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Subsidies for the development of an epistemological reference model for the understanding of the concept of ordinary differential equations

Subvenciones para el desarrollo de un modelo epistemológico de referencia para la comprensión del concepto de ecuaciones diferenciales ordinarias

Subventions pour le développement d'un modèle de référence èpistémologique pour la compréhension du concept d'équations différentielles ordinaires

Subsídios para o desenvolvimento de um modelo epistemológico de referência para a compreensão do conceito de equações diferenciais ordinárias

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Abstract

The special issue of the journal Educação Matemática Pesquisa (EMP) presents the question: How to develop an Epistemological Reference Model (ERM) for the teaching of Calculus? To help answer this question, we present an excerpt from ongoing doctoral research on Ordinary Differential Equations. As factors to be considered in an ERM, we highlight the contributions of Critical Mathematics, which highlights an analysis of school failure, with a specific focus on mathematics, and how this is related to curricular practices and dominant underlying epistemological models, which emphasize the transmission of knowledge in a passive and decontextualized way. The central idea is that these models should not be passively accepted, but rather questioned and constantly revised. In this study, some historical elements on the subject were considered, as well as difficulties and advances in the teaching and learning process, taking the results of some research. The evolution of the concept of Ordinary Differential Equations over time is intrinsically linked to the contributions of mathematicians in their definition and understanding. Modeling and the theory of Registers of Semiotic Representation made it possible to compose a scenario that evidenced the necessary knowledge

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for the teaching of ordinary differential equations. The technological knowledge present in the research comprises the knowledge of GeoGebra that helped in the teaching and learning process of this theme. Preliminary studies of a Dominant Epistemological Model (DEM) on a mathematical object, in this case Ordinary Differential Equations, may have relevant implications, as it motivates the construction of an ERM.

Keywords: Mathematics education, Teaching of ordinary differential equations, Critical mathematics, Epistemological reference model.

Resumen

El número especial de la revista Educação Matemática Pesquisa (EMP) presenta la pregunta: ¿Cómo desarrollar un Modelo Epistemológico de Referencia (MER) para la enseñanza del Cálculo? Para ayudar a responder esta pregunta, presentamos un extracto de una investigación doctoral en curso sobre Ecuaciones Diferenciales Ordinarias. Como factores a considerar en un MER, destacamos los aportes de la Matemática Crítica, que destaca un análisis del fracaso escolar, con un enfoque específico en las matemáticas, y cómo este se relaciona con las prácticas curriculares y los modelos epistemológicos subyacentes dominantes, que enfatizan la transmisión del conocimiento de manera pasiva y descontextualizada. La idea central es que estos modelos no deben ser aceptados pasivamente, sino cuestionados y revisados constantemente. En este estudio se consideraron algunos elementos históricos sobre el tema, así como dificultades y avances en el proceso de enseñanza y aprendizaje, tomando los resultados de algunas investigaciones. La evolución del concepto de Ecuaciones Diferenciales Ordinarias a lo largo del tiempo está intrínsecamente ligada a las aportaciones de los matemáticos en su definición y comprensión. Modelización y la teoría de los Registros de la Representación Semiótica permitió componer un escenario que evidenció los conocimientos necesarios para la enseñanza de las ecuaciones diferenciales ordinarias. El conocimiento tecnológico presente en la investigación comprende el conocimiento de GeoGebra que ayudó en el proceso de enseñanza y aprendizaje de este tema. Los estudios preliminares de un Modelo Epistemológico Dominante (MED) sobre un objeto matemático, en este caso las Ecuaciones Diferenciales Ordinarias, pueden tener implicaciones relevantes, ya que motivan la construcción de un MER.

Palabras clave: Educación matemática, Enseñanza de ecuaciones diferenciales ordinarias, Matemática crítica, Modelo epistemológico de referencia.

