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The concept of rational numbers in secondary and higher education: a systematic review of the literatura in latin america

El concepto de números racionales en la educación secundaria y superior: una revisión sistemática de la literatura en américa latina

Le concept des nombres rationnels dans l'enseignement secondaire et supérieur : une revue systématique de la littérature en Amérique Latine

O conceito de números racionais nos ensinos médio e superior: uma revisão sistemática de literatura na américa latina

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Abstract

This article brings results from a doctoral research, focusing on teaching and learning of the concept of rational numbers, relevant both in the school context and in everyday living activities. The complexity of the subject has aroused the interest of several researchers over the years, considering that high school and college students still do not understand the concept, although it has been introduced in the early grades. The main objective of this research is to conduct a Systematic Review of the Latin American Literature in the last ten years, on the concept of rational numbers after its study in elementary school, that is, in high school and in higher education. Among the purposes of the research is the pursuit to identify which themes are present in the investigations, with the intention of knowing the nature of the issues perceived in the classroom while using the concept, as well as the gaps left and recent trends in the field of mathematics education research. By observing the stages of a Systematic Review of Literature, the analyzed corpus consists of 45 articles. With this analysis it was possible to identify 4 main themes: Ratio and Proportional Reasoning; Rational Number Operations;

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Rational Number Representations; and The Concept of Rational Numbers and their Interpretations.

Key words: Systematic review of literature, Rational numbers, Fractions, High school, Higher education.

Resumen

Este artículo trae resultados de una investigación doctoral, centrada en la enseñanza y el aprendizaje del concepto de números racionales, relevante tanto en el contexto escolar como en las actividades de la vida cotidiana. La complejidad del tema ha despertado el interés de varios investigadores a lo largo de los años, ya que los estudiantes de secundaria y universitarios aún no comprenden el concepto, aunque ya ha sido introducido en los grados iniciales. El principal objetivo de esta investigación es realizar una Revisión Sistemática de la Literatura de la producción latinoamericana de los últimos diez años, sobre el concepto de números racionales luego de su estudio en la escuela primaria, es decir, en la secundaria y en la educación superior. Entre los propósitos de la investigación está la búsqueda de identificar qué temas están presentes en las investigaciones, con la intención de conocer la naturaleza de las problemáticas percibidas en el aula al utilizar el concepto, así como los vacíos dejados y las tendencias recientes en el campo de la investigación en Educación Matemática. Observando las etapas de una Revisión Sistemática de la Literatura, el corpus analizado consta de 45 artículos. Con este análisis fue posible identificar 4 temas principales: Razonamiento Racional y Proporcional; Operaciones con Números Racionales; Representaciones de Números Racionales; y El concepto de números racionales y sus interpretaciones.

Palabras-chave: Revisión sistemática de la literatura, Números racionales, Fracciones, Escuela secundaria, Educación superior.

Résumé

Cet article apporte des résultats d’une recherche doctorale axée sur l’enseignement et l’apprentissage du concept des nombres rationnels, important tant au contexte scolaire que dans des activités quotidiennes. La complexité du thème a suscité l’intérêt de divers enquêteurs au cours des années, car des élèves poursuivent dans l’enseignement secondaire et supérieur sans comprendre la théorie, même si elle a été introduite dans les premières années de l’école. L’objectif principal de l’étude est de réaliser une Revue Systématique de Littérature de la production de la recherche à l’Amérique latine à propos du concept des nombres rationnels après son apprentissage à l’école élémentaire, c’est à dire, à l’enseignement secondaire et

supérieur, dans les 10 dernières années. Parmi les propos de l'enquête, on cite l'identification des thématiques présentes dans les récentes investigations. L'intention est de connaître la nature des problèmes aperçus en salle de cours lorsque le concept est utilisé, ainsi que les lacunes laissées et les tendances actuelles au champ de la recherche en éducation mathématique. En observant les étapes d'une Revue Systématique de Littérature, le corpus analysé comprend 45 études. Avec l'analyse des articles, 4 thématiques ont été identifiées: Raisonnement et Pensée Proportionnelle; Opérations avec les Nombres Rationnels; Représentations des Nombres Rationnels; et Le Concept de Nombres Rationnels et leurs Interprétations.

Mots-Clés : Revue systématique de littérature, Nombres rationnels, Fractions, Enseignement secondaire, Enseignement supérieur.

Resumo

Este artigo traz resultados de uma pesquisa de doutorado, sendo que o foco é o ensino e aprendizagem do conceito de números racionais, relevante tanto no contexto escolar quanto em atividades cotidianas. A complexidade do tema tem despertado interesse de diversas pesquisas ao longo dos anos, pois alunos do ensino médio e do ensino superior continuam sem compreender o conceito, embora esse tenha sido introduzido já nas séries iniciais. O objetivo principal da pesquisa é realizar uma Revisão Sistemática de Literatura da produção da América Latina nos últimos dez anos, acerca do conceito de números racionais após seu estudo no ensino fundamental, ou seja, no ensino médio e no ensino superior. Dentre as finalidades da pesquisa, está a busca por identificar quais temáticas estão presentes nas investigações, com a intenção de conhecer a natureza das questões percebidas em sala de aula ao utilizar o conceito, bem como as lacunas deixadas e as tendências recentes no campo da pesquisa em educação matemática. Observando as etapas de uma Revisão Sistemática de Literatura, o corpus analisado constituiu-se de 45 artigos. Com essa análise, foi possível identificar 4 temáticas principais: Razão e Pensamento Proporcional; Operações com Números Racionais; Representações dos Números Racionais; e Conceito de Números Racionais e suas Interpretações.

