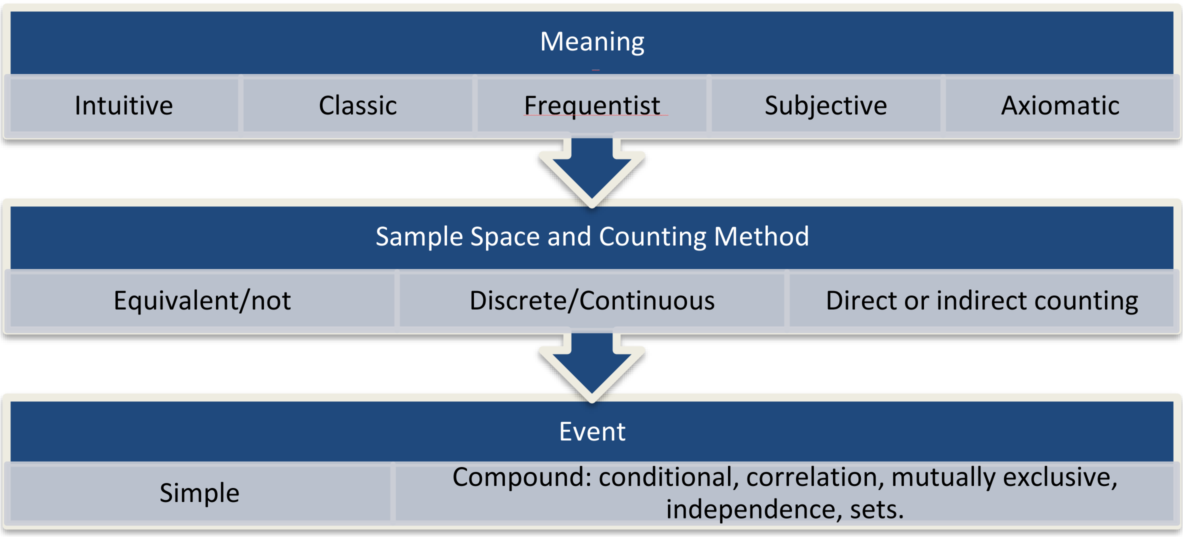


Figure 6

Schematic of the data analysis method (The authors)

Thus, we used the frequency of occurrence of the criteria and sub-criteria as a basis for the analysis to lead the discussion thread of the qualitative analysis, dialoguing with research already carried out and with the main points defended in the main curricular guideline for Secondary Education, which is the NCCB.

Results and discussions

To analyze the textbooks, we mapped them page by page, looking for probability problems and categorizing them according to the criteria and sub-criteria established.

1) Meanings of Probability

Initially, we analyzed the number of problems by meaning in the different collections analyzed (Table 2).

Table 2.

Percentage of probability meanings per collection (Research data)

COLLECTION	MEANING (%)					D.N.F. (Does not fit
	Intuitive	Classic	Frequentist	Subjective	Axiomatic	
Collection A	9,3	61,3	20,0	4	4	1,4
Collection B	-	53,7	29,3	12,2	-	4,8
Collection C	18,9	54,1	24,3	-	-	2,7
Collection D	1,2	56,6	31,3	-	2,4	8,5
Collection E	1,5	73,9	9	4,5	8,2	2,9
Collection F	1,5	91,0	-	-	-	7,5
Collection G	2,3	95,3	2,3	-	-	0,1
Collection H	-	97,6	-	-	-	2,4
Collection I	3,7	89,0	1,2	-	-	6,1
Collection J	2,9	82,9	2,9	-	-	11,3

Table 2 shows a high concentration of probability activities with the classical meaning in all collections. This concentration can lead to a limited view of the concept of probability, leading students to understand it only as a ratio between the number of favorable cases and the number of elements in the sample space. Research such as that of Silveira (2021) and Silva (2023) has already pointed out the high density of problems in textbooks on classical probability.

Although the NCCB encourages more in-depth study of probability in secondary school, with an emphasis on the frequentist sense, there is a small percentage of this sense in the books, with two collections (F and H) proposing no activities.

In relation to intuitive probability, collection C stands out, presenting the highest percentage of activities with this meaning. The subjective meaning of probability is also absent in most collections (seven out of ten), although the NCCB encourages discussion of probabilistic risks and decision-making based on data. In this regard, Batanero (2005) and Gal (2004) have already highlighted the need for probability education, both at the basic and higher levels, to include the articulation of different meanings to give breadth to students' probabilistic knowledge.

