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Trajectory, resources knowledge for teaching mathematics in early career

Trayectoria, recursos y conocimientos ara la enseñanza de las matemáticas en el inicio de la docencia.

Trajectoire, ressources et connaissances pour l'enseignement des mathématiques en début de carrière

Trajetória, recursos e conhecimentos para ensinar matemática no início da docência

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Abstract

This article presents some results of post-doctoral research that aimed to: create, implement, analyze and disseminate resources to recompose learning after the Covid-19 pandemic. We investigate processes of knowledge building to teach mathematics. Looking to understanding their relationships of their resources' creation and modification. We worked on a qualitative approach that involved monitoring and interviewing teachers during their participation in the research project. We focus on teachers work in the beginning of their career. To analyze the data, we articulated the Documentational Approach to Didactics and mathematical knowledge for teaching frameworks. In this context, we analyzed data from our work with one mathematical teacher that was actively implicated in our project. We analyze data from her trajectory and her participation in the project. Teacher's trajectory analysis presented that initial training is based on the construction of knowledge centered on academic mathematics. Teacher's participation in the project had huge consequences in her knowledge to teach mathematics stimulated critical reflections about teacher's practices, discovering of new resources and generating knowledge to teach mathematics.

Keywords: Documentation work, Mathematics teacher knowledge, Teachers training, Documentational trajectory.

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Resumen

Este texto presenta algunos resultados de una investigación posdoctoral que busca crear, implementar, analizar y difundir recursos para restaurar el aprendizaje matemáticas después de la pandemia de Covid-19. En este artículo investigamos los procesos de construcción de conocimiento para enseñar matemáticas y sus relaciones con el trabajo de creación y adaptación de recursos por parte de una docente que inicia su carrera docente. Para ello, se trabajó con un enfoque cualitativo que implicó monitorear y entrevistar a los docentes durante su participación en la investigación. Para analizar los datos, articulamos el Enfoque Didáctico Documental y el marco de conocimiento para la enseñanza de contenidos. En este contexto, analizamos el proceso vivido por una docente colaboradora, combinando el análisis de su trayectoria y su participación en el proyecto. El análisis de su trayectoria presentó e indicó otros resultados de investigación que muestran que la formación inicial se fundamenta en la construcción de conocimientos centrados en la matemática académica. En el caso del proyecto, inferimos que propició un momento de reflexión crítica sobre la propia práctica y el aprendizaje de nuevos recursos, desencadenando la construcción de nuevos conocimientos para enseñar matemáticas.

Palabras clave: Trabajo documental, Saber docente de matemáticas, Educación continua, Trayectoria documental.

Résumé

Ce texte présente quelques résultats d'une recherche postdoctorale qui vise : créer, mettre en œuvre, analyser et diffuser des ressources pour rattraper les apprentissages des mathématiques après la pandémie de la Covid-19. Dans cet article, nous étudions les processus de construction des connaissances pour enseigner les mathématiques et leurs relations avec le travail de création et d'adaptation de ressources réalisée par une enseignante débutante au début de sa carrière d'enseignant. Pour cela, nous avons travaillé sur une approche qualitative qui consistait à suivre et à interroger les enseignants lors de leur participation à la recherche. Pour analyser les données, nous avons articulé l'approche documentaire du didactique et le cadre de connaissances pour le contenu pédagogique. Dans ce contexte, nous avons analysé le processus vécu par une enseignante collaboratrice du projet, combinant l'analyse de son parcours et sa participation au projet. L'analyse de sa trajectoire a présenté que la formation initiale repose sur la construction de connaissances centrées sur les mathématiques académiques. Dans le cas du projet, nous déduisons qu'il a encouragé un moment de réflexion critique sur sa propre pratique, l'apprentissage de nouvelles ressources en déclenchant la construction de nouvelles connaissances pour enseigner les mathématiques.

.***Mots-clés*** : Travail documentaire, Savoir pédagogique en mathématiques, Formation continue, Trajectoire documentaire.

Resumo

Esse texto apresenta alguns resultados de uma pesquisa de pós-doutoramento que busca criar, implementar, analisar e divulgar recursos para recompor aprendizagens de matemática após a pandemia da COVID19. Nesse sentido, investigamos os processos de construção de conhecimentos para ensinar matemática e as suas relações com o trabalho de criação e adaptação de recursos por uma professora em início de docência. Para tanto, trabalhamos em uma abordagem qualitativa que envolveu acompanhar e entrevistar professoras durante a participação na pesquisa. Para análise dos dados articulamos a Abordagem Documental do Didático e o quadro de conhecimentos para ensinar um conteúdo. Nesse contexto, analisamos o processo vivido por uma professora colaboradora, combinando a análise da sua trajetória e da sua participação no projeto. A análise de sua trajetória apresentou e indicou outros resultados de pesquisas que evidenciam que a formação inicial é pautada na construção de conhecimentos centrados na matemática acadêmica. No caso do projeto, inferimos que este favoreceu um momento de reflexão crítica sobre sua própria prática e a conhecer novos recursos, desencadeando a construção de novos conhecimentos para ensinar matemática.

Palavras-chave: Trabalho documental, Conhecimentos do professor de matemática, Formação continuada, Trajetória documental.

Trajectory, resources, and knowledge for teaching mathematics in early career

The beginning of a career is generally a defining moment for teachers. Each beginning is marked by many personal and professional choices that will influence the actions and decisions of the teacher. Elias (2021) discusses initial teacher education and advocates for training that is practice-based. The author presents the context of his research by mobilizing his site of speech and articulating it with research on initial education. He proposes a discussion about the devaluation of primary teachers and the overvaluation of academic mathematics. To this end, he distinguishes between school mathematics and academic mathematics and relates them to initial teacher education. School mathematics includes all the pedagogical aspects of teaching certain concepts. Academic mathematics corresponds to the presentation of results accepted by a particular scientific community through demonstrations and arguments based on logical-deductive reasoning. In addition, the author points out the distance between these two mathematics and how this affects the training of mathematics teachers. He concludes that initial training has focused on a strong background in academic mathematics without addressing key issues in mathematics education.