Palavras-chave: Revisão sistemática de literatura, Números racionais, Frações, Ensino médio, Ensino superior.

The concept of rational numbers is considered to be one of the most complex that children face in their early years of basic education (Charalambous & Pita-Pantazi, 2005).

Many obstacles in the teaching-learning process of this concept follow students throughout their school life and have a distinct nature, related to: i) the complexity of the concept; ii) the formal deductive thinking processes of Mathematics; iii) the teaching processes; and iv) the cognitive development and the affective and emotional attitude of students towards Mathematics (Dodera et al, 2014).

The complexity of the concept refers to: i) the multiple interpretations it takes on, depending on the context in which it is inserted: part-whole, measure, quotient, operator and ratio, - referred to by some authors as a "multifaceted concept" (Behr et al, 1983); ii) the multiple ideas associated with it: partitioning, unitizing, equivalence and order, notion of quantity, multiplicative notion and structures for adding and subtracting (Martinie, 2007; Ventura, 2013; Wheeldon, 2008); iii) the multiple representations (figural - continuous and discrete, symbolic - fractions, decimals and percentages, and natural language) (Paraol & Rodrigues, 2018). Thus, the concept of rational number can only be understood broadly, in the cognitive articulation of the various representations, interpretations and associated ideas (Martinie, 2007).

The second obstacle refers to the fact that students apply properties and procedures from natural numbers to make inferences about rational numbers, known as natural number bias (Van Hoof et al, 2014). Rational numbers comprise a new numerical field and, for this reason, "involve breaking with the ideas students have built up about natural numbers and, therefore, [their study] requires time and an appropriate approach" (Brasil, 1997, p. 67).

The third obstacle refers to the teaching-learning process, which usually takes place in a mechanized and procedural way in the early years. Mathematics teaching has failed to enable students to build "a concept of rational number that will allow them to use it later on. Operations with rational numbers are, at best, mechanized around a few basic rules that are often confused with each other" (Brolezzi, 1996, p. 1).

Finally, the fourth obstacle takes up a particular area of students in institutional contexts, related to feelings of tension, anxiety and fear of failure that generate blockages of affective origin and have repercussions on academic performance in Mathematics (Dodera et al., 2014).

Despite the great difficulties presented during the teaching-learning process of this concept, its importance can also be noted from different aspects: educational, mathematical, practical and psychological (Behr et al., 1983; Damico, 2007).

In terms of education, the importance of rational numbers is notorious in the proposed curricula for basic education. In our current curriculum, it is one of the cross-cutting topics in almost every year of elementary school (Brazil, 2017).

As for the Mathematical aspect, its importance is evidenced by: i) underpinning the development of proportional reasoning and, later, topics of advanced Mathematics, including Algebra, Geometry and Probability (Behr et al., 1983; Clarke & Roche, 2009); ii) allowing students to acquire a deeper and more mature understanding of the conceptual knowledge of the number system (Stelzer et al., 2016); iii) maintaining its essential character in problem solving; that is, many problems in Mathematics and other areas of knowledge (such as Engineering, Physics and Probability) are intrinsically related to ideas about rational numbers, 2016); iii) maintain its essential problem-solving character; that is, many problems in Mathematics and other areas of knowledge (such as Engineering, Physics, Chemistry and Biology) are intrinsically related to ideas about rational numbers and can provide an ideal context for the acquisition of new concepts (Behr et al., 1983; Cabello & Mercado, 2015; Muller et al., 2020).

From a practical point of view, understanding and manipulating rational numbers is important, as they arise in a wide variety of everyday situations, such as those requiring measurements. Countless professionals use this type of number in their work, such as building masters, cooks, nurses, shopkeepers, among others. Scientific concepts such as speed, temperature and density are represented by rational numbers.

Lastly, as far as the psychological aspect is concerned, "rational numbers provide a rich arena in which children can develop and extend the mental structures necessary for continued intellectual development" (Behr et al., 1983).

Given its complexity and importance, the concept of rational numbers has been the subject of several studies in the field of Mathematics teaching, since difficulties with this type of number last throughout school life. The main aim of this article is to present the results of a systematic review of literature over the last ten years on the concept of rational numbers, focusing on their study in secondary and higher education.

Analyzing the production of indexed articles reflects the theoretical development of the field of research on the concept of rational numbers. In the case of this research, with a focus on secondary and higher education. In this way, it is possible to recognize, among the main

themes addressed in the research, the nature of the difficulties presents in the use of the concept at school, answering the following question: What thematic trends about the concept of rational numbers, in secondary and higher education, are present in the research published from 2011 to 2021? The data was collected in the first semester of 2021, so only publications from the first semester of 2021 are considered, as described in Autor (2022).

We consider a systematic review like this to be relevant because of the potential implications it could have for the teaching-learning process of the concept of rational numbers in the early years.

Next, we will briefly describe what a systematic review of literature is and the procedures for carrying it out.

