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# Logic notes from the pamphlet "some popular fallacies about vivisection" by Lewis Carroll: approaches between mathematics and literature

# Notas de lógica a partir del panfleto “algunas falacias populares sobre la vivisección» de Lewis Carroll: aproximaciones entre matemáticas y literatura

# Notes de logique à partir du pamphlet "quelques sophismes populaires sur la vivisection » de Lewis Carroll : approches entre mathématiques et littérature

# Notas de lógica a partir do panfleto "algumas falácias populares sobre vivissecção" de Lewis Carroll: aproximações entre Matemática e Literatura

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## Abstract

In order to promote greater integration between Literature and research in Mathematical Education, we present in this study an analysis of the text "Some Popular Fallacies about Vivisection", a pamphlet published in 1875 by Lewis Carroll. In this work, Carroll employs principles of Classical Logic to question opinions about the practice of vivisection. We discuss reasons justifying the relevance of literature in research in the field of Mathematical Education and provide a general overview of the content and structure of Carroll's text, also offering a brief historical contextualization of the pamphlet's theme, vivisection. Finally, we exemplify how Classical Logic can be employed as an alternative approach to interpreting Carroll's work, thus demonstrating the intersection between Mathematics and Literature in this study.

***Keywords:*** Lewis carroll, Logic, Mathematics and literature.

## Resumen

Con el fin de promover una mayor integración entre la Literatura y las investigaciones en Educación Matemática, presentamos en este estudio un análisis del texto "Some Popular

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Fallacies about Vivisection", un panfleto publicado en 1875 por Lewis Carroll. En este trabajo, Carroll emplea principios de la Lógica Clásica para cuestionar opiniones sobre la práctica de la vivisección. Discutimos razones que justifican la relevancia de la literatura en investigaciones en el campo de la Educación Matemática y abordamos de manera general el contenido y la estructura del texto de Carroll, ofreciendo también una breve contextualización histórica del tema del panfleto, la vivisección. Finalmente, ejemplificamos cómo la Lógica Clásica puede ser empleada como un enfoque alternativo para interpretar la obra de Carroll, demostrando así la intersección entre Matemáticas y Literatura en este estudio.

***Palabras clave***: Lewis carroll, Lógica, Matemáticas y literatura.

**Résumé**

Dans le but de promouvoir une plus grande intégration entre la Littérature et la recherche en Éducation Mathématique, nous présentons dans cette étude une analyse du texte "Some Popular Fallacies about Vivisection", un pamphlet publié en 1875 par Lewis Carroll. Dans ce travail, Carroll utilise des principes de la Logique Classique pour remettre en question les opinions sur la pratique de la vivisection. Nous discutons des raisons justifiant la pertinence de la littérature dans la recherche dans le domaine de l'Éducation Mathématique et donnons un aperçu général du contenu et de la structure du texte de Carroll, offrant également une brève contextualisation historique du thème du pamphlet, la vivisection. Enfin, nous exemplifions comment la Logique Classique peut être utilisée comme une approche alternative pour interpréter l'œuvre de Carroll, démontrant ainsi l'intersection entre les Mathématiques et la Littérature dans cette étude.

***Mots-clés***: Lewis carroll, Logique, Mathématiques et littérature.

**Resumo**

Com o intuito de promover uma maior integração entre a Literatura e as investigações em Educação Matemática, apresentamos neste estudo uma análise do texto "Algumas falácias populares sobre vivissecção", um panfleto publicado em 1875 por Lewis Carroll. Neste trabalho, Carroll utiliza princípios da Lógica Clássica para questionar opiniões sobre a prática da vivissecção. Discutimos motivos que justificam a relevância da literatura em pesquisas na área da Educação Matemática e abordamos de forma geral o conteúdo e a estrutura do texto de Carroll, fornecendo também uma breve contextualização histórica sobre o tema do panfleto, a vivissecção. Por fim, exemplificamos como a Lógica Clássica pode ser empregada como uma

abordagem alternativa para interpretar a obra de Carroll, demonstrando assim a interseção entre Matemática e Literatura neste trabalho.

***Palavras-chave:*** Lewis carroll, Lógica, Matemática e literatura.

## Logic notes from the pamphlet "some popular fallacies about vivisection" by Lewis Carroll: approximations between mathematics and literature

Lewis Carroll's[3] prolific literary work has enchanted and continues to enchant generations of readers around the world for more than a century and a half. Famous for "Alice's Adventures in Wonderland" and "Through the Looking-Glass, and What Alice Found There," among others, as a writer, Carroll has, over time, attracted the attention of many researchers interested in exploring content aspects that underlie his writings beyond the foreground plot. Among these aspects, the relationships between Mathematics and Literature have been guiding our studies centered on Carrollian fiction for a long time (since Montoito (2007)).

A new path, however, has been fueling our enthusiasm recently: exploring Carroll's productions little known in Brazil, not always fictional literature, many of which until recently had not been translated into Portuguese, to better understand his polyfaceted mind. Aiming to disseminate and expand research based on it, we brought together in Montoito (2023) the consolidation of a project that included the collaboration of several researchers in the field of Mathematics Education to publish eight unpublished texts by Carroll in Portuguese. One of these texts, the article *Some popular fallacies about vivisection*[4] (Minks & Montoito, 2023) takes center stage in this article. It is a literary-scientific article published in 1875 in a London periodical and also in pamphlet format, where the author talks about the subject of vivisection (a controversial practice that consists of the dissection of live animals), bringing together thirteen arguments published in periodicals of the time alluding to the positions of the academic and laic community on the moral admissibility of vivisection. Carroll notices and points out in the arguments listed logical defects that allow classifying such arguments as fallacies.

To this end, Carroll mobilizes elements of Classical Logic to present his point of view, but, in our understanding, he does so indirectly, rarely explaining logical concepts in full detail. Our intention is precisely to explore in this Carrollian writing the concepts and definitions of Classical Logic we see underlying the text. We will present an analysis of five of the thirteen statements presented by Carroll as fallacious — in the Aristotelian sense of the term (Zanoni,

[3] Pseudonym of the British Charles Lutwidge Dodgson (1832-1898), who was, in addition to being a writer, an Anglican reverend, and professor of Mathematics at the University of Oxford, also the author of extensive works on mathematics, religion, and other topics (Cohen, 1998). For convenience, in this article, we will always refer to Dodgson by his renowned pseudonym, Carroll.

[4] Published by Lewis Carroll in 1875 (Carroll, 1939, pp. 1071-1082). The research that produced this article involved the complete translation of the pamphlet into Portuguese in an exercise of "translation topology" (Montoito & Dalcin, 2022) and composes the book *Alimentar a mente (e outros textos de Lewis Carroll): tradução* (Carroll, 2023). Although the methodological aspects and references specific to the translation process are a crucial stage in our research into this text by Carroll, considerations about the translation developed are beyond the scope of this article and will be addressed on another occasion.

Bitencourt & Farina, 2016) — and explore possible hidden premises in the reasoning listed by the author and propose different reconstructions of the original statements in logical-mathematical structures.