Another noteworthy point is the approach to axiomatic probability in three of the collections analyzed, even though it is not explicitly covered in the NCCB. According to Batanero (2005), this meaning plays an important role in higher education because of its formalization and mathematical structuring, which gives consistency to the mathematical object of probability. It should also be noted that the approach of the textbooks does not include mathematical demonstrations or proofs of theorems, but rather contextualization of the basic rules of Kolmogorov's Axioms.

There are still problems classified as N.S.A., which means Not Suitable, because they deal with situations in which the student has to elaborate the problem situation.This converges with the NCCB skill EM13MAT312 (Brasil, 2018, p. 529), which recommends that the student *"solve and elaborate problems that involve calculating the probability of events in successive random experiments"*. Here, we emphasize the importance of this type of activity, since these problems not only develop the student's creativity, but also mobilize important concepts and become tools for the teacher to look at the student/knowledge differently.

By this, we mean that the distribution of the meanings of probability is similar in the different collections, and that there is still some way to go, provided the evidence pointed out by other research and the normative document itself about the importance of teaching that considers all meanings.

2) Analysis of sample space characterization and counting methods

To analyze the sample spaces, each question was analyzed independently in two categories: the type of sample space and the counting method used to solve the problem (Table 3).

Table 3.

Percentage of problems by type of space and counting method (Survey data)

COUNTING	**SAMPLE SPACE (%)**			**TOTALS**
	Discrete	*Continual*	*Not applicable*	
Qualitative	2,5	-	1,1	3,6
Direct quantitative	71,0	0,32	1,7	73,0
Indirect quantitative	15,4	1,8	0,1	17,3
Not applicable	0,8	-	5,3	6,1
TOTAL	89,7	2,1	8,2	100

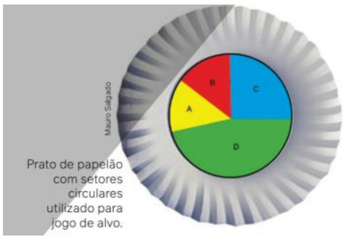
The data indicate a preponderance (89.7%) of sample spaces with discrete measures (Figures 7), confirming trends found in the NCCB and in previous research (Silva, 2023); however, this type of measure is limited in terms of an individual's mastery of the number of elements in a sample space. When we look at examples from everyday life, such as the weather forecast, the stock market, investment risks, among others, this no longer serves as a parameter. Thus, even if this is a hint already present in several studies (Batanero, 2005; Gal, 2004, 2019), the collections still fall short in terms of extending the concepts of continuous spaces to which most decisions are linked.

26 Foram formados todos os anagramas da palavra SAPO. Sorteando um desses anagramas, calcule a probabilidade de ele ser diferente de SOPA.	1. Um dos estudantes de sua turma foi sorteado aleatoriamente e não foi você. Logo em seguida, haverá um segundo sorteio, que escolherá outro nome. Qual é a probabilidade de você ter o nome sorteado no segundo sorteio?

Figures 7.

Discrete space problems (Collection A, p.126; Collection D, p.131)

Figures 7 and 8 represent problems found in the collections where the calculation of probabilities involves counting elements within the (discrete) sample space, either explicitly (in the case of anagrams) or implicitly/algebraically (in the case of the class draw). However, when we look at continuous space problems, not only do they take up little space (2.1% of the total), but they also have contextual limitations and their applications are focused on geometric probability (Figure 9).



Para pensar e discutir

1. Em qual setor é mais provável que o grão de milho caia? Justifique!
2. É menos provável que o grão de milho caia no setor A do que no setor C. Isso é correto? Justifique.
3. O que representa o espaço amostral desse experimento?
4. Como você pode determinar o espaço amostral e também os eventos: *A* (cair no setor A), *B* (cair no setor B), *C* (cair no setor C) e *D* (cair no setor D)? **1.** É mais provável que caia no setor D, porque ele ocupa a área maior na região colorida (o círculo com os quatro setores).

Figure 8.

Continuous space problem (Collection C, p.144)

We can see that the problem deals with a technique (calculating the area) and then expresses the probability, i.e. a comparison between them (in the case of point 2). In any case, the reasoning used to solve the problem revolves around probability, understood as a proportion between the partitions of the sample space discussed.

When we examine the meanings of probability involved for these sample spaces, the classical and frequentist meanings have a predominance of discrete cases (74.4% - 450 cases). These data confirm the principles of Laplace and Bernoulli, since the expression of probability is presented as a ratio, the origin of the data being either the simple count of the elements of the entire sample space or the sample related to the repetition of the experiments.