A systematic review of literature model

A systematic review of literature is characterized by specific procedures and rules and is "a type of research that follows protocols and seeks to understand and give some logic to a large corpus of documents" (Galvão & Ricarte, 2019, p. 2). Thus,

> a systematic review is a type of scientific investigation that aims to gather, critically evaluate and synthesize the results of multiple primary studies. It also aims to answer a clearly formulated question, using systematic and explicit methods to identify, select and evaluate relevant research, collect and analyze data from studies included in the review (Cordeiro et al., 2007, p. 429).

Fink (2014) highlights the following characteristics of this type of review: systematic, a key aspect, as it follows a well-defined methodological approach; explicit, as the procedures used must be very clear; comprehensive, as it aims to include all relevant material already published; and reproducible, so that others who wish to follow the same approach can reproduce it.

In this way, we understand that the Systematic Review of Literature fulfills two main roles: a retrospective role, in the sense of pointing out and mapping what has already been researched on a given topic; and a systematic role, in the sense of having well-established predefined steps.

In this type of review, the objects of investigation are the primary studies selected systematically and defined in advance. We will point out here the procedures adopted to search for and organize the primary studies in this research.

The procedures needed to search for and organize the studies on a given topic used in this research, based on the reading of some authors (Cordeiro et al., 2007; Creswell, 2010; Donato & Donato, 2019; Okoli, 2019; Sampaio & Mancini, 2007), are summarized in Table 1.

Table 1.

Procedures for a systematic literature review

Planning	**I) Delimitation of the topic**	Identify the topic, define the research question and the objective.
		Identify the keywords; choose the descriptors that will be used in the searches and the databases that will be used.
Selection	**II) Data Collection**	Select the computerized databases that will be used in the searches.
		Establish inclusion and exclusion criteria, in order to select only the most appropriate texts.
		Perform searches in libraries and computerized databases.
	III) Data Selection	Read the titles and abstracts initially and make a selection beforehand.
		Select the most relevant texts to answer the research question, applying the inclusion and exclusion criteria.
Extraction	**IV) Descriptive analysis**	Summarize the selected texts.
		Organize a map of the literature.
Execution	**V) Presentation of the Systematic Literature Review**	Bring together in a structured and organized way by the most relevant concepts or in a thematic way the ideas present in the selected texts. Bring authors into dialogue.
		Suggest how this study can be an addition to the literature.

Even though there are differences between the authors' proposals for the stages indicated here, we highlight the importance of documenting the procedures adopted as an extremely important aspect for maintaining rigor (Okoli, 2019).

The delimitation of the topic includes defining the topic, the research question and the objective, which are essential steps for the proper development of any investigation and are essential for reaching the main objective (Okoli, 2019). When drawn up at the beginning, they have influence on the development of the research.

For the data collection stage, it is necessary that both the search and the selection of articles are carried out well, so that it is possible to identify and analyze the most relevant research on the subject in question. At this stage it is vital to define how and in which databases the searches will be carried out. Creating a prior protocol is crucial and an important step in ensuring the quality of the research (Donato & Donato, 2019; Okoli, 2019). This protocol includes the descriptors, databases, inclusion and exclusion criteria and the stages to be

followed. The inclusion and exclusion criteria cover "appropriate search time, target population, interventions, measurement of the outcomes of interest, methodological criteria, idiom, type of study, among others" (Sampaio & Mancini, 2007, p. 86).

The data selection phase involves deciding which studies will be considered for the systematic review of literature and "should be broad enough to include a sufficient number of studies that can satisfactorily answer the research question" (Okoli, 2019, p. 19). The purpose of this stage is to reduce the number of studies to be analyzed to a tangible number, as data collection usually generates a volume of potentially eligible references (Donato & Donato, 2019; Okoli, 2019). At this point, the inclusion and exclusion criteria are put into practice for the selection of texts, which can initially be carried out by titles, abstracts and keywords. This is followed by a more careful reading of the selected texts.

At last, it is necessary to check which types of data will be extracted from each of the selected studies, so that they are relevant to the research objective. This stage consists of descriptive analysis and involves reading the articles in full, followed by organizing and analyzing the selected data. This process also includes a descriptive stage of the research in general and it is important to create tables with the most relevant data (Conforto et al., 2011; Donato & Donato, 2019). Next, it is necessary to synthesize the data and then answer the initial research question. In other words, the aim is to interpret the data collected from the various texts into relevant and logical conclusions. We call this stage theme analysis, i.e. the stage in which the themes are defined on the basis of a literature map, which consists of "a visual summary of the research that has already been conducted by others and is typically represented by a figure" (Creswell, 2010, p. 61). We will now return to the route we took.

Procedures of a systematic review of literature adopted in this research

The procedures adopted in this research begin with the delimitation of the topic and the identification of the following keywords: rational numbers; fractions; decimal numbers; secondary education; higher education and their respective Spanish translations. We chose the terms "secondary school" and "higher education" because we are interested in research that targets these audiences, as many of the difficulties with this content persist throughout schooling.

The descriptors used in the database searches were: "rational numbers" or "fractions" or "decimal numbers"; synonyms for "secondary education" and "higher education"; "rational numbers" or fractions or "decimal numbers"; and "secondary education" or "higher education"

or "undergraduates" or "initial training" or "university students". The results obtained were: "high school" (421), "higher education" (213), "undergraduates" (53), "initial training" (181), "university students" (75).