Résumé

Le numéro spécial de la revue Educação Matemática Pesquisa (EMP) présente la question suivante: Comment développer un modèle de référence épistémologique (MRE) pour l'enseignement du calcul? Pour aider à répondre à cette question, nous présentons un extrait d'une recherche doctorale en cours sur les équations différentielles ordinaires. En tant que facteurs à prendre en compte dans MER, nous soulignons les apports des mathématiques critiques, qui mettent en évidence une analyse de l'échec scolaire, avec un accent particulier sur les mathématiques, et comment cela est lié aux pratiques curriculaires et aux modèles épistémologiques sous-jacents dominants, qui mettent l'accent sur la transmission des savoirs de manière passive et décontextualisée. L'idée centrale est que ces modèles ne doivent pas être acceptés passivement, mais plutôt remis en question et constamment révisés. Dans cette étude, certains éléments historiques sur le sujet ont été pris en compte, ainsi que les difficultés et les avancées dans le processus d'enseignement et d'apprentissage, en prenant les résultats de certaines recherches. L'évolution du concept d'équations différentielles ordinaires au cours du temps est intrinsèquement liée aux contributions des mathématiciens dans leur définition et leur compréhension. La Modélisation et la théorie des Registres de Représentation Sémiotique a permis de composer un scénario qui mettait en évidence les connaissances nécessaires à l'enseignement des équations différentielles ordinaires. Les connaissances technologiques présentes dans la recherche comprennent les connaissances de GeoGebra qui ont aidé dans le processus d'enseignement et d'apprentissage de ce thème. Les études préliminaires d'une Modèle Épistémologique Dominant (MED) sur un objet mathématique, en l'occurrence les équations différentielles ordinaires, peuvent avoir des implications pertinentes, car elles motivent la construction d'un MRE.

Mots-clés : Enseignement des mathématiques, Enseignement des équations différentielles ordinaires, Mathématiques critiques, Modèle de référence èpistémologique.

Resumo

O número especial da revista Educação Matemática Pesquisa (EMP) apresenta a questão: Como elaborar um Modelo Epistemológico de Referência (MER) para o ensino de Cálculo? Para colaborarmos com a resposta a essa questão, apresentamos um excerto de uma pesquisa de doutorado em andamento sobre Equações Diferenciais Ordinárias. Como fatores a serem consideradas em um MER ressaltamos as contribuições da Matemática Crítica que destaca uma análise sobre o fracasso escolar, com um foco específico na matemática, e como isso está relacionado às práticas curriculares e aos modelos epistemológicos subjacentes dominantes, os quais enfatizam a transmissão de conhecimento de forma passiva e descontextualizada. A ideia central é que esses modelos não devem ser aceitos passivamente, mas sim questionados e revisados constantemente. Neste estudo foram considerados alguns elementos históricos sobre o tema, assim como dificuldades e avanços do processo de ensino e de aprendizagem tomando resultados de algumas pesquisas. A evolução do conceito de Equações Diferenciais Ordinárias ao longo do tempo, está intrinsecamente ligada às contribuições dos matemáticos na sua definição e compreensão. A modelagem e a teoria dos registros de representação semiótica possibilitaram compor um cenário que evidenciasse os conhecimentos necessários para o ensino de equações diferenciais ordinárias. O saber tecnológico, presente na pesquisa, compreende o conhecimento do GeoGebra que auxiliou no processo de ensino e de aprendizagem deste tema. Estudos preliminares de um Modelo Epistemológico Dominante (MED) sobre um objeto matemático, no caso Equações Diferenciais Ordinárias, podem decorrer em implicações relevantes, na medida em que motiva a construção de um MER.

Palavras-chave: Educação matemática, Ensino de equações diferenciais ordinárias, Matemática crítica, Modelo epistemológico de referência.

The Dominant Epistemological Models (DEM) and the Reference Epistemological Models (REM) represent different approaches to understanding and constructing knowledge in a given area, such as mathematics. Reflection on these two models and their relationships is fundamental to understanding how knowledge is constructed and transmitted in the educational context.