Thus, in general, the sample spaces presented in textbook problems are consistent with the NCCB recommendations in the sense that most opt for discrete counting methods. However, they are still inadequate when it comes to continuous spaces, since they do not provide a contextual diversification of the problems, nor do they discuss the corresponding theoretical elements.

3) Composition of events

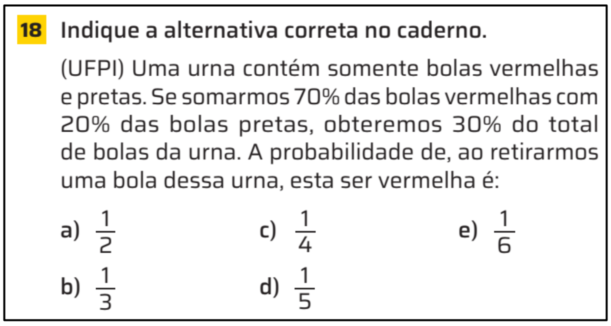
To determine greater objectivity in the analysis of the composition of the events in the study collections, we cut out the problems, excluding those in which the meanings are not predetermined and the configuration of the event is left to the discretion of the student. It should be noted that this total of 48 problems from the 10 collections includes problems in which the students have to create their own problem situation. Thus, 710 questions were analyzed in terms of situations and events (Table 4).

Table 4.

Percentage of problems per situation/event (Survey data)

EVENT	MEANING OF PROBABILITY (%)					TOTAL
	Int.	**Class.**	**Freq.**	**Subj.**	**Axiom.**	
Simple	2,3	39,3	7	-	2,1	50,7
Mutually Exclusive	0,3	7,2	0,4	-	0-	7,9
conjunctive	0	8	2	0,3	0,1	10,4
Conditional	0,3	10,7	0,7	1,7	-	13,4
Independence Analysis	-	16,8	0,8	-	-	17,6
TOTAL	2,9	82	10,9	2	2,2	100

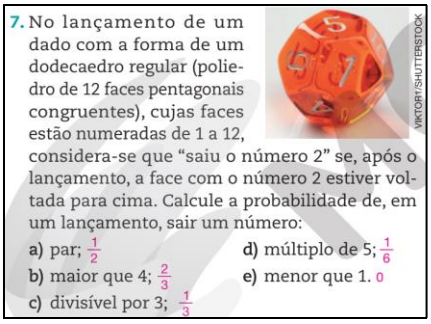
When dealing with simple events, the collections focus their proposals on the classic meaning, since the analyses include situations in which there is no parallelism to analyze the chances of events occurring. Thus, a striking feature of this type of event is the analysis of games of chance, such as ballot boxes and throws of coins and honest dice (Figures 9). Research such as that by Silveira (2021) and Silva (2023) already corroborates these results, by bringing up this same situation among textbooks; however, in addition to this, there is the predominant presence of simple events for the intuitive and axiomatic meanings.


18 Indique a alternativa correta no caderno.

(UFPI) Uma urna contém somente bolas vermelhas e pretas. Se somarmos 70% das bolas vermelhas com 20% das bolas pretas, obteremos 30% do total de bolas da urna. A probabilidade de, ao retirarmos uma bola dessa urna, esta ser vermelha é:

a) $\frac{1}{2}$ c) $\frac{1}{4}$ e) $\frac{1}{6}$

b) $\frac{1}{3}$ d) $\frac{1}{5}$


7. No lançamento de um dado com a forma de um dodecaedro regular (poliedro de 12 faces pentagonais congruentes), cujas faces estão numeradas de 1 a 12, considera-se que "saiu o número 2" se, após o lançamento, a face com o número 2 estiver voltada para cima. Calcule a probabilidade de, em um lançamento, sair um número:

a) par; $\frac{1}{2}$
b) maior que 4; $\frac{2}{3}$
c) divisível por 3; $\frac{1}{3}$
d) múltiplo de 5; $\frac{1}{6}$
e) menor que 1. 0

Figures 9.

Questions on simple events (Collection A, p.102; Collection B, p.97)

In this sense, we would also like to point out the limitation of this type of work with axiomatic probability, as it does not justify some important theorems of probability theory, even through problem-based induction. In contrast, for intuitive probability questions, this type of approach is sufficient for measuring chances through experience and intuition.

When it comes to joint and mutually exclusive events, the collections maintain homogeneity in the quantitative sense of problems and their concentration around classical meaning (Figure 10). In this sense, there is theoretical coherence and alignment with the foundations of the High School curriculum (NCCB), meeting the theoretical assumptions that govern probability. Even so, even though they are not highlighted in the NCCB, the articulation of the composition of these events with assessments of subjective meaning (*a priori and a posteriori* probabilities) would be possible, provided there was a demand for a distinction between numerical and qualitative quantification.