Elias' (2021) account provides us with access to part of his trajectory, showing how certain events and contexts shaped his decisions and teaching career. In this case, we find a university professor whose initial training focused on working with academic mathematics, which led him to begin his teaching career inspired by the pedagogical practice he had in school rather than at university. According to him, "handouts and textbooks were my best friends for a year as a teacher in the last years of primary and secondary school, reproducing the same traditional teaching I had had in my school days" (Elias, 2021, p. 5). This text prompts us to reflect on the first years of teaching and its relationship to teachers' careers. As Elias (2021) points out, a large proportion of initial teacher education courses do not prepare teachers for the start of teaching, which leads many to use their schooling as a basis for creating their lessons.

Many courses are still based on the perspective that **knowing the content** is enough to teach it (Shulman, 1988), ignoring a body of research in mathematics education that has shown that there is a **range of other knowledge that is needed to teach a piece of content** (Ball, Thames & Phelps, 2008). In fact, teaching content requires the teacher to create and implement activities in the classroom. This type of work is referred to by Gueudet and Trouche (2015) as documentation work, which ranges from finding resources to preparing and implementing them in the classroom. These authors highlight the importance of the creation of resources by the teacher as the core of their professional activity. This gives rise to the **Documentational**

Approach to Didacts (Gueudet and Trouche, 2008, ADD), which seeks to reflect on the training of teachers through the resources they create and use. This process of creating resources in initial teacher education can be very challenging, as during their undergraduate studies, students are confronted with few situations in which they plan lessons or discuss resources that help explain various concepts.

The gap between research and practice has been highlighted at various levels of education (D'Ambrosio, 2012). Our postdoctoral research was influenced by this issue, and to understand the research presented here, we believe it is fundamental to understand the context in which it was developed. In this article, we reflect on the experiences of early career teachers who participated in a research project. However, the focus of this project was to create and use activities with students to recompose learning that had been damaged by the COVID-19 pandemic.

In relation to the concept of learning recovery, we draw upon the discourse proposed by Abe (2022), who delineates the distinction between "recovery" and "reinforcement" of learning. The term "reinforcement" is used to denote the process of reinstating or restoring a concept or skill that has previously been taught, while "recovery" signifies the establishment of connections between the concepts or skills that are currently being taught and those that have not been extensively covered in the past. In Mato Grosso do Sul (MS), the learning recovery project (PRA-MS) was initiated in 2022. This state government initiative aims to mitigate the social inequalities caused by the pandemic (Rolim, 2022). Consequently, the present project endeavors to contribute to the creation of resources, strategies, and technologies for the learning recovery of students in Mato Grosso do Sul.

During the creation of the activities, we had a researcher teacher who taught 6th to 9th grade students. This researcher conducted the research and interacted with a master's student who taught at a private school and with two undergraduate mathematics students who were about to begin graduate studies. While creating and using the activities and reflecting on the students' difficulties, the teacher-researcher noticed a side effect of the project that had not been considered in its design. She noticed that at some points during the project, the collaborators discovered resources and questioned themselves about key issues related to the process of teaching the proposed concepts. Therefore, in this article we propose to analyze these moments that articulated the construction of knowledge necessary to teach a content and the documentation work based on the different resources proposed in the project, discussing two issues:

- What impact does initial training have on building a set of resources and knowledge for teaching?
- How can certain resources contribute to building the knowledge needed to teach mathematics?

To discuss these two issues, our article is divided into the following sections: (1) reflections and theoretical foundations, presenting some important concepts to understand the proposed work, (2) research paths, discussing some choices that influenced the creation and organization of the research; (3) trajectory and experience before teaching, highlighting events that impacted the initial training of the teacher analyzed; (4) experiences during teaching and their relationship with the project carried out, discussing resources and knowledge under construction; (5) conclusions and final considerations, reflecting on the text presented here.

Reflections and theoretical foundations

In this section, we discuss elements of our theoretical framework that inform our actions in the project and our data analysis. First, we look at some of the principles that guide our understanding of teaching and learning, both in terms of how students recompose their learning and in terms of our interactions with project staff. We then discuss the knowledge needed to teach content, based on Ball, Thames, and Phelps (2008). In addition, we introduce some concepts from the Documentational Approach to Didactics that show how documentation work is at the heart of the teaching profession. Finally, we discuss the articulation between the concepts presented and their relationship to the work of teachers at the beginning of their teaching careers.

Our intention to create resources that would contribute to the mathematics recovery learning led us to think about what would guide our actions in the research and what would help the collaborators of the project to understand our proposal. Thus, our actions are based on some principles that we have explained. They come from different theoretical frameworks of mathematics education that are interested in the processes of teaching and learning mathematics.

- **(Principle 1) Learning is adaptive:** We try to deal with situations in which students are placed in an environment of contradictions, difficulties and imbalances, understanding that the search for adaptation to this environment generates learning (Brousseau, 2008).

- **(Principle 2) Learning is creative and engaging:** we encourage work with problem-solving that stimulates students' creativity, seeking learning with pleasure and meaning.
- **(Principle 3) Learning mathematics requires the articulation of different representations:** we pay special attention to the articulation of different mathematical representations, suggesting articulation between different registers (Duval, 1993).
- **(Principle 4) Learning more with different resources**: we believe in mobilizing different digital or non-digital resources to contribute to students' mathematical conceptualization (Gueudet and Trouche, 2008).
- **(Principle 5) Teaching requires the articulation of different scenarios**: we try to work with different learning scenarios by working on exercises, problem solving, play activities, and real and semi-real situations. In this conception of teaching and learning, we believe that there is no single model that guarantees student learning (Skovsmose, 2000), but that some care must be taken when creating and applying activities.
- **(Principle 6) Teaching requires the use of different resources**: we aim to contribute to the teacher's documentation work by proposing different resources for organizing, preparing and disseminating the work done in the classroom (Gueudet and Trouche, 2008).
- **(Principle 7) Learning involves using our mistakes as opportunities**: we agree with SPINILLO et al. (2014, p. 12) when the author states, "if there were no mistakes, there would be no learning because everything would be learned and known beforehand". Thus, our work with students seeks to use mistakes as opportunities to learn more.