With this, we aim to contribute to the investigation of the potential in the creation of mathematical records when a text originally in a mother tongue is transferred to a language organized in structures specific to Mathematics (in the case highlighted in this text, structures from Classical Logic). As far as students and teachers are concerned, following this path is justified by the urgency of thinking about interdisciplinary approaches to teaching Mathematics, which represent and assume human thought as something complex and, with regard to the area of Mathematics Education, for continuing to constitute a movement to expand research spaces and practices that recognize the interrelationship between Literature and Mathematics as potentially didactic.

Thus, in the following lines, we make some considerations about the topic (vivisection) and its historiographical contextualization made by Carrol; Classical Logic and how it can be applied when scrutinizing arguments; and examples of how Classical Logic can be mobilized as another way of interpreting Carroll's text to reflect on its insertion as a powerful weapon against the deceptions and argumentative falsehoods that permeate contemporary times in the most varied themes of human activities.

## Vivisection, a controversial topic

Carroll's work on which we now focus to investigate the logical expedients is the pamphlet *Some Popular Fallacies about vivisection*, which, although not a work of fiction, is strongly marked by the Carrollian literary style. The text, published in 1875 and signed with the author's civil name, — Charles Lutwidge Dodgson, professor at Christ College at the University of Oxford, and not his renowned literary pseudonym Lewis Carroll — brings at its heart ethical discussions regarding the practice of vivisection, which is the dissection or surgical operation on <u>living</u> animals, to study some anatomical and physiological phenomena. We assume that the study of this pamphlet falls within the context of research in Mathematics Education that considers a literary text both as a historical source capable of enhancing discussions on the History of Mathematics (Montoito, Dalcin & Rios, 2021) and as a genre of discourse whose "compositional construction, content theme and style," (Almeida, 2016, p. 46) "when used in the classroom, when planned appropriately, can offer an opening for discussion on issues of interest to society" (Almeida, 2016, p. 49-50). In this sense, we present brief historiographical notes on vivisection to contextualize Carroll and his pamphlet.

Records of vivisection being used as an investigative method of biological characteristics can be found as early as the Roman Empire, associated with the famous physician Galen (130-210), Greek by origin but based in Rome (Guerrini, 2003, p. 18). However, according to Carvalho and Waizbort (2014, p. 202), only from the 16th century onwards did this technique of scientific investigation begin to develop systematically, catalyzed by post-scholasticism, with emphasis on the mechanical philosophy of René Descartes (1596-1650) and the empiricism of Francis Bacon (1561-1626).

Ethical concerns about the pain and suffering inflicted on animals gained relevance in European philosophical thought in the following centuries. Among others, the philosopher Henry More (1614-1687) and the natural philosophers Robert Boyle (1627-1691) and Robert Hooke (1635-1703) were against vivisection as a natural investigative technique due to the intense suffering inflicted on animals. At the end of the 18th century, Jeremy Bentham (1748-1832), an English philosopher and jurist who was one of the founders of the utilitarian doctrine, spoke out vigorously in defense of animals, considering the torture intrinsic to the practice of vivisection to be unjustifiable, opposing the thought of the German philosopher Immanuel Kant (1724-1804), who postulated as true only relationships between rational beings, relegating animals to the category of mere objects (Carvalho & Waizbort, 2014).

Despite the growing anti-vivisectionist demonstration among the European *intelligentsia*, the dissection of live animals continued to gain ground among experimental physiologists. However, when the 19th century dawned, England remained reticent to the "whooping" of vivisection as scientific research in continental Europe.

> Contrary to what happened in France — and Germany — in British lands, experimental physiology still did not mobilize much interest among doctors or scientists in the 1850s or 1860s. Historian Richard French [...] partly attributes this British "backwardness" to the deep aversion that the British had against experiments conducted with live animals due to their cultural tradition as a land of animal lovers. From the end of the 1860s onwards, however, the social, intellectual, and educational factors responsible for the conservatism of the medical profession in England had been progressively giving in to demands related to the concern of achieving the supremacy of European medicine (Carvalho & Waizbort, 2014, p. 207).

At the same time that experiments on live animals were finally spreading throughout laboratories in England, those opposed to the practice grew in number, and the anti-vivisectionist debate began to deeply permeate lay society, going beyond the hitherto predominant university debate to the extent that abuses perpetrated by vivisectors were denounced in the newspapers, gaining public discussion. Among the people engaged in the anti-

vivisection cause, the journalist and writer Frances Power Cobbe stood out, an activist for various social causes whose efforts, according to Carvalho and Waizbort (2014, p. 209),

> led her to found and lead the *Victoria Street Society for the Protection of Animals Liable to Vivisection* (VSS) in 1875, the most powerful anti-vivisectionist organization in Great Britain and the world during the Victorian Era [...]. Integrated with the aristocracy, parliamentarians, and members of the clergy, the VSS would sue physiologists for abuses and articulate *lobbies* and bills in defense of animals and against excesses in the practice of vivisection [...]. Furthermore, the anti-vivisectionist cause had the support of some press outlets.

It was precisely at the height of the public discussion about vivisection, with the establishment of the *Victoria Street Society* in 1875, that Lewis Carroll, in the same year, published his pamphlet *Some popular fallacies about vivisection*. Attentive to the issues of his time, — just as he once, with ardor and creativity, rejected the new geometry teaching books that were willing to replace Euclid's *Elements* (Carroll, 2015) — Carroll keeps his mathematician's gaze on scrutinizing the arguments, published in periodicals of the time, by vivisectionists and their staunch opponents. In his diary, Carroll recorded the occasion his text was accepted for publication:

> 1875, May 19. (W). Heard from Mr. John Morley, editor of *The Fortnightly Review*, that he accepts my article on 'Some Popular Fallacies about Vivisection' [where it was published on June 1, and subsequently 150 copies were printed for private circulation. Included in Nonesuch Omnibus, pp. 1071-82.][5] It had been sent to him through Miss F. P. Cobbe, having been first offered to the *Pall Mall*, but declined on the ground of the fallacies being unheard of, though eight out of the thirteen came from a *Pall Mall* article! (Carroll, 1953, p. 339).

As in other of his writings (Carroll, 1939a), Carroll constructed this pamphlet with a textual/mathematical architecture, where the mother tongue and mathematical language intertwine to support his arguments, a writing practice he enjoyed (Montoito, 2020) and that allows, as a consequence, that his texts are studied, appropriated and given new meanings in the field of Mathematics Education —it is worth highlighting that, in this article, it is to Classical Logic, among the translated statements, that a more accurate investigative look is directed.

As we will see, the author reveals the fallacies one at a time, exposing his arguments, sophisms, and semantic subterfuges. Although the characteristic resources of Classical Logic are hidden in the foreground (Carroll unmasks the fallacies in an essay way and not through

[5] The commentary in brackets is by Roger Lancelyn Green, editor of Carroll's diaries in the 1953 compilation.

syllogisms, schemes, or relational diagrams), they are present between the lines of the text, almost as if Carroll wished to present his arguments twice, in a creative redundancy: first, directly in the construction of the discourse; second, as a watermark visible only to those attentive to the elements of Classical Logic, through mathematical language.

