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Teaching Polyhedra through a Potentially Significant Teaching Unit Mediated by the Flipped Classroom Methodology

Una Unidad Didáctica Potencialmente Significativa mediada por el Aula Invertida para enseñar Poliedros

Une unité d'enseignement potentiellement significative médiatisée par la classe inversée pour enseigner les polyèdres

Uma Unidade de Ensino Potencialmente Significativa mediada pela Sala de Aula Invertida para ensinar Poliedros

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Abstract

Polyhedra are objects of knowledge of geometry, fundamental to the process of teaching and learning mathematics. For this, it is expected that students, throughout their schooling, develop spatial visualization, logical reasoning and problem solving skills. Therefore, this study was developed as qualitative research and is characterized as a case study. It is an excerpt from a master's degree research which sought to evaluate the potential of applying a Unit of Potentially Meaningful Teaching (UPMT) on Polyhedra in a High School class, based on the

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methodological work carried out by the Flipped Classroom. The application took place in a second grade high school class, in the first two months of 2023, with the participation of 14 students from the state education system. The data were produced from the elaboration, application and analysis of the results of a UPMT, considering, for this work, three of the categories selected: technological dimension and applied resources (videos, texts, slides, exercises), diversified activities (games, investigative experiments, etc.) and students' knowledge of Polyhedra. The data collected indicates significant advances in student learning, highlighting student engagement, protagonism, autonomy and understanding of the topic, in addition to challenges such as limited internet access. In conclusion, this research suggests that the active methodology of the Flipped Classroom can be an effective alternative for basic education teachers to minimize traditional teaching practices.

Keywords: High school, Polyhedra, Flipped classroom, Potentially meaningful teaching unit.

Resumen

Los poliedros son objetos de conocimiento en geometría, fundamental para el proceso de enseñanza y aprendizaje de las matemáticas. Por lo tanto, se espera que los estudiantes, a lo largo de su escolarización, desarrollen habilidades de visualización espacial, razonamiento lógico y resolución de problemas. Este estudio fue desarrollado como una investigación cualitativa, se utilizó como metodología el estudio de caso. Es un segmento de una investigación de maestría que tuvo como objetivo evaluar el potencial de aplicar una Unidad de Enseñanza Potencialmente Significativa (UEPS) sobre poliedros en una clase de secundaria, basada en el trabajo metodológico realizado por el Aula Invertida. La aplicación tuvo lugar en una clase de segundo año de secundaria en el primer trimestre de 2023, con la participación de 14 estudiantes del sistema educativo estatal. Los datos se produjeron a partir del diseño, implementación y análisis de los resultados de la UEPS, considerando tres categorías seleccionadas para este trabajo: dimensión tecnológica y recursos aplicados (videos, textos, diapositivas, ejercicios), actividades diversificadas (juegos, experimentos de investigación, etc.) y el conocimiento de los estudiantes sobre poliedros. Los datos recopilados indican avances significativos en el aprendizaje de los estudiantes, destacando el compromiso, la autonomía y la comprensión de los alumnos sobre el tema, a pesar de los desafíos como el acceso limitado a internet. En conclusión, esta investigación sugiere que la metodología activa del Aula Invertida puede ser

una alternativa eficaz para que los profesores de la escuela primaria minimicen las prácticas de enseñanza tradicionales.

Palabras-clave: Secundaria, Poliedros, Aula invertida, Unidad didáctica potencialmente significativa.

Résumé

Les polyèdres sont un objet de connaissance en géométrie, fondamental pour les processus d'enseignement et d'apprentissage des mathématiques. Par conséquent, on s'attend à ce que les élèves, tout au long de leur scolarité, développent des compétences en visualisation spatiale, raisonnement logique et résolution de problèmes. Cette étude a été menée en tant que recherche qualitative, caractérisée comme une étude de cas. Il s'agit d'un segment d'une recherche de maîtrise visant à évaluer le potentiel de l'application d'une Unité d'Enseignement Potentiellement Significative (UEPS) sur les polyèdres dans une classe de lycée, en s'appuyant sur le travail méthodologique réalisé par la Classe Inversée. L'application a eu lieu dans une classe de seconde année de lycée au premier trimestre de 2023, avec la participation de 14 élèves du système éducatif public. Les données ont été produites à partir du design, de la mise en œuvre et de l'analyse des résultats de l'UEPS, en tenant compte de trois catégories sélectionnées pour ce travail : dimension technologique et ressources appliquées (vidéos, textes, diapositives, exercices), activités diversifiées (jeux, expérimentations de type investigatif, etc.) et connaissance des élèves sur les polyèdres. Les données collectées indiquent des avancées significatives dans l'apprentissage des élèves, mettant en avant l'engagement, l'autonomie et la compréhension des élèves sur le sujet, malgré les défis tels que l'accès limité à Internet. En conclusion, cette recherche suggère que la méthodologie active de la Classe Inversée peut être une alternative efficace pour les enseignants de l'éducation primaire afin de réduire les pratiques d'enseignement traditionnelles.

Mots-clés : Lycée, Polyèdres, Classe inversée, Unité d'enseignement potentiellement significative.

