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The emergence of didactic obstacles in the contractual teacher-student- knowledge relationship in the teaching of probabilities

La aparición de obstáculos didácticos em la relación contractual profesor- alumno-conocimiento en la enseñanza de probabilidad

L'émergence d'obstacles didactiques dans la relation contractuelle enseignant- élève-savoir dans l'enseignement des probabilités

A emergência de obstáculos didáticos na relação contratual professor-aluno-saber no ensino de probabilidade

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Abstract

This paper is an excerpt from a master's research project that aimed to analyze how contractual relationship can influence the emergence of didactic obstacles in the teaching of probability in the final years of elementary school. According to the French theorist Guy Brousseau, the didactic contract concerns the clauses established, especially implicitly, from the relationship between teacher and student in the face of knowledge pertaining to the didactic situation. The idea of a didactic obstacle, in turn, refers to the teacher's choices when conducting the didactic situation, reflected in his/her actions, speeches and gestures, which potentially hinder the student's appropriation of knowledge. The mathematical knowledge contemplated in the research was Probability, a field characterized by indeterminism, which has as its object the study of chance. The data of the master's research in question were produced from two stages: the semi-structured interview and the classroom observation, in which two Mathematics teachers from a public school in the city of São Lourenço da Mata - Pernambuco participated. The results indicate that the emergence of didactic obstacles occurs, above all, at the time of negotiation of the clauses of the didactic contract, but they also pointed out that they can also

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emerge in other elements that characterize the Didactic Contract, such as expectations, rules and rupture.

Keywords: Didactic contract, Didactic obstacle, Probability.

Resumen

El presente trabajo es un extracto de una investigación de maestría que tuvo como objetivo analizar cómo la relación contractual puede influir en el surgimiento de obstáculos didácticos en la enseñanza de la probabilidad en los últimos años de la Educación Primaria. Según el teórico francés Guy Brousseau, el contrato didáctico se refiere a las cláusulas establecidas, especialmente de forma implícita, a partir de la relación entre profesor y alumno frente a conocimientos pertenecientes a la situación didáctica. La idea de obstáculo didáctico, a su vez, refiere a las elecciones del docente al conducir a la situación didáctica, reflejadas en sus acciones, discursos y gestos, que potencialmente dificultan la apropiación del conocimiento por parte del estudiante. El conocimiento matemático abordado en la investigación fue la Probabilidad, campo caracterizado por el indeterminismo, cuyo objeto es el estudio del azar. Los datos para la investigación de maestría en cuestión fueron producidos a partir de dos etapas: la entrevista semiestructurada y la observación de clase, en la que participaron dos profesores de Matemáticas de una escuela pública del municipio de São Lourenço da Mata - Pernambuco. Los resultados indican que el surgimiento de obstáculos didácticos se da, sobre todo, en la negociación de las cláusulas del contrato didáctico, pero también señalan que también pueden surgir en otros elementos que caracterizan el Contrato Didáctico, como expectativas, reglas y ruptura.

Palabras clave: Contrato didáctico, Obstáculo didáctico, Probabilidad.

Résumé

Le présent travail est un extrait d'une recherche de maîtrise visant à analyser comment la relation contractuelle peut influencer l'émergence d'obstacles didactiques dans l'enseignement des probabilités dans les dernières années du primaire. Selon le théoricien français Guy Brousseau, le contrat didactique concerne les clauses établies, notamment implicitement, fondées sur la relation entre enseignant et élève face à des savoirs appartenant à la situation didactique. L'idée d'obstacle didactique, quant à elle, fait référence aux choix de l'enseignant lorsqu'il mène à la situation didactique, reflétés dans ses actions, discours et gestes, qui entravent potentiellement l'appropriation des connaissances par l'élève. La connaissance mathématique couverte par la recherche était la probabilité, un domaine caractérisé par

l'indéterminisme, dont l'objet est l'étude du hasard. Les données pour la recherche de master en question ont été produites à partir de deux étapes : l'entretien semi-structuré et l'observation en classe, à laquelle ont participé deux professeurs de mathématiques d'une école publique de la municipalité de São Lourenço da Mata - Pernambuco. Les résultats indiquent que l'émergence d'obstacles didactiques se produit surtout lors de la négociation des clauses du contrat didactique, mais ils soulignent également qu'ils peuvent également apparaître dans d'autres éléments qui caractérisent le contrat didactique, comme les attentes, les règles et la rupture.

Mots-clés : Contrat didactique, Obstacle didactique, Probabilité.

Resumo

O presente trabalho é um recorte de uma pesquisa de mestrado que objetivou analisar de que forma a relação contratual pode influenciar o surgimento de obstáculos didáticos no ensino de probabilidade nos Anos Finais do Ensino Fundamental. Segundo o teórico francês Guy Brousseau, o contrato didático diz respeito às cláusulas estabelecidas, sobretudo de maneira implícita, a partir da relação entre professor e aluno diante de um saber pertencente à situação didática. A ideia de obstáculo didático, por sua vez, refere-se às escolhas do professor ao conduzir à situação didática, refletidas em suas ações, falas e gestos, que, potencialmente, obstaculizam a apropriação do saber pelo estudante. O saber matemático contemplado na pesquisa foi a Probabilidade, campo caraterizado pelo indeterminismo, que tem por objeto o estudo do acaso. Os dados da pesquisa de mestrado em questão foram produzidos a partir de duas etapas: a entrevista semiestruturada e a observação de aula, nas quais participaram dois professores de Matemática de uma escola pública do município de São Lourenço da Mata - Pernambuco. Os resultados indicam que a emergência de obstáculos didáticos acontece, sobretudo, nos momentos de negociação das cláusulas do contrato didático, mas também apontaram que também podem emergir em outros elementos que caracterizam o Contrato Didático, como a expectativa, as regras e a ruptura.

Palavras-chave: Contrato didático, Obstáculo didático, Probabilidade.

The emergence of didactic obstacles in the teacher-student-knowledge contractual relationship in the teaching of Probability

The didactic scenario constituted by teacher, student and the management of knowledge, is conducive to the emergence of several phenomena that can be defined as didactic phenomena. Such phenomena, often not perceived by the didactic partners of the relationship, may or may not favor the appropriation of knowledge by the student. What particularly interested us in our master's dissertation was the teacher's management of didactic situations related to the teaching of probability, which, at the time of negotiation of the didactic contract, produced -or would potentially produce-didactic obstacles (Brousseau, 1983).

The theoretical foundation of this study is in the Didactics of Mathematics of French influence. Research in this area has brought and still brings great contributions, one of the main factors in the constitution of this field, the creation, at the end of the 60s, of the IREM-Instituts de Recherche sur L'enseignement des Mathématiques (Institutes of Research in the Teaching of Mathematics), located in France (Brito Lima, 2006).

Important theories continue to be developed and contemplated by research to the present day, with emphasis on research carried out in France, Spain, as well as in Brazil, a country with considerable production in the field of Mathematics Didactics. The French researcher Guy Brousseau (1998), considered by many to be the father of Mathematics Didactics, was the one who theorized about the notions that interest us in this study, within the scope of the Theory of Didactic Situations (TSD): the didactic contract and the didactic obstacles.

When we focus on the relationships which are established in the classroom, involving the teacher, the students and the knowledge that should be taught and learned, it is possible to observe that such intrinsic situations are crossed by a series of influences, rules and conditions, which are not usually foreseeable in the idealization of a didactic situation. These aspects are linked to the didactic contract, described by Brousseau (1996) as the set of rules, expectations and conditions inherent to the functioning of the didactic relationship, most of which are implicit.

This contractual relationship is, as stated by Chevallard, Bosch & Gascón (2001, p.49), "the touchstone of the entire school organization". From this perspective, the elements of a didactic contract point out important aspects which can contribute to the understanding of the student's learning process, as well as to the reflection on possible obstacles that are instituted in the didactic process.

