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Os esquemas de estudantes surdos em uma situação de combinatória

Schemes of deaf students in a combinatorial situation

Los esquemas de los estudiantes sordos en una situación combinatoria

Les schémas des étudiants sourds en situation combinatoire

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Resumo

O objetivo deste estudo foi analisar os esquemas de ação de dois jovens estudantes surdos em situações de combinatória, com base no constructo de esquema da Teoria dos Campos Conceituais de Gérard Vergnaud, na Língua Brasileira de Sinais (Libras) e no potencial comunicativo e cognitivo dos gestos. Trata-se de uma abordagem qualitativa no design de estudo de caso de cunho descritivo, fundamentada numa análise microgenética associada à videografia. Os surdos mobilizaram esquemas no espaço, associando gestos e sinais em Libras, expressando os conceitos de adição e correspondência biunívoca (sinal-a-dedo). As escolhas linguísticas da intérprete de Libras influenciaram na resolução de cada surdo. Os resultados obtidos neste estudo destacam a importância de refletir, em ambientes linguísticos traduzidos, sobre a elaboração de enunciados de situações-problema tanto em Língua Portuguesa como em Libras, levando em conta a competência linguística de cada estudante surdo. A descrição detalhada dos esquemas mobilizados pelos surdos pode ampliar a compreensão do seu estágio de desenvolvimento e beneficiar o professor de Matemática no ensino desse conceito no contexto educacional inclusivo.

Palavras-chave: Esquemas, Gestos, Estudantes surdos, Língua de sinais, Combinatória.

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Abstract

The objective of this study was at analyzing the action schemes of two young deaf students in combinatorial situations, based on the scheme construct of Theory of Conceptual Fields by Gérard Vergnaud, in Brazilian Sign Language (Libras) and communicative and cognitive potential of gestures. This is a qualitative approach in design of a descriptive case study, based on a micro genetic analysis associated with videography. The deaf students mobilized schemes in space, associating gestures and signs in Libras, expressing the concepts of addition and one-to-one correspondence (finger-sign). Linguistic choices by the Libras interpreter influenced the resolution of each deaf person. Detailed description of the schemes mobilized by the deaf students may broaden the understanding of their stage of development and benefit the mathematics teacher in teaching this concept in an inclusive educational context.

Keywords: Schemes, Gestures, Deaf students, Sign language, Combinatorics.

Resumen

El objetivo de este estudio fue analizar los esquemas de acción de dos jóvenes estudiantes sordos en situaciones combinatorias, a partir del constructo de esquemas de la Teoría de Campos Conceptuales de Gérard Vergnaud, en la Lengua de Señas Brasileña (Libras) y el potencial comunicativo y cognitivo de los gestos. Se trata de un abordaje cualitativo en el diseño de un estudio de caso descriptivo, basado en un análisis microgenético asociado a la videografía. Los sordos movilizaron esquemas en el espacio, asociando gestos y signos en Libras, expresando los conceptos de suma y correspondencia uno a uno (signos dactilares). Las elecciones lingüísticas del intérprete de Libra influyeron en la resolución de cada persona sorda. La descripción detallada de los esquemas movilizados por las personas sordas puede ampliar la comprensión de su etapa de desarrollo y beneficiar al profesor de matemáticas en la enseñanza de este concepto en un contexto educativo inclusivo.

***Palabras clave*:** Esquemas, Gestos, Estudiantes sordos, Lengua de signos, Combinatoria.

Résumé

L'objectif de cette étude était d'analyser les schémas d'action de deux jeunes étudiants sourds dans des situations de combinatoire, en se basant sur la construction de schémas de la théorie des champs conceptuels de Gérard Vergnaud, sur la langue des signes brésilienne (Libras) et sur le potentiel communicatif et cognitif des gestes. Il s'agit d'une approche qualitative dans la conception d'une étude de cas descriptive, basée sur une analyse microgénétique associée à la vidéographie. Les personnes sourdes ont mobilisé des schémas dans l'espace, associant des gestes et des signes en Libras, exprimant les concepts d'addition et de correspondance biunivoque (signe-doigt). Les choix linguistiques de l'interprète Libras ont influencé la solution de chaque sourd. Les résultats obtenus dans cette étude soulignent l'importance de réfléchir, dans des environnements linguistiques traduits, à l'élaboration d'énoncés de problèmes à la fois en portugais et en libras, en tenant compte de la compétence linguistique de chaque élève sourd. Une description détaillée des schémas mobilisés par les élèves sourds peut élargir notre compréhension de leur stade de développement et profiter aux professeurs de mathématiques lorsqu'ils enseignent ce concept dans un contexte éducatif inclusif..