Considering recomposing student learning after the Covid-19 pandemic led us to ask how to create situations that could contribute to this end. These situations try to consider these principles, which were explicitly and implicitly worked on, with the participants in our research. In our research, recomposing the learning means going beyond traditional math activities, not just, for example, considering remedying difficulties by giving extra lists of exercises. Reconstructing students' learning means looking for students who are active and enjoy learning in school.

Mobilizing these principles requires the development of knowledge about how to teach mathematics. In this context, our work with staff sought to contribute to the construction of the **Content Knowledge for Teaching** (Ball, Thames & Phelps, 2008). Teaching involves the construction of a particular kind of knowledge to teach content. Ball, Thames, and Phelps (2008) discuss examples that show how some knowledge for teaching mathematics can

influence the management of some difficulties in teaching the content. The authors illustrate that many people can determine whether a person has gotten a problem wrong or right. However, to understand the origin of the error or what actions can be taken to turn it into a learning opportunity, specialized teaching knowledge is needed.

In order to study the knowledge needed to teach mathematics, we will mobilize the categories proposed in this theoretical framework: **common content knowledge (C-Content),** which is related to the issue of mathematical knowledge and skills used in non-teaching situations**; specialized content knowledge (C-Specialized**), which is intrinsic to the act of teaching mathematical concepts, as it is directly related to teaching situations, i.e., **knowledge of content and students (CC-Student**), which makes it possible to explore, investigate and use interaction with the student and for learning to take place, taking into account-specific questions about mathematical learning; **Knowledge of content and teaching (CC-Teaching)**, which combines knowledge of teaching and mathematics that will underlie all the organization and decisions of teaching situations to teach a given concept; **Knowledge of content and curriculum (CC-Curriculum),** which includes knowledge of the interactions with the different content and skills involved in the school environment.

This knowledge is fundamental for teachers to manage and create instructional situations in their profession. Creating situations to teach content involves creating, adapting, and combining different resources. In this sense, the **teacher's documentation** work is central to this profession. In ADD, the focus is on the **resources** that nourish the teacher's activity (Gueudet & Trouche, 2008). In this context, a **resource** is anything that can be made explicit by the teacher and that is used or created to carry out their professional activity. When teachers add knowledge to these resources, they become **documents** for them (resources + knowledge = documents), which is the process of **documentational genesis**. Because of this process, these resources are combined with others and incorporated into their resource system. The **resource system** is more than a list of resources used by teachers; it is a more complex web of relationships, interactions, and classifications related to the purpose of each resource and its status for action.

Throughout their professional life, teachers create and adapt their resources in interaction with the content and students, thus developing their personal repertoire. Various events take place during their career which contribute to the transformation of their resource system. Rocha (2023) was interested in studying these events and how they impact the teacher's activity in the classroom. Rocha (2023) conceptualizes professional events as a moment made explicit by the teacher or inferred by the researcher that has an impact on their documentation

work. In this sense, two concepts have been proposed within ADD to analyze the professional activity of mathematics teachers: **documentational experience and trajectory**.

The **documentational trajectory** is defined by the combination of events and transformations in the teacher's documentation work over time. The identification of certain events and their consequences in the teacher's practice helps us to understand certain knowledge used to teach the content. These professional events can be identified by the teacher (reflective events) or by the researcher (inferred event). Rocha (2021) also observed that when analyzing a set of events, some restructured a large part of the teachers' resource system and reoriented their documentation work. These events are called **symbolic transition events** (STE). Identifying these events allows us to understand many of the decisions made by teachers to carry out their work. The **documentational experience** is what is appropriated by the subject throughout their history, creating and adapting resources. Two teachers can experience the same event, but the experience will not be the same. Each subject is marked differently by the situations they live through, and this is what makes each subject's trajectory unique.

The research we present in this article works from a formative perspective so that our actions seek to influence the teachers' documentation work during their participation in the project. In this sense, we consider participation in the project as a professional event in the teachers' documentational trajectory. During this event, we present in the analysis some moments that we conclude have contributed to the documentation work. Therefore, we propose the concept of **micro-event** to denote the moments of documentation work within an event. To understand this term, we can think of it as using a magnifying glass to delve into the events present in a documentational trajectory.

We articulated the ADD (GUEUDET, TROUCHE, 2008) and the knowledge approach (BALL; THAMES, PHELPS, 2008) to look at the relationship between initial training and the start of teaching for the project staff. The creation of resources such as plans, activities, and assessments for a teacher starting to teach without an initial repertoire is very complex. After all, the documentation work of these teachers is starting from scratch and at the same time creating their own professional identity. In addition, the lack of experience with school organization, curriculum, and interaction with students can make decision-making difficult. Thus, by asking the collaborators to create resources, use them, and discuss their effects, our research proved to be challenging for the collaborators and provided moments of knowledge building for teaching mathematics.

In light of this discussion, we return to our questions after the theoretical explanation:

- What events during initial training have an impact on the construction of the system of resources and knowledge for teaching?
- How could documentation work, following the principles proposed in the research, contribute to the construction of the knowledge needed to teach mathematics?

In the next section, we will discuss the methodological choices that were made and some elements of our data analysis.

Research paths: discussing the context and some methodological choices

In this section, we discuss methodological issues related to our work with teachers and the production of data. We present the *formative intervention* (Engeström, 2011) and elements related to our qualitative research approach (Bogdan & Biklen, 1994; Goldenberg, 1998).