Resumo

Poliedros é um objeto de conhecimento da geometria, fundamental ao processo de ensino e aprendizagem de matemática. Para tanto, espera-se que os alunos, ao longo da escolarização, desenvolvam habilidades de visualização espacial, raciocínio lógico e resolução de problemas. Dessa forma, este estudo foi desenvolvido como pesquisa qualitativa e se caracteriza como estudo de caso. É um recorte de uma pesquisa de mestrado que buscou avaliar quais as

potencialidades da aplicação de uma Unidade de Ensino Potencialmente Significativa (UEPS) sobre Poliedros em uma turma de Ensino Médio, a partir do trabalho metodológico realizado pela Sala de Aula Invertida. A aplicação ocorreu em uma turma da segunda série do Ensino Médio, no primeiro bimestre de 2023, contando com participação de 14 alunos da rede estadual. Os dados foram produzidos a partir da elaboração, aplicação e análise dos resultados de uma UEPS, considerando, para este trabalho, três das categorias selecionadas, *a priori*: dimensão tecnológica e recursos aplicados (vídeos, textos, slides, exercícios), atividades diversificadas (games, experimentos de cunho investigativo etc.) e conhecimento dos estudantes sobre Poliedros. Os dados coletados indicam avanços significativos na aprendizagem dos alunos, destacando-se o engajamento, protagonismo, autonomia e a compreensão dos estudantes sobre o tema, para além dos desafios, como a limitação no acesso à internet. Em conclusão, esta pesquisa sugere que a metodologia ativa da Sala de Aula Invertida pode ser uma alternativa eficaz para professores da educação básica minimizarem práticas de ensino tradicional.

Palavras-chave: Ensino médio, Poliedros, Sala de aula invertida, Unidade de ensino potencialmente significativa**.**

Teaching Polyhedra through a Potentially Significant Teaching Unit Mediated by the Flipped Classroom Methodology

This text is an excerpt from the master's research conducted by the first author[5]. The research aimed to evaluate the potentialities of applying a Potentially Significant Teaching Unit (PSTU) on Polyhedra in a high school class, based on the methodological work carried out by the Flipped Classroom Methodology (FC). The application of the PSTU took place in a second-year high school class during the first semester of 2023, with the participation of 14 students from a State School located in the City of Estância, in the State of Sergipe, Brazil.

The guiding educational documents consulted, such as the National Curriculum Parameters (PCN – Brazil, 1998, 2000), the National Common Curricular Base (BNCC – Brazil, 2018), and the Sergipe State Standards (Sergipe, 2022) state that the subject Polyhedra is fundamental in the study of geometry and, consequently, mathematics. Therefore, the educational process must focus on the development of spatial visualization, logical reasoning, problem-solving, and understanding of real-world phenomena and situations. It is expected that "these skills are acquired throughout the years of Elementary School", so that students can refine them at the end of their schooling – the stage corresponding to High School (Sergipe, 2022, p. 512-584).

According to the established guidelines in the aforementioned documents, it is necessary for high school education that the deepening of the subject be approached from a contextualized and interdisciplinary perspective. This type of teaching will allow for a greater connection between concepts and properties and the students' reality, such as exploring problem-solving involving area and volume measurements.

Thus, initial learning in the study of Polyhedra, starting from the early years of Elementary School, is an important subsumption for the learning proposed for High School. According to Moreira (2011, p. 14), a subsumption is specific and relevant knowledge to a new learning, a prior knowledge. The assignment of new meanings depends on the interactive process between what is already known and what will be learned.

According to Ausubel (2003), Novak (2000), and Moreira (2011), meaningful learning of new knowledge relies heavily on the prior knowledge available in the cognitive structure.

[5] The master's thesis was defended in the Graduate Program in Science and Mathematics Teaching at the Federal University of Sergipe, under the guidance of the second author. Available at: https://ri.ufs.br/jspui/handle/riufs/18889.

Therefore, it is important, from the early stages of schooling, to present mathematical concepts with significance to children so that, over time, they become subsumptions for new learning.

It is worth noting that, according to Santos et al. (2021), throughout history, the teaching of geometry has faced challenges and transformations. Souza (2021) discusses how teaching conceptions vary according to curricular proposals over time. There was a period when geometry concepts were taught under two aspects, according to gender: for girls, through crafts and embroidery; for boys, in an abstract manner, through classical scholarly teaching. In the mid-20th century (between the 1950s and 1970s), the Modern Mathematics Movement (MMM) was a milestone in the teaching of mathematics as a whole. This movement prompted curricular reforms from the perspective of algebraic structures, making geometry teaching more aligned with these structures.

Kaleff (1994), Lorenzato (1995), and Pavanello (2009) discuss that, with the MMM in Brazil, there was an attempt to algebraize geometry, but this approach was unsuccessful. As a result, the teaching of geometry faced a process of neglect, despite public policies maintaining curricular guidelines over these decades, encouraging the teaching of geometric objects through geometric constructions using tools such as rulers and compasses. However, the proposed algebraization did not rekindle students' interest, as the emphasis in the initial teacher education programs for mathematics teachers was on algebraic structures, particularly in Mathematics Teaching degrees.

Beyond aspects of modernity, in the 1970s, the educational legislation governing Brazilian schools favored freedom regarding the curriculum implemented in schools, allowing their projects to be oriented toward the students' reality[6]. Unfortunately, this was not an effective alternative; each school had its own standards, and mathematics teachers, in turn, followed the textbooks available to them. Such textbooks presented subjects related to geometry in the final chapters, which eased the justification of lacking time to teach and consequently study them (Pavanello, 2009; Souza, 2021).