***Mots-clés* :** Schémas, Gestes, Élèves sourds, Langue des signes, Combinatoire.

Schemes by deaf students in a combinatorial situation

Average performance in Mathematics of Brazilian students has remained lower than the average achieved by countries in the Organization for Economic Cooperation and Development (OECD). According to the International Student Assessment Program (Inep, 2022), socioeconomic factor may be an aggravating factor for low performance.

In Elementary School, diagnostic research by Castro Filho, Santana and Lautert (2017) points out insufficient performance in resolving situations in multiplicative field. This panorama suggests the need for research that address the teaching strategies development, aiming at expanding students' mastery of multiplicative conceptual field.

In a comprehensive educational context, the situation of young deaf students enrolled in school in Brazil is no different, as they have not yet satisfactorily mastered concepts related to the additive conceptual field (Fávero & Pimenta, 2006; Peixoto; Cazorla, 2011), nor the multiplicative one (Queiroz, 2011; Fávero & Pina Neves, 2012; Peixoto, 2015).

Somehow, these unfavorable results are understandable, since deaf signers are a minority in a hearing world. Recognition of Libras (Brazilian sign language) as a legal means of communication and expression for deaf people, through the Law number 10.436/02, as well as the right to a Sign Language Interpreter (SLI) in the classroom by means of Decree number 5.626/05, and inclusion of Bilingual Education as a modality of Basic Education with the Law number 14.191/2021 are achievements that need to be consolidated in schools in our country.

Educational SLI is a character who enters the teaching process not just to translate or be a teacher, but also to encourage learning by the deaf student, mediating communication between teachers and deaf students. However, SLI often has difficulties in interpreting some subjects which are not part of initial training, bringing another obstacle to be overcome within the inclusive educational context (Muniz, 2018).

Regarding deaf people, lack of interaction with other subjects who share the same language since childhood may limit informal experiences with Mathematics and harm performance in Mathematics schooling (Nunes et al., 2008; Borges; Nogueira, 2015). It is important to highlight that such performance is assumed not only as an assimilation of algorithms transmitted by the school, but also from the perspective of conception and development of operational procedures, supported by mental schemes guided by diversity in thinking, aiming at validating production of mathematical knowledge inside school and beyond it.

In that regard, Fávero and Pimenta (2006, p. 225) affirm that “difficulty faced by deaf people when dealing with Mathematics problems comes from the schooling process, which focuses on the acquisition of rules for solving procedures, to the detriment of conceptual acquisition”. About conceptual understanding of mathematical tasks, research by Borges and Nogueira (2015) demonstrated that deaf students still do not understand activities and concepts covered in the classroom, even with an interpreter present. For the same authors, the so-called “comprehensive” school needs to rethink curricula, methodologies and teacher training, among other aspects that involve deaf and hearing people in the same space.

To contribute to development of more elaborate resolution strategies by deaf people in multiplicative situations, Queiroz (2011, p. 8) recommends mathematical statements that consider linguistic specificities and daily experiences of deaf people, associating representation supports (concrete material, paper and pencil records): “[...] thinking about alternative teaching routes in comprehensive classrooms is necessary for mathematical concepts acquisition by deaf people”.

Segadas-Vianna et al. (2016) developed combinatorial analysis activities for deaf students. The authors state that interpretation of mathematical statements in texts written in Portuguese may be a barrier for hearing students, even harder for deaf students who use Libras as their first language and Portuguese as their second one. Thereby, they recognize that, for learning, considering their linguistic competence is necessary. These authors developed the activity called Path to School, which was developed at National Institute of Education for the Deaf (INES in its Portuguese acronym), with 24 9^{th} grade students at Elementary School; in a Pre-University Exam class with 6 students; and with 12 3^{rd} grade High School.