Since our research is based on a qualitative approach, our data "aim to gain a more profound understanding of certain social phenomena based on the assumption of the greater relevance of the subjective aspect of social action" (Goldenberg, 1998, p. 49). Thus, we tried to look at the initial training and the beginning of teaching of some subjects to understand this process. In this approach, it is significant to discuss the paths, choices, and difficulties faced to understand the results presented. Bogdan and Biklen (1994, p. 50) argue that researchers using this type of approach are interested in the different ways in which people make sense of their lives. In this context, we sought to gain an in-depth understanding of the case of some teachers in terms of their relationship with the resources presented in the project and their knowledge of how to teach mathematics.

Our work with the collaborators is in line with the formative intervention methodology (Engeström, 2011), which considers a subject with its agency. In this framework, the researcher is not considered the holder of knowledge and solutions to be applied by the teachers, but rather as part of a community seeking a solution to a problem in their activity. Engeström (2011) presents four important aspects of this methodology: from the start, the researcher does not know all the problems and solutions to be applied in the activity; the training is a process and everything is negotiated during it, as the participants have the power to act and are involved in the search for solutions; the results are new concepts that "can be used in other contexts as frameworks for creating new locally appropriate solutions". A key outcome of formative interventions is the agency of the participants" (ENGESTRÖM, 2011, p. 606); regarding the role of the researcher, it is to stimulate, support, share, and bring about change.

In this sense, our work with the three members of the project was based on a collaborative perspective, proposing an active participation in the decisions to create and apply

the activities. In our project, we worked with two undergraduates who did a scientific initiation for eight months and the teacher, who was a scholarship holder, for ten months.

In this article, we present our work with Giovana[3], who is a teacher in a private school and a master student in a postgraduate program. As mentioned above, our research aims to create activities to apply and disseminate possibilities for recomposing mathematical learning. Giovana played an active role in the project, coordinating the construction of the website, creating activities, participating in the after-school meetings, and coming in on Wednesdays to monitor the work in the classroom. Our main concern was to create resources, and she actively participated in this process. During various interactions, we noticed some of Giovana's fears and reactions to the activities proposed in the project. Many of these occurred during meetings or informal interactions about the activities.

During these interactions, we began to realize that the project was having an impact on Giovana's view of teaching and possibly contributing to the construction of knowledge about teaching mathematics. Recognizing this possibility, we began to wonder what impact our research practice was having on this teacher's practice. Thus, we conducted a semi-structured interview to explore this possible contribution. We then transcribed the interview, which took place at the end of the first year of the project, to examine her career and the impact of the project on her work in the classroom. In addition, we have videos of some of the meetings that show this teacher applying the activities in the after-school hours and some plans published on the project's website. Many interesting moments of our interaction with Giovana were not recorded because they were informal moments of discussion, but when they are mentioned in the transcript, we will describe our version of that event.

Our data analysis followed the following steps: identification of **events** from initial training that will guide their relationship with the profession, resources and/or knowledge (denoted: E1, E2, ...); identification of **micro-events**, a term we use to identify episodes within their participation in the project that allow us to understand the knowledge built at the beginning of their teaching career (denoted: ME1, ME2, ...); identify stable resources, a term we use to designate the resources that make up your resource system in your first year of teaching; identify possible potential resources, a term we use to designate the resources that have been assimilated from the project experience as a possibility for future use in your classes. For each event, micro-event, stable resource, and potential resource, we tried to relate the data to the construction of the knowledge needed to teach.

[3] Fictitious name.

In the next two sections, we present our analysis of the data. First, we will analyze the general events presented by Giovana since her first training. Finally, we will analyze the micro-events and resources explored during her participation in the project.

From initial training to starting teaching: investigating the documentational trajectory and the knowledge developed

Giovana is a teacher who has studied in public schools all her school life, she had an easy time with mathematics and helped her classmates. She initially thought of studying engineering at the encouragement of her parents, she told us:

> My father used to say, "Professor, right?" (In a derogatory tone) Let us go into engineering! I don't know what... But I knew I wouldn't get a good grade. Because I didn't feel safe with the school's teaching, yes, because it's not the public school's fault. Somehow, public schools don't prepare you that well in terms of entrance exams, things like that.

We noticed that in her speech, her father expressed his disbelief in the profession of teaching. However, Giovana noted that her grade in the selection process to enter engineering could be low, so she decided to study for a degree in mathematics. In this context, she studied for a degree from 2017-2022, which was part of her education during the COVID-19 epidemic.

Giovana argues that during her undergraduate studies she had many difficulties with mathematical knowledge:

> I suffered a lot at university because I didn't have an excellent foundation. As simple as I was, I didn't see everything I should have seen. (...) There was actually a moment when I thought about quitting college. I think it was when I failed Calculus twice, Calculus 1. So I thought, it's not possible; I'm not going to make it. Then I felt like giving up. I just said, "No! I got to the middle; I think I was almost halfway through. I said, "No! Since I'm here, I'll go to the end.

Participating in the calculus subject (E1) was a difficult moment for her teaching career, calling into question her choice to study for a degree. This subject essentially works with academic mathematics with the following topics: functions of a real variable, limit, continuity, derivatives, applications, and indefinite integrals. Calculus 1 usually has a strong focus on proving and applying theorems. After failing this subject twice, she **discovered a new video assessment methodology (E2)**. In fact, the **video resource** was used by the teacher as an assessment method and made her regain interest in the subject and perform well in it. This event sparked her interest in research in Mathematics Education and led her towards a master's degree to discuss the topic of assessment, thus contributing to the construction of **knowledge**

of content and teaching, as she was able to learn about another method of assessing mathematical knowledge. This event was essential in Giovana's trajectory, even affecting the direction her career would take, giving rise to new resources and events. In this context, we infer that it is a **symbolic event of transition.**

The moments of evaluation during her undergraduate studies were remarkable for this teacher, always causing her a lot of anguish and fear. However, during the pandemic, assessments were adapted and carried out in different ways and, in general, students had more time to solve the proposed questions. This had a complete impact on the results and on Giovana's relationship with this resource. She told us

> You had time to evaluate. Therefore, these weren't questions that you had already solved. You had to find a way, like solving a list. It was crazy because you didn't know whether you were right or not, but I had time, and that time gave me a sense of calm. You know why? Occasionally, I was too nervous to solve the test in two hours. So all these exam questions, and I don't know what... It got me interested in the subject of assessment. To talk about assessment.