The Dominant Epistemological Model (DEM) refers to the approach that prevails at a given moment in educational history and practice. It represents the prevailing conceptions about how knowledge is produced, validated, and transmitted. In the context of mathematics, for example, the Dominant Epistemological Model may be associated with a more formalistic or content-based view, where the emphasis is placed on memorizing formulas and procedures, without necessarily exploring conceptual connections and practical applications.

On the other hand, the Epistemological Reference Model (ERM) is an alternative approach, which seeks to question and expand the conceptions established by the dominant model. It represents a more critical and reflective view of knowledge, emphasizing the importance of contextualization, interdisciplinarity and the active construction of knowledge by the student. In the case of mathematics, an Epistemological Reference Model can promote a more inquiring approach, where students are encouraged to explore mathematical concepts through real-world problems, group discussions, and experimentation.

Gascón (2014) indicates that a ERM is a set of ideas, principles and approaches that can be considered to compose theoretical or methodological frameworks for the study of a specific area of knowledge, in this case, the teaching of Mathematics.

The relationships between these two models are dynamic and complex. In many cases, the Epistemological Reference Model emerges as a response to the limitations perceived in the dominant model, seeking to overcome its weaknesses and promote a more inclusive and meaningful approach to knowledge. However, the dominant model may resist change and challenge the adoption of new pedagogical practices, especially if they represent a break with established traditions.

It is important to highlight that epistemological models are not necessarily mutually exclusive. They can coexist and interact in different ways in the educational context, influencing both teachers' practice and students' learning experience. An effective approach to improving the teaching and learning of mathematics, involves critically reflecting on these models, recognizing their potentialities and limitations, and seeking to integrate elements of both to create a richer and more meaningful educational experience.

In this aspect, we can consider Skovsmose (2001) who offers a critical analysis of school failure, with a specific focus on mathematics, and how this is related to curricular practices and the underlying epistemological models. Although the author does not directly address the terms "dominant epistemological model" and "epistemological reference model" in his work, he does discuss concepts and approaches that are related to these models.

Skovsmose (2001) argues that school failure is not just an individual issue, but a complex social and cultural phenomenon, influenced by several factors, including educational practices and conceptions of knowledge and learning. In this sense, he criticizes traditional approaches to teaching mathematics that are based on Dominant Epistemological Models, which emphasize the transmission of knowledge in a passive and decontextualized way.

By proposing a critical analysis of curricular reforms in mathematics, Skovsmose (2001) highlights the need to question and reformulate the epistemological models underlying these educational practices. He advocates for a more reflective and contextualized approach, which recognizes mathematics as a culturally situated activity and promotes a more inclusive and democratic view of mathematical knowledge.

The "Critical Mathematics" approach proposed by Ole Skovsmose is closely linked to the analysis and criticism of both the Dominant Epistemological Model and the Epistemological Reference Model in mathematics education.

DEM in mathematics education is often based on a traditional view of mathematics as a neutral and objective discipline, where knowledge is transmitted in a hierarchical way by the teacher to the students. This model tends to emphasize the memorization of formulas and procedures, often disregarding the conceptual connections and practical applications of mathematics. Skovsmose (2001) challenges this view, arguing that mathematics is a culturally situated activity, shaped by dominant values, beliefs, and powers.

On the other hand, the ERM in mathematics education proposes an alternative and critical approach to the dominant model. It seeks to question established conceptions about mathematical knowledge and promote a more inclusive and contextualized view of the discipline. In this sense, Skovsmose's "Critical Mathematics" can be considered an expression of this ERM, which values the critical analysis of mathematical and pedagogical practices, as well as of social and political relations.

Therefore, Skovsmose's Critical Mathematics approach represents an alternative to both DEM and ERM in mathematics education. It challenges the assumptions underlying these models, promoting a more reflective, inclusive, and socially engaged view of the discipline. This approach has the potential to empower students to become critical thinkers and agents of change in their communities.

Skovsmose (2001) also proposes the work with modeling in mathematics education, inspired by Rodney Bassanezi, as a possible way for the democratic aspect to be presented in the classroom.