According to Pais (2019), it is necessary to refer to the idea of didactic obstacles when there are actions in the didactic plan that can hinder the evolution of learning. Cury (2013), in

turn, points out that it is necessary to analyse the errors that occur within the classroom in order to understand how knowledge, which has been poorly generalized or wrongly applied to a new situation, is expressed. This reflection is directly related to what Brousseau defined as a didactic obstacle (Brousseau, 1983). This researcher was inspired by the idea of epistemological obstacles, proposed by Gastón Bachelard, which referred to the obstacles related to the evolution of knowledge itself. In the specific case of the classroom and the management of didactic knowledge, Brousseau argues that there is another nature of obstacles that is revealed, which were named by him as didactic obstacles.

For Brousseau (2008), these obstacles can be understood by the manifestation of errors that may be related to a way of knowing, or even a previous knowledge that was successful, but that in a new situation ends up constituting an obstacle to the evolution of knowledge.

Therefore, the search for understanding the elements that permeate the contractual relationship and the production of didactic obstacles, in this research, will contemplate the knowledge of Probability, as we had already announced, since some research (Brum & Silva (2015), Almeida & Farias (2018) and Cavalcante, Lima & Andrade (2021)) reveals the problematizing scenario of the teaching and learning of probabilistic concepts. One of the arguments that support this choice and that is discussed in the research to which we refer, concerns the culture of seeing Mathematics only from the bias of determinism and accuracy, as opposed to the ideas of uncertainty and chance, objects of study in the mathematical field chosen by us.

In addition, other problematic issues involve this field, such as the structure of the problems used in classes and the limitation of studies to equiprobable sample spaces. We also draw attention to the fact that there is no adequate emphasis on this knowledge in the initial and continuing education courses for Mathematics teachers, which contribute to a gap in the "probabilistic way of thinking" of many teachers. (Rufino & Silva, 2015) and development of probabilistic thinking (Figueiredo, Lima & Bianchini,2023).

Based on the assumptions presented, this article deals with an excerpt from a Master's dissertation that aimed to analyse how the contractual relationship can influence the emergence of didactic obstacles in the teaching of probability in the Final Years of Elementary School. Specifically, it sought to identify the contractual clauses that govern the relationships between the three elements of didactic situations (teacher-student-knowledge) and to point out the didactic obstacles most commonly identified in the field of Probability.

The contractual relationship in the didactic scenario

Guy Brousseau proposed a theory that we can consider as the one that inaugurates the Didactics of Mathematics: The Theory of Didactic Situations (TSD) (Brousseau, 2008). In it, he makes an important reflection on the didactic triad, formed by the teacher, the student (or group of students) and knowledge. For Brousseau, the didactic relationship would then be configured as a triangular relationship, since the didactic game that is established between teacher and student(s) is intermediated by knowledge. This idea was, in a certain sense, revolutionary, because pedagogy had already been focused for a long time on the dual-teacher/student relationship, but it had not yet placed knowledge as an important and structuring element in this relationship, as Brousseau did.

According to this author, the didactic relationship is established when the relationship between two is transformed into a relationship between three: the teacher, the student and the knowledge that the student is expected to learn. Knowledge, in turn, enters the classroom at the moment when the teacher organizes the environment (milieu), creating a didactic scenario permeated by situations that make knowledge work and promote its evolution in the didactic context. (Brousseau, 1986)

The didactic situation is marked by the didactic intention, which characterizes the relationship between the human elements (teacher and student), based on the management of knowledge, constituted by a tripod that Brousseau called "triangles of didactic situations". (Brito Lima, 2006)

The entry of knowledge, as one of the poles of this triad, meant an advance in the studies of Psychology and Pedagogy in the second half of the twentieth century. The triangular conFiguretion allows us to analyse the relationship in pairs-teacher/student, teacher/knowledge, student/knowledge - as well as to look at the triangle as a whole, having, at the heart of this relationship, the didactic contact. Almeida (2016) proposes a scheme of the triangle of didactic situations that we consider quite complete, since it brings the conFiguretion of the milieu, in addition to other elements that cross it.

Figure 1.

The triangle of didactic situations and the milieu (middle)

According to this same author, the student's state of knowledge is intentionally modified, but not controlled, in its entirety, based on this structuring. It also reflects that, in the same way that the historical, cultural and social contexts interfere in the singularity of the teacher and the student, as subjects crossed by the individual and collective constructions of the society in which they participate, they also directly influence knowledge, from their scientific research base to the schools.

Directing a more particular look at each of the poles of the triangle of didactic situations, we will initially focus on the student. According to Pais (2019), it is necessary that the situation provides conditions for the student to assume the role of protagonist. This occurs above all in what Brousseau (1986) called didactic situations, that is, those organized by the teacher, as a space for the student's intellectual autonomy and, consequently, for his active participation in the situation. The moment when the student accepts the task as his/her own, when the teacher proposes a certain situation, characterizes what Brousseau called didactic feedback. Accepting the accomplishment of the task as being his/her responsibility makes the student get involved in his/her learning. Learning would no longer be under the teacher's control, as it was previously supposed. It would be up to him to organize the scene, based on the constitution of the milieu and the didactic choices established by him.

In view of this scenario, we corroborate Almeida (2016) and return to the reflections based on figure 1. There is a dependence, in the act of learning, which establishes the student's involvement with the structured didactic environment, with a view to the construction of knowledge. Therefore, the teacher is responsible for creating the situations and providing a scenario of student protagonism, based on the creation of strategies, appreciation of attempts and understanding of mistakes. It is in this sense that we must also look at being a teacher. According to Brousseau:

The teacher's work consists, then, in proposing to the student a learning situation so that he elaborates his knowledge as a personal response to a question, and makes it work or modifies it as a response to the demands of the environment and not to a teacher's desire. (Brousseau, 1996, p.55)

Therefore, the teacher's mission goes beyond executing the action of teaching in its most common sense (passing on knowledge). Their role involves gestures of donation and sharing. It is knowing how to share the protagonism of the scene and understanding that this sharing

should be one of the most important moments within the scenario of educating. This leads us to the idea that knowledge (mathematical, in the case of our specific interest) needs to be negotiated with the students, in which each teacher and student will play certain didactic roles, that teacher and student will know what they can expect from each other, and that there will be a division of responsibilities and clauses to be fulfilled by both partners in the relationship. This set of elements and characteristics, listed here, Brousseau called the didactic contract.

To sum up, the didactic contract is characterized by clauses/rules, mostly implicit, which will be negotiated by the teacher with his students, and which will determine which roles will be assumed and which obligations must be fulfilled by the partners in the relationship, one in relation to the other and both in relation to knowledge, and which will be constantly negotiated, broken and renegotiated.

Thus, for Brousseau (1996), the didactic contract is "the rule of the game and the strategy of the didactic situation". However, this researcher points out that it is not something closed and immutable, that is, the contract changes according to the paths taken in the didactic game.

Jonnaert (1994) proposes a synthesis and adds other elements to the understanding of what the didactic contract is: in addition to the idea of division of responsibilities and the consideration of the implicit; it is necessary to consider the dimension of the relationship with knowledge. In this respect, it advocates that the contract must take account of the asymmetry of the relationship with the knowledge at stake. As the situation progresses, this asymmetry will tend to be reduced to the point where the student is as close to knowledge as the teacher himself (Brito Lima,2006).

In this regard, Eloi & Andrade (2020) discuss that sometimes the teacher is in a more direct relationship with knowledge, sometimes he moves away, so that the student assumes this role, as illustrated in Figure 2.