Giovana argues that the pandemic has made her studies easier because if she had had to take some subjects in the classroom, she believes she would have accumulated more retention, "but I don't think I stayed longer because of the pandemic, right? I just think I would have stayed longer. I wouldn't have passed the first analysis. (...) We were able to consult." On the one hand, the pandemic allowed her more time for the assessment; on the other, it prevented her from doing two supervised internships in high school. As a result, she felt discouraged from teaching in secondary schools; she said, "I didn't do the last two internships. So I had no contact with students, and to this day, I'm afraid of secondary school, perhaps because I didn't do a secondary school internship". Giovana considers the elementary school internship to be important as it allowed her to get to know this level of education better. These two aspects show us that the pandemic has left its mark on Giovana's initial training.

Giovana was also a **scholarship holder in the teaching initiation project - PIBID (E3)** and **a scholarship holder in the pedagogical residency project (E4),** which helped to create experiences about teaching mathematics. However, only in PIBID was she able to go to school and develop activities, as her participation in the pedagogical residency project was during the pandemic.

Then I went to PIBID; I always liked it because I didn't have the responsibility of teaching. I started to get a feel for it, and then I kind of realized, wow, it's not that easy, right? But I still

thought it was easy because I was there in one day. Did I have that ideal plan? I applied the idea; everything worked out, like that.

In fact, the PIBID experience, although it serves as an introduction to teaching, is still far from the real life of a teacher who works 20 to 60 hours a week. The time a student has to prepare a plan and collaborate with colleagues is not the same as when they are responsible for a classroom.

In addition to these two project experiences, another important event was her **voluntary participation in a science induction program (E5).** The calculus teacher led this experience. Giovana felt motivated by the video assessment experience to do more research on this methodology. However, she was advised to work on the subject during her **Master's degree** (E6) and began to research the use of Google Forms with undergraduates in a course.

The events identified up to her initial training allow us to make some partial reflections on this period of Giovana's preparation for teaching. First, we observed the problem of the devaluation of primary school teachers (Elias, 2021), as she points out that her father devalues the fact that she chose to be a teacher and that he would prefer her to do engineering. In addition, during the interview, she did not show how the subjects dedicated to mathematics education contributed to building **specialized content knowledge.** Another aspect is that her initial training was strongly based on working with academic mathematics, common knowledge of the content, which almost led her to abandon her teaching career. Finally, we see a defining moment in her documentational trajectory, which is the discovery of the video assessment methodology, **content knowledge and teaching**, which triggered several other events, such as the start of a scientific initiation and, later, a master's degree.

At the beginning of 2022, when Giovana started her Master's degree, she was called to be a teacher in a school in Mato Grosso do Sul, a so-called authoring school, which is a project where the student stays at school all day, attending classes and also doing projects. This was not an easy time for Giovana because the proposal was that the students would choose a topic, and she would prepare a lesson on that topic. She explains

> Then I joined, right? Right at the beginning of the year, along with my master's degree, so I was under a lot of pressure. Master's and school, teaching, my God. And there, (I taught an) elective subject. And it's a school of authorship. The students had to choose the subject of the elective, and I had, like, several classes, because it was one elective class a week. Right? One class, and then they had to choose the subject, so I don't know, 6th grade chose cooking. So prepare a menu so you can give them cooking to get involved in math. I said, What? I don't even know what to teach 6th grade. Of course, I've read the BNCC, but you can tell what it's like in practice. (...) Sometimes I had difficulty with some basic content because I didn't go into it in depth at school and

> college. (...) I said, man, what now? (...) I said, no, I'm going to get a master's degree; I'm not going to stay here; it was still time to select the scholarship.

We describe this event as **starting and leaving teaching in a comprehensive public school (E7),** and we can see in Giovana's speech very important aspects relating to her lack of resources and knowledge to teach at the start of her career. Firstly, we see that she believes that difficulties with basic content and **common content knowledge** were a factor that hindered her in this first experience. In addition, we see that the lack of **knowledge of the content on the horizon** appears as another factor when she says that she "doesn't know how to teach the basics of 6th grade." In addition, we see that the **resource of** the Common National Curriculum Base is pointed out as known **knowledge of the content and the curriculum**, but as theoretical knowledge and not practiced being mobilized in the classroom. Finally, we see that Giovana's speech is in line with the discussion proposed by Elias (2021) that many undergraduate courses have not prepared teachers to teach school mathematics, leading them, in many situations, to rely on their experiences as students to think about their teaching.

After leaving school, she was awarded **a scholarship for her master's degree** (E8), which allowed her to take part in various subjects and projects related to research, giving her more time to devote to her dissertation. However, due to some events in her personal life, she had to seek employment and abandon the scholarship, thus leading her to two professional events: returning to **teaching in a private school** (**E9)** and becoming a **teacher in a school tutoring company** (**E10)**. This tutoring space has a traditional method, offering lists of exercises that lead students to fix concepts through repetition. She didn't have many outstanding characteristics about this experience, but points out that the opportunity helped her financially. So when the grant to take part in our project came along, she left that space and dedicated herself to the activities we proposed. This brought us to the event of **participating in the project at the public school to create resources (E11),** which we will look at in more detail in a magnifying glass effect. In this sense, in the next section we will analyze event E11, identifying some micro-events that show transformations in Giovana's resource and knowledge system. These micro-events show us important characteristics of Giovana's documentation work at the start of her career.