> [...] It is extremely important for students to learn about building models, and the best way to learn this is by building models. The pragmatic tendency is built on a philosophical assumption about mathematics, which states that an essential aspect of mathematics is its usefulness (completely contrary to structuralist and formalist philosophy, which states that the essential aspect of mathematics is its "logical architecture") (Skovsmose, 2001, p. 40)

The development of ERM can be considered as an alternative to the dominant epistemology in mathematics teaching. He proposes a more critical and reflective approach, where not only the mathematical content itself is analyzed, but also the processes of didactic transposition, that is, how this knowledge is transformed and transmitted in the classroom.

Epistemological models are essentially assumptions or structures of thought that underlie how mathematical knowledge is constructed and transposed. The central idea is that these models should not be accepted passively, but rather questioned and reviewed constantly.

One of the keys to this model is the ability to critically analyze official documents, teachers' and students' conceptions, textbooks, study plans, and other pedagogical practices. This allows for an initial construction of the model that is more aligned with the needs and challenges faced in the specific educational context.

In this text we present an excerpt from ongoing doctoral research on Ordinary Differential Equations (ODE), in which some historical elements are considered, as well as difficulties and advances in the teaching and learning processes, taking results from some research and that can be considered as subsidies for the construction of a ERM.

The evolution of the concept of ordinary differential equations over time is intrinsically linked to the contributions of mathematicians in defining and understanding them. As the concept of differential equations has expanded and refined, it has also opened up new horizons of application in other fields. Historical research can allow the analysis and adjustment of particular epistemological models of a specific area of mathematical practice (Gascón, 2014). Understanding the historical context allows you to obtain information about the essence of the concept, how it affects its teaching and learning, and helps in the construction of ERM.

The DEM represents the practices and conceptions of mathematics teaching that prevail in each historical and cultural context. In the case of ODE, the DEM emphasizes algebraic approaches and the use of standardized solving methods, where students focus on solving equations without necessarily understanding the relationship between the solutions and the context that originated the equation. Thus, teaching tends to be decontextualized and focused on the passive transmission of techniques.

The ERM proposes an alternative that values the active construction of knowledge by students, incorporating contexts and practical applications. The construction of ERM for the teaching of ODE can be based on a historical analysis, in which the evolution of the concept over time and its various applications in physics, engineering and other sciences are considered. This model values the use of technologies such as GeoGebra and mathematical modeling, enabling an investigative approach that contextualizes concepts and helps students

The theoretical framework of the research was based on Duval's (1993) theory of Semiotic Representation Registers, which can help to compose a scenario that evidences the knowledge that conditions the construction of the knowledge necessary for the teaching of Ordinary Differential Equations and bring subsidies for the development of ERM. The technological knowledge present in the research comprises the knowledge of the interface, functionality and characteristics of GeoGebra that can help in the teaching and learning processes of this theme, also inspiring for the development of ERM.

By articulating these elements with the teaching of Calculus, it is perceived that the ERM allows a critical and inclusive view, where mathematical knowledge is understood as a dynamic and culturally situated process. Duval's Theory of Semiotic Representation Registers complement this approach as it allows students to transition between different representations (algebraic, graphic, numerical) and understand the concept of ODE in an integrated way.

Therefore, the construction of ERM for the teaching of Calculus with a focus on ODE requires overcoming the traditional DEM. It should include historical elements, mathematical modeling, and technological resources, promoting an education that encourages critical interpretation and the practical application of knowledge, expanding students' mathematical understanding and making it more meaningful in the educational context.

Preliminary studies of a Dominant Epistemological Model (DEM) on a mathematical object, in the case of Ordinary Differential Equations, may have relevant implications, as it motivates the construction of an Epistemological Reference Model (ERM) for the exploration not only of the mathematical content itself, but also for the processes of didactic transposition.

Historical elements and research results on Differential Equations

A DEM can present some historical elements, as well as difficulties and advances in the teaching and learning process, taking results from some research, to be considered as subsidies for the construction of an ERM.