Before delving into the hidden layers of *Some popular fallacies about vivisection*, we present in the following section a collection of some definitions from the field of Logic that are convenient for a better understanding of the arguments expressed in the Carrollian pamphlet and for the construction of logical-mathematical structures that we will propose as another way of reading the text.

## Brief notes on Logic

To be of logical interest, an argument must be composed of statements, understanding that "statements are linguistic segments that have complete meaning and can be true or false, while arguments are a set of statements that combine in the form of premises and conclusions" (Nahra & Weber, 2009, p. 91-92).

Statements can be simple: (a) Alice asks, (b) The March Hare answers. There are two statements, (a) and (b); they have complete meaning and can be true or false. We can combine simple statements to produce compound statements by manipulating **logical operators**[6] between the simple statements; each operator has conditions under which the resulting compound statement will be true or false. For example, applying the conjunction operation to the suggested statements (a) and (b), we obtain "Alice asks and the March Hare answers." This compound statement will be true only if both simple statements are true.

Thus, statements, whether combined or not by logical operators, when related in the form of propositions that support a conclusion, constitute what is called an **argument**, such as the following syllogism:

(a) If Alice asks, then the March Hare answers.
(b) Alice asks.
(c) The March Hare answers[7].

[6] There are basically the following logical operations between two simple statements (a) and (b): **conjunction** [(a) and (b)], **disjunction** [(a) or (b)], **conditional** [if (a) then (b)], and **biconditional** [(b) if and only if (a)]. An utterance can still be **denied** [not (a); (a) denied].

[7] In terms of classical logic, this argument can be called a **deductive argument** since the conclusion can be drawn directly from the given premises. Deductive arguments presented in three declarative propositions (two premises and a conclusion) classically constitute a **syllogism**, whose theory was presented by the Greek philosopher Aristotle (2016). However, if syllogisms are understood only as deductive arguments expressed in a series of

Given premises (a) and (b), elapses the conclusion (c). Premise (a) is a statement composed of two simple statements combined by the logical **conditional** operator. This logical operator always involves an antecedent that necessarily entails the consequent[8] (although the consequent may be true even if the antecedent is false). Premise (b) indicates the veracity of the antecedent; therefore, the conclusion is the veracity of the consequent. This argument is said to be **logically valid**.

Analyzing a variation:

(a) If Alice asks, then the March Hare answers.
(b) The March Hare answers.
(c) Alice asks.

Premise (b) is exchanged for conclusion (c) in relation to the previous argument. This time, the argument is **logically invalid** as the conclusion may be false in light of the given premises. This occurs because the logical operator of the conditional present in premise (a) only guarantees that, given the antecedent (Alice asks), we will have the consequent (the March Hare answers). But this logical operator does not necessarily require the antecedent for there to be the consequent (the March Hare can answer even if Alice has not asked).

In the lines above, we refer *en passant* to notions of true and false statements and logically valid and invalid arguments. Therefore, it is convenient to distinguish between Truth and Validity. "Truth and falsehood may be predicated of prepositions, but never of arguments. And the Properties of validity and invalidity can belong only to deductive arguments, never to propositions" (Copi, 1961, p. 9). There is a synergy between truth (or falsehood) and validity (or invalidity), and the connection can be complex.

> There are valid arguments with false conclusions, as well as invalid arguments with true conclusions. Hence the truth or falsehood of its conclusion does not determine the validity or invalidity of an argument. Nor does the validity of an argument guarantee the truth of its conclusion. There are perfectly valid arguments which have false conclusions —but any such argument must have a least one false premiss. The term "sound" is introduced to characterize a valid argument all of whose premisses are true. Clearly the conclusion of a sound argument is true. A deductive argument fails to establish the truth

propositions, it is possible to have deductive arguments that are not syllogisms (more than three propositions), although these can also be analyzed by logical expedients, sometimes decomposing the argument into chained syllogisms.

[8] In the conditional operation, if (a) then (b), the statement (a) becomes called the antecedent, and whenever (a) occurs, we will have the event (b) as its cause, said to be the *consequent.*

> of its conclusion if it is unsound, which means either that it is not valid, or that not all of its premisses are true. To test the truth or falsehood of premisses is the task of science in general, since premisses may deal with any subject matter at all. The logician is not so much interested in the truth or falsehood of propositions as in the logical relations between them, where by the "logical" relations between propositions we mean those which determine the correctness or incorrectness of arguments in which they may occur. Determining the correctness or incorrectness of arguments falls squarely within the province of logic. The logician is interested in the correctness even of arguments whose premisses might be false (Copi, 1961, p. 10-11).

These concepts can be elucidated with an example taken from Copi (1961, p. 9):

All spiders have six legs.

All six-legged creatures have wings.

Therefore, all spiders have wings.

In this argument, both the premises and the conclusion are false, but it is a logically valid argument because the conclusion elapses from the premises, and if the premises were true, its conclusion would also have to be true. The Validity of an argument can be formalized according to Nahra and Weber (2009, p. 92) as follows:

An argument is defined as *valid* when:
1. It is impossible that if the premises are true, its conclusion is false.
2. It is impossible that considering the premises to be true, the conclusion cannot be immediately deduced from these premises.

An argument will be *invalid* when:
1. Assuming the premises are true, the conclusion may be false.
2. Although the premises are considered true, the conclusion cannot be deduced from these premises (because the opposite conclusion is logically possible or logically necessary).

Among the logically valid arguments, we can divide them, for convenience, in light of the *truth judgment* of the propositions involved, into **good arguments** and **bad arguments**. A good argument is the same as a *sound argument*. That is, its premises and conclusion are all true. An argument that is not solid, that contains premises that are or could be false, is a bad argument. This is another nomenclature that can appear in contexts of logical argumentation (Nahra & Weber, 2009). However, a bad argument is different from an invalid argument. A bad

argument is, by definition, a valid argument with false propositions (premises or conclusion) — such as the syllogism involving spiders presented previously.

This article is interested in a class of arguments whose invalidity or falsity are not very obvious, appearing convincingly like valid or true arguments. It is a set of **fallacies**. When coming across them with an inattentive look, we can take them for solid arguments when, in fact, they are not. It is pertinent to explore the logical concept of fallacy to strengthen the understanding of the pamphlet *Some popular fallacies about Vivisection*.

Deception is typical of this class of arguments, given psychological and persuasive aspects, but it is not simple to define the concept of fallacy in all contexts in a single statement.

> The word "fallacy" is used in various ways. One perfectly proper use of the word is to designate any mistaken idea or false belief, like the "fallacy" of believing that all men are honest. But logicians use the term in the narrower sense of an error in reasoning or in argument. A fallacy, as we shall use the term, is a type of incorrect argument. Since it is a *type* of incorrect argument, we can say of two different arguments that they contain or commit the *same* fallacy. Many arguments, of course, are so obviously incorrect as to deceive no one. It is customary in the study of logic to reserve the term "fallacy" for arguments which, although incorrect, are psychologically persuasive. We therefore define a fallacy as a form of argument that *seems* to be correct but which proves, upon examination, no to be so (Copi, 1961, p. 52).