Documentation work to build knowledge for teaching mathematics

Our research began in February 2023, and in April, Giovana started as a fellow in the project. Giovana's responsibility was to participate in the creation of resources and, in particular, to collaborate with the development of a website to disseminate resources. Our first

request to Giovana was to find and create creative activities in which the student could be active in the construction of knowledge. Giovana found the opportunity interesting, as she had been teaching since February at a private school and was finding it difficult to create activities. However, when she started the project, she was worried about this responsibility of creating activities. She said

> Then I went, Oh my God, activity, he's going to tell me to make that gigantic plan again, and then I'm going to suffer once more. So I kind of went back to that idea from college of innovative activities. When you discussed creative and innovative activities, I said, people, I only solve; I can barely solve an exercise. An exercise that's in the workbook with the students. How am I going to do an innovative activity? And that gave me a lot of insecurity at first, you know? Even about the website itself (which we developed in the project). It was a bit of a challenge for me at first, because I didn't know how to use it. I said, let's go to the Google site; I know it, I know how to use it.

This excerpt from Giovana's speech is essential to understanding how she experiences this moment when she starts teaching. In fact, the feeling of insecurity due to her lack of **common content knowledge** of school mathematics has a strong impact on the **specialized content knowledge** needed to teach mathematics. In addition, she has a negative view of the time she was given in college to create a plan. Creating an innovative plan for a teacher at the start of their career requires a lot of documentation work, requiring the mobilization of a great deal of knowledge. Evaluating her participation in the project, Giovana said, "I really liked this creative approach, I'm very inside the box. With each activity proposed in the project, we always discussed how the student could get involved and enjoy what they were doing. Of course, sometimes it's not possible, but it was a constant care and search that we had. Giovana also said that "I'm the teacher who teaches with a handout because I'm not sure if what I'm going to say is right or not. Because I have no experience". From the perspective of documentation work**,** this type of practice saves the teacher research time; however, the teacher is less reflective and critical in relation to the activities and methodologies applied. In this sense, our project was a moment in which Giovana was able to reflect on new methodologies for working in the classroom. We noticed in her speech that during her teaching at the private school, the textbook was a **stable resource** in the teacher's practice.

A **micro-event** that took place in the project was the **use of the Cuisenaire scale**[4] (ME1). Giovana got to know this resource during the project, as we used it to work on the part-

[4] This material is made up of colored wooden bars that represent rectangular blocks with a volume of one cubic decimeter to 10. Available at: https: //wp.ufpel.edu.br/obeducpacto/files/2019/12/Escala-Cuisinaire.pdf (Accessed April 11, 2024).

whole relationship associated with the concept of fraction with students. These activities were carried out after school with all the classrooms in the final years of elementary school, with the aim of recomposing GUEUDET learning in mathematics. About this material, she said, “It helped me, even with the question of fractions. We saw so much about fractions, I even bought the cuisenaire scale (...) And then, in the tutoring classes, I used this scale with the pupils.” This type of manipulative resource can help them understand the relationship between parts and wholes and also explain fractions greater than one. This type of discussion with Giovana nurtured her **specialist knowledge of mathematics teaching,** showing her new resources that can be used in her interactions with students and also contributing to her **knowledge of the content and of the students**. We therefore consider the cuiseraire scale to be a **potential resource** that can be used by Giovana in her classes.

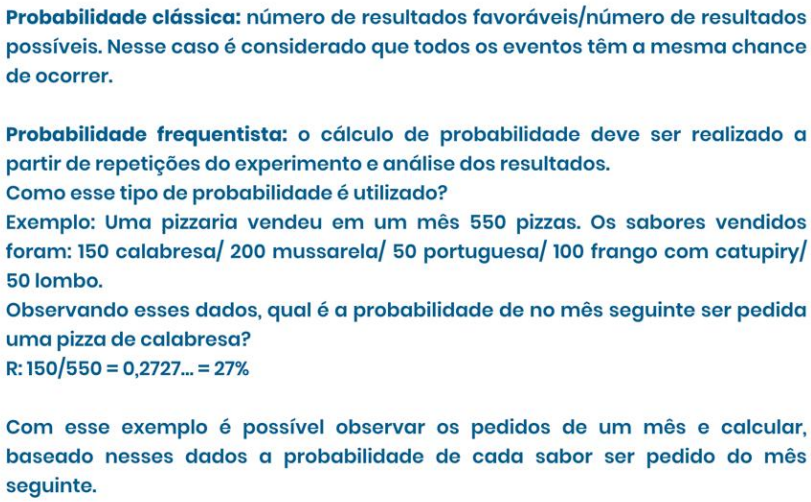
Another **micro-event** was the **use of golden material in teaching decimal numbers** (ME2). During one of the lessons observed by Giovana, the teacher-researcher used the golden material to work with sixth grade students on decimal representation. During this lesson, Giovana was really intrigued by the teacher's explanation and came to question her informally about the material. For a while, the teacher-researcher explained the concepts of decimal numbers, articulating the numerical representations and the division algorithm using this material. Giovana found the explanation interesting and said that she really understood how she could illustrate the use of the comma representation to the student. She argues

> I think I understood the part you said, but I need to study more to tell them. The decimal number part because that's something I didn't know how to explain. Yes, like, add zero, why do you have to? I don't even know why, you know? So when you said that in the material, I said, “Guys, I need to teach this way”.

ME2 shows us another moment that triggered the construction **specialized content knowledge**, when resources were discussed that could be used to answer some students' questions. Ball, Thames, and Phelps (2008) argue that this type of knowledge is inherent to the act of teaching because, in addition to knowing how to do calculations (CC-Content), the teacher has to know why that help teach the content (CC-Specialized), which will contribute to the interaction and learning of the students (CC-Students). During the project, there were many moments when we created or discussed resources and situations that would allow us to understand the why. In this sense, we believe that the fact that the golden material has favored her learning makes it a **potential resource** for Giovana to use in the classroom.