A historical perspective for a comprehensive understanding of OD is necessary to be carried out, as it can highlight the evolution of knowledge, the influences received and the debates that have taken place over time.

The emergence of differential equations is related to the development of calculus in the late XVII and plays a crucial role in many of the fundamental achievements in mathematics and science. The history of differential equations begins with the work of two prominent mathematicians: Isaac Newton (1642-1727) and Gottfried Wilhelm Leibniz (1646-1716), who developed the concepts of derivative and integral, albeit in different ways, creating the foundations of what would come to be known as differential and integral calculus.

Newton used differential equations primarily to describe mechanical problems, such as the motion of planets and other bodies under the influence of gravity. He introduced the concept of rate of change and formulated laws of motion that could be expressed mathematically through differential equations.

Leibniz, independently, developed calculus techniques and applied them to various problems of tangents and areas, which also led to the development of differential equations. The notation he used for differential calculus is widely used to this day and facilitates the calculation of derivatives and integrals

During the XVIII century, differential equations became an essential tool in physics and engineering. Mathematicians such as Leonhard Euler (1707–1783), and Joseph-Louis Lagrange (1736–1813) expanded the use of differential equations, applying them to more problems in mechanics.

According to Boyer and Merzbach (2012), the Swiss mathematician and physicist Euler contributed to the methods for solving linear and nonlinear differential equations that are found in differential equation courses, and many of the problems found in textbooks can be traced back to the great treatises that Euler wrote on calculus.

Joseph-Louis Lagrange was a XVIII century Italian mathematician and astronomer whose contributions had a significant impact on the formulation and solution of differential equations, especially in the field of mechanics. Lagrange is known for his work in analytical mechanics, which reformulated Newton's classical mechanics that employs vector quantities such as force, velocity, and acceleration, introducing a new formalism that utilizes scalar quantities such as energy and generalized coordinates.

The development of differential equations took a significantly new direction with the contribution of Henri Poincaré (1854-1912) in the late XII and early XX centuries. Poincaré pioneered the qualitative approach to differential equations, which focuses more on the general properties of solutions and the geometry of trajectory trajectories in phase space than on exact and specific solutions.

Many problems involving motion, growth rates, electricity, and other physical or biological phenomena that exhibit rates of change are often modeled using Ordinary Differential Equations (ODE). Therefore, the study of these equations establishes connections between Mathematics and other sciences, creating an environment conducive to learning through contextualization.

Another aspect to be considered is the inclusion of the discipline of Differential Equations in the curricular matrices of higher education institutions, which vary according to the Pedagogical Project of each course. Differential Equations is often offered as a stand-alone course that covers fundamental concepts of ordinary and partial differential equations, as well as analytical and numerical methods of solving and their applications in various areas. However, in courses such as Engineering, this discipline can be integrated with Differential and Integral Calculus, dealing with the topics of differential equations in the broader context of functions, limits, derivatives and integrals. This choice depends on the philosophy of the course, the specific objectives of the program and the needs of the students and can result in a more detailed and in-depth approach when offered independently, or more contextualized and applied when integrated into calculus.

In some Higher Education Institutions, the curricular components are offered every semester, and the course is organized in sequential periods, where students advance from one period to the next semester.

In the bachelor's degree in civil engineering at the Palmas Campus of the Federal Institute of Tocantins (FITO), the ODE content is integrated as part of the discipline of Differential and Integral Calculus II (DIC II). This discipline, offered in the second period of the course, has a total workload of 60 hours. According to Brasil (2023), the curriculum planning of this institution is structured based on prerequisites, requiring students to complete the DIC I course, also with 60 hours, before progressing to DCI II.

At the Federal University of Bahia (FUBA), in the Mechanical, Chemical and Civil Engineering courses, the Ordinary Differential Equations (ODE) are addressed as the only topic of the Calculus C discipline, offered in the third period of the course. Similar to the model adopted by the Federal Institute of Tocantins (FITO), the Federal University of Bahia (FUBA) structures its curriculum based on a logic of prerequisites. To enroll in Calculus C, students must first complete the disciplines of Calculus B and Linear Algebra, both offered in the second period. In addition, Calculus B requires students to have already taken Calculus A and Analytic Geometry, which are taught in the first term of the course. All these disciplines have a workload of 102 hours.