R8. Em uma pesquisa realizada com 50 pessoas foi perguntado:

- Você ouve rádio?
- Você ouve *podcast*?

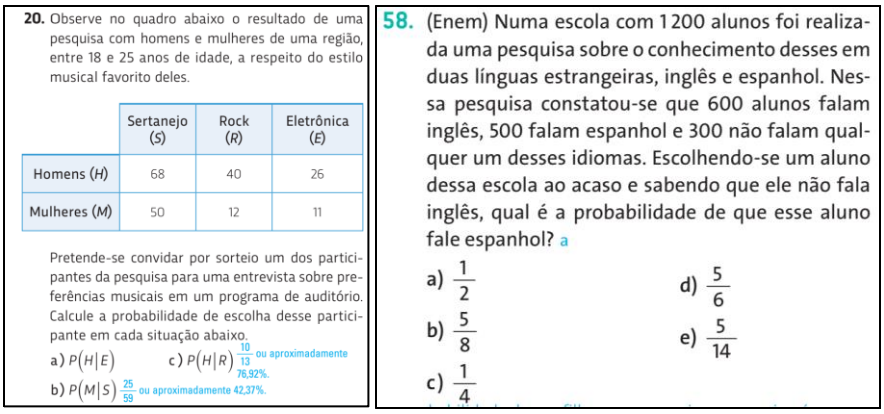
Os resultados indicaram que 25 pessoas ouvem rádio, 20 pessoas ouvem *podcast* e 20 pessoas não costumam ouvir rádio nem *podcast*. Calcular a probabilidade de, ao selecionar uma dessas pessoas, ela ouvir rádio e *podcast*.

Figure 10.

Joint/mutually exclusive probability problem

In addition to the above observations, we would like to highlight the lack of visual resources for these problems. While theorists such as Batanero (2005) and Ross (2015) present problem models using diagrams, the books analyzed only have 3 proposals with the same configuration. We understand that a facilitating part of the assimilation process is left aside when this type of representation is ignored.

When it comes to conditional probability (Figures 11), the collections deal with problems in which the initial probability P(A) is calculated using a probabilistic algorithm or is given without establishing the criteria for this. Regarding the possibilities for this type of problem, the subjective meaning of probability is set aside when the event analyzed a priori does not depend on different methods of quantification, i.e. when the opinion of an expert is considered (Batanero, 2005).



20. Observe no quadro abaixo o resultado de uma pesquisa com homens e mulheres de uma região, entre 18 e 25 anos de idade, a respeito do estilo musical favorito deles.

	Sertanejo (*S*)	Rock (*R*)	Eletrônica (*E*)
Homens (*H*)	68	40	26
Mulheres (*M*)	50	12	11

Pretende-se convidar por sorteio um dos participantes da pesquisa para uma entrevista sobre preferências musicais em um programa de auditório. Calcule a probabilidade de escolha desse participante em cada situação abaixo.

a) $P(H|E)$ c) $P(H|R)$ $\frac{10}{13}$ ou aproximadamente 76,92%.

b) $P(M|S)$ $\frac{25}{59}$ ou aproximadamente 42,37%.

58. (Enem) Numa escola com 1 200 alunos foi realizada uma pesquisa sobre o conhecimento desses em duas línguas estrangeiras, inglês e espanhol. Nessa pesquisa constatou-se que 600 alunos falam inglês, 500 falam espanhol e 300 não falam qualquer um desses idiomas. Escolhendo-se um aluno dessa escola ao acaso e sabendo que ele não fala inglês, qual é a probabilidade de que esse aluno fale espanhol? a

a) $\frac{1}{2}$

b) $\frac{5}{8}$

c) $\frac{1}{4}$

d) $\frac{5}{6}$

e) $\frac{5}{14}$

Figures 11.

Conditional probability problems (Collection F, p.111, Collection B, p.123)

In this sense, most of the questions deal with the quantification of a priori events based on data from fictitious statistical surveys, with the production of double-entry tables to size and cut out the sample spaces (Figure 11), and selections for partitioning the sample space using set theory (Figure 11). However, Fernandes and Braga (2023) have already pointed out in their research that students have difficulties interpreting conditionality and its impact on calculating probabilities, and also the influence of the independence observed or not in these cases. Even so, no such movement is noticeable in the questions proposed in the textbooks.