The next **micro-event** was the creation of a **plan to work on the concept of probability** (ME3). To post the activities on the website, we always looked for research or articles that

could help us disseminate research results. Giovana had found an article in a magazine that presented some activities. To post them on the site, we had to include some comments for the teacher who was going to use the activity. In the proposed activities, there were some questions that sought to get students to differentiate between classical probability and frequentist probability. Giovana had put the two definitions in the plan, and the teacher-researcher asked her to find an example to explain the difference between the two. At the end of an informal lesson, she said she had difficulty understanding the difference and giving an example of what the definition meant. In this sense, the teacher-researcher discussed some ideas with her that exemplified this difference. In Figure 1, we can see the final result in the plan posted on the website.


Probabilidade clássica: número de resultados favoráveis/número de resultados possíveis. Nesse caso é considerado que todos os eventos têm a mesma chance de ocorrer.

Probabilidade frequentista: o cálculo de probabilidade deve ser realizado a partir de repetições do experimento e análise dos resultados.
Como esse tipo de probabilidade é utilizado?
Exemplo: Uma pizzaria vendeu em um mês 550 pizzas. Os sabores vendidos foram: 150 calabresa/ 200 mussarela/ 50 portuguesa/ 100 frango com catupiry/ 50 lombo.
Observando esses dados, qual é a probabilidade de no mês seguinte ser pedida uma pizza de calabresa?
R: 150/550 = 0,2727... = 27%

Com esse exemplo é possível observar os pedidos de um mês e calcular, baseado nesses dados a probabilidade de cada sabor ser pedido do mês seguinte.

Figure 1.

Extract from Giovana's planning[5].

We can infer that this moment of discussion involves understanding these concepts (**common content knowledge**) and knowing how to explain them (**specialized content knowledge**). She discussed the fact that she thought she had put it in the plan, but hadn't understood it before interacting with the researcher

[5]Classical probability: number of favorable outcomes/number of outcomes
possible. In this case, all events are considered to have the same chance of occurring.
Frequentist Probability: The probability must be calculated by repeating the experiment and analyzing the results.
How is this type of probability used?
For example: A pizzeria sold 550 pizzas in a month. The flavors sold were: 150 pepperoni / 200 mozzarella / 50 Portuguese / 100 chicken with catupiry / 50 sirloin.
Given this data, what is the probability that a pepperoni pizza will be ordered next month?
R:150/550= ,2727...= 27%
With this example, you can look at the orders for one month and calculate the probability of each flavor being ordered the following month.

> I even prepared it; I put it there (in the planning), and then when you said it (the example of one pizza being more popular than another in a certain pizzeria), I said guys, that makes perfect sense; it was so obvious, I said wow, that's great. Then I got this idea, and it gave me more confidence.

ME3 shows us how the interaction with research allowed Giovana to create resources that could potentially contribute to her teaching practice. With each plan she created and posted on the site, there were many discussions about content, teaching, and relationships with students. This planning is therefore a **potential resource** for Giovana's classroom practice. Her talk and the excerpt from the plan tell us that Giovana went through a process of **building expertise in the conten**t of probability education.

The proposal to work with different resources with Giovana was seen by her as enriching; she told us, "The things I said helped me a lot, I have a knowledge of the material, like the question of opening the view and saying, oh, there is, and it makes a difference that you use a material". Another thing she explained because of working on the project is that she tries to ask open-ended questions in her assessments so that students can justify their answers more. She said

> And what happens with the tests, I couldn't do any more tests with x grades. I already had something, I couldn't do it because I kept thinking, how do I know that this student understands? No, because here we followed the development of the student a lot by looking at the project. And I didn't get much there. Anyway, it wasn't like the project. So I started asking open questions to see. I would say, oh, you have to justify what you're thinking.

In this excerpt from the interview, we see that the teacher highlights an impact on her interaction with the students by seeking to look at the student's development, **knowledge of the content and student**. In addition, the changes in her conception of how to assess students transform the resources used to assess, **content knowledge for teaching**. Furthermore, when analyzing her participation in the project, she says

> (...) to watch your classes. It was excellent for me because occasionally, I saw the ease that you brought to the classes (...). When we see the teacher, we would like to learn; we want to learn more because it was one thing when I was in PIBID at school, I looked at the teacher as if I were a student. Today I looked at the teacher, and I'm like, how does he do it?

We can see in this statement that being present in the school observing and participating in the project was something that Giovana believes was more connected to her current reality and that this contributed to her learning. She has a critical view of her own work that was triggered by the project. She says

> I kept thinking, the students can't take it anymore. I kept going from beginning to end, talking non-stop. I talked, I talked, I talked, I talked, I talked like this, backwards, forwards, backwards in the middle, jumping, like, I talked a lot? I thought, I have to talk in all kinds of ways to see if he'll understand? But then I saw your classes and the dynamics of the project. I said, it's not like that, the student learns more if the teacher talks less.

This reflection by Giovana is a consequence of the principles on which we base our activities. This type of reflection reveals a lot about the process of knowledge building that she started in the project. The three micro-events presented here are examples of moments that triggered these reflections and show how the documentation work proposed in the project contributed to Giovana's construction of some knowledge. In the next section, we will offer some more global reflections on the research presented here and discuss our research questions.

Considerations and conclusions

The first research question we proposed was to investigate: **what impact does initial training have on the construction of a set of resources and knowledge for teaching?**

In Giovana's case, we observed that her initial training was permeated by many difficulties related to the knowledge of the content of academic mathematics. We can conclude that she spent a lot of time studying this mathematics and that the experience of teaching practice subjects took a back seat. We also have to consider that half of her initial training took place during the pandemic, which made it impossible for her to do a supervised internship in high school, for example, and affected her participation in the pedagogical residency project. However, she did have classroom experience, an elementary school practicum, and participation in the PIBIB project. Even with these experiences, we saw that Giovana felt unprepared in terms of school mathematical knowledge.