The activity was delivered in Portuguese and later explained in Libras. Findings confirm lack of understanding the statement in Portuguese and difficulty for them to understand that situation of activity (Way to School) was not personal in nature. Furthermore, they also highlighted that the statement interpretation in Libras was crucial, as it helped clarifying unknown terms for deaf students. Figures played an important role in the process, and developing more than one drawing was necessary. In summary, the authors concluded that deaf students learning was influenced by the statements’ formulation and their writing and illustration characteristics.

Research by Peixoto and Cazorla (2011) with 3 deaf young people (two of them 6^{th} and 7^{th} grade Elementary school, and one 1^{st} grade High School) on understanding additive and multiplicative situations highlighted unsatisfactory performance in multiplicative reasoning, both in relational calculation, understanding relationships involved in the situation, and in numerical calculation.

Thereunto, understanding how deaf students present their resolution strategies or schemes is necessary, aiming at understanding consolidated, unconsolidated or in consolidation phase concepts, i.e., "[...] better understanding knowledge in action, potentialities, incompleteness, deviations and shortcuts, resignifications, errors and obstacles almost always present in mathematical productions in the classroom" (Muniz, 2009, p. 115).

In that regard, the schema construct, proposed by Piaget and retaken by Vergnaud (2009) in the Conceptual Field Theory, presents as a tool for both researcher and teacher may glimpse the complex network of concepts mobilized in the student's activity.

Then, this paper focuses the analysis of action schemes of two young deaf students in division situations, specifically combinatorial ones, based on the schema construct of Conceptual Field Theory by Gérard Vergnaud, in Brazilian Sign Language (Libras) and in communicative and cognitive potential of gestures. This study is a clipping of a doctoral thesis. Although data were collected during the mentioned thesis preparation, analysis of schemes in situation of combinatorics relies on new elements arising from more recent research and experiences, with greater depth level, which justifies new and fruitful discussions.

This situation was chosen considering that high school students need to face different situations that promote the ability "(EM13MAT310) Solving and elaborating counting problems involving different grouping types of elements through multiplicative and additive principles, using different strategies such as the tree diagram" (BNCC, 2018, p. 529). It is important that these situations are part of everyday problems which involve application of mathematical concepts and make sense to the deaf student.

Schemes in mathematical activity: combinatorial situation

When facing a situation, students leave some traces in their activity records. The various records of their action, whether verbal, written, bodily-gestural, must be examined. To "understand the nature of students' thinking and obtain their most essential constitutions" (Muniz, 2009, p. 116), the teacher must analyze the various records made by students in a proposed activity:

> In fact, what develops during the experience is *a wide repertoire of organization forms of human activity: gestures, affections and emotions, language, relationships with others, knowledge and scientific and technical skills [savoir-faire]* (Vergnaud, 2009, p. 41, emphasis by the author).

Organization ways of human activity reside in individual skills already developed not only in the school context, but also in their social life.

For the author, *scheme* is an "invariant organization of activity and behavior for a given class of situations", definition inspired by algorithm theory. Like the algorithm, the scheme refers to a class of situations, "and the invariant is not the procedure itself but its organization" (Vergnaud, 2009, p. 44). Schemes contents are operational invariants which form

> The properly epistemic part of scheme (and representation): they consist of categories (concepts-in-action) and in propositions considered to be true (theorems-in-action), whose function is precisely collecting and selecting pertinent information, in addition to processing it, to infer objectives, anticipations and rules (Vergnaud, 2009, p. 45).

Much of this content underlies student behavior, its validity is less than formal theorems, and they may even be wrong (Vergnaud, 1983, p. 146). It is up to the teacher finding these relationships in students' procedures and explain them to help producing more schemes for new situations, expanding this conceptual field. Analysis of "students' intuitive strategies" constitutes a path to "help them transform intuitive knowledge into explicit one" (Magina et al., 2008, p. 17).

According to Vergnaud (2009), gestures may integrate schemes. In that regard, Goldin-Meadow (2014) affirms that they may provide information not found in speech, for example, in a count of objects, when a child points, he or she is indicating understanding "one-to-one" correspondence, a central concept underlying the conservation of number, which does not appear in the speech.