There are so-called **formal fallacies**, in which the instrumentality of symbolic logic[9] is sufficient to unveil mistakes, whether intentional or not, in reasoning. In these cases, as the symbolic treatment of the logical structures involved already indicates, it is not necessary to delve into the content of the statements to verify the invalidity of the argument that takes them as propositions.

However, there is another class of fallacies, **non-formal fallacies**, which, although often can be presented by valid logical structures, still encompass incorrect reasoning. On such occasions, it is impossible to avoid investigating the contents of the statements involved if one wants to attack the argument. So, a strictly formal, symbolic treatment, simply stating and correlating symbols, is not enough. Therefore, this class of arguments is called "non-formal fallacies" because the referent of each proposition starts to matter. These are the ones we come

[9] As classical logic is interested in *the relationships* between statements and not, a priori, in their content, it is usual to use a mathematical notation to represent and operate the statements. We have already subtly done this, in part, when we named by (a) the statement "Alice asks" and (b) the statement "the March Hare answers." We can denote the logical operators applied between two simple statements in a usual but not unique notation: **conjunction**, a ^ b; **disjunction**, a v b; **conditional**, a → b; **biconditional**, a ↔ b. As for the **negation** of a statement ((a) negated), we have ~a.

across most frequently, often without understanding the deceit in them, in the varied human activities with argumentative discourse.

Non-formal fallacies are on the border between Logic and other knowledge, especially Ethics. According to Nahra and Weber (2009, p. 126), "This discipline begins where Logic stops. It will be able to say not whether the arguments are valid or invalid, but whether the arguments, decisions and especially actions are good or bad," and even guide the formulation of counter-arguments that require pointing out scientific mistakes. These authors emphasize that "the study of informal fallacies[10], with regard to the object of analysis, lies beyond Logic, but, as it still involves a process of derivation, such a study must, to some extent, still be done through Logic" (Nahra & Weber, 2009, p. 135).

The field of non-formal fallacies is vast. There are many conveniently named after the different tricks they contain. It is beyond our scope to list and detail them all. Some examples, however, are given in the following section, in which we investigate their occurrences in *Some popular fallacies about vivisection*. However, as a prelude, it is worth classifying non-formal fallacies into similarity groups. According to Copi (1961, p. 53), "We may divide informal fallacies into fallacies of *relevance* and fallacies of *ambiguity"*.

Regarding fallacies of relevance, it can be stated:

> Common to all arguments which commit fallacies of relevance[11] is the circumstance that their premisses are *logically irrelevant* to, and therefore incapable of establishing the truth of, their conclusions. The irrelevance here is logical rather than psychological, of course, for unless there were some psychological connection, there would be no persuasiveness or *seeming* correctness. (Copi, 1961, p. 53).

The fallacies of ambiguity "occur in arguments whose formulations contain ambiguous words or phrases, whose meanings shift and change more or less subtly in the course of the argument and thus render it fallacious" (Copi, 1961, p. 73-74).

Regarding the arguments, the notes presented in this section can be summarized: in a **valid argument**, the conclusion necessarily follows from the premises, and the judgment of validity is formal; that is, it is only interested in the logical relationships between the propositions and not the content of the statements that constitute the propositions; a **good argument** is a valid argument that has true or certain premises and conclusion (judgment of truth); a **bad argument** is one that, although valid, has at least one false or wrong premise; a

[10] *Informal fallacies* and *non-formal fallacies* are expressions referring to the same concept.
[11] Except for a type of fallacy classified as *Petitio Principii* or Petition of Principle (Copi, 1978, p. 74).

**fallacy**[12] is a bad argument, but it is psychologically persuasive and tries to pass it off as a good argument. "It is profitable to study such arguments, for familiarity and understanding will help keep us from being misled by them. To be forewarned is to be forearmed" (Copi, 1961, p. 52).

As an educator, Carroll (1977), when thinking of Logic as a fundamental element not only for learning Mathematics but also for a more lucid understanding of the world, shares the conviction that being versed in the tools of Logic enables us to scrutinize the deceptions typical of cunning reasoning:

> Once master the machinery of Symbolic Logic, and you have a mental occupation always at hand, of absorbing interest, and one that will be of real *use* to you in *any* subject you may take up. It will give you clearness of thought – the ability to *see your way* through a puzzle – the habit of arranging your ideas in an orderly and get-at-able form – and, more valuable than all, the power to detect *fallacies*, and to tear to pieces the flimsy illogical arguments, which you will so continually encounter in books, in newspapers, in speeches, and even in sermons, and which so easily delude those who have never taken the trouble to master this fascinating Art. *Try it*. That is all I ask of you (Carroll, 1977, pp. 52-53).

As will be seen in the next section, Carroll puts into practice his own advice for applying the *machinery of Logic* and demonstrating its *usefulness* in *finding a way* through popular *fallacies* about vivisection. However, more than one way can be found to debate the fallacies brought by Carroll. Thus, we will reflect on the different ways of following these paths, using Logic as a compass and aiming to approach it — in a movement within reflections on Mathematics Education — with potential teaching practices that seek, in the most distinct textual productions, relevant mathematics elements or relevant mathematical interpretations.

The preamble made with aspects of classical logic to guide the analysis of arguments, with emphasis on fallacies, aimed to provide the reader with the knowledge to better study the text on vivisection. Throughout his pamphlet, Carroll questions other people's arguments, some against and others in favor of vivisection, in most cases without explaining Classical Logic. However, it is clear that such entities permeate the author's reasoning underlying the text. We are interested in explaining this reasoning. The interpretation we propose is certainly not unique but coherent.

## Some popular fallacies about vivisection

[12] If we want high rigor, this definition describes *non-formal fallacies* as bad arguments that can be logically valid. As seen previously, *formal fallacies* are revealed directly in formal terms, and it is unnecessary to delve into the content of the statements; these constitute invalid arguments. The classifications of *valid* or *invalid* are from the strict universe of classical Logic, while the classifications of good or bad (coming from the judgment of truth) reside on the border of Logic – especially Ethics.

Initially, to deal with the fallacies listed by Carroll in his writing, it is necessary to make a note of the *way* in which the author brings these arguments: he states them in a form that we propose to call a *condensed argument*, in a single sentence (for example, the first fallacy, among others, discussed in detail below, is presented singularly in the following proposition: *the infliction of pain on animals is a right of man, needing no justification*). At first glance, there is no explicit chain of premises leading to a conclusion. In fact, this is the usual way in which the act of arguing occurs in human activities. Because,

> in general, when arguing, some premises of the arguments are not made explicit. Sometimes, it may even be on purpose, with bad intentions, and other times because, if we were to make all our premises clear, it would take us a long time to state our conclusion. [...] [However, in logical scrutiny,] it is essential to clarify all the premises of the argument, especially because, often, if we do not make the premises clear, the argument may seem invalid from a logical point of view, when in reality it is not. It is necessary to clarify the hidden premises of the argument in question to verify if it is, in fact, invalid (Nahra & Weber, 2009, p.118).