Studies by Gazire (2000), Magalhães et al. (2012), Rosa et al. (2020), and Santos (2021) also corroborate these statements by showing that gaps in pedagogical practices still persist today, and the current generation of teachers often lacks the proper training to teach geometry as prescribed in the standards. Souza (2021) highlights this issue by reporting the low performance in national assessment systems, which also reveals that the performance of Basic Education students is characterized by low scores concerning geometric subjects.

[6] Guidelines and Bases Law – LDB No. 5,691/1971.

Thus, there is a need to explore innovative approaches that can contribute to active and meaningful learning for students regarding geometric concepts in Basic Education. In this context, the desire emerged to teach Polyhedra in association with the Potentially Significant Teaching Unit (PSTU) – a tool for meaningful learning – and the Flipped Classroom (FC) – an active methodology.

The PSTU is a pedagogical strategy aimed at making the teaching and learning process more significant for students. It is an approach designed to engage students in the learning process. According to Moreira (2011a), it is a potentially significant material. In turn, FC has gained prominence in recent years and has been used both in Basic Education and Higher Education. According to Bergmann and Sams (2021), it is an approach that involves reversing the traditional teaching dynamic. In other words, students initially study the content at home, and subsequently, classroom time is used to apply and deepen knowledge through practical activities and discussions.

According to Valente (2018, p. 30), this teaching method is commonly used in the Humanities. Renowned universities have applied this strategy to explore technological advances, reduce dropout rates, and improve failure rates. “The difficulty of the inversion occurs especially in the exact sciences, where the classroom is used to convey already accumulated knowledge.” However, similar strategies are used without recognizing that the Flipped Classroom is a teaching methodology.

The mapping conducted by Schreiber et al. (2018) revealed that there are few publications regarding the use of FC in mathematics teaching. However, these authors inform us that the few studies located related to Higher Education practices indicate that “these works have achieved satisfactory results in the flipped classroom, demonstrating that it can be an alternative” to “traditional teaching, as it enables the development of students' protagonism and autonomy in the classroom” (ibid., p. 233).

Silva et al. (2023, p. 8) also conducted a systematic literature review on the application of a Potentially Significant Teaching Unit (PSTU) through the Flipped Classroom (FC) to teach Polyhedra. In this review, “based on the search for articles in national journals indexed in Google Scholar,” these authors found that there were no publications in the search field and period they defined. Thus, it underscores the need to reflect on the applicability of the PSTU associated with FC in teaching Polyhedra, considering its potentialities and limitations within the public-school context.

In line with Moreira (2011a), who states that teaching and learning are intrinsic and inseparable processes, we aim to contribute to discussions on the possibilities offered by active methodology in the field of geometry teaching in High School, through the design, application, and analysis of evidence of meaningful learning and conceptual evolution. Guided by the principles of a potentially significant material, we also explore the potentialities of the PSTU through the FC.

Therefore, in this paper, we present the results obtained from the application of the Potentially Significant Teaching Unit (PSTU) on Polyhedra in a high school class, using the Flipped Classroom (FC) methodology. For organizational purposes, the text outlines the methodological approaches, the characterization of the PSTU in an FC environment, the analysis of the results, and, finally, the concluding reflections.

Methodological Approaches

For this proposal, we conducted a qualitative study based on Minayo (1994). This type of study seeks to answer questions and understand relationships that are not quantifiable. Thus, considering the PSTU as a tool for active engagement in the teaching and learning process, this research falls within a case study approach, as defined by Yin (2003), who states that:

> In general, case studies are the preferred strategy when addressing 'how' and 'why' questions, when the researcher has little control over events, and when the focus is on contemporary phenomena within some real-life context. These *explanatory*[7] case studies can be complemented with two other types – 'exploratory' and 'descriptive' studies" (Yin, 2003, p. 19).

Therefore, the case study focuses on the application of a PSTU through the Flipped Classroom (FC) aimed at facilitating learning about Polyhedra amongst high school students. It is worth noting that the researcher was also the mathematics teacher for the class during the research period, which aligns the investigation with the principles of action research.

Action Research, as outlined by Tripp (2005), is a methodological approach aimed at continuously improving educational practice. This process is cyclical and involves stages of planning, action, observation, and reflection. In the educational context, both teachers and students actively participate in this process, contributing to the ongoing improvement of teaching practice. This approach is participatory and collaborative, encouraging the contribution of all involved in education. Additionally, according to the same author, action

[7] Our emphasis.

research effectively integrates theory and practice, using theoretical foundations to inform and enhance the pedagogical strategies applied in the classroom.

As previously mentioned, the study involved 14 students, aged between 16 and 22 years, from a second-year high school class at a state school with a full-time schedule, located in the City of Estância, approximately 70 km from Sergipe state capital, Aracaju. For this study, the research project was approved by the Research Ethics Committee of the same university to which the Master's program is affiliated, under Opinion No. 5,531,812/2022. Thus, under the Committee's criteria and in respect of anonymity, the participants are identified as students Student 1 through Student 14.

For the application of the PSTU, two modalities were defined, with both in-person and virtual sessions, as provided by the Flipped Classroom Methodology. Therefore, in-person sessions included lectures using slides created in Canva and PowerPoint, as well as group activities and exercise resolution. For the virtual sessions, videos recorded by the teacher and available on YouTube were provided; gamified activities created in Wordwall; on Padlet and Google Forms; in addition to PDF files sent in the class WhatsApp group, with the purpose of engaging students and assessing their learning.