For McNeill (1992, p. 245), "gesture, alongside the language, helps to constitute thought, and gestures reflect the mental image representation activated at the speaking moment". Gestures and speech are unitary systems, but they differ semiotically, a gesture "is global, synthetic, instantaneous and not specified by conventions" (McNeill, 2006, p. 1). Based on oral narratives, David McNeill classified the gestures into

> *Iconic gestures:* have a direct relationship with semantic discourse, i.e., there is an isomorphism between the gesture and the entity it expresses. However, understanding them is subordinated to the discourse that accompanies them. *Metaphorical gestures:* are reflections of an abstraction, in which the content is an abstract idea, rather than a concrete object, an event or a place. These gestures are externally like iconic ones but refer to abstract expressions. *Rhythmic gestures (beats)*: are short and quick and accompany speech, giving special meaning to a word, not because of the object it represents but because of the role played in the speech. *Deictic gestures:* the subject uses them to touch or directly indicate an object, person, place or event (McNeill, 1992, p. 12-18).

According to Alibali and Nathan (2012, p. 252), these categories were considered as dimensions by McNeill because “individual gestures often incorporate elements from multiple categories. For instance, a representational gesture may be performed over an object or location; this gesture is both iconic and deictic at the same time”.

Deictic gestures may be observed in the act of pointing to a triangle to refer to it and iconic gestures may be observed when a triangle is drawn in the air to represent this figure (Alibali & Nathan, 2012). Deictic gestures occurrence is common in the action of counting a collection by children, when they point to each object making a one-to-one correspondence, central concept underlying the conservation of number (a scheme), which does not appear in their speech (Goldin-Meadow, 2014). Combined gestures may occur, for example, when the student points three fingers to each finger (3-1, 3-1, 3-1, 3-1) to count 3 x 4 = 12, we consider that deictic-metaphorical gestures were performed.

In mathematical activity and communication, Alibali and Nathan (2012, p. 251) argue that “mathematical cognition is embodied in two main senses: it is based on perception and action and is grounded in the physical environment”. Considering communication between teachers and students in mathematical activity, they argue that gestures reveal embodied knowledge in three ways:

> (a) Pointing gestures reflect foundation of cognition in the physical environment, (b) Representational (i.e., iconic and metaphorical) gestures manifest mental simulations of action and perception, and (c) some metaphorical gestures reflect body-based conceptual metaphors (Alibali &Nathan, 2012, p. 252).

In teaching activity, Alibali et al. (2014) show that gestures integrate the teacher’s mathematical communication and may help students learn better.

McCleary and Viotti (2011) studied the mutual influence between linguistic and gestural elements in Libras based on a narrative by a deaf adult, observing everything, from the formation of the lexicon to the discourse organization. Findings suggest that gesture and language interact in signed languages. However, the “sign” is agreed upon by a social group and is part of grammatical structure. Nevertheless, in oral languages, the form of the word is not modified by gesture, but in sign languages, “gestures and linguistic components may go together as a complementary resource to establish directionality in the signed space and to insert absent referents in the signed speech” (Correa, 2007, p. 57).

Study by Goldin-Meadow et al. (2012) found the use of gestures in agreement with American Sign Language when solving mathematical tasks by 40 deaf children. The authors highlighted that the use of teaching strategies based on gestures may help deaf students. Peixoto

(2015b, p. 359) analyzed the performance of three deaf young people in multiplication situations and identified that these students' schemes were composed by gestures and signs (in Libras), but "gestures went beyond the function of communication and began to integrate the cognitive action of these students":

> Deictic gestures were related to counting (counting from), with the concept of cardinal, bijection and the correspondence scheme (sign-to-signal and sign-to-finger), metaphorical gestures highlighted the composition of natural numbers and the concept of multiplication as an addition of equal portions, a concept also evidenced by rhythmic gestures (Peixoto, 2015b, p. 381).

Analysis of students' schemes and their contents (concepts in action and theorems in action), articulating various language resources, such as gestures and signs in Libras, may benefit the teaching activity in expanding and mastering the Multiplicative Conceptual Field by deaf students, especially mastering the concept of division.

Before entering school, hearing and deaf children may deal with intuitive situations involving the concept of division, but deaf children may present "difficulties in mastering mathematical knowledge socially transmitted and informally acquired by hearing children" during the same period (Nunes et al., 2008, p. 265).