A condensed argument inherently contains, as alluded to by Nahra and Weber (2009), "hidden premises." Given the need to bring these premises to light to investigate the validity of the arguments that incorporate them, we propose calling an argument that explicitly displays the hidden premises of a primary *condensed argument* an *expanded argument*. Certainly, exposing possible hidden premises involves an interpretative effort by those who analyze a given argument. However, unraveling such premises "is one of the most fascinating jobs a logician can do, and allows him to discover many things about the human soul" (Nahra & Weber, 2009, p.118). It is clear that this interpretative component in logical analysis, unavoidable in many cases, must be rigorous when aggregating to an argument certain premises that were not initially "stated clearly." To this end, it is essential to observe coherence with the context, cause and effect relationships, and intentionality of the issuer of the argument, among other aspects.

Our main objective in this section is to propose and interpret the possible unveiling of the premises that we consider hidden in Carroll's text, transforming condensed arguments into expanded arguments using syllogisms and other logical-mathematical structures. When a text by Lewis Carroll is involved, this interpretative investigation takes on a richer shape. Master of Nonsense[13] as his literary style, Carroll tensions the conceptions of logicality through unusual

[13] Nonsense literature is the structure that supports all Carrollian literary works. The product of nonsense emerges from perfectly linked logical sentences, unlike what happens with absurdist literature. It is also a closed system in itself, like a game with its own rules.

paths that weave unforeseen but robust connections towards conclusions that are often surprising but logical. Note, however, that the conclusion can be uncertain since "nonsense is a message-in-a-bottle[14], that is, a message that the sender, when sending, is not sure when and if it will be received by an interlocutor, not even when and how it will be understood" (Montoito, 2019a, p. 41).

The interpretation of a Carrollian text is also an exercise in creativity, considering that.

> the logic of nonsense is not understood by automatic and mechanical interpretations but by "twists" and "contortions" of thought that, if in principle seem impossible, in the end, the reader will be able to do it. The thoughts of anyone who studies the logic of nonsense will turn around on themselves, like the crocodile capable of walking on its own head that the brothers Sílvia and Bruno encounter. This is how Carroll achieves his goals as an educator, even if in a subliminal way — and, depending on the text, quite discreet and introductory (Montoito, 2019a, p. 35-36).

In this sense, when entering *popular fallacies about vivisection,* we become aware of the *nonsensical* game that makes it possible to broaden the interpretations in Carroll's texts. The reader will notice our movement like a crocodile walking over our own heads. This is to say that the analyses proposed in the following lines are not impervious to other perspectives and other versions. Carroll, back in 1875, compiled from printed periodicals 13 opinions that he considered most popular on the subject of vivisection and gave his version, in metaphorical prose, of why they consisted of fallacies. Now, a century and a half later, we dust off this old text and revive the debate by proposing to do what Carroll did not do at length (at least explicitly): scrutinize *popular fallacies* by associating them with known classes of fallacies in the field of Logic, as well as, when possible, present views in mathematical terms as an analytical resource to reach the conclusion.

Although it is recognized that mathematical language and mother tongue are in symbiosis, assuming that there is an absolute equivalence between them "cannot be more than a naive stance that must be transcended" (Machado, 2001, p. 74) and, to problematize these discussions, we intend to work on tensioning the limits of both, studying the approximations and distances between their forms, symbols and signs, aiming at the construction of a hermeneutic — among the various possibilities already worked on in the area of Mathematics Education (Garnica & Salandim, 2014; Montoito, 2019; Montoito & Rios, 2019) — which, based on literary texts, contributes to the teaching of Mathematics and helps students to develop skills that allow them to transition between the two languages.

[14] The expression message in a bottle (*Flaschenpost*) is coined by Theodor W. Adorno in *Philosophy of New Music* (Ávila, 1996) and concerns artistic production.

Without delay, we propose to highlight aspects of Logic present in fallacies 1 to 4 and fallacy 6 contained in the original 1875 pamphlet[15]. As will be seen, the controversy of opinion that produces the arguments listed is centered on the *suffering* caused to animals subjected to the practice of vivisection. Thus, making Carroll's voice ours:

> I begin with two contradictory propositions, which seem to constitute the two extremes [of vivisection subject], containing between them the golden mean of truth:
> *1. That the infliction of pain on animals is a right of man, needing no justification.*
> *2. That it is in no case justifiable* (Carroll, 1939, p. 1071).

We will use the numbering structure used by Carroll to refer to the arguments when investigating them; for example, when analyzing "Fallacy 1" we will be referring to the proposition "*That the infliction of pain on animals is a right of man, needing no justification*."

Our investigation begins with the first fallacy[16]. We perceive in this proposition elements typical of an *Argumentum ad baculum* (recourse to force[17]) – even though Carroll does not allude to this in his text–, which "is the fallacy committed when one appeals to force or the threat of force to cause acceptance of a conclusion" (Copi, 1961, p. 53-54), and it is an example of a non-formal fallacy of relevance.

This assumption is coherent since the original proposition does not allude to any alleged "origin" for the so-called "right of man" to cause pain in animals. Nor does Carroll invest his words in denuding this detail, preferring to attack the moral problem of the need for justification. In a possible interpretation, to highlight the hidden premises of this fallacy, we consider that *ad baculum*, a fallacy summarized in the aphorism "might make right" (Copi, 1961, p. 54), fits the case in question.

Therefore, for Fallacy 1, we propose the following *expanded argument*:

(a) The stronger has the right to subjugate the weaker, even causing pain, **and** there is no need for justification.

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[15] We chose to analyze here, among the thirteen collected by Carroll, fallacies 1 to 4 because they are the first presented in the pamphlet and therefore introduce the author's reflections on vivisection, and because they can be classified into classes of fallacies of relevance well defined. Fallacy number 6 was brought up since its analysis using Euler-Venn diagrams favors our intended approaches of the text to Mathematics. The other fallacies were not analyzed here as they would be too profuse for the space of an article. A complete analysis of the thirteen fallacies will be presented on another occasion.

[16] It is worth clarifying that we agree with Carroll that the arguments enumerated by him in his pamphlet, according to the *condensed argument* structure that we expose, and analyzed here in an expanded way in a possible interpretation, in fact, constitute **logical fallacies** – as we seek to demonstrate.

[17] By "strong" we do not mean exclusively an attribute of physical strength, although it may be. In a broad sense, it is the notion of predicates that allow one being (or a set) to subjugate another in a certain context. The expression of strength, among humans, can "also be money, power, voters, means of communication" (Nahra & Weber, 2009, p. 145).

(b) Man is stronger[18] than animals.
(c) Man has the right to inflict pain on animals, **and** he does not need justification.