The analysis of the data obtained was based on the participants' responses according to the categories defined based on the research hypotheses and questions. However, for this segment, three analytical categories were selected to illustrate the results by which we sought to evaluate the potentialities related to the application of the PSTU developed for this research: technological dimension and applied resources (videos, texts, slides, exercises), diversified activities (games, investigative experiments, etc.), and the knowledge of the research participants about Polyhedra.

Consequently, the results were analyzed and discussed according to the central objective of the research, aiming to provide reflections on the potentialities of combining the PSTU with the active methodology of the Flipped Classroom for teaching Polyhedra.

Thus, the study adopted an interactive and multimodal methodology to collect and analyze data on students' access to and use of digital technologies. Initially, a questionnaire was distributed via Google Forms to understand the group's profile and assess the quality of their internet access and familiarity with digital tools. During the activity, students' motivation and engagement were monitored, as well as their interactions with the resources and among themselves.

For qualitative analysis, moments recorded in video, photos, and dialogues occurring during discussions were selected. There was a need to administer additional questionnaires emphasizing students' understanding of geometry and knowledge about Polyhedra, aiming to identify meaningful learning based on conceptual evolution. Students' perceptions of the process and the research instrument used to enhance their knowledge were also considered and analyzed.

PSTU in a Flipped Classroom Environment

Flipped Classroom is a form of blended learning that has become established in the educational landscape. The main idea of this methodology is that students study the subject at home and, in the classroom, they already have a basic understanding of the topic proposed (Bergmann and Sams, 2021). Thus, the in-person time becomes a period of exchange between teacher and students for group tasks and discussion of questions related to the subject matter at hand.

For the data collection of this research, one of the stages involved the development of a PSTU guided by the directives of the curriculum documents for secondary education. The planning aimed to associate a tool that promotes meaningful learning with a methodology that encourages active student participation in school tasks. According to Ribeiro (2015), the PSTU is a tool capable of fostering conceptual development, proposing a logical sequence of activities utilizing diverse resources and methods, based on the principles of Meaningful Learning Theory (MLT).

The proposal aimed to create a break from what the class was accustomed to, fostering active participation in the evolution of learning. Consequently, the proposal integrated both online and in-person activities following the steps that define a PSTU according to Moreira (2011a), as follows:

1. Define the Specific Topic to be Addressed: Within the realm of spatial geometry, we chose to focus on Polyhedra. The proposal was presented to the students by the mathematics teacher[8], and then a diagnostic assessment was conducted in person to check prior knowledge regarding basic geometry concepts (intuitive elements, names of geometric figures, net diagrams, concept of regular polygons). Additionally, a questionnaire was provided via Google

[8] As previously mentioned, the researcher was the teacher of this class at the time of the research, which is why we use the term "teacher" in the third person to highlight the pedagogical work carried out.

Forms to profile the class, which served as one of the online data collection moments for the research.

2. Seeking Prior Knowledge: Based on the initial information, an introductory activity was developed with the aim of externalizing prior knowledge related to Polyhedra. These situations act as advance organizers, serving as "cognitive bridges" between what the learner "already knows and what they need to know" (Moreira, 2011, p. 30). In this step, students were asked to create a photo panel on Padlet; they were instructed to photograph objects representing the geometry present in their surroundings and assemble a panel at the provided link. The goal was to identify geometric solids in the photos and classify them as either flat or spatial figures.

In the in-person class, the panel was presented and discussed to assess whether students could recognize, identify, and name a geometric figure based on its overall appearance. This is one of the basic skills related to geometric thinking, which needs to be developed from the early school years and deepened in the final years of Elementary School (Brazil, 2018; Souza, 2021).

3. A Problem Situation: A problem situation was presented to the students as a group work proposal, considering the prior knowledge identified in the previous activities. Each group of students was tasked with constructing a trash bin in the shape of a geometric solid. To complete the construction, each group had to name the chosen solid and its characteristics, as well as identify the amount of material used in its creation and the capacity of the trash bin. This marked the beginning of observing prior knowledge related to area and volume measurement concepts. These were the guiding questions for the discussion generated in the class, without starting to teach the PSTU topic, as outlined by Moreira (2011a). It was an introductory moment for addressing the subject matter (Polyhedra).

For this moment, the planning focused on developing two of the skills established in the National Common Curricular Base: EM13MAT309 and EM13MAT504. The first skill involves solving and creating problems related to area and volume, while the second involves investigating the processes for determining the volume of geometric solids (Brazil, 2018). The proposal, therefore, was to raise questions, propose research, and reflections so that students would be prepared to present their results at a later stage, encompassing another phase of the PSTU.

After discussing how students should approach the activity, a PDF file with a link to a text on the initial ideas of Polyhedra was sent to the class's WhatsApp group, requesting a mind map or summary. The link to a video recorded by the teacher and a playlist of YouTube videos

selected by her on the general approach to the study of Polyhedra were also shared. Additionally, two gamified activities on Wordwall were provided to consolidate learning.

For the video recording, the teacher prepared and used a booklet containing theory and application exercises, which was made available to the students as study material. With this video, booklet, and other provided materials, the research participants had resources to understand the topic, beginning the stage of knowledge proper and comprehending the homework assignment.

In the classroom task, the tasks from step 3 (problem situation) aimed for students to perform activities in a rotational dynamic with investigative elements. The objective was to experimentally consolidate the concept of Euler's relation, the calculation of area and volume measures, and to understand that the volume of a pyramid is one-third of the volume of a prism with the same height.