If teaching at school prioritizes learning algorithms instead of investing in valuing students' own schemes, it will not guarantee a broad understanding of division meanings in different situations and numerical contexts. Circumstances from students' daily lives may be addressed in teaching to enrich their conceptual repertoire, and combinatorics category may illustrate everyday experiences to benefit deaf students (Queiroz, 2011). Furthermore, Magina, Santos and Merlini (2011, p. 5) affirms that "linguistic aspects greatly interfere with students' performance when asked to solve multiplicative problems", especially with an expression not congruent with the requested operation.

Conceptual field of multiplicative structures "comprises all situations which may be analyzed as problems of simple and multiple proportions, for which multiplication or division is generally necessary" (Vergnaud, 1988, p. 141), or a combination of these operations. When classifying problem situations whose solutions admit multiplication or division, Vergnaud (1983, 1988) distinguished the types: (a) measurement isomorphism, (b) case of a single measurement space, (c) measurements product and (d) multiple proportion.

Within the scope of this study, we will address situations type (c), in the domain of natural numbers, this type of situation, called *Measurement product* may be defined as a structure based on a Cartesian composition of two magnitudes, M_1 and M_2, to find a third one

M_3. It covers situations of area, volume, Cartesian product and other physical concepts. As it involves three variables, it is not convenient to represent it by a simple correspondence table, as used in the case of measurement isomorphism, but by a double-entry table. These situations are resolved by multiplication or division (Vergnaud, 1983).

For instance, for the situation "What is the area of a rectangular ballroom that is 15 meters length and 10 meters wide?" M_1 = [wide], M_2 = [length] and M_3 = [area], the solution consists of a multiplication of two quantities in dimensional and numerical aspects: area (m^2) = length (m) × wide (m) = 10 m × 15 m = 150 m^2, e.g., solution is a measurement product. If we change the question "The area of a rectangular ballroom is 150 m^2, its length is 15 meters, what is the width of this room?" the solution is a division area (m^2) ÷ length (m) = width (m). Figure 1 shows a scheme that represents this situation (Vergnaud, 1983, p. 135).

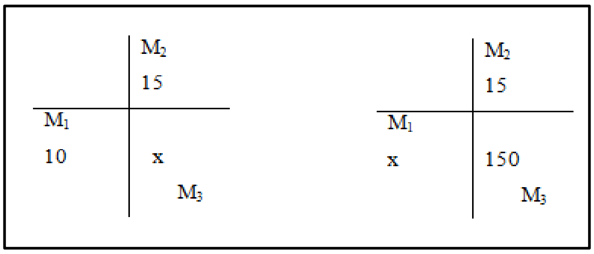

Figure 1.

Scheme of multiplication and division as a measurement product (Vergnaud, 1983, p. 135)

Let's look at an example relating to the notion of Cartesian product[3] of two disjoint sets ($M(girls) \cap R(boys) = \emptyset$), present in situations associated with the idea of combinatorics. For instance, "Four boys and three girls are dancing at a ball, each boy wants to dance with each girl and each girl wants to dance with each boy. How many different couples of a boy and a girl may be formed?". M = [m_1, m_2, m_3], R = [r_1, r_2, r_3, r_4]. This situation may be solved by multiplying 3 × 4 = 12 and easily verified by a double-entry table, according to the Table 1 (Vergnaud, 1983).

Table 1.

Double entry table: $M \times R = \{(mi, ri)/mi \in M, ri \in R, i=1, 2, 3, 4\}$ (Vergnaud, 1983, p. 135)

Boys (R) / **Girls (M)**	**r_1**	**r_2**	**r_3**	**r_4**
m_1	(m_1,r_1)	(m_1,r_2)	(m_1,r_3)	(m_1,r_4)
m_2	(m_2,r_1)	(m_2,r_2)	(m_2,r_3)	(m_2,r_4)

[3] Given two sets A and B, the Cartesian product of A and B, denoted A×B, is the set of all ordered pairs (a, b), where a ∈ A and b ∈ B. Notation: A × B = {(a, b)/a ∈ A and b ∈ B}.

m_3	(m_3,r_1)	(m_3,r_2)	(m_3,r_3)	(m_3,r_4)

In all parallel columns, it is observed that the number of couples is proportional to the number of girls, when the number of boys is constant. For example, *m* is the number of girls and *4m* is the number of couples, when $r = 4$, it is $4m/m = 4$ (constant of proportionality). The number of couples is also proportional to the number of boys when the number of girls is constant, observed in all parallel lines. For example, *r* is the number of girls, 3*r* is the number of couples, when $m = 3$, it is 3r/r = 3 (constant of proportionality), according to the Table 2 (Vergnaud, 1983).