Our analysis indicated that she felt unprepared both in terms of school mathematics and resources for teaching mathematics. In this context, her initial training was somewhat disconnected from what happened in her classes, leading to a feeling of insecurity and a strong attachment to the textbook. Add to this the fact that she had a basic education that she considered weak, and this feeling of insecurity became more and more pronounced. Some events during her initial training, such as failing in calculus, were related to this feeling and even led her to want to drop out of the course. Overall, we can conclude that Giovana's initial

training did not provide her with significant opportunities to build her knowledge of how to teach mathematics.

The study of a case like Giovana's is not enough to draw general conclusions, but it does alert us to the importance of supporting teachers at the beginning of their career. It should also be noted that Giovana's profile is very specific, as she participated in two projects during her undergraduate studies and is currently studying for a Master's degree in Mathematics Education. Because of this different profile, we see that Giovana is always reflecting and taking a critical stance on her practice. Given this differentiated profile, we wondered about the documentation work of teachers who exclusively live the disciplines related to academic mathematics and go directly to school. In the case of Elias (2021), we also see that the author's personal history took this route and that this made his practice traditional and without much reflection. We therefore argue that initial training must offer more moments of documentation work for the teaching of mathematics. In addition, we also advocate monitoring teachers during their first years of teaching because, as we saw in Giovana's case, when she felt responsible for a classroom, many questions arose and her participation in the project helped her to reflect on them. In this sense, the great difference in working with Giovana was that the researcher and the teacher worked together in a kind of documentation partnership, involving exchanges and discussions. This kind of work also reinforces the importance of working together with other colleagues at the beginning of teaching.

The second research question we proposed was to investigate: **how can certain resources contribute to building the knowledge needed to teach mathematics?**

Creating plans was a challenge for Giovana because she was in a private school where she consumed everything in the textbook. Following a workbook is like following a recipe for a cake: if everything works, it's no problem. But if something doesn't work, you may not know how to fix it. That's what happened when she couldn't explain some whys. This material didn't help her understand the mathematical concepts. Here we show you some potential resources for your practice, such as the Cuisenaire scale, the gold material, and the plans posted on the website. Using these resources, we showed that the teacher developed some knowledge about how to teach mathematics. This also happened informally with other resources during the 8

months we worked together. On the website that we developed together with Giovana, we made available several resources that were discussed in the project and tried to expand the network of teachers who use these resources. The resources we used in the project aim to contribute to students' mathematical learning and to support mathematics teaching. In addition, we share on the site resources that can help teachers in their documentation work, both for their organization and for the preparation of their lessons.

Our research involved Giovana and two other undergraduate mathematics students and sought to move from this local experience to a more global one. In this sense, the website is an important result of the project that we will try to keep active on a voluntary basis. We are committed to the dissemination of practices and resources because sharing classroom experiences is an enriching act for in-service teacher training. How many research projects and mathematics teaching materials exist that remain within research groups and are not shared with the community? Our research joins those that seek to build bridges between the world of research and classroom practice.

From a theoretical point of view, we have tried to link the documentational approach to didactics (GUEUDET; TROUCHE, 2008) with Ball, Thames, and Phelps' (2008) approach to the knowledge needed to teach mathematics. We can see in Giovana's case that the documentation work with some resources in the project contributed to the construction of knowledge in all the categories proposed by Ball, Ball, Thames, and Phelps (2008). The analysis of her documentational trajectory helped her to understand why her teaching is based on what is proposed in the workbook, and how some experiences in her initial training contributed to increasing her uncertainty about her knowledge of the content. In an intertwined way, the lack of knowledge about how to teach mathematics has an impact on her documentation work, leading her to look for resources to reduce the choices she has to make and to guide her work in the classroom. In general, the articulation between these two theoretical frameworks has allowed us to look at the phenomenon of a teacher starting to teach and to try to contribute to the debate on teacher training.

From the perspective of methodological choices, our research took place in a public school and was led by a teacher-researcher who was a teacher in the school and who, together

with the staff, sought solutions to work on the recomposition of learning. Working in an elementary school classroom and doing research at the same, time is a setting that seemed very conducive to analyzing, creating, and developing new activities. However, this dynamic also had an impact on the time we had to prepare and implement the activities. Many of the activities we proposed did not make it onto the website and need to be refined. Many of the resources we worked on with Giovana were not available at the time of the interview, and we weren't able to keep an active log of the activities. Being constantly involved with the school we were researching, articulating the work with the academics, the extracurricular activities, and the development of the website was very challenging.

What's more, working with collaborators was a side effect of our project that we believe we realized too late for data collection and analysis. Many interesting moments occurred without formal data collection and could not be included in the analysis. However, our interviews with staff allowed us to present some micro-events and potential resources that exemplify the impact of the work being done. These two concepts, which we have introduced in this article, seem interesting to us when analyzing an ongoing event in the documentational trajectory of a subject. In Rocha (2023), we presented an analysis of the documentational career of a French teacher, proposing this new concept of analyzing the transformation of the system of resources over time and the events that triggered them. In the case of the French teacher, events related to collective work with a document partner and involvement in collective work were deduced as important for reflection on her practice. In the case of our partnership with the Brazilian teacher, we again inferred that partnerships are fruitful, and so is her connection to the collective of project collaborators. For this reason, the time she spent participating in the project in parallel with her work in the classroom was fundamental, as she was able to connect research and classroom practice.

In conclusion, our text has tried to highlight some results of an ongoing postdoctoral project (2023-2024), showing the complexity of the documentation work and the construction of knowledge by a teacher just starting out as a teacher and collaborating in the project. This work has reinforced our ideas about the importance of a project that takes place in the school and links research and practice. Many questions remain about this project, such as the impact

on student learning and how to continue the work of feeding the site with plans. Finally, the discussions proposed here have allowed us to contribute to discussions that link the beginning of teaching, resources, and knowledge built.

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