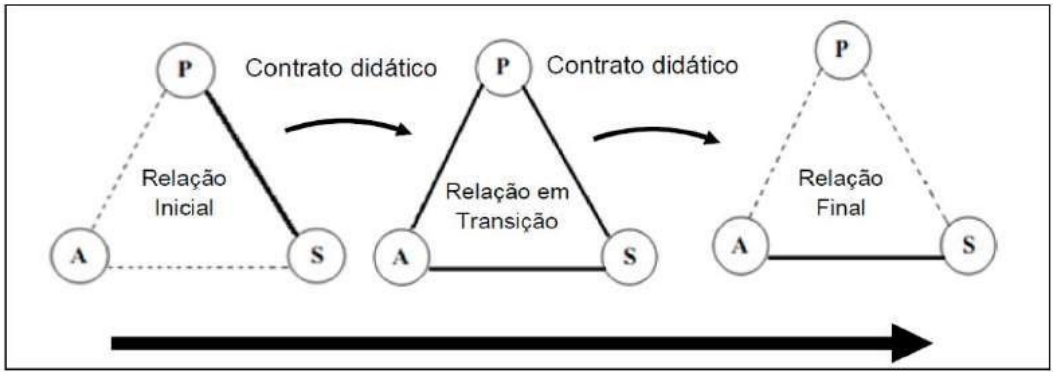

Figure 2.

Representation of asymmetry in relation to knowledge

Thus, initially, the student does not have such an adequate relationship with knowledge, while the teacher has a more adequate relationship. As negotiations, ruptures and renegotiations occur, these relationships change and the distance and inadequacies between student and knowledge tend to decrease. Jonnart & Borght (2002) state that this modification function is related to the type of Didactic Contract established.

The ideas defended by these authors reveal a kind of dialectic of the didactic contract, that is: the didactic contract is not static; it is always in motion, configuring and reconfiguring itself. It is not, either, perennial; on the contrary, it grows old and becomes obsolete, requiring other contracts to be established. The didactic contract is not intended to be rigid; it breaks when it no longer makes sense in the situation or when one of the parties (especially the student) does not accept it. Brousseau (1983) goes even further and reveals that it is in the denial of the contract and its rupture that the great leaps of learning take place. Thus, the good contract is not the one that is not broken, but the one that can intentionally be broken.

We can reflect, in line with what Brousseau proposed, and seeking to advance with our own speculations, that the more fragile the position of a teacher is, in the face of knowledge, the more he feels secure by establishing a rigid contract that he believes to be immutable. Allowing the didactic contract to be dialectical, flexible, fluid, requires from the teacher a mastery and management of knowledge and the classroom, which allow him to organize and reorganize situations, accept and encourage contractual ruptures, put in check previously established expectations.

We understand that when Brousseau (1983) states that there are effects of contract (effects that he discussed in some of his important writings, but which we will not dwell on in detail in this article), such effects often appear as a function of the professor's attempt that the didactic situation does not fail. For him, many times, the breach of contract-contrary to what Brousseau proposed -would indicate the failure of the relationship. To sustain the unsustainable, the teacher produces effects and creates situations that can hinder students' learning.

This is the question that fundamentally interests us: we do not want to dwell on the didactic contract that works as it is desired, we want to understand what happens to the contract whose negotiated clauses can lead the student to error, can create an obstacle to his learning. That said, the question that led to this study was: is it possible that due to the negotiations and establishment of the didactic contract, in the classroom, obstacles to learning are produced?

For this, we searched the literature for a mathematical field whose errors and didactic obstacles seem more evident, and we chose Probability, which, by working with chance and

uncertainty, breaks with one of the most common representations that are held among teachers and students: that mathematics is precise, exact, that there is no place in this field for the uncertain.

The following topic will deal with the discussion about didactic obstacles, the other central theoretical framework for our study.

Didactic obstacles in the teaching of Mathematics

As already discussed, the student brings with him a whole representation of the set of experiences and conceptions belonging to a history and culture. In this way, the didactic game will be crossed by these experiences, even if this factor is a source of possible learning difficulties. What can occur, as one of the consequences of these relationships, is the emergence of obstacles during this process. To understand the causes and effects of these errors in a didactic situation, we must first understand the conceptualization of learning. Therefore, we agree with Brousseau (1998, p.119), when he states that "learning is done by attempts at successive, temporary and relatively good concepts, which he (the student) would reject or transform into a true new genesis at a time".

In the same direction, Bachelard (1996) states that the learning process can be understood as the deconstruction of a poorly established previous knowledge, even if it has been successful in past contexts. Brousseau (1983) clarifies that the errors of ancient knowledge do not disappear instantly or completely, since it is a complex process and, even with the belief that it will be overcome, it tends to reappear, depending on the circumstances.

However, it is not coherent to imagine that these errors and difficulties intrinsic to the process learning systems are instituted only from ignorance or uncertainty, but one must think of knowledge that previously had some meaning, but in other situations is unadaptable, hindering the evolution in the process of knowledge construction, which Brousseau (1983) called an obstacle.

Brousseau (1983) was responsible for introducing the notion of obstacles in the teaching of Mathematics. Although inspired by Bachelard, he contradicted some of his notes on this theme in Mathematics and particularly highlighted the idea of a didactic obstacle. We can highlight three different origins for the emergence of an obstacle (Brito Lima, 2006): ontogenetic, epistemological and didactic, the latter being of interest to our research.

Obstacles of ontogenetic origin are those that arise from the limitations of the subject in his cognitive development (neurophysiological, among others). Obstacles of epistemological origin, in turn, are inherent to the epistemological nature of knowledge and can be identified in the history of the evolution of a concept, based on some rupture or radical change. Obstacles of

didactic origin, on the other hand, arise from the need to observe more carefully the consequences, within the teaching and learning processes, of the teacher's choices. Each teacher chooses strategies, mechanisms, methods and resources that he or she believes are the most appropriate for his or her class. When reflecting on these issues, Gomes (2002) points out:

> This obstacle becomes evident to the extent that the teacher transmits knowledge as being dogmatic, making it impossible to question, to discuss ideas, to elaborate hypotheses, since, being dogmatic, it is seen as the only and absolute truth. Even unconsciously, the teacher (who also had the same training) reproduces this teaching, since for him that is the necessary and true knowledge, presenting itself in a very calm and easily acceptable way, and therefore arguments, inquiries or questions are unnecessary. (Gomes,2002, p.7)

In this sense, we understand that the identification of obstacles is one of the greatest challenges to overcome them. With this in mind, Brousseau (1988), based on the idea that epistemological obstacles can be studied at the historical level and in educational practices (Bachelard,1996), develops a research method based on three stages: I) finding systematic errors and conceptions around which these errors are grouped; Il) to find obstacles in the history of mathematics; III) confront historical obstacles with obstacles in learning.

In the study we propose, we seek to consider the above elements, pondered by Brousseau. Finally, we seek to consider two important elements about which Brousseau theorized: the didactic contract and the didactic obstacles, but which we do not find in the literature of this author or of those who develop research anchored in their reference, any study that establishes a relationship between them. The result of our efforts is presented in this article, but before moving on to the results, we will discuss, albeit briefly, the field of knowledge chosen by us, for the reasons we have already mentioned.

Probability and its historical, curricular and didactic perspectives

O Probabilistic knowledge, which has chance as its object of study, presents several important views and applications to be investigated from an epistemological and didactic point of view. Understanding chance is the first step to delving into the world of uncertainty. Viali (2008, p.144) defines chance as "a set of forces, in general, not determined or controlled, which individually or collectively play a preponderant role in the occurrence of different results of an experiment or phenomenon".

According to Laplace (2010), it is a paradox to question "what is Probability?", because there are few things in our knowledge that we can know for sure, most of them are only probable, even in the mathematical sciences. It has not been a short time since society has been researching

uncertainty, because the idea of chance is very old, being understood from the mystical and divine dimension.

These studies and revolutions in the history of probability led to the identification of four approaches: classical (Lapacian), geometric, frequentist and Baysian (subjective). However, it was the French mathematician Pierre-Simon Laplace (1749-1827) who founded the most widely used definition today when teaching Probability in Basic Education, based on the classical approach.