For data collection, we restricted ourselves to Google Scholar, as it is a database in the field of Mathematics Education research that encompasses the others to be considered. We relied on Silva (2020) and his comparative analysis of statistics education research in Latin America between ten databases and Google Scholar, concluding that the latter "has a greater ability to locate texts than the 10 sources combined" (Silva, 2020, p. 55).

As a data collection and selection strategy, we developed the following inclusion and exclusion criteria: 1) search time: from 2010 to 2020; 2) languages: Portuguese and Spanish; 3) target population: secondary and higher education; 4) type of text: journal articles; 5) availability: texts available in full format in electronic support and free of charge; 6) mathematical content: concept of rational numbers or fraction or decimal numbers; 7) methodology employed: didactic-pedagogical interventions or individual ability tests; 8) type of study: qualitative, quantitative or mixed. And the following exclusion criteria: 1) duplicate articles; 2) articles focusing on textbook analysis or ENEM questions; 3) articles with a target audience in youth and adult education; 4) articles conducted with teachers; 5) articles in Spanish from Spanish journals.

Table 2.

Description of the steps taken to select the data

Steps	Description	Portuguese	Spanish	Total
1ª	Read the title and keywords, applying the inclusion and exclusion criteria listed.	136	147	283
2 ª	Read the abstracts, applying the inclusion and exclusion criteria listed.	67	120	187
3 ª e 4 ª	Categorize the texts. Exclude texts that were not from Latin American magazines, that is, from Spanish journals.	67	87	154
5 ª	Exclude theses, dissertations and texts from conferences and the like, because at the end of these stages we still had a total of 154 texts	28	29	57
6 ª	Read the texts in their entirety. Delete 11 theory texts.	22	23	45

To carry out the searches on Google Scholar, combinations of all the descriptors mentioned were made, obtaining a total of 943 entries in Portuguese and 1326 entries in Spanish.

Table 2 describes the steps taken to select the data, according to the application of the inclusion and exclusion criteria, from the 2269 texts in Portuguese and Spanish.

After carrying out all the steps, we arrived at a total of 45 articles.

Outcomes: descriptive analysis

In this section we present the descriptive analysis, which consists of showing the main characteristics of the studies included in this systematic literature review, in order to finally identify the topics of greatest interest in research into Mathematics education in secondary and higher education.

Regarding the years in which the articles were published, we noticed a greater volume of publications between 2015 and 2020, as shown in Figure 1. As the searches were carried out in the first half of 2021, we cannot consider that there has been a decrease in the number of articles this year, because the search did not cover the entire year.

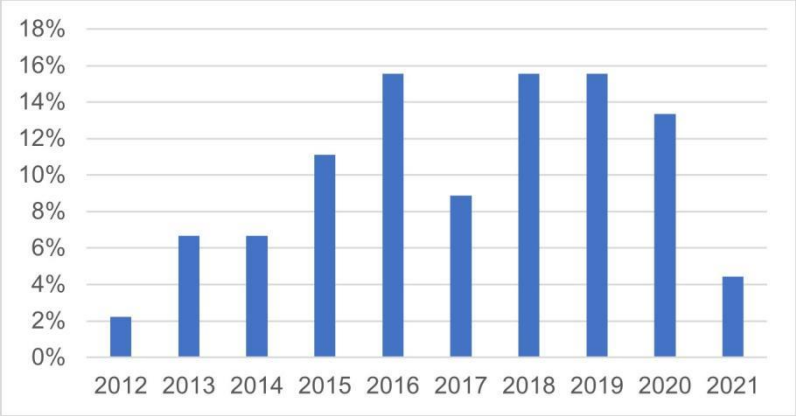

Figure 1.

Relative frequency distribution according to the year of publication of the articles

Concerning the journals in which the articles were published, the total of 32 outlets indicates a considerable variety. Only one journal had a larger number of articles, which was *Acta Latinoamericana de Matemática Educativa* (Alme), with 9 articles (20%), followed by *Revista Eletrônica de Educação Matemática*, with 3 articles (7%). All the others had 1 or 2 publications.

As for the countries in which the magazines are published, half, or 16 magazines, are Brazilian, followed by 5 Mexican, 3 Argentinian, 3 Colombian, 1 Peruvian, 1 Chilean and 1 Nicaraguan.

Regarding the quality criteria of the journals, all of them are indexed and the articles published are submitted to peer review. For this classification we used the Sucupira platform, from the 2013-2016 quadrennium, evaluation area "teaching". Some journals did not have Qualis (a Brazilian ranking system) in the "teaching" area, but did in the Education or interdisciplinary area and were included in the table as well.

Table 3.

Frequency distribution by qualis

Qualis	**Articles (absolute number)**	**Articles (%)**
A1	2	4
A2	5	11
B1	8	18
B2	11	24
B3	2	4
B4	1	2
B1 (Education)	1	2
B4 (Interdisciplinary)	1	2
B5 (Interdisciplinary)	1	2
C (Education)	1	2
N (Latin American journals that do not have qualis)	12	27
Total	**45**	**100**

A considerable number of journals (27%) have not been evaluated here in Brazil and are therefore not listed in Qualis CAPES. They are journals edited and published in other Latin American countries.

Our decision, which was restricted to Google Scholar as the database for searching the texts, resulted in a broad representation of all types of Qualis, with a predominance of Qualis B1 (8) and B2 (11). The greater number of Qualis B2 studies was due to *Revista Alme* being the journal with the largest number of articles.