Another situation to note regarding conditional probability problems is their applicability to different meanings of probability (Figure 12). By concentrating 80% of the problems on classical probability and only 13% on subjective probability, the possibility of justifying the use of subjective probability is suppressed, since it is from there (in a particular case of Bayes' Theorem) that this meaning took shape and became, according to Bussab and Morettin (2017), an instrument capable of updating the opinion about an event.

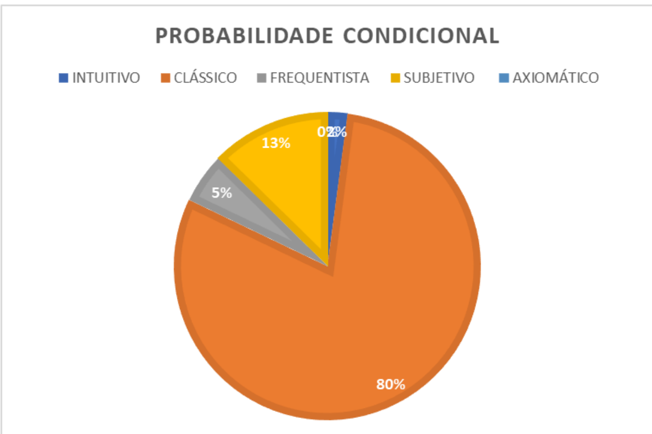


Figure 12.

Distribution of conditional probability problems (Survey data)

Another point to note is the lack of problems of this type in terms of axiomatic meaning, failing to justify the formulas used in this type of calculation, such as $P(A|B) = \frac{P(A \cap B)}{P(B)}$), which is used in all the collections analyzed. Furthermore, considering the guidelines of the NCCB and some research (Batanero, 2005; Gal, 2004, 2019; Silva, 2023), this meaning has unique importance in the context of decision-making and probabilistic risks, since it analyzes a single event through all the subsequent updates.

As for the independence analysis problems, we can see the variation in how the textbooks propose their use, from intuitive perceptions to those that are justified through the probabilistic calculation $P(A \cap B) = P(A) \times P(B)$. Of the 125 problems that deal with this topic, 119 (95.2%) are focused on the classical meaning, again showing the trend towards symmetrical sample spaces and situations in which probability is interpreted as a ratio in binomial probability.

Conclusion

The aim of this article was to analyze the perspective of teaching probability in the probability textbooks approved by the NTBP 2021 for the Brazilian secondary education. In each of the 10 volumes, all the activities were analyzed, page by page.

As far as the meanings of probability are concerned, we found patterns already pointed out by other researches, although we already noticed developments in line with the determinations of the NCCB, such as the deeper development of frequentist probability and the use of problems dealing with continuous spaces. Nevertheless, the concept of classical probability is in the majority, with extensive use of experiments whose sample space configuration is compromised by the abuse of symmetry in their structuring. Another observation lies in the fact that axiomatic probability is not present in the NCCB, but in the collections that adopt contextualized approaches to classical laws of probability theory.

In this context, we believe that it is a wise move for the textbooks to propose these activities in the form of examples, since formal demonstrations would be premature for this level of schooling. It is worth noting that theorists in the field have already mentioned this possibility in high school, arguing that it is part of the necessary abstraction and corroborates the precepts of the NCCB itself when it states that students should deepen their knowledge of probability in high school.

As for the shape of the sample spaces, the books remain faithful to most of the guidelines in the NCCB, such as the broad development of discrete spaces. However, for continuous cases, there is a clear limitation in reducing these problems to geometric views of probability. In the quantitative sense, the collections have diversified approaches within the possibilities, dealing with direct and indirect counting when the quantification is numerical. In the context of measuring quantitative probability, there are no problems that sufficiently address the possibilities for applications in meanings of probability that depend on this, such as subjective and intuitive.

In this way, there is also a movement that goes against what various researches in the field recommends regarding this conceptual view of probability, when the space for decision-making, opinion-forming and risk management is affected and students no longer rely on this type of tool in their daily lives.

Furthermore, if we analyze the way in which events are developed throughout the problems, we find that there is a variability of approaches, which goes against the basic rules of probability calculation. However, despite this variability, the textbooks are weak in the contextual insertion of these events, sometimes failing to conceptualize important notions in different meanings of probability, which is something that the NCCB demands throughout basic education.

In this sense, we believe that, since the textbook is one of the greatest tools to support the teaching action in the classroom, the current NTBP 2021 collections, despite meeting the skills required by the NCCB, still have gaps that can create epistemological obstacles in the construction and consolidation of this knowledge by students. We believe that the autonomy and critical sense of the teachers should prevail in the use of these books so that they can guide and complement the content whenever necessary, with a view to the integral development of the students.

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