The theory of chance consists in reducing all events of the same kind to a certain number of equally possible cases, so that we are equally undecided about their existence, and in determining the number of cases favourable to the event whose probability is desired. The relation between this number and that of all possible cases is the measure of this probability, which thus corresponds to a fraction whose numerator is the number of favourable cases, and the denominator is the number of all possible cases. (aplace,2010, p.46)

Understanding chance as the object of study of Probability, we consider here the idea of epistemological rupture, addressed by Vergnaud & Cortes (1986). A rupture of this nature is considered the passage from one dimension of mathematical thought to another, in which it is necessary to reformulate conceptions and appropriate new mathematical objects. Although, this concept is widely studied in the transition between Arithmetic and Algebra, we appropriate it in order to consider the rupture that exists in the immersion in probabilistic knowledge, since, as a

we are conditioned since the first years of Basic Education to treat Mathematics as a field of accuracy, Probability, which is one of the mathematical fields, is on the reverse: uncertainty.

Probability was inserted in the Basic Education curriculum in the block called Information Treatment, after the promulgation of the National Curriculum Parameters (PCN) (Brazil, 1997, 1998). The didactic guidelines addressed in this document allow the student to understand that most of the events that occur in everyday life are of a random nature, and that there is the possibility of identifying possible results and making estimates about them.

In addition to this idea, the PCN + state that Probability "should be seen as a set of ideas and procedures that allow the application of Mathematics in real-world issues, quantify and interpret a set of data or information that cannot be quantified directly or exactly" (Brazil, 2002, p. 126).

In the National Common Curricular Base, uncertainty and data processing are objects of study in the thematic unit Probability and Statistics. The BNCC (Brazil, 2018) suggests the development of skills in the instances of daily life, science, and technology. Regarding Probability, the afore mentioned document, in the guidelines for Elementary School (EF) - Early

Years (1st to 5th grade) - proposes the understanding of the existence of non-deterministic events, with the development of the notion of randomness and the beginning of the construction of a sample space. In the Final Years (6th to 9th grade), in turn, the study must be expanded and deepened, in order to carry out experiments and simulations that confront results and appropriate the enumeration capacity of the sample space.

For High School, the BNCC (Brazil, 2018) lists five specific competencies (SC) of Mathematics and its technologies, namely: use strategies, concepts and mathematical procedures for interpretation in the most varied contexts (CE1); articulate mathematical knowledge for research and decision-making (CP2); interpret, build models and solve problems in the specific fields of Mathematics (CE3); develop mathematical reasoning from the understanding and use of flexibility and fluidity (CE4); investigate and conjecture different concepts and mathematical properties (CE5). For each specific competency, the BNCC (Brazil, 2018) establishes some skills, and among them, three stand out, in which Probability is mentioned.

However, to understand how Probability is inserted in Mathematics classes, it is necessary to transcend the reading of official documents, it is necessary to go to the reality of the teaching and learning processes of this knowledge on the classroom floor. Despite the relevance from probabilistic knowledge to the process of educational, social and critical construction, Rufino and Silva (2018) point out two aspects that should be reflected on regarding possible difficulties for the teaching of Probability: for most teachers, Probability was not presented as an object to be taught; one can identify a lack of the "probabilistic way of thinking", because Mathematics is still directly related to a traditionalism based on determinism and accuracy, which limits the view of Probability. Figueiredo, Lima and Bianchini (2023), in the same direction, propose that it is necessary to reflect on cognitive competencies and teaching situations that favour the development of probabilistic thinking.

On the first point, we can turn to the study by Pietropaolo, Silva & Campos (2015). This research, carried out with 23 teachers who worked in the Final Years of Elementary School, found that there is an inadvertence in the teaching of Probability, when compared to other topics. In an attempt to justify it, the teachers stated that there is a difficulty in dealing with the didactic time. In interviews with these teachers, these authors identified that most of the teachers participating in this study conceptualize probability directly from fractional representation.

This points, in our understanding, directly to the didactic contract. From the point of view of the contractual rules for the teaching of mathematics, there is a culture of mathematics as an "exact" field, whatever the mathematical contents contemplated. In addition, another rule that is

common in classes is that the answer to a problem should always be a precise number (Brito Lima, 2006), as if in mathematics there was no place for uncertainty.

These are some elements that invite us to investigate the contractual elements and their relationship with the production of didactic obstacles.

Data construction and recording

In this section, we present the methodological paths traced in the Master's research. The research was carried out in a public school in the municipality of São Lourenco da Mata, located in the state of Pernambuco, which we call Instituicão Alfa (IA), which serves the Final Years of Elementary School (6th to 9th grade), also with the Youth and Adut Education (EJA) modality. The school operates in three daily shifts and has 14 (fourteen) classes, totalling 376 (three hundred and seventy-six) students, distributed in four classes from 6th grade to 8th grade, classes A and B in the morning shift and C and D in the afternoon shift; two classes of the 9th grade, with class A working in the morning and class B in the afternoon.

The choice to carry out the research in the Elementary School stage was due to the need to follow the expansion and deepening of concepts in the field of Probability, as indicated by the organization proposed by the BNCC (Brazil, 2018). In addition, this grade was chosen because the students have already attended all years of Elementary School, and that it is a decisive year, since it marks the end of a long school stage, as well as the preparation for immersion in a new stage, High School.

Two teachers with a degree in Mathematics who teach the subject of Mathematics for the Final Years of Elementary School at Instituicão Alfa participated in the study. One of them teaches in the morning shift and the other in the afternoon shift.

As an instrument for the construction of data, we opted for a semi-structured interview, based on a script, but which allowed a freer approach to the proposed theme, and the observation of classes. To record the constructed data, an audio and video recorder were used.

Three stages of the research were contemplated. The first stage of a theoretical nature was characterized by the mapping of possible didactic obstacles in the teaching of Probability, based on the literature review carried out with studies in this field. Dissertations, theses, articles and works in events were mapped in the period between 2005 and 2024.From the identified studies, a priori categories were established about the possible obstacles to be identified, related to the learning of Probability. This review was based on the digital repositories of theses and dissertations of Brazilian universities, the scientific productions of Graduate Programs in Science Teaching and the works published in nationally recognized events and journals.

The second stage consisted of conducting interviews with the participating teachers. The interview questions were divided into three parts: professional and academic profile; mathematical knowledge about Probability; therefore, three questions guided this interview: for you, what is Probability? What are the main concepts of Probability? How do you organize and work Probability in the classroom?

The first participating teacher, whom we named Lucca, has 30 years of experience as a Mathematics teacher in Basic Education, Elementary School and High School. When asked what probability means to him, he replied: "it's the attempt of something". He complemented by citing the roll of a six-sided dice as an example. Regarding the second question, he stated that the most important thing in Probability is the "question", justifying it by stating that "So the student will have to know how to interpret the question to answer".

Regarding his teaching methodology (third question), he replied "First we research, is that right? These searches are through the internet, I go to YouTube and look for it see the best possibility of passing this on to the students. We must see the student's level to get where we want".

The second participating teacher, who we named Maria, has 1 year of experience, exclusively as an active teacher in Elementary School. From the interview, the professor tells us that probability can be defined as the chance that something must happen and the most relevant concept in this field is the sample space. For the third question, the teacher answered that she applies the concept and takes a game to her class, she added by stating: "I always try to bring something visual to them at first, so that they can visualize it first before I apply the concept. And let's suppose an example, I bring a box of balls that has ten balls and has different colours in it, and then we get to work, right? This question of how many the balls there are, how would it be, right? How many if I have three yellow balls, for example, out of ten, what is the probability of coming out?"

The third stage was the observation of the classes of the participating teachers in their classes of the 9th grade of Elementary School (class A and class B). It should be noted that the results presented in this article will refer exclusively to the third stage of the research, since it was in this stage that the elements of the contractual relationship and the production of didactic obstacles could be evidenced.