Concerning the level of education of the research participants, we included those whose participants were High School and College students, meeting the inclusion criteria. Among

those selected, some also included Elementary School. The frequency distribution by participants' level of education is shown in Table 4:

Table 4.

Distribution of frequencies by level of education of the participants

Educational level	**Articles (absolute number)**	**Articles (%)**
Higher education	25	56
High school	9	20
Elementary and high school	3	7
Elementary school, high school and higher school	3	7
Elementary school and higher education	3	7
High school and higher education	1	2
Preparatory course	1	2
Total	**45**	**100**

The highest frequency corresponds to higher education, with the following distribution of courses: Pedagogy (52%), Degree in Mathematics (26%), Engineering (6%), Logistics (3%), Medicine (3%), Degree in Physics (3%), Accounting (3%) and others (3%).

It's not surprising that a greater number of studies have Pedagogy students as participants (16), as these future teachers will be responsible for introducing the concept of rational numbers in Elementary School. This justifies the interest of Mathematics educators in investigating issues related to the topic of "rational numbers" in these professional training settings. Next, and with a similar justification to the previous one, the Mathematics degree course stands out, with 8 articles presenting research results on numbers. We also see this frequency as evidence of the interest of Mathematics Education in investigating the training of teachers who will teach the concept of rational numbers.

With regard to the theoretical and methodological characteristics, we explored the types of studies that have been produced, the methodological approaches, the data collection procedures and the theoretical references.

The research methodology (qualitative, quantitative or combined) revealed a predominance of qualitative research (40), followed by 5 combined studies and none with a purely quantitative approach.

The four categories defined to describe the methodological approach of the research are: instrumental and/or tool analysis; documental analysis; historical analysis; or teaching-learning

(Silva et al., 2015). Due to the inclusion and exclusion criteria of this literature review, all the research selected and analyzed focuses on "teaching and learning".

In this context, we identified four categories in the research approaches adopted by researchers to investigate the processes of teaching and learning rational numbers: pedagogical intervention; diagnosis; proposal and analysis of didactical activities; and didactical-pedagogical process (Silva et al., 2015). Table 5 summarizes the incidence of articles in each of these categories:

Table 5.

Distribution of frequencies of scientific production according to the categories of the methodological approach "teaching and learning"

Categories of the teaching and learning methodological approach	**Articles (absolute number)**	**Articles (%)**
Diagnosis	27	60
Didactical-Pedagogical Process	8	18
Pedagogical Intervention	4	9
Proposal and Analysis of the didactics activities	4	9
Diagnosis and Proposal and Analysis of the Didactic Activity	2	4
Total	**45**	**100%**

More than half of the research aims to draw up a diagnostic profile of Secondary School and Higher Education students on topics relating to the concept of rational numbers.

It is interesting to note that a large part of the research involving future teachers (Pedagogy and Mathematics undergraduate students) - 24 studies - did not focus on the teaching process, but rather on the conceptions of these future teachers regarding the concept of rational numbers. The importance of these investigations focusing on conceptions is justified by the fact that they contribute with results on knowledge that must be reconstructed by future teachers who are going to teach the concepts. This understanding is evident when we realize that only 8 studies focus on the didactical-pedagogical process, i.e. they take the professional performance of the future teacher into the foreground.

In terms of methodological procedures for obtaining data (interviews, case studies, questionnaires, bibliographical research, etc.), we had the following distribution, as shown in Table 6:

Table 6.

Distribution of frequencies of scientific production according to classification by the way in which data were obtained

Procedures for obtaining data	**Articles (absolute number)**	**Articles (%)**
questionnaire or diagnostic test or phased test	18	40
sequence of activities (problem solving, teaching sequence, etc.)	9	20
training meetings, guided discussions and collaborative groups	8	18
use of resources (game or technology)	5	11
questionnaire + interview	3	7
Interview	1	2
interview + sequence of Activities	1	2
Total	**45**	**100%**

As expected, since most of the research has a diagnostic focus, the predominant data collection procedure was the use of questionnaires, diagnostic tests or phased testing (40%).

As for the theoretical frameworks used, we highlight the diversity of perspectives adopted, as well as their authors. Among the most cited specific theoretical frameworks on rational numbers are those developed by Kieren (1976, 1980) and Behr et al. (1983), as described in Table 7:

Table 7.

Frequency distribution of scientific production according to the classification of the main theoretical framework

Main Theoretical Framework	**Articles (absolute number)**	**Articles (%)**
Rational Numbers - various authors	11	24
Teachers' specialized knowledge – Shulman, Ball et al	10	22
Ontosemiotic Approach – Godino, Batanero and Font	4	9
Problem Solving – multiple authors	3	9
Technologies – various authors	3	7
None	3	7
Register Theory of Semiotic Representation – Duval	2	4
Use of Games – various authors	1	2
Algorithmic Intuitive Aspects – Fischbein	1	2
Anthropological Theory of Didactics – Chevalard	1	2
Mathematical Activity – Vygotsky	1	2
Advanced Mathematical Thinking – Dreyfus and Tall	1	2
Theory of Conceptual Fields – Vergnaud	1	2
Communities of Practice – Wenger	1	2
Realistic Mathematics – Puig	1	2
Teachers' specialized knowledge and Problem Solving	1	2
Total	**45**	**100%**

From the analysis of the predominant theoretical currents, the "Specialized knowledge of the Mathematics teacher" stands out, followed by the ontosemiotic approach, problem solving and technologies.