At the Federal University of Piauí (UFPI), located in Teresina, the bachelor's degree in mathematics and mathematics' degree courses are organized according to a logic of prerequisites, as detailed in the 2017 Pedagogical Course Projects (PCP). Both courses share a similar structure in the core mathematics disciplines, such as Differential and Integral Calculus, Analytic Geometry, Linear Algebra I and Elements of Mathematics I, all with a workload of 90 hours.

In the Mathematics Degree course at the State University of Goiás, located at the Cora Coralina Campus, in the city of Goiás, the curricular matrix adopted in 2015 structures its disciplines, except for those related to the Internship and the Course Completion Work, with a standard workload of 60 hours. A notable point of this course is that it does not follow a system of prerequisites for the subjects, differentiating itself from many other higher education institutions that adopt this practice. The ODE is studied in the discipline "Applied Differential Equations", which is offered in the sixth period of the course.

The analysis of the curricula of Mathematics Teaching Degree and bachelor's degree courses in higher education institutions reveals that ODE are generally introduced in the last periods. This curricular structure is adopted due to the need for students to have a solid foundation in Calculus, acquired in previous disciplines, essential for a better understanding of such equations. In these courses, they are often treated as a stand-alone discipline, highlighting their complexity and importance.

In contrast, in Engineering courses, fundamental subjects such as Differential and Integral Calculus I and II or Calculus A, B and C are taught in the first periods. In these contexts, ODE is often incorporated within these disciplines of Calculus, either as an isolated topic or in conjunction with other concepts.

As Boyce (2020) points out, the structuring of courses that include Ordinary Differential Equations (ODE) must consider a solid background in Calculus and Linear Algebra. The author emphasizes that the contents of Calculus are fundamental and should be taught over two to three semesters to adequately prepare students. In addition, Linear Algebra is highlighted as essential due to its use in the matrix approach to solving systems of differential equations.

In the Higher Education Institutions examined, most follow the practice of establishing prerequisites, with the exception of the mathematics degree course at the State University of Goiás, which adopts a more flexible approach. In addition, calculus disciplines, considered essential by Boyce, are a constant in all the courses analyzed, reinforcing the importance of these mathematical foundations as suggested by the author.

Research such as Dullius, Veit and Araujo (2013) and Alvarenga, Dorr and Vieira (2016) point out that the approach adopted in the teaching of ODE is predominantly algebraic, where students learn resolution methods without analyzing the behavior of the solutions obtained. This results in students failing to properly interpret the terms of the equations, not recognizing the connection between the ODE and the real systems they model, and not performing a qualitative analysis of the problem that originated the equations.

A possible scenario for ERM on Ordinary Differential Equations

Raymond Duval's theory of Semiotic Representation Registers can support the construction of a scenario that evidences the knowledge that conditions the teaching of ODE, indicating subsidies for the development of a ERM. This scenario can also involve technological knowledge, such as the use of GeoGebra to understand its functionalities and characteristics and assist in the teaching and learning process of this topic. Mathematical Modeling, too, can contribute as a methodological strategy.

According to the Theory of Semiotic Representation Registers,

> Semiotic representations are productions constituted using signs belonging to a system of representation, which have their own restrictions of meaning and functioning (Duval, 1993, p. 39).

There are three cognitive activities that characterize the semiotic representation registers, the first being the formation of an identifiable representation that can be determined through an understandable sentence, a drawing, a figure, a formula, among others. The second, the treatment of a representation, which is the transformation of this representation in the record itself, such as algebraically resolving the ODE.

When the representation of a mathematical object is transformed into another representation in another register, conversion occurs, as for example, when there is a graphical representation of a function obtained from an algebraic representation. Therefore, the processing takes place within the same registry and the conversion between different registries.