In addition, in addition to analysing all the observed classes, we deepened the investigation of two episodes which we call "the paper dice throw" and the "kick" game, which occurred in the classes of Professor Lucca and Professor Maria, respectively. The analysis of the episodes will point out in more detail the contractual elements and the didactic obstacles in

the didactic situations highlighted, and it is possible to establish the relationships in the sense of the objective outlined for this study.

When referring to the didactic situation, we agree with Brousseau (2008), who defines it as a context intentionally planned by the teacher, involving the interaction between student, teacher and educational system.

Criteria for the analysis and discussion of constructed data

Based on the theoretical assumptions included in the present research about the notion of didactic contract described by Brousseau (1983), we define the following categories, which we call contractual elements (CE), for the analysis of the contractual relationship, which will occur during the observation stage:

Table 1.

Categories for analysis of the didactic contract

Categories I-Contractual Elements (CE)	*ECI-Expectations*	The waiting among human elements of the contractual relation of certain behavior of the teacher or the student when the knowledge is at play.
	EC2-Negotiations	The construction of functions and the set of destined rules to the roles of teacher or student into knowledge at stake. Negotiations can be done one by one (unilateral) or more people (joint they can be carried out explicitly or, above all, implicitly.
	*EC*3- Rules	Explicit or implicit agreements between the teacher and the student in the face of a didactic situation based on the relations to knowledge.
	*EC*4- Ruptures	Breaking the rules by the human elements of contractual relationship, breaking expectations and opening space for a new negotiation.
	*EC*5- Renegotiations	Construction of new rules from ruptures. The negotiation must be carried out from a new perspective created under the events that had occurred.
	*EC*6- Effects	Consequences of the events of the contractual relationship established between the teacher, the student and knowledge, with the objective of the situation does not fail.

The first stage of the research made it possible, as already mentioned, to create categories for the analysis of possible didactic obstacles that could emerge in the Observed Probability class. Based on this mapping, considering in-depth research in the literature, we established the following a priori categories, called obstacle factors (FO), presented in the table below.

It should be noted that, although we are talking about didactic obstacles, the pole of knowledge is intrinsically linked to the didactic dimension, including according to the Didactics of Mathematics, the theoretical field in which we are anchored. Thus, some factors here called obstacles are very close to the dimension of knowledge (for example, FO1), establishing a tenuous limit, in our understanding, between the epistemological and the didactic.

Table 2.

Categories for the analysis of didactic obstacles

Categories Il-Factors Obstaclers (FO)	*FO*1- Determinism	Related to the belief in exactness and the determinism on Mathematics an in all the fields.
	*FO*2- Reductionism	Application of didactic resources and contexts which don't explore, in their entirety, the real probabilistic applications.
	*FO*3- Uniqueness of approach	Exploration of only one probabilistic approach to the conceptualization and problem solving.
	*FO*4- Curricular isolation	Probability approach decontextualized and dissociated from other areas of knowledge.
	*FO*5- Illusion of equiprobability	Perspective that sees any and all probabilistic problem to equality of the probability of all its sample points.

Analysis of the contractual relationship and didactic obstacles in the management of knowledge probability

This section will address the analyses carried out from the observation of the class of the participating teachers I and IV, who were named Lucca and Maria, respectively. The first classes that were observed were conducted by teacher Lucca, who allocated two classes to contemplate the content of Probability for the 9th grade class A.

-Teacher Lucca

Upon arriving in the classroom, Lucca wrote the date and his name on the board and reminded the students of the subject of the class (Probability), as if the new content had already been combined in the previous class. Promptly, he stated that the content would be easy and exemplified the probability with the weather forecast, as can be seen in the excerpt below.

Teacher Lucca: Today we are going to see the probability. It's an easy subject. I'm telling you that it's easy because it's easy. Sometimes, when you are listening to the reporter, there is someone who says "the probability of..." Isn't that true? The people from the interior say: "boy, the gang wants to know more than God!! You don't know, no!!" But I'll explain. But the girl (referring to the reporter) doesn't say it's going to rain, she says: "The probability of rain tomorrow is very high" because she... She uses the machines. So, let's go. If you want, you can copy it, please.

Figure 3.

Excerpt 1 from Teacher Lucca's class

From this speech by Lucca, we can verify propositions that we consider relevant. First, Professor Lucca believes in the "ease" of the Probability content and, considering his statement: "I'm telling you that it's easy because it's easy", the teacher seems to need students

to also believe in this premise, which can generate an expectation in both, with regard to the dimension (it is worth remembering that expectation is one of the elements that make up the didactic contract). After this moment, a short dialogue leads us to a reflection that we consider equally relevant:

Student A: Teacher, does this content have a lot of calculation? **Teacher Lucca:** No. **Student A:** Then, it will be fine.

Figure 4.

Excerpt 2 from Teacher Lucca's class

From this excerpt, it is worth analysing some aspects and their influences: 1. The teacher states that Probability does not have much calculation; 2. The student believes that he will be better because he does not have much calculation (would calculation make learning difficult?);3. Expectations in relation to Mathematics. In other words, we have here a rule of the didactic contract established between Professor Lucca and his class: not having "too much calculation" in the content to be taught is a good thing. Beyond this contractual rule, from the answer attributed to the student, a certain relief seems evident, because there is already an expectation related to Mathematics itself: mathematics means calculating.

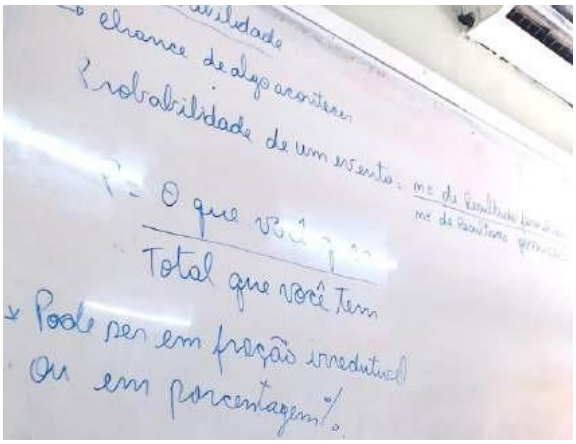

Figure 5.

Record 1 of Teacher Lucca's class

Starting his explanation from what he had recorded on the board, Lucca proposes to the class two alternatives to present the answer to a problem that involves Probability (irreducible fraction and percentage).During his explanation, he leads the students to follow a contractual rule, which is to present the answer as he wants it to be done, as can be seen in the excerpt below.

Professor Lucca: if I ask, if I ask (emphasizes in a louder tone), in the exercise I want in percentage, then you will have to find the percentage.

Figure 6.

Excerpt 3 from teacher Lucca's class

Lucca establishes a negotiation with the students, putting the answer in percentage of a possible Probability question in the background, being acceptable only when it is requested by him to do so (present the answer in percentage terms). This explicit rule of the contract will directly influence the understanding of probabilistic concepts.

After his verbal explanation, Lucca addressed some issues that involved coin toss, flipping a six-sided dice, drawing a number from a roulette wheel (odd and even numbers, multiples and primes). One of the questions asked for the probability of falling face 2 when rolling a dice. Then Lucca throws the dice upwards, expecting the object to fall with the face representing the digit 2 (two) facing up. He makes three attempts and in none of them what he expected occurs. Promptly, he argues "this data is flawed", adding that it is not easy to happen to the desired situation. It is perceived that Lucca's expectation is broken, so he gives an answer that escapes the mathematical dimension, but that mitigates his frustration, due to the rupture of his Expectation. The teacher then resorts again to the classical formula and arrives at 1/6 (one sixth) as a result, affirming to the students the ease of that path.