One point to note is that six of the ten articles on "Specialized Knowledge of Mathematics Teachers" deal with the topic of ratio and proportional thinking, which may point to a difficulty for future teachers in the process of teaching (or learning) this content.

A categorization of the main contents investigated is organized in Table 8, based on the distribution of their frequency as thematic content in each article in the selected research corpus:

Table 8.

Distribution of frequencies according to the main content of the articles

Content	**Articles (absolute number)**	**Articles (%)**
concept of fraction, rational numbers, decimals and percentage	17	38
subconstructs of the concept	8	18
proportional thinking	7	16
representations	6	13
operations and equivalence	6	13
density and order of rational numbers	1	2
Total	**45**	**100**

Seventeen articles, which corresponds to a large part of the research, do not make it clear in their objective which aspects of rational numbers are being investigated. They vaguely state an intention to investigate the "concept of fraction" or "rational numbers", etc. We interpret it as a possibility that, by adopting a naturalistic and qualitative methodological approach, the authors expect specific aspects of the concept to emerge during the course of the research. These studies were included in Table 8 in the category called "fraction concept, rational numbers and decimal numbers".

The other studies state from the outset which specific aspects of the concept of number they intend to investigate. The specification of the subconstructs developed in Kieren (1976) and Behr et al. (1983) guided a representative number of these studies, demonstrating the scope and importance of these studies in Mathematics education research. These studies are followed, in terms of volume of publications, by studies on proportional thinking.

As for the keywords, we translated them from Spanish into Portuguese in order to calculate the frequencies. We obtained a total of 166 keywords, which are organized in Table VII, grouped by words we consider to be synonyms:

Table 9.

Keywords separated by groups

Keyword Group	Articles (absolute number)	Articles (%)
Rational numbers, fractions, and decimals	36	22
Target audience	28	17
Several	11	7
Teaching and learning	10	6
Teacher's Knowledge	9	5
Subconstructs	9	5
Mathematics and Mathematics Education	9	5
Performance, errors, and difficulties	7	4
Technology	7	4
Resources	7	4
Problem Solving	6	4
Proportional Reasoning	6	4
Semiotics	5	3
Operations	4	2
Attitudes	4	2
Other Mathematics topics	4	2
Methodology	2	1
Percentage	2	1
Total	**166**	**100**

We noticed that, for the most part, the keywords were related to the school context of the research participants. Other topics that appeared frequently were related to the objective of the research and the main focus (subconstructs, rational numbers, proportional reasoning, etc.), the type of methodology adopted (technology, problem solving, etc.), the theoretical framework (teacher knowledge, semiotics, etc.), the type of methodology used to carry out the research (group approach, phased tests) and the broad area of Mathematics, such as Mathematics teaching or Mathematics Education.

The role of keywords in an article is to represent the content and, for this reason, they must be chosen carefully. They are important because they are search terms in databases, i.e. when chosen well, they make it easier to find articles on specific topics. However, what we have noticed is that many articles use very generic and broad keywords - for example, words like "Mathematics", "Mathematics Education" and "Mathematics teaching", which have a very

broad meaning, or other words that don't really specify the focus, such as "resources", "didactics", etc.

We reiterate the importance of choosing representative keywords for a systematic literature review, as they can provide central themes for the literature map.

Thematic trends: literature map

By contrasting and comparing the different studies selected in this systematic review of literature, with the characteristics listed in the previous section, we organized the articles that had common characteristics, taking into account their main themes. In this way, we were able to divide them into four themes and answer the central question of this research, which we describe in this section. We then analyzed the specific features of each theme, distinguishing and bringing them together, and identifying sub-themes. The literature map is the result of this analysis.

In addressing the central question of this research: "What topics on the concept of rational numbers in Secondary and Higher Education are present in the research?", we identified:

i) Ratio and Proportional Thinking;

ii) Operations with Rational Numbers;

iii) Representations of Rational Numbers;

iv) The concept of rational numbers and their interpretations.

Furthering the analysis and contrasting the research included in each of the themes, the related sub-themes were highlighted, as described in Table 10.

Table 10.