4. Dialogued Presentation: According to Moreira (2011a), the fourth step of the Potentially Significant Teaching Unit (PSTU) involves presenting the knowledge to be taught/learned, starting with more general aspects and then addressing the more specific ones (progressive differentiation), providing an initial overview before delving into the details.

This stage was dedicated to a lecture planned based on perceptions gathered during the application and resolution of the learning exercises provided in the booklet created and shared by the teacher. At this point, it became necessary to review the calculation of the area of plane figures.

5. Revisiting Structural Aspects: At this stage, the goal is to revisit the concepts studied through problem-solving situations, thus encouraging group work and student interactions. Moreira (2011) suggests that this fifth step of the PSTU should revisit the structural aspects of the subject matter present in the teaching unit but at a higher level of complexity.

For our study, this was the moment when each previously organized group presented their respective trash bin construction related to the chosen geometric solid. Following this, there was a discussion about the research they conducted on plane figures, focusing on relating the experiment to the subject of study – Polyhedra.

6. Continuing the Process of Progressive Differentiation Seeking Integrative Reconciliation: To address this step, exercises were proposed to be completed both at home and in the classroom, using the provided material. Progressive differentiation is crucial for meaningful learning because, through "[...] successive interactions, a given subsumption progressively acquires new meanings, becoming richer, more refined, more differentiated, and

better able to serve as a foundation for new significant learning" (Moreira, 2011, p. 20). According to Moreira, the process of assigning meaning occurs simultaneously with the integration of new knowledge to resolve inconsistencies and is defined as integrative reconciliation.

7. Assessment of Learning: This step of the PSTU involves concluding the unit by revisiting the most relevant features of the content from an integrative perspective, through the proposal of new, more complex problem-solving situations (Moreira, 2011). In our study, in addition to the proposed activities, an individual assessment was also conducted to cover the entire subject studied, to analyze student learning.

8. Evaluation of the PSTU: This stage occurred concurrently with the previous stages, as adjustments were made based on observations and perceptions throughout the implementation. According to Moreira (2011), the evaluation should focus on capturing meanings, understanding, and the development of problem-solving skills progressively, with an emphasis on the process in search of evidence of meaningful learning.

Another important aspect was the evaluation conducted by the research participants, for which a questionnaire was provided via Google Forms. The aim was to gather feedback on the appreciation, opinions, and perceptions of the participants regarding the evolution of their respective learning experiences throughout the application of this teaching approach for the research.

According to Ausubel (2003) and Novak (2000), to evaluate the occurrence of meaningful learning (ML), it is necessary to conduct activities that demonstrate understanding through different languages and diverse contexts, which are distinct from the originally encountered learning material. Demonstrating meaningful learning can be challenging, as genuine understanding involves having clear, precise, differentiated, and transferable meanings.

Thus, in this final moment, a gamified activity was conducted in person. Students were invited to form two groups and compete by answering questions about the topic studied throughout the research period. It was a very participative and engaging moment.

Analysis and Results

The data collected from the implementation of the PSTU will be presented below. The interpretation of the predefined categories helped to evaluate the potentialities of applying a PSTU on Polyhedra in a high school class, based on the methodological work carried out by the Flipped Classroom. It is worth noting that, for this paper, the research as a whole was narrowed down, and the analysis of three predefined categories was presented.

Technological Dimension and Applied Resources

In this category, we aimed to analyze the contribution of the available and applied resources to the learning of the students participating in the research on Polyhedra through a Google Forms survey, while also questioning the use of technological tools. All 14 participants unanimously agreed that these tools and applied resources contributed to their understanding of the subject matter. For these participants, the type of approach used was a positive factor in the development of the proposed activities.

Among the responses, 8 participants highlighted that the videos were very important. This aligns with Bergman and Sams' (2021) findings, which also concluded in their studies that videos support the Flipped Classroom approach. However, it is important to note that limited internet access is a point of limitation for the use of the FC. In our proposal, the fact that not all students had internet access at home highlighted the need, as noted by Sanches et al. (2019), to redirect the planning to include all students—in our case, the research participants.

> It is observed that the time required for the preparation and selection of materials for the Flipped Classroom is an aspect to be considered when adopting this approach. Since these materials must be well-suited to the methodological planning, actions directed towards them must be carried out with care and attention. Strategies for online activities should consider the students' technological knowledge and the quality of their internet access; for in-person activities, among other aspects, it should be taken into account that some students or groups may find the tasks easier than others, and the teacher needs to ensure that no one's development is hindered (Sanches et al., 2019, p. 480).

Given this reality, PDF files were printed, and the textbook was used to include other students. These were alternatives the teacher considered to include all students in the planned proposal, in an attempt to achieve the objectives.

Diversified Activities (games, investigative experiments, etc.)

In investigating how students identified which proposed activities were applied and how contributive they were to their learning, we found that the responses varied slightly. While 2 students were indifferent to the gamified Wordwall activities, 5 reported that these activities contributed significantly, and 7 said they contributed. It is observed that one of the important aspects of the Flipped Classroom, highlighted by Cunha et al. (2019), is the possibility of feedback. In this study, the teacher-researcher was always attentive to making connections between the lessons, showing students the association between the activities being developed.