The emphasis on the ease of the formula and the preference for conducting the whole class by the mathematical application of the Probability formula, from the classical approach, demarcate indications that represent the determinism category (FO1), since their actions, their speeches, the examples and the issues addressed, can lead the student only to the path of accuracy, making it difficult, for example, the determination of the degree of predictability of random experiments.

In addition, the situations contextualized by Lucca in the questions presented point in the direction of Reductionism (FO2), since the activities proposed and worked on are elective events that, despite being the most addressed in textbooks, can hinder probabilistic applications to the real contexts of life. In addition, the contexts used did not contemplate an interdisciplinary approach, which directs the situation to the categorization in Curricular Isolation (FO4), as it tends not to allow its understanding from applications in other areas such as Genetics, Economics and Meteorology and is isolated only in the mathematical fields/contents.

We want to highlight a point that fundamentally refers to the idea of didactic contract. By repeatedly using the words "easy", "ease", the teacher seems to have the expectation that students will be more engaged in the activity if they are "easy", which seems not to be very

common in mathematics, because there are many calculations (as evidenced in the dialogue with the student).

Another proposed question asked for the probability of rolling an even number when spinning a roulette wheel numbered from 1 to 20. By correcting this question coercively, Lucca writes all the natural numerals from 1 to 20 on the board, and when solving the question by applying the classical formula, he simplifies the fraction found and argues that the question "shows right away" the possibility of "cutting the zeros".

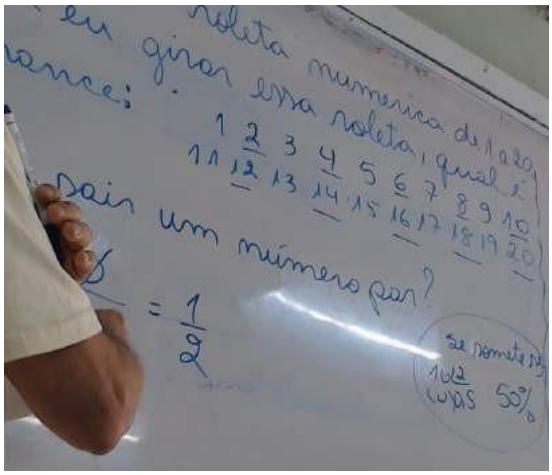

Figure 7.

Record 3 of teacher Lucca's class

This practice of Lucca corresponds to the **expectation** that students already knowhow to "cut the zeros" from the fraction and, consequently, also understand the meaning of this action. Again, he performs the division to arrive at the percentage terms and emphasizes the agreement: "write as if it were a percentage". Still from this moment, it is considered that Lucca's didactic choice, about the non-explanation and exploration of the sample space, since he writes all the possibilities (natural numerals from 1 to 20) and does not talk about what that set is, it is a situation that points in the direction of possible errors associated with the meanings attributed to the predictions.

The **non-description of the sample space** can be considered a factor that can constitute a didactic obstacle (F06), a factor that is not found in the mapping carried out, but which we believe can interfere in the understanding of probabilistic concepts, since the solution of a probabilistic problem begins with the analysis of possibilities, that is, it is from this recognition that it is possible to quantify and estimate values (Cavalcante, 2021).

Another recurring fact in Lucca's class is the exploration of probabilistic experiments that are only equiprobable. This fact refers to the category **illusion of equiprobability** (FO5), which can be developed, in addition to the unique approach of equiprobable contexts, due to

the lack of analysis of the sample space, the misunderstanding of the concept of randomness and the work on event dependence.

During the correction of the exercises, Lucca chose to use only the classical Probability. We identified that this situation represents the category of **uniqueness of approach (FO3)** and, as stated by Corrêa (2010), this idea does not favor overcoming the dichotomous logic of yes/no,and can constitute an element that hinders the understanding of what uncertainty is, the field where Probability operates.

Teacher Maria

Teacher Maria arrived to start her class; however, some students were still outside the room. She, then, invited them into the room and greeted them. Upon entering the room, he asked the students to organize their desks in rows and explained to them that the content to be worked on in that class would be Probability.

Looking at her notes on her cell phone, the teacher began to write on the board and asked her students to copy them in their respective notebooks. When she realized that the room was still making noise and that some students had not yet opened the notebook to copy, Maria stressed that she was tired, promoting the following dialogue:

Teacher Maria: I'm tired today, see? TIRED! (*emphasizing in a louder tone*). Let's go, student B, let's go!
Student B: Miss, I can't even find the subject, let alone do it.
Teacher Maria: sad end
Student B: I found it now because you said it.
Teacher Maria: really?

Figure 8.

Excerpt 1 of teacher Maria's class

The dialogue above expresses an expectation of the teacher regarding the empathy of her students for their fatigue. This expectation does not necessarily refer to the didactic contract (since it has nothing to do with mathematical knowledge), but points to the engagement of the class in the class, by respecting their fatigue. For her, saying that being tired and emphasizing this adjective would make her students show a quiet behaviour in class. Even so, student B said he couldn't find where he should write in the notebook and then Maria replied by saying: "sad end", trying to end the dialogue and turning back to the blackboard.

A few minutes later, the teacher began her verbal explanation. Maria recalled again that the content to be seen in that class was Probability and before having directly assigned any definition, she argued that this content is seen in everyday life, mentioning the lottery as an example.

Teacher Maria: Oh, the content that we are going to see today is the content of Probability. Probability is a content that you see in your daily life, I will give you an example. The lottery, who here would like to match the lottery number? Who doesn't? How many millions! But the lottery involves probability, why? It's the chance of something happening, for example, you choose six numbers out of sixty, so there's a probability of those six numbers coming out, right? Each number in that one has a very small probability, so the Probability content is involved in the lottery.

Figure 9.

Excerpt 4 of teacher Maria's class

By citing the example of the lottery and her students commenting in a low tone or making expressions that they showed they remembered this context, she argued that the lottery, what the students naturally remembered, involves Probability. In a rhetorical question (she asked the question and answered it herself), she stated that Probability is the chance of

something happens, and, going back to the example, he said that each number of that has a "very small" probability of coming out, using the decrease to emphasize the low chances of hitting the numbers in lottery draws, "as in Mega da Virada", as one student recalled.

The students were focused on the class and seemed excited about the discussion. Professor Maria then attributed another example, this time related to betting on football games, which were routinely being carried out, from the creation and dissemination of new digital platforms for betting on games.

Teacher Maria: another example, who here has already bet on life? Gambling betting, for example.
Student C: I have!
Teacher Maria: For example, the Brasileirão games. The platforms, there's the Beta, I don't know what...that you go there and bet on the team.For example, Corintinhans versus Palmeiras, then you look at the table of the Brasileirão, you see probability. Inside what? Within goal balances. Look, if Corinthians is the leader of the Brasileirão and Palmeiras is in sixth place, who has the greater probability of winning?
Students: Corinthians! (most students respond)
Student C: Palmeiras (everyone is laughing)
Teacher Maria: the greatest probability is Corinthians.
Student B: they are ahead of the other.
Teacher Maria: yes, they are ahead. But can Palmeiras win?
Students: they can! (most students respond)

Figure 10.

Excerpt 5 of Teacher Maria's class

During the dialogue, Teacher Maria says, based on her example, that it is possible to look at the table formed by the goal difference of the teams in some championship and "see the probability". She complemented her speech by exemplifying a game Corinthians versus

Palmeiras, stating that the first team is the leader of the Brasileirão (Brazilian football championship) and the other team is in sixth place. Then, only from this information did she ask which team the students would bet on.

Most students answered "Corinthians", based on the information given by the teacher. A student, however, when wanting to show his support for the other team, replied: "Palmeiras". Having noticed that her stud ents understood the idea of Probability as the measure of chance, Maria agreed with her students' answer, questioning "can it happen that Palmeiras wins?", and the majority again answered yes.