Map of Literature

Thematic Focus (Some surveys have been allocated to more than one subtheme, so the sum of the percentages obtained is a number greater than 100%)	**Sub-themes**
THEME 1: Reason and proportional thinking (18%)	**Proportional thinking and teachers' specialized knowledge** (Astudillo et al., 2018; Buforn & Fernández, 2014; Camacho & Guerrero, 2017; Riquelme & Vergara, 2019; Silva, Cândido et al., 2018; Soto, 2017)
	Strategies for Using Proportional Thinking (Silva et al., 2018; Trevisan & Domingues, 2017)
	Proportional Thinking and Its Use in Real-Life Situations (González et al., 2019)
THEME 2: Operations with rational numbers (26.5%)	**Games and Virtual Aids as an Alternative for Revising the 4 Operations with Rational Numbers** (Barbosa et al., 2018; Braga & Lima, 2020; Pérez, 2012)
	Performance and Difficulties of High School and Higher Education Students in the Four Operations with Rational Numbers (Carvalho et al., 2021; Etcheverria et al., 2019; Justulin, 2017; Oliveira et al., 2020; Paraol & Rodriguês, 2018; Retana & Muñoz, 2018; Sanchez-Luján & Moreno, 2018; Nardoni et al., 2015)
	Operations and Specialized Knowledge for Teaching (Proença, 2015; Retana & Muñoz, 2018; Silva, Carvalho et al., 2018)
THEME 3: Representations of rational numbers (13%)	(Abrahão, 2016; Silva & Dias, 2015; Flôres et al., 2020; Moura & Mattos, 2014; Paraol & Rodriguês, 2018; Santana et al., 2015; Parra-Zapata et al., 2021)
THEME 4: The concept of rational numbers and its interpretations (53%)	**Rational numbers as part-whole** (Carpes & Carpes, 2020; Etcheverria & Amorim, 2020; García & Cabañas-Sánchez, 2013; Justulin, 2017; Menegazzi & Dalcin, 2016; Paraol & Rodriguês, 2018; Salazar et al., 2016)
	Rational numbers as quotient (Cabello & Mercado, 2015; Carpes & Bisognin, 2019; García, 2019; García & Cabañas-Sánchez, 2013; Konic, 2019; Menegazzi, 2013; Mercado et al., 2016; Muller et al., 2020)
	Rational Numbers as Measures (Abrahão, 2016; Amaya et al., 2018; Medina, 2013; Menegazzi, 2013; Oliveira & Araman, 2017; Retana & Muñoz, 2018)
	Rational Numbers as Operators (Carpes & Bisognin, 2019; Godoy & Barreto, 2016; Reyes & Prieto, 2015; Vecino et al., 2015)

For the first three themes, the identification of the sub-themes considered the theoretical framework adopted, the methodologies and the approaches used. The following are the characteristics of the sub-themes.

The topic of *Ratio and Proportional Thinking* is the focus of 8 studies in our corpus (corresponding to 18% of the total). Participants and the context in which the research on this topic was carried out were mostly higher education students, with 6 studies involving students from Pedagogy courses and 1 including students from a Medicine course. Only 1 study involved high school students (Trevisan and Domingues, 2017). Regarding to theoretical references, the theoretical frameworks on teachers' specialized knowledge, developed by Deborah Ball and Lee Shulman, are the most used.

A shared objective of the six studies involving future educators was to identify teachers' knowledge of ratio and proportional thinking. While recognizing their particularities, these studies characterize our first sub-theme: Proportional Thinking and Specialized Knowledge for Teaching. Five out of six articles were written in Spanish, involved participants from other countries and were published in foreign journals. Only the research by Silva, Cândido et al. (2018) was carried out in Brazil and is written in Portuguese.

The results of these investigations indicate that a large proportion of future teachers have standard content knowledge on the subject of "ratio and proportional thinking", but they do not have specialized content knowledge or pedagogical content knowledge (Astudillo et al., 2018; Buforn & Fernández, 2014; Riquelme & Vergara, 2019; Silva, Cândido et al., 2018). In other words, prospective teachers are able to solve problems involving ratio and proportional thinking; however, they are unable to explain the procedures adopted to teach them. Many solutions are carried out mechanically or using algorithms - without a clear understanding of the justifications for the processes applied - without knowing how to diversify the resolution strategies, restricted to purely procedural knowledge and without knowing how to explain the processes involved.

Two research focus on the strategies used by students when using proportional thinking to solve tasks and characterize the second sub-theme: Strategies for Using Proportional Thinking (Silva et al., 2018; Trevisan & Domingues, 2017). The authors conclude that the strategies most used by students to solve problems with ratios and proportions reveal a greater occurrence of the unit rate and not the rule of three.

A single study involved students on a medical course and is included in the sub-theme: Proportional Thinking and its Use in Real Life Situations.

The topic *Operations with Rational Numbers* was addressed in 13 publications, which corresponds to 29% of the total selected research corpus, and characterizes 3 sub-themes.

Three studies carried out with secondary school students investigated the use of games or virtual aids to revise the content of operations with rational numbers and characterize the first subtheme: *Games and Virtual Aids as an Alternative for Revising the 4 Operations with Rational Numbers* (Barbosa et al., 2018; Braga & Lima, 2020; Pérez, 2012). They all point to the use of games or virtual aids as potential alternatives for revisiting some basic knowledge of rational numbers, which is essential for acquiring new concepts.

Eight studies analyze the performance and main difficulties that students have in problems with the four operations with rational numbers and characterize the second sub-theme: *Performance and Difficulties of High School and Higher Education Students in the Four Operations with Rational Numbers* (Carvalho et al., 2021; Etcheverria et al., 2019; Justulin, 2017; Oliveira et al., 2020; Paraol & Rodriguês, 2018; Retana & Muñoz, 2018; Sanchez-Luján & Moreno, 2018; Nardoni et al., 2015). One of the highlights of this sub-theme is that all of the studies use diagnosis as a methodological approach and seek to analyze students' performance and main difficulties in relation to the four operations with rational numbers, in their fractional and decimal forms. The results of these studies show a procedural conception characterized by the use of algorithms that have no meaning for the students and are sometimes easily distorted, which usually results from the mechanical memorization of procedures.