> The fact of integrating Digital Information and Communication Technologies in teaching does not guarantee learning; it is necessary to have a philosophical foundation

> to understand how the learning process occurs, in which theoretical or practical activities must always be mediated by the teacher. Thus, for better utilization of the class, one can initially focus on resolving doubts and clarifying misunderstandings about the content previously delivered. (Cunha et al., 2019, p. 144)

Another noteworthy aspect is that all students in the class expressed enjoyment for the experimental activities, as well as for the expository lessons (both video-based and in-class explanations). For them, consolidating the study of Polyhedra by comparing the volume calculation of a prism with that of a pyramid made it possible to perceive more concretely and tangibly the relationship between the volume measurements of the Polyhedra used. Only 2 participants indicated that the proposal was indifferent; the others said it contributed greatly to their learning.

Here, as in Santos's (2017, p. 74) study, it was observed that the goal was achieved, since students, starting from intuitive ideas, reached a "broad, generic, and inclusive" conclusion about the concept of volume, characterizing progressive differentiation according to Moreira (2011).

Regarding the application of diversified activities, the students were also asked if these activities contributed to improving their ability to work in groups, self-learning, and practical application of the knowledge. Among the responses, we selected those that highlighted positivity:

> ***Student 2:*** *I think so because most of us could help each other in groups.*
> ***Student 3:*** *Yes. Because with the videos, it makes learning easier.*
> ***Student 5:*** *I believe so, after I understood the topic, it became better.*
> ***Student 7:*** *Yes. With this method, I find it easier to apply my knowledge.*
> ***Student 10:*** *Yes, I can manage my time better by watching video lessons. [...] I think that the question-and-answer games bring competitiveness, which is good in a way.*
> ***Student 11:*** *Yes. Because we are already prepared after watching the videos.*
> ***Student 13:*** *Yes. Because students come to class with prior knowledge*[9].

The students expressed positive opinions about group learning and using videos as a facilitating tool. They highlighted the importance of understanding the content for a more fruitful experience. Time management was mentioned, as well as positive competitiveness. The fact that students had prior knowledge about the topic before it was taught was also valued.

[9] Translator's Note: The authors aimed to maintain the original language presented in the results, thus, writing it in italics. The corrections are pointed out in brackets. The translator preferred to adapt the language to a formal style for better understanding.

Thus, it is considered that the application of the Flipped Classroom had a positive effect, as 50% of the participants highlighted the use of videos prior to the classroom explanation of the subject as an important point for learning. Even though they are used to the more common classroom model, where the information is directly provided by the teacher (in this case, the teacher/researcher), with the Flipped Classroom, they stepped out of their comfort zone and considered it positive to be more prepared for the class; to come to the classroom with prior knowledge; to develop group activities and help each other.

In other words, we observed the positive effects of the Flipped Classroom application, with comprehension activities through different languages and varied contexts; we observed indications of meaningful learning (ML) among our students (Novak, 2000; Ausubel, 2003). Here are the positive points they also highlighted in their responses regarding the lessons on Polyhedra, though there are reservations about some negative points among them.

> ***Student 2:*** *Positive points: More interaction with the student and teacher. Negative points: perhaps a bit difficult to understand.*
> ***Student 3:*** *Positive points: ease of learning and made learning fun. Negative points: for those who don't have an adequate device or even any type of device, it becomes extremely difficult to carry out the proposed activities.*
> ***Student 4:*** *Positive points: I learned much more about Polyhedra and geometry.*
> ***Student 5:*** *I learned things that I didn't know before, and the negative points are that the subject is quite long.*
> ***Student 8:*** *Positive point: the ease.*
> ***Student 10:*** *My difficulties are greater than I thought, but they are not impossible.*
> ***Student 11:*** *Improvement in my learning, the content became simpler.*
> ***Student 13:*** *Greater ease in understanding the subject, a bit complex because everything is very similar.*

However, we cannot forget that there are students who do not understand these contexts and have only pointed out negative aspects, such as:

> ***Student 1:*** *Having only the activities online can be both a good and a bad idea because of those who live far away and only use mobile data.*
> ***Student 4:*** *No, because the classroom doesn't cooperate, and any topic is already a reason for ruckus*[10].
> ***Student 8:*** *Negative point: a bit of difficulty.*
> ***Student 11:*** *[There are] people without access to a cell phone and/or the Internet, and these people cannot benefit from the new methodology.*

[10] Translator's note: The original word "zuada" means the presence of a lot of noise in a given environment; a term widely used in Sergipe, meaning a noisy and disorderly environment.

These student responses reflect a variety of experiences related to using different teaching and learning methods. While they highlight positive aspects, they also emphasize the challenges involving individual difficulties and diverse contexts, such as limited internet access. These challenges present significant disadvantages regarding the use of the FC approach.

We observe that these testimonies should be considered so that teachers wishing to use FC can think carefully about their planning. Even though the class, in general, appears enthusiastic and motivated to carry out the activities, we found that the participants themselves revealed it is not always possible to fully achieve the planned objectives. The teacher's planning and organization are crucial in these aspects to balance the diversity of activities. Therefore, during the implementation, adjustments were necessary to make the planning flexible while considering the research objectives.

It is essential to ensure that students, by participating in all activities, understand the significance of the subjects they are learning in each class. Therefore, it becomes important to be diligent and careful in always connecting the activities to show students that having prior knowledge facilitates new learning. In the next category, we aim to illustrate what they reported learning about Polyhedra.