The problem of the validity of the conclusion of the original argument, as well as its expanded version by the syllogism proposed, revolves around the need for justification for the infliction of pain on animals. Countless contexts differ in justification because, for example, it is not the same thing to hurt, in self-defense, a rabid dog that attacks someone and to hurt a dog simply because one likes cats. Both cases involve causing pain to an animal, but they have different ethical aspects. To overcome the ethical problem regarding the justification for causing pain, this argument invokes the supposed right of man over animals, a right based on the implicit premise that men are stronger than animals. However, this thesis is insufficient (perhaps irrelevant), although persuasive, to overcome the problem of justification in causing pain, which makes the argument fallacious, no matter how much the conclusion derives from the premises. In Carroll's words (1939, p. 1071): "All who recognize the difference of right and wrong must admit, if the questions be closely pressed, that the infliction of pain is in *some* cases wrong."

Fallacy 2 (*That it is in no case justifiable)*, in turn, opposes the thesis of Fallacy 1, constituting a debate between, according to Carroll, "two contradictory propositions, which seem to constitute the two extremes [regarding the issue of justification for causing pain], containing between them the golden mean of truth" (Carroll, 1939, p. 1071). We propose rewriting Fallacy 2 as the following expanded argument in the form of a syllogism:

(a) An absolute evil is in no case justifiable.
(b) Inflicting pain is an absolute evil.
(c) Inflicting pain is in no case justifiable.

We consider this construction of argument in light of Carroll's statement (in his text) that Fallacy 2 "has been assumed by an Association lately formed for the total suppression of Vivisection, in whose manifesto it is placed in the same category with Slavery, as being an absolute evil, with which no terms can be made" (Carroll, 1939, p. 1071).

[18] Same as the previous footnote.

It is possible to make a *reductio ad absurdum*[19] to face this fallacy: if we assume this argument as *good*, in the example of the rabid dog attack, it would be unjustifiable to cause pain to the dog, even to defend ourselves from the possibility of serious injuries and a lethal disease, which will cause pain. As pain is an absolute evil, it is also not justifiable for the dog to attack someone, however, as the predicates of rationality escape the dog, it would be absurd to expect attributes of sensibility from it. Many times, life imposes itself. In the example given, and in other situations in which *pain* inflicted on a human or an animal is a variable of the event, it is possible that a human action constituting itself as causing pain does not occur as the execution of a conscious, reflected act. In pressing, unexpected events, typical of accidents and attacks, the unconscious reflex often prevails, the instinct for action to preserve life; they are organic actions that do not obey abstract concepts. In such cases, a condemnatory ethical judgment may not be simple, even considering *pain an absolute evil*.

The ethical judgment here cannot avoid considerations of *intentionality* and *causality*. Fallacy 2, listed by Carroll, is one of the arguments of those categorically against vivisection. Nowhere in the pamphlet does Carroll make explicit his inclusion in the "antivivisectionist" group, but it is clear from the text that this is the case. Even so, the author criticizes the argument based on *absolute evil* for pure intellectual coherence. Saying that *in no case is the infliction of pain justifiable* definitively removes the attributes of intention and cause, as well as pure contingency, which can lead to misleading conclusions.

From another perspective, if we consider that the argument that constitutes Fallacy 2 brings with it pity and compassion for the suffering of others as a basis for definitively ruling out — despite the circumstances — the possibility of causing pain, then the argument falls into the fallacy of relevance *Argumentum ad Misericordiam*[20] (appeal to pity), which "is the fallacy committed when pity is appealed to for the sake of getting a conclusion accepted" (Copi, 1961, p. 58).

[19] Latin expression for "reduction to absurdity," a method used in Logic to deny the truth of an argument by demonstrating that its premises lead to absurd or ridiculous consequences.
[20] "This argument is frequently encountered in courts of law, when a defense attorney may disregard the facts of the case and seek to win his client's acquittal by arousing pity in the jurymen" (Copi, 1961, p. 58).

Regarding these first two fallacies, one opposed to the other, Carroll (1939, p. 1071) summarizes: "I think I may assume that the proposition most generally accepted is an intermediate one, namely, that the infliction of pain is in some cases justifiable, but not in all."

These first two fallacies have at their core the issue of the **right and justification to inflict pain**. Advancing the controversy, some of Carroll's contemporaries branched out this issue to relate it to the supposed **right and justification to kill** (considering the context, an animal), as did "Mr. Freeman[21], in an article on Field Sports and Vivisection, which appeared in the *Fortnightly Review* for May, 1874, (...) when he classes death with pain together, as if they were admitted to be homogeneous" (Carroll, 1939, p. 1072). Now, according to Carroll, the following fallacy arises in the debate:

> 3. That our right to inflict pain on animals is co-extensive with our right to kill, or even to exterminate a race (which prevents the existence of possible animals) all being alike infringements of their rights (Carroll, 1939, p. 1072).

Understanding **coextensive** as the attribution of the same amplitude or extension value, this argument creates equality between the right to kill and the right to cause pain to an animal and considers that both acts violate animal rights, thus creating a (supposed) conflict of rights between species. The conception of this conflict makes it, in Carroll's opinion, "one of the commonest and most misleading of all the fallacies" (Carroll, 1939, p. 1072) that permeate the theme of vivisection.

Carroll uses the method of reduction to absurdity, together with the conclusions produced when attacking the previous fallacies, to demonstrate that death and pain are neither homogeneous nor coextensive, as there can be death without pain and pain without death:

> In discussing the "rights of animals," I think I may pass by, as needing no remark, the so-called right of a race of animals to be perpetuated, and the still more shadowy right of a non-existent animal to come into existence. The only question worth consideration is whether the killing of an animal is a real infringement of right. Once grant this, and a *reductio ad absurdum* is imminent, unless we are illogical enough to assign rights to animals in proportion to their size. Never may we destroy, for our convenience, some of a litter of puppies – or open a score of oysters when nineteen would have sufficed – or light a candle in a summer evening for mere pleasure, lest some hapless moth should

[21] This is probably Edward Augustus Freeman (1823-1892), an English historian and politician interested in the subject of vivisection, author of the article *Surgery and vivisection* (Freeman, 1885), published in the London newspaper *The Times*, on January 16, 1885, and also publicized by the *Victoria Street Society for the Protection of Animals from Vivisection* in partnership with the *International Association for the Total Suppression of Vivisection.*

> rush to an untimely end! Nay, we must not even take a walk, with the certainty of crushing many an insect in our path, unless for really important business! Surely all this is childish. In the absolute hopelessness of drawing a line anywhere, I conclude (and I believe that many, on considering the point, will agree with me) that man has an *absolute* right to inflict death on animals, without assigning any reason, provided that it be a painless death, but that any infliction of pain needs its special justification (Carroll, 1939, p. 1072-1073).