Finally, the third subtheme is characterized by three studies aimed at identifying and analyzing prospective teachers' content knowledge about operations with rational numbers: *Operations and Specialized Knowledge for Teaching* (Proença, 2015; Retana & Muñoz, 2018; Silva, Carvalho et al., 2018). The results of the research share an understanding that common content knowledge is familiar to the vast majority of future teachers, although specialized content knowledge and pedagogical content knowledge are under development. This follows from the analysis of the responses of the future teachers, who had difficulties justifying the calculations made and proposing different strategies for solving certain problems.

The topic *Representations of Rational Numbers* was covered in seven articles, equivalent to 15.53% of the total (Abrahão, 2016; Silva & Dias, 2015; Flôres et al., 2020; Moura & Mattos, 2014; Paraol & Rodriguês, 2018; Santana et al., 2015; Parra-Zapata et al., 2021). Due to the small number of studies that make it up, we chose not to organize them into sub-themes without studying their specificities and to focus on what identifies them. The results of the research show that high school and college students' difficulties in understanding the same rational number in its different registers are persistent throughout their school life. Furthermore,

students' conceptions of the representations of a rational number seem to be restricted to a single register. On the other hand, difficulties in interrelating representations remain, even during high school and university, and a predominance of the numerical register is evident (Abrahão, 2016; Santana et al., 2015). This is confirmed when we see the numerous difficulties with conversions, of which the one that represented the greatest obstacle was the conversion from the figural or fractional register to the decimal register (Paraol & Rodriguês; 2018; Santana et al., 2015).

The theme *The Concept of Rational Numbers and its Interpretations* was covered in 22 publications (corresponding to 49% of the total).

Most of the research is included in this theme, and the general interest is centered on the knowledge, conceptions and difficulties that high school and college students, mainly teachers-to-be, have about the concept of rational numbers.

Several of these studies were based on the subconstructs of the concept of rational number. For this reason, the subthemes are confused with the subconstructs: part-whole, measure, quotient and operator.

The varying levels of learning of the concept of rational numbers that have been identified are due to the differing interpretations that students have of each of them (Muller et al., 2020). Particularly, students show greater proficiency in the notion and use of the part-whole and operator subconstructs and show limitations with the measure and quotient subconstructs (Menegazzi & Dalcin, 2016; Muller et al., 2020).

In addition, their conceptions of rational numbers are often partial and limited, and are not reoriented throughout schooling.

Final considerations

At the end of this Systematic Review of Literature, it was possible to observe what has been worrying mathematics educators in terms of what obstacles with rational numbers persist into higher education. In the four themes - Ratio and proportional thinking; Operations with rational numbers; Representations of rational numbers; and Concept of rational numbers and their interpretations - it was possible to identify various conceptions of students in secondary and higher education, of different natures. To conclude, we consider it relevant to mention the nature of these conceptions pointed out by the research.

The results for the topic of Ratio and Proportional Thinking indicate that the knowledge of most would-be teachers and high school students is characterized by a pre-proportional conception, as it incorporates a notion of the concept restricted to the execution of procedures.

This means that the concept is centered on qualitative relationships. The nature of this conception may be related to the emphasis on procedural skills rather than conceptual understandings in the teaching-learning process throughout elementary school.

Research on *Operations with Rational Numbers* generally concludes that the success rates on questions involving this topic at all levels of education were unsatisfactory (Etcheverria & Amorim, 2020; Paraol & Rodriguês, 2018; Retana & Muñoz, 2018; Sanchez-Luján & Moreno, 2018). Of the studies analyzed, four were aimed at higher education, which indicates that many difficulties in this content still persist at this stage of education. The nature of the difficulties with the operations of rational numbers listed in the research was also the emphasis on procedures. In other words, they usually occur as a result of mechanical memorization of procedures; the use of algorithms generalized from special cases or inferred from procedures valid in other contexts; or the non-relational use of the concept of rational numbers and operations.

The topic Representations of Rational Numbers points out that students' conceptions of the representations of a rational number are restricted to a single register. In other words, difficulties in interrelating representations remain until university and a predominance of the numerical register is evident (Abrahão, 2016; Santana et al., 2015).

Finally, the theme *Concept of Rational Numbers and their Interpretations* encompassed a total of 22 studies that sought to analyze knowledge, understanding, performance and difficulties regarding the concept of rational numbers. The conceptions identified in this sub-theme originate from: i) a teaching-learning process restricted to the part-whole subconstruct. The crux of the matter is not the introduction through this subconstruct, but the emphasis placed on it, to the detriment of experience with other equally important subconstructs; ii) prior knowledge of natural numbers (natural number bias), which can give rise to some inconsistent conceptions. We conjecture that the teaching-learning process as it is being developed has not allowed the transition from the set of natural numbers to the set of rational numbers; iii) a centrality on procedures, algorithms and rules can lead to limited conceptions of the concept of rational number.

In conclusion, it is necessary to recognize that understanding the concept of rational numbers and all the relationships between its main ideas involves a long learning process, as it requires experiences with notions involved in a network. Which means comprehending: the multiple interpretations (sub-constructs); the multiple associated ideas; the multiple representations; the symbols and formal language of rational numbers; the properties involved and the operations using different models of representation.

From drawing up the literature map and identifying the research topics, the nature of the issues that have interested researchers emerges, perceived by educators as obstacles in the process of teaching and learning the concept of rational number, which persist throughout the academic life of students up to university.

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