This theory provides support both in cognitive terms, as it allows us to understand how knowledge acquisition occurs, and in didactic/methodological terms, when it addresses the way learning is processed.

To contribute to the teaching and learning of ODE, Mathematical Modeling can be a methodological strategy because it performs, among other things, an interaction between the "real world" and mathematics, as Bassanezi emphasizes:

> Mathematical Modeling is a dynamic process used to obtain and validate mathematical models. It is a form of abstraction and generalization for the purpose of trend forecasting. Modeling essentially consists of the art of transforming reality situations into mathematical problems whose solutions must be interpreted in the usual language (Bassanezi, 2022, p.24).

The research by Dullius, Veit and Araujo (2013) brings evidence that Mathematical Modeling can be a facilitator of learning because teaching activities in a modeling environment brings out several mathematical and extra-mathematical concepts, favoring learning.

Complementing this teaching methodology, the use of Information and Communication Technologies (ICT) can also be considered. According to Marin (2008, p. 138), it is in the development of classes that "[...] it is identified that ICT allows carrying out activities that would be impossible to be done only with the use of paper pencils, providing the organization of pedagogical situations with greater potential for learning".

Gravina and Santarosa (1998, p.01) report that learning "depends on actions that characterize 'mathematical doing': experimenting, interpreting, visualizing, inducing, conjecturing, abstracting, generalizing and finally demonstrating [...]", thus, ICT can contribute to students modeling, analyzing simulations, conducting experiments, conjecture, confronting and refining their ideas.

Blum and Niss (1991) highlight the importance of technologies in Mathematical Modeling, highlighting the following possibilities they offer:

• Complex problem solving: Technologies make it possible to deal with problem situations that would be unapproachable without the aid of numerical or graphical simulations due to their complexity or the need for an advanced theoretical basis.

• In-depth analysis and understanding: They facilitate the understanding and analysis of problem situations through the variation of parameters and the performance of numerical, algebraic and graphic studies.

• Focus on the modeling process: Technologies allow learners to focus more on modeling processes, as they reduce the manual workload required in calculations, allowing for a more efficient and in-depth exploration of mathematical models.

In the ongoing doctoral research, the GeoGebra software was used, developed for the teaching and learning of Mathematics at various levels of education (from basic education to university education). It can be found for free on the internet and is available in Portuguese, favoring its accessibility and use. It was created in 2001 by Markus Hohenwarter because of his PhD thesis at the University of Salzburg in Austria (Hohenwarter & Preiner, 2007).

According to Gerônimo, Barros and Franco (2010), the fundamental characteristic of this software brings together Geometry, Algebra and Calculus resources, thus, it is possible to build and manipulate tables, graphs, functions, points, vectors, lines, line segments, polygons, among others, with the ease of using them in the same environment. In this way, GeoGebra has the didactic benefit of the simultaneous presentation of several representations of the same object.

One of the advantages of using GeoGebra is that the constructions are dynamic, that is, without the loss of geometric links. This allows the user to experiment with geometric propositions that allow him to construct geometric propositions (Gerônimo, Barros & Franco, 2010, p.11).

GeoGebra initially presents two different regions for the presentation of mathematical objects, these being the viewport and the Algebra window. In this way, the representations of the same object are dynamically linked and are automatically updated with the changes made in any of the windows.

One possibility of proposing an activity, in the context of Modeling and considering a real problem, could be that, after the students recover concepts about derivation and integration, start the theme about ODE, indicate the construction of a field of directions on graph paper and then use the command SlopeField and SolveODE in GeoGebra (Figure 1) where they can compare the resolutions and recognize the functionalities provided by the software.