Although Carroll does not indicate examples of cases that fall, in his view, into the so-called "special justification" for inflicting pain on animals, it is possible to infer from the context of the text — and his biography — that he rejects any action that causes pain for free. It is as if Carroll was saying between the lines that *the infliction of pain (to animals or humans) should be avoided whenever possible.* Thus, in the excerpt, by recognizing that human action on earth will cause inevitable impacts on the lives of animals, the author expresses, in our view, the conviction that considerations of *intentionality* and *causality*— considerations that are also present in the analysis conducted for Fallacy 2 — matter centrally when it comes to inflicting pain.

It is worth remembering that one of the main aspects surrounding the topic of vivisection is the controversy regarding the pain and suffering caused to animals in this practice. It is clear that the thoughts present in Fallacies 1 and 3 (which presumably express the point of view of people favorable to the practice) seek to invalidate the pain caused to specimens as a reason to oppose vivisection.

Fallacy 4, in turn, approaches the controversy from another perspective, no longer endorsing the supposed "rights of man" but considering the possibility of obtaining benefits to human life arising from knowledge acquired through physiological studies from vivisection[22]:

> 4. That man is infinitely more important than the lower animals, so that the infliction of animals suffering, however great, is justifiable if it prevent human suffering, however small (Carroll, 1939, p. 1073).

The premise "*man is infinitely more important than the lower animals*" aims to justify the argument. But we consider it unnecessary for the logical analysis of the suggested reasoning. As stated before, fallacies often bring unnecessary or irrelevant premises from a strictly logical point of view inserted into the argument for persuasion and psychological appeal. Therefore,

[22] Understanding vivisection as an act of scientific investigation in the field of Biology, the underlying thought behind Fallacy 4 (also present in other Fallacies listed by Carroll in the pamphlet and not analyzed in this article) evokes the conviction that scientific knowledge always aims, to some extent, the well-being of humanity. However, Fallacy 4 (and others) goes further to admit that, in scientific practice, the ends would always justify the means.

by explaining the relational essence between the statements involved to point out the implicit conditional operator, the argument becomes: **If** the infliction of pain on animals (however big) prevents human suffering (however small), **then** the infliction of pain on animals is justifiable.

The quantifiers "however big" and "however small" cancel each other out, as this detail of Fallacy 4 can be read as "it doesn't matter how much pain is inflicted on animals as long as the intention is to prevent any human pain/suffering." We can then rewrite the argument by making the hidden pro-vivisection premise explicit in the following syllogism:

(a) **If** the infliction of pain on animals prevents human suffering, **then** the infliction of pain on animals is justifiable.
(b) Vivisection (which inflicts pain on animals) aims to prevent human suffering.
(c) Vivisection is justifiable.

What we did to reach the syllogism presented for Fallacy 4 is indicative of how intricate it can be to extract from a fallacious argument foreign to a rigorously logical chain. Carroll confronts this fallacy in two distinct ways. In the first, he questions the irrelevance attributed to suffering quantifiers, as thousands (he believes) would be "ready to assure the vivisectors that, so far as their personal interests are concerned, they are ready to forego any prospect they may have of a diminution of pain, **if** it can **only** be secured by the infliction of so much pain on innocent creatures" (Carroll, 1939, p. 1073, our emphasis). The underlined words indicate a biconditional operation — they are equivalent to the meaning of *if and only if* proper to this logical operator — which supports the counter-argument that, in reality, suffering quantifiers are necessary elements to support whether or not it is justifiable to cause hardship to an animal. It is possible to express this line of thought by Carroll also in the form of a syllogism, proposed below:

(a) Mitigating minor human suffering is not justifiable if it causes great suffering to animals.
(b) The reduction of human suffering (through scientific knowledge arising from vivisection) happens *if and only if* great suffering is perpetrated against animals.
(c) The reduction of minor human suffering is not enough to consider vivisection justifiable.

In turn, the second way in which Carroll attacks Fallacy 4 considers the implicit proposition that vivisection aims to prevent human suffering from being *false* and, given the falsity of one of the premises, the argument is *bad*. According to Carroll (1939, p. 1074),

vivisectionists do not want to admit that "the lust for scientific knowledge[23] is [their] real guiding principle. The lessening of human suffering is a mere dummy set up to amuse sentimental dreamers."

Finally, it is time to analyze the sixth fallacy brought by Carroll to problematize comparisons made between *vivisection* and *sports* that involve some possible infliction of pain on animals:

> I now come to another class of fallacies – those involved in the comparison, so often made, between vivisection and field-sports. If the theory, that the two are essentially similar, involved no worse consequence than that sports should be condemned by all who condemn vivisection, I should be by no means anxious to refute it. Unfortunately the other consequence is just as logical, and just as likely, that vivisection should be approved of by all who approve of sport.
>
> [...] *6. That the pain inflicted on an individual animal in vivisection is not greater than in sport.*
>
> I am no sportsman, and so have no right to dogmatize, but I am tolerably sure that all sportsmen will agree with me that this is untrue of shooting [24], in which, whenever the animal is killed at once, it is probably as painless a form of death as could be devised; while the sufferings of one that escapes wounded ought to be laid to the charge of unskilful sport, not of sport in the abstract. Probably much of the same might be said of fishing: for other forms of sport, and especially for hunting, I have no defence to offer, believing that they involve very great cruelty (Carroll, 1939, p. 1074-1075).

There is a possible reinterpretation in formal logical terms for fallacy *6. That the pain inflicted on an individual animal in vivisection is not greater than in sport.* Firstly, the expression that constitutes the fallacy implicitly assumes that the practice of sport necessarily implies the infliction of pain on the animals involved, if any. This implicit assertion is the sophistical core that allows laying bare the fallacy in logical terms. To better explain the mathematical structure underlying the excerpt, the elements of Classical Logic mobilized for the symbolic reconstruction of the argument are highlighted in bold. We can analyze the logical argument of Carroll based on the **extension** of **terms** that are representatives of the **concepts**

[23] The noun "lust" can be understood both as "great/strong/powerful desire" and as "craving/lasciviousness/concupiscence." It is possible that Carroll chose the term to purposely give a double meaning to the sentence since the entire paragraph from which this excerpt originates seems to contain a certain irony and a tone of reprimand.

[24] Carroll considers the terms *shooting* (in the sense of "target shooting") and *hunting* to have different connotations when approaching sports that involve the slaughter of animals. The distinction, it seems, is due to the fact that certain shooting sports against small animals — such as ducks and other birds — usually lead to instant death of the specimen shot, while when hunting large animals — for example, wild boars and deer — the animal often remains alive and suffering for a long period after being shot.

involved in its chain. By "concept," we understand the notion or mental representation of an object and, by "term," the symbolic expression of a concept. Thus, let the terms X, Y, and W be related to the concepts explained below (whereas the terms X and W make up disjoint sets):

X = animals subject to the practice of sport
Y = animals that feel pain
W = animals subjected to the practice of vivisection

The **extension of a term** is the set of entities to which that term can be applied. The famous Swiss mathematician Leonhard Euler (1707-1783) demonstrated the existence of only five possible relationships, in logical terms, between two terms (for example, X and Y): complete equality between X and Y; X belongs to Y; Y belongs to X; partial interaction between X and Y, complete differentiation between X and Y.