Students' Knowledge of Polyhedra

To assess learning, questions focused on the progress in their geometry knowledge, specifically regarding Polyhedra. Among the students, 50% responded to the questionnaire very succinctly, with vague answers: just "Yes" (Student 3, Student 4, Student 7, Student 8, Student 9, Student 10, and Student 12) and one "I consider it average" (Student 1). The others detailed their responses, showing whether they had prior knowledge about this geometric object, how much they needed to study, or what they still need to delve deeper into to better understand this subject.

> ***Student 2:*** *To be honest, not so much. I'm still developing my understanding of this subject.*
> ***Student 5:*** *Yes, I did study a lot about the subject.*
> ***Student 6:*** *Yes, I learned a little, I had a lot of difficulty.*
> ***Student 11:*** *Yes. Because I didn't know much about the topic of "polyhedra" and now I understand the subject much better.*
> ***Student 13:*** *Yes. I had minimal knowledge about Polyhedra before the start of the lessons, now I feel comfortable doing activities on the subject.*
> ***Student 14:*** *I feel some difficulties regarding this subject.*

Obtaining such responses is important not only as a research result but also for the students themselves to reflect on their self-knowledge. Among the competencies established in the BNCC, self-knowledge and the development of student autonomy in relation to their learning are highlighted. In this sense, questioning students during and after the application of activities or even assessing the object of study allows them to develop reflection on how much they have learned or need to improve in terms of responsibility and commitment to their studies for their self-development. It is a way for them to demonstrate and, at the same time, recognize whether or not they are prepared for an evaluation.

However, we observed responses that expressed individual perspectives, providing us with insights into the direction of more effective pedagogical practices, considering the students' needs and different levels of knowledge. In other words, educational practices that promote students' autonomy and decision-making in their learning process. Thus, the participants in our research, when asked whether there were advantages or disadvantages regarding the lesson model they experienced, provided different justifications:

> ***Student 1:*** *Yes. The disadvantage is that some people do not have access to the internet. The advantage is that it is easier to understand.*
> ***Student 3:*** *Yes. Advantages: it improved my understanding of the content and allowed us to work better in groups. Disadvantages: (for now, none).*
> ***Student 4:*** *Yes, the slides and videos helped a lot in understanding the topics.*
> ***Student 7:*** *Advantage, as everyone has their own way of focusing and learning better.*
> ***Student 10:*** *The advantage is that we learn more easily. The disadvantage is that it gets a bit disorganized.*
> ***Student 11:*** *The new methodology helped me understand the subject better, and so far, I haven't seen any disadvantages.*
> ***Student 13:*** *It helped a lot, but the students were more distracted in class.*
> ***Student 14:*** *Yes, it helped a lot with development.*

The students' perspectives reveal a variety of advantageous experiences, including ease of understanding, improved comprehension of the subject studied, and the possibility of group work. In other words, responses such as "helps a lot" and "better understand and comprehend the subject" indicate that these research participants had a favorable experience with the new methodology, which valued aspects like challenge, dynamics, and the innovative approach, as highlighted below.

> ***Student 1:*** *What I liked was the slight sense of challenge.*
> ***Student 3:*** *Mainly the new teaching methodology.*

Student 5: *I liked everything. [...] The idea of the flipped classroom and environment were very good.*
Student 10: *No, I thought the dynamics were cool.*
Student 11: *There was nothing I didn't like. [...] I particularly liked everything.*
Student 14: *It was cool, a different experience.*

For these participants, the activities contributed to their learning, providing a different experience in a positive, dynamic, and challenging way. In this context, others also revealed that the Flipped Classroom strategy enabled an environment of: "*Student interaction*" (Student 4 and Student 6); *cooperation* (Student 7); "*competitiveness*" (Student 10); "*student independence*" (Student 13); and "*development for all*" (Student 14).

This aligns with the previously discussed assumptions of the Flipped Classroom. According to Moreira (2011), when a teacher focuses on creating activities that build cognitive bridges through successive interactions with their students, a subsumption knowledge allows these students to revisit structuring aspects of the subject matter being taught. Thus, using the Flipped Classroom, although it may appear chaotic compared to more traditional classroom models, provides students with the opportunity to acquire new meanings. Consequently, their knowledge becomes richer and serves as a foundation for new meaningful learning, both for the current subject and for other new concepts that may be related and for other subjects they will study in the future.

However, there is another side where disadvantages are also evident, as mentioned at other times; the limited access to the internet is notably highlighted, and there is also the potential for disorganization. This disorganization implies the force of habit, as they are used to experiencing more routine classroom models, often with the teacher as the main figure who announces, explains, and exemplifies the subject matter without even questioning what the students may know about the subject.

From these students' perspective, it seems possible that the class should take place in an "*organized and quiet*" environment, as they state: "*I didn't like the chaos*" (Student 1); "*Out-of-context conversations and unnecessary fights*" (Student 4); "*Lack of attention from some*"

(Student 7); "*Sometimes students didn't have a complete understanding of the subject*" (Student 13).

Based on these excerpts, a conception emerges where the teacher is the center of the educational process, considered by several authors as a traditional teaching view. Firstly, it is the teacher who explains the concepts, presents examples, and the student listens quietly, copying what is presented on slides or on the board. Then, the student responds to exercise lists based on models presented by the teacher or found in their textbook. If they respond without help from peers or the teacher, they come to understand that they have learned, potentially succeeding in evaluations. However, in later situations, they may not remember the studied concepts or even make significant connections with their daily lives or other areas of knowledge. Such situations are often reflected in issues related to geometry teaching in research, as teachers usually do not contextualize or associate geometric objects with each other.