Although Maria's statement referred to goal balances, the teacher's didactic choice, by stating that it is possible to see Probability in the action of looking at the table, can lead the student to the interpretation that there is no need for a more in-depth analysis to make predictions and make decisions based on probabilistic concepts in other contexts. Therefore, this situation can lead to a FO2-Reductionism.

The dialogues established between teacher Maria and her students seemed to be fluid and constant in her classes, that is, the students accepted the responsibility of answering the questions asked by teacher Maria, in a kind of negotiation. In addition, Maria led the class based on her questions, building ideas and starting from her examples to, together with her students, present definitions, which generated great expectation in the answers attributed by the students, and from them, the class planning was reorganized or not.

At other times, the teacher repeated the same habit: always reproducing the students' speeches and her own, to conclude what was being said. This validation of the teacher seemed to be expected by the students and happened all the time. If she repeated it, then the assigned answer was correct, as another implicit rule of the contract.

To define the idea of equiprobability in a sample space, Professor Maria returned to the example of soccer games. To this end, she stated that some aspects arranged for both teams were equal and, therefore, they had the same chances in a match. He emphasized the equality of the number of players in both teams and the uniqueness of the instrument of the game (the ball).

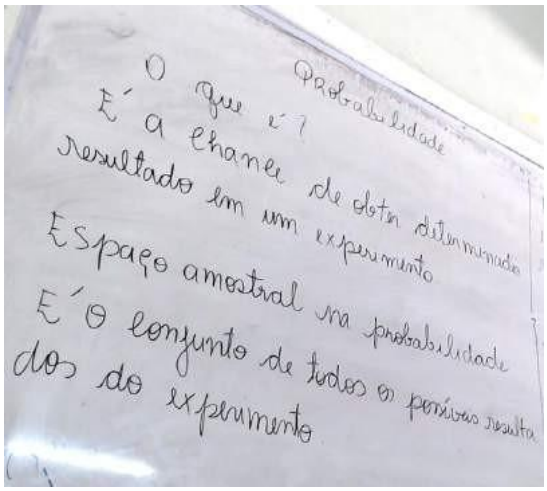

Figure 11.

Record 1 of teacher Maria's class

After having made the discussion in the room, about the probabilistic concepts, Maria continued for the next step of the lesson: the realization of a betting game, involving the launch of two dice. According to her, the objective of the game was to "better fix" the explanations given by her during class. To find out if the students had already played a similar game, the teacher asked the class if everyone had already played Adedonha or Stop. Then, a student asks if it is the one that involves names, that is, a game that involves a draw of a letter of the alphabet and all participants must write, in the shortest possible time, words that relate to the categories already established at the beginning of the game (for example: name, place, object, etc.),which starts with the letter drawn. After everyone writes their words or a participant who has already completed the game says "stop" out loud, and the round ends, the score is made as follows: if the participant did not write any words in that category, he earns zero points; if the participant wrote a word also written by another participant(s), he gets five points; If the participant has written a word that no opponent has used, he gets ten points. The teacher says it's another game, but it has that name too. In this case, it is not words, but numerical guesses, based on the sum obtained from the faces of the dice, after the simultaneous roll of two six-sided dice, in which each guess has a score, ranging from zero to three.

The invitation to play the game was made as can be seen in the dialogue presented above: "what are we going to do?". The emphasis given to the term "us" refers to the idea of

everyone's participation, including the teacher Maria, by inviting her students to the game, may include herself in the action, creating the expectation of a dynamic and participatory game.

The teacher led the students, informing them where and how the game table should be written; she asked everyone to write individually in their respective notebooks and use a pen, which was the condition for the realization of the game. As for the rules written on the blackboard, she stated that it was not necessary to write in the notebook, since he had written on the blackboard only to visualize the rules. The rules of the game, shaped by the terms established by the teacher, constitute a negotiation between Maria and her students, that is, it would only be possible to play if each student committed to: writing the table in the notebook, using pens, following the rules of the game.

For the award, Maria withdrew four chocolates from the bag, leaving this decision (the distribution of the number of chocolates to the winner(s)) with the class. Then, she asked the class if all the chocolates would go to whoever got the most points or if they would make another type of distribution, such as, for example, two chocolates for those who came in first place and two chocolates for those who came in second. This attitude of the teacher, based on the ideas suggested by her and by some students in the class, demonstrated a negotiation made in class in relation to the award of the game. Therefore, most students chose to be from the following Shape: two chocolates for first place, one chocolate for second place and one chocolate for third place.

The students were making the guesses and making the moves. Maria chose, due to the management of class time, to perform five rounds. After the game ended, he copied a table on the board that expressed all possible sums by rolling two dice. Then, he asked the students which numbers appeared the most and went around to count together.

Episode analysis: "the roll of a paper die"

The first episode deals with an event in Professor Lucca's class: the throw of a paper dice and the expectation of the exit of face two falling face up. This event began with the discussion of a question proposed in his class (ifl roll a dice, what is the probability of face2 coming out?).In an attempt to demonstrate the experiment, Lucca took a piece of paper data

and began to make entries,making some weightings in each of them, as can be seen in the excerpt below.

Teacher Lucca: We have the number 1, 2, 3, 4, 5 and 6. But he wants it to get only the number 2. If you take this one and throw it like this...
-we got 5, did not get 2 (first throw);
-Let's see now, 6 (second throw);
-6 Again, boy, this dice is rigged (third roll).

Figure 12.

Clip 1 of episode 1

The conclusion established by Lucca after making three rolls (the dice is flawed) may point in the direction of some misunderstandings in the understanding of probabilistic concepts. The first point concerns an obstacle factor already observed in Professor Lucca's class: he chooses to use a probabilistic approach, not going through the frequentist bias. However, the methodological paths of his class lead directly to the application of the classical approach (number of favorable cases divided by the number of possible cases). This fact is reinforced when Lucca goes to the board to apply the formula, after not occurring what was expected in the throws made, as observed in the excerpt below.

Teacher Lucca: So, notice that it's not easy, then it happens... See how easy it is (if you go back to the board and start writing the fraction corresponding to the classical probability) the possibilities, the probability is as follows. He only wants the number 2; he only has 1. The number 2 only has 1, and the quantity is 6. So, it's going to end here, oh, one-sixth (completes writing the fraction on the board).

Figure 13.

Clip 2 of episode 1

From this situation and from the expectation that the students have in relation to the teacher, that is, to trust that his conclusions and statements are true, we realize that the understandings generated from Lucca's attitude are: it is only necessary to roll the dice three times, but if you do not obtain the desired result, the dice are flawed; there is no link between the performance of the experiment and the numerical response that can be found when applying it to the classical formula; It is not easy to carry out the experiment and wait for the desired event, it is easier to resort directly to the application in the classical formula.

In addition, it is inferred that, from the arguments used by the teacher, in order not to be considered difficult, the desired event should occur in the first throws, bringing to light a randomness explained much more from the idea of luck and/or bad luck. Therefore, Lucca's

didactic choice makes it difficult to understand probabilistic phenomena, as well as their interpretation.

One of the contractual elements that draws our particular attention in this episode deals with negotiation. When Lucca creates the expectation of the exit of face 2 (two), makes three rolls, infers conclusions (for example, the dice is flawed), returns to the board to resort to the classic formula and emphasizes, as in other moments of the class, the ease of this application, he negotiates clauses of the contractual relationship.

These attitudes of the teacher point in the direction of didactic obstacles and may work as an obstacle to the evolution of probabilistic thinking, since there was an emphasis on mathematical determinism and the exclusion of other forms of approach to probability. In this sense, it is observed that in this episode there is a direction for the emergence of didactic obstacles, since their idealization. In this way, the planned situation was not potentially good, since its planning.