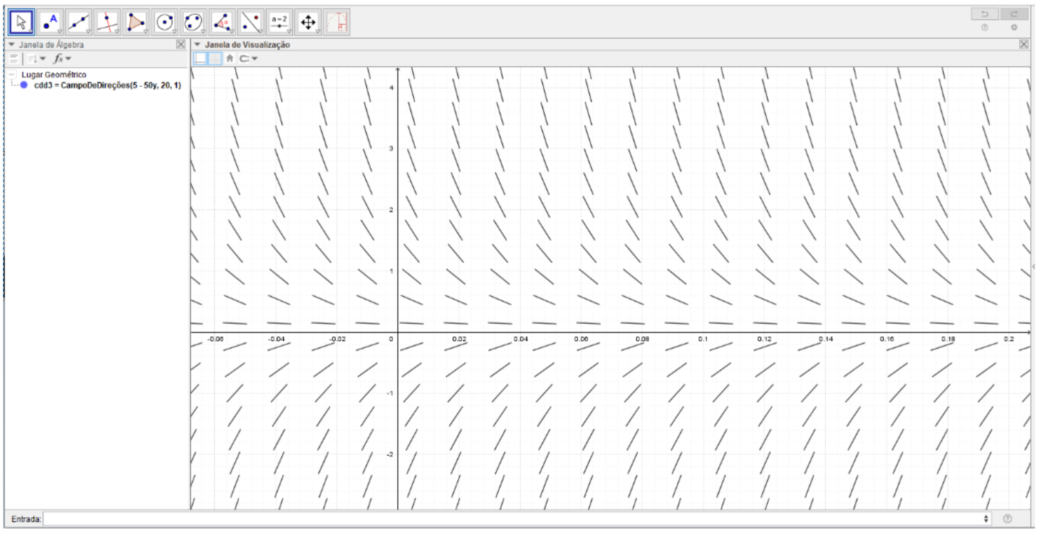

Figure 1.

Direction field generated by GeoGebra

With these GeoGebra functionalities, it is possible to obtain several representations of a mathematical object, meeting what is recommended by Raymond Duval's Theory of Semiotic Representation Registers (TSRR), when considering that learning occurs when the individual makes articulations between the different representation registers of the same mathematical object.

Final thoughts

The Dominant Epistemological Models (DEM) and the Reference Epistemological Models (REM) represent different approaches to understanding and constructing knowledge in each area, such as mathematics. They can coexist and interact in different ways in the educational context, influencing both teachers' practice and students' learning experience. Reflection on these two models and their relationships is fundamental to understanding how knowledge is constructed and transmitted in the educational context.

An effective approach to improving the teaching and learning of mathematics involves critically reflecting on these models, recognizing their potentialities and limitations, and seeking to integrate elements of both to create a richer and more meaningful educational experience. In this way, we highlight the contributions of Ole Skovsmose's Critical

Mathematics, which highlights factors related to curricular practices and the dominant underlying epistemological models, which must be constantly questioned and revised

To support the construction of a ERM, we present an excerpt from an ongoing doctoral research on Ordinary Differential Equations that suggests elements of a scenario for the construction of an ERM, such as the evolution of the concept of this theme over time and difficulties and advances in the process of its teaching and learning considering the results of some researches.

The theory of Semiotic Representation Registers contributed to composing this scenario because it evidenced the necessary knowledge for the teaching of ordinary differential equations through Semiotic Representation Registers. Technological knowledge, which included the knowledge of GeoGebra to assist in the teaching and learning process of this theme was also present.

Still with the aim of contributing to the teaching and learning of ODE, Mathematical Modeling can be an interesting methodological strategy

The development of ERM for the teaching of Calculus, with a focus on ODE, requires a rethinking of traditional teaching practices. While DEM emphasizes the transmission of techniques and methods, ERM proposes a more inclusive and investigative approach, valuing historical context, modeling, and the use of educational technologies. The integration of these practices, articulated by the Theory of Semiotic Representation Registers, allows students to develop a deeper and more critical understanding of ODE transforming the learning experience into something meaningful and connected to reality.

We conclude that preliminary studies of a Dominant Epistemological Model (DEM) on a mathematical object, in the case of Ordinary Differential Equations, may have relevant implications, to motivate the construction of ERM.

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