This traditional thinking contradicts what is established in the current curriculum document – the BNCC (National Common Curricular Base) – regarding the skills to be developed by high school students in mathematics. In this proposal, we highlight two skills mentioned earlier (EM13MAT309 and EM13MAT504). These are skills that involve the study of Polyhedra through problem-solving and the investigation of processes for calculating area and volume measures, for example.

Thus, based on the collected data, we can see that the teaching proposal using the Flipped Classroom (FC) approach achieved its goal of providing students with prior contact with the material, leaving class time for content deepening and interactive activities, as proposed by Bergmann and Sams (2021). The responses obtained, highlighting both advantages and disadvantages, indicate that this pedagogical practice can also foster a reflective movement among students.

Given the three categories selected for this analysis, the positive impact of FC is evident, from how much participants enjoyed the applied activities, pointing out pros and cons, to their reflections on their own knowledge about the study of Polyhedra. Among these aspects, it is

noteworthy how they appreciated having some knowledge about the subject before it was taught in class.

However, the challenges faced by some students in accessing the material sent by the teacher, due to not having a cellphone and/or internet access at home, remain a point of concern. This was a significant limitation as it compromised the fundamental aspects of implementing the Flipped Classroom.

Our proposal reflects and reaffirms Ribeiro's (2015) studies on how the Potentially Significant Teaching Unit (PSTU) provides logical and interactive material that engages and mobilizes students to learn. Below are our final reflections.

Final Thoughts

Based on the students' conceptions, we sought to evaluate the potential of applying a PSTU on Polyhedra in a high school class, utilizing the Flipped Classroom (FC) methodological approach. This involved planning and implementing a sequence of activities grounded in the principles of TAS, using various digital resources and diversified strategies. According to Moreira (2011a) and Valente (2018), this methodology requires well-structured and planned material that adheres to both the FC and PSTU models.

By combining the PSTU with the Flipped Classroom in teaching Polyhedra, we observed contributions to the educational process on this topic, such as contextualization, autonomy, and better utilization of face-to-face class time. The PSTU facilitated a more meaningful, sequential, and contextualized learning experience for students, while the Flipped Classroom allowed them to access information prior to the formal instruction of the knowledge object. These actions enabled students to reflect on the topic of Polyhedra and make connections with their daily lives before delving into the concepts, properties, and theorems related to the study.

Students, feeling motivated, took responsibility for seeking the information necessary for their learning. This allowed them to develop skills such as research, selection of relevant information, problem-solving, and time management. We understand that combining these two approaches provided students with the opportunity to experience an innovative, dynamic, and

interactive learning environment. Although this strategy may not capture all ideas, it served as a reference to define steps and organize the material, as well as the methodological paths to follow.

It is also important to highlight the limitations found in this pedagogical practice. Flipped Classroom requires students to have access to electronic devices and internet to read the subject before class and to be motivated to do so in preparation for in-person sessions. This was a significant limitation; the essential aspects of implementing the Flipped Classroom were compromised. There were instances of students without cell phones or access to the school's internet.

The solution was for the teacher to provide these students with a mathematics textbook covering the subject, print the activities being sent in the WhatsApp group, and make available equipment and internet access at the school for those without access to complete the questionnaires. The students' self-study habits are something to be thought through.

The Flipped Classroom (FC) may be unfeasible for those who have difficulty learning independently or lack resources. The students may not feel motivated to study theoretical material outside the classroom, which can hinder the implementation of the Potentially Significant Teaching Unit (PSTU). This was also observed during the application. Another issue is that FC assumes students have learned the subject before class, which may not be true for everyone. This can create learning gaps that are difficult to identify and fill during the practical activities of the PSTU.

Therefore, we observe that, in order for this methodology to work, we need to be more flexible regarding expectations and deadlines for students, extending the teaching process. For the teacher's planning, it is necessary to prepare the didactic materials and make them available in advance, closely monitor progress, and provide timely feedback to guide the next steps. This involves a series of activities that go beyond in-person and/or online sessions.

This combination aimed to promote active student engagement in learning, which is an important aspect of PSTU and FC. We consider that we achieved our objective, as part of the class felt challenged and viewed the proposed activities as an interesting practice that fostered greater integration among students and between them and the teacher. The personalization of

learning was also observed as a potentiality, allowing students to learn theoretical content at their own pace; they had more time to reflect and assimilate the material before in-person sessions.

From the observations and analyses, we understand that this pedagogical proposal generated an understanding of the study object – Polyhedra. It contributed to student protagonism in knowledge construction. The activity involving the construction of a trash can demonstrated that these students were able to non-arbitrarily and substantively relate new concepts to their prior knowledge. Thus, we consider that during the implementation of the PSTU, both the chosen methodological aspects and the theoretical assumptions of the Theory of Meaningful Learning contributed to the successful construction and assimilation of concepts.

Our aim is not to generalize the results, as this is a case study. We believe we have confirmed our hypothesis that the FC model combined with PSTU positively contributed to the conceptual advancement related to learning about Polyhedra. We hope that this study will continue by other teachers, including expanding to more subjects, different educational levels, and various school realities, to deepen analyses and discussions on the cognitive processes that PSTU associated with the Flipped Classroom can provide.

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