Evaluating the terms selected to express the concepts of the fallacy under analysis, we discover that defenders of vivisection, in their argument, recognize that W belongs to Y (all animals subject to the practice of vivisection feel pain – *That the pain inflicted on an individual animal in vivisection is no greater than that in sport*), but they also postulate, underlyingly, that X belongs to Y (that all animals subject to the practice of sport feel pain – *That the pain inflicted on an individual animal in vivisection is no greater than the [pain inflicted on an animal] in sport*). However, in reality, Carroll considers as true the argument that only some animals subject to the sport, but not all, feel pain. Conversely, among the animals that feel pain, some, but not all, are subject to the practice of sport (animals can feel pain, for example, when attacked by a predator). In other words, there is a "partial interaction between X and Y," and there is a consensus that "W belongs to Y." We can denote these relationships using Euler-Venn diagrams and thus expose the Carrollian conclusion:

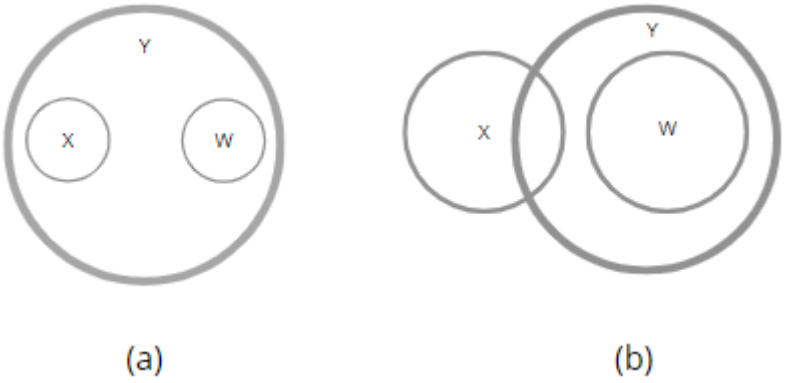


Figure 1.

*The Carrollian conclusion*

*(a) vivisectionists' argument (fallacious). (b) Carroll's argument*

From the excerpt, is possible to identify that Fallacy 6 is a rhetorical attempt by vivisectionists or their defenders to "legitimize" the practice of vivisection by associating its most controversial predicate (the infliction of pain on animals) with field sports involving animals. However, the pro-vivisection argument postulates, underlyingly, as stated, that the practice of sports involving animals necessarily implies the infliction of pain on the specimens, which is, as we have seen, a **false** argument. Therefore, through the logical structure organized and summarized in Figure 1, we demonstrated, once again and in another language, Carroll's logical confrontation with the assertion that he judged to be fallacious about the relations between vivisection and sports.

With this example, we drive away, for now, to our foray into the Logic present in Carroll's text presented here, in which, in the last example, we emphasized the perception of symbolic and formal reasoning typical of Mathematics, which we verified in the previous paragraphs. The correlation between Mathematics and Logic is indisputable because,

> starting with premisses which would be universally admitted to belong to logic, and arriving by deductions at results which as obviously belong to mathematics, we find that there is no point at which a sharp line can be drawn, with logic to the left and mathematics to the right (Russell, 1920, p. 195).

As often happens in Mathematics, there may be different paths to reach the same result or the same conclusion. Each path has nuances, charms, obstacles, difficulties,

interrelationships, and surprises. The proof of a theorem is a logical development that is unpredictable to the mathematician, something impossible to fully understand before being actually completed. The logical unfolding of the fallacies discussed suffered the same fate. The paths taken are certainly not the only ones, nor do they exhaust the subject. We proposed just a few syllogisms as discreet beams of light to illuminate certain regions of Carroll's thought hidden between the lines of his pamphlet on vivisection. Much more is possible to discover.

## From the premises, the conclusion

Caroll expresses in *Symbolic Logic* (Carroll, 1977) — his last published work, with its original edition dating from 1896, which can be taken as his "mature work" on the theme of Logic — in parallel to the studies he conducted to systematize Logic as a content, his pedagogical intentions on this topic. Carroll considered that "any one, who has to superintend the education of young people (say between 12 and 20 years of age), must have realised the importance of supplying them with healthy mental recreations" (Carroll, 1977, p. 45) such as recreations that have the nature of games and puzzles, in which the tools of Logic can be exercised. Thus, starting from playful movements to gradually entering sophisticated concepts and touring more "robust" materials,

> (...) the accomplished *Logician* has not only enjoyed himself, all the time he was working up to that position, fully as much as the Champion-player has done; but he finds himself, when that position is won, the holder of an "Open Sesame!" to an inexhaustible treasure-house of varied interests. He may apply his skill to any and every subject of human thought; in every one of them it will help him to get *clear* ideas, to make *orderly* arrangement of his knowledge, and more important than all, to detect and unravel the *fallacies* he will meet with in every subject he may interest himself in (Carroll, 1977, p. 46).

Caroll expressed this belief several times in his writings almost as advice, and he ratified it in his own intellectual practice when writing *Some popular fallacies about vivisection* with a logical chain that allowed him to unveil the fallacies on this topic that certainly interested him. His lecture on the use of Logic in the most varied aspects of life gains special relevance today, in times of profusion of fake news, especially on recurring issues of great polarization of ideas.

In this sense, this article sought to favor discussions that present Classical Logic not only as academic content but also as a structure of thought to be developed for the reflection and insertion of the individual in the world, serving as support for making decisions that escape classic fallacies, which compromise actions. This article is the continuation of a research effort that, placing Literature in dialogue with references from Mathematics Education and other knowledge, aims to provide students with learning spaces that do not ignore or diminish their subjectivities, as it is necessary to understand that Literature can help to understand and think about social practices (Bauman & Mazzeo, 2020), in addition to being a powerful school of life (Morin, 2004).

Among the more concrete aspects of the analysis presented, it is pertinent to note that some of the logical operations discussed in this article are important for the development of computational thinking and for the understanding of modern digital technologies, as they underlie programming languages and, thus, constitute a very current subject of educational interest. Furthermore, it is worth pointing out that identifying logical operators and logical structures in texts written in natural language and the subsequent transfer of textual meaning to mathematical language can be suitable in studies and practices of mathematical literacy.

In addition to elements of Mathematics and logical analysis discussed here, this Carrollian text also proves to be interesting, in a historiographical approach, for other topics it addresses: the author's opinions on scientific experiments, on ethics in dealing with animals, and on religious aspects (Cohen, 1998), as well as the question of the historical evolution of the debate on vivisection, a topic still on the agenda in bioethics (Carvalho & Waizbort, 2014). These questions point to other possible works that can be conducted using *Some popular fallacies about vivisection* as an object of study. From the glimpses of the content of the pamphlet, we discovered that it is possible to raise (from it), inside and outside the classroom, discussions outside the subject of Mathematics since developing an ethical stance in scientific practice and respecting and preserving the animal life continues to be issues of social relevance.

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