Episode analysis: "the "guess" game"

The second episode deals with the use of the term "guessing" in the realization of the game that teacher Maria took to her class. At the beginning of the game, she stated to the students that this stage of the class had the purpose of "doing better", referring to the explanation of the content given in the previous minutes. Promptly, she began to copy the rules of the game on the board and established a board with three columns (round, guess and points) and eleven lines (referring to the nine rounds of the game, the titles and the total score), as can be seen in the following image.

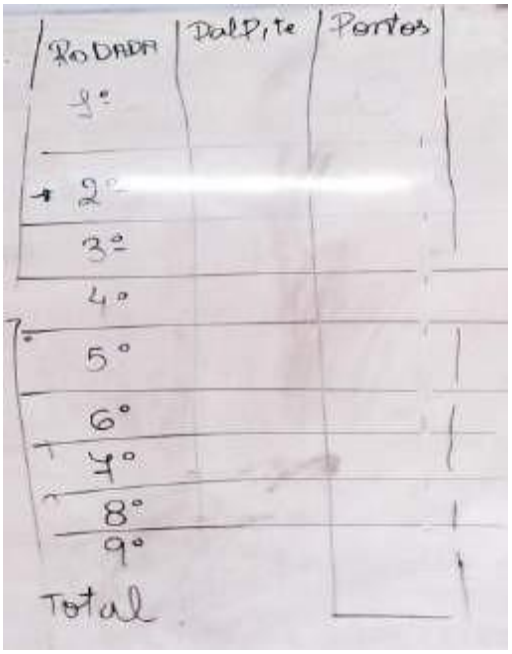

Figure 14.

Episode 2 Log 1

When the teacher went to explain the rules of the game, she used the term "kick" in her explanation to designate the action that the students should perform in each round. Then, she added that they should "make a guess", then carrying out a test round, as shown below.

Teacher Maria: oh, look, what's the idea of the game? It's you trying... I'm going to roll the dice, the two dice, oh, pay attention, I'm going to roll this dice twice, right? Then here you will guess, you will make a guess. You're going to say, "Oh, I think the sum of the two dice is going to be 6." Ah, I rolled the dice...tell me.
Student C: 6.
Student B: 10.
Student D: 9.
Student A: 2.
Teacher Maria: Calm down! Oh, 9, he guessed 9, then I'll rotate here (performs the rolls of the two dice). I spun, then it dropped 2 and 1, how much did we get?
Students: 3. (*some students respond*)
Teacher Maria: 3! Two and one makes 3!

Figure 15.

Clip 1 of episode 1

The use of the expression "guess", as we can see in the excerpts "here you will guess" or "Take a guess", does not enhance the creation of ideas that provide reflection on game strategies, in the dimension of probabilistic thinking. This fact is because the term "guess", in the colloquial sense, refers to the attempt to attribute an answer, without having knowledge of the subject, pointing to the trial-and-error strategy.

This fact can lead to the emergence of didactic obstacles, which may be related to the way the teacher conducted the class. It is important to understand the existence of a contractual rule already established in the educational context, which concerns the relevance of attending to the teacher's commands. Students, generally, when they believe in the veracity of the information brought by their teacher, follow the direction determined by him. Therefore, if the teacher said that she should guess, that's what they will do, associating it with what they commonly known to be a guess.

Therefore, the fact that, throughout the game, the students guessed the numbers referring to the sum of the data points, demonstrates that the way Maria negotiated the situation does not induce reflection based on probabilistic thinking. In addition, at the end of the rounds, Maria drew a table that indicated all the possible sums in the simultaneous rolls of two dice (see figure 16). From that moment on, Maria asked her students which number was most repeated within the board, making circles around those that were indicated by the class.

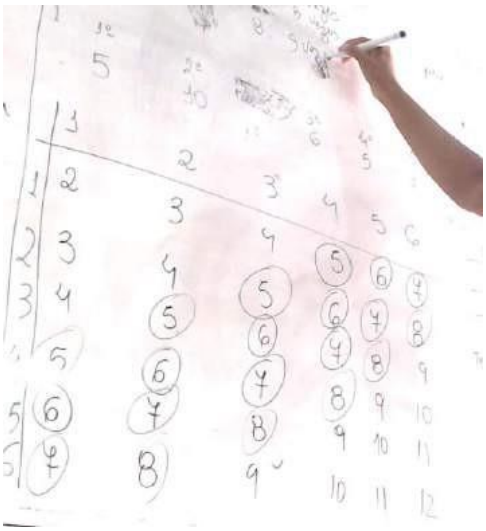

Figure 16.

Episode 2 Log 2

In addition to what we have mentioned, using the expression "random" as a form of luck or misfortune does not contribute to the development of probabilistic thinking and hurts the dimension of chance. The expression "guess" follows the same paths, since, when the student guesses an answer, this being the rule explained by the teacher for the execution of the game, she is not being instigated to reflect on, nor to associate the concepts contemplated in class, with what is being worked on in the game.

In this case, unlike episode 1, the didactic situation planned and experienced by Maria was potentially good, in relation to the development of probabilistic thinking, but the choices made by the teacher during its execution point to the possibility of the emergence of didactic obstacles. This occurs when the situation, despite presenting good planning and having the potential to contribute to the understanding of Probability, does not do so, due to the teachers' choices, when conducting the situation.

Final considerations

The relationships established between the phenomena that emerge in a didactic situation can explain, to a large extent, the origin of difficulties that are perpetuated for a long time, whether related to the teaching or learning process. The theoretical and methodological paths traced in this research were intended to deepen these reflections, not only with the intention of presenting the difficulties, but with the intention of pointing to the roots of the problems and, consequently, being able to think about their possible solutions.

The idea of a didactic contract and a didactic obstacle, despite being idealized by the same researcher, Guy Brousseau, were not proposed jointly in research of this nature. Thus,

the results found in the present study, in addition to presenting a new proposal for understanding the phenomena, in the interrelation of these two concepts, extend the views for future investigations.

The analyses carried out from the data constructed in this research reveal that there is a close relationship between the didactic contractual elements and the factors that can favour the emergence of didactic obstacles, particularly in the context of knowledge probability. Therefore, it is possible to think that some obstacles (especially when dealing with epistemological obstacles) can influence the negotiation of the didactic contract, but, above all, itis highlighted that negotiations may or may not enable the emergence of obstacles, which was more evident in our study.

The observations of the classes allowed us to understand that the phenomenon that we denounce the emergence of didactic obstacles occurs, above all, in the way the teacher negotiates the situation. That is, we can say that it was in the nuances of the element contractual "negotiation" that we found aspects more relevant to the emergence of didactic obstacles, at least in the classes that were observed.

In addition to what we have proposed, it is worth reflecting that other factors related to the proposition of a didactic situation can also explain the starting point of the emergence of these obstacles. In this research, for example, we were able to identify a situation that favoured the emergence of obstacles since its idealization (teacher Lucca); It is a situation that did not favour the emergence of obstacles, but that, due to the way the teacher negotiated the realization of the activity, factors that could favour didactic obstacles were identified.

Still in relation to the didactic obstacles, it is emphasized that the results of this research revealed a new factor that did not appear in the initial mapping. The non-description of the sample space in Professor Lucca's class. This hypothesis that we raise, to be considered as one of the factors, constitutes, in our understanding, one of the most important contributions of this work, since it points to new developments in relation to the process of teaching and learning probability.

Although our research is directed to the context of Basic Education, specifically to the last year of Elementary School, the discussions made indicate that it is necessary that we focus on the curriculum of initial training and on continuing education, which need to be spaces that lead teachers to reflect on the knowledge that will be taught and on their didactic choices.

Still on the particularities of this knowledge, our discussions bring a reaffirmation of what has been discovered by other research works, in the sense of understanding the way of

thinking about probability as one of the challenges in mathematics teaching. This aspect occurs since general mathematical thinking and probabilistic thinking, belonging to this larger field, have specificities that need to be understood and conceptions that need to be resignified for the learning of knowledge Probability: determinism and chance.

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