Table 2.

Proportional relationship between the number of couples and the number of boys or girls, depending on whether one or the other remains constant (Vergnaud, 1983, p. 135)

Number of Boys / Number of Girls	**1**	**2**	**3**	**4**	**5**	**...**	**r**
1	1			4			
2	2	4	6	8	10	...	**2r**
3	3	6	9	12	15	...	**3r**
...						...	
m			3m	4m			**mr**
	Number of couples						

Changing the question, "At a ball, 12 different pairs of girls and boys were formed. As there were 4 boys, how many were there girls?", three multiplicative procedures may be identified: (i) inverse operation of multiplication ($4\times m = 12$, $m = 12\div3 = 4$) involving the theorem-in-action "dividing 12 couples by 4 boys to find the number of girls"; (ii) search for the missing factor by trial and error involving the theorem-in-action "what number multiplied by 4 is 12?" and (iii) use the double entry table or form ordered pairs (r, m) by trial and error, using the theorem-in-action "With 4 boys, how many girls are needed to form 12 different couples?".

Combinatorics is a primordial structure to expand the potential of thought, providing the opportunity to initiate hypothetical-inductive or formal thinking. In contrast to concrete operations, which rely on objects, their meetings or classes, relations, enumeration, this structure "engenders a new logic", allows the liberation of thought in relation to objects, or rather, frees “relations and classifications from their concrete or intuitive ties” (Piaget & Inhelder, 2012, pp. 118-119), Furthermore, it enables the construction of

> Any relations and any classes, whatever they may be, bringing together 1 to 1, or 2 to 2, 3 to 3, etc. any elements. This generalization of classification operations or order relations results in what is called combinatorics (combinations, permutations etc.), the simplest of which is constituted by the operations of combinations themselves, or classification of all classifications.

Daily and school problem situations provide the content to develop the concept of division which, according to Vergnaud (2009), is linked to different meanings. For this author, Mathematics is an indispensable tool for analyzing differences between these classes of problems. Teacher must use it to understand the procedures that students use and, above all, help them recognize the problem structure to find increasingly more appropriate procedures to solve it.

Methodological procedures

This research takes a qualitative approach in the design of a descriptive case study, based on a micro genetic analysis associated with videography. To understand learning processes and reveal “the flexible and circumstantial character of elaborate representations” in mathematical activity, Meira (1994, p. 59-71) recommends combining micro genetic analysis and videography:

> Videography (study of activity through video footage) and micro genetic analysis (detailed study of the evolution of relationships between agents and situations) combine to form a data collection and analysis model that allows for a robust and consistent interpretation of mechanisms psychological aspects underlying human activity (Meira, 1994, p. 59).

This excerpt contains the analysis of schemes by two deaf students aged 18, who were studying the high school 1st grade at a public school in a city in Bahia countryside. Their fictitious names are Fábia and Frank. Initially, the researchers interviewed the students to know them and about their interests, school trajectory, difficulties, skills and knowledge of Libras.

Based on this information, the following situation was worked out: “At a ball it was possible forming 12 different couples to dance. As there were 4 boys and everyone danced, how many girls were there?”. This situation was presented to each student in Libras by an SLI, in a meeting in the school’s multifunctional resource room, where Specialized Educational

Assistance (AEE in its Portuguese acronym)[4] is provided. This meeting was filmed with due clarification and consent from the students, in accordance with the ethical standards of research[5]. Dynamics involved presentation of situation in the sequence: (i) only in Libras, without any illustration, leaving marker, eraser and blackboard available for student records; (ii) If the student had difficulty understanding the statement, the question statement in Libras was repeated twice more; (iii) If there were still doubts, the illustration in Figure 2 was used to help after interpreting in Libras.

Figure 2.

People dancing at the ball, four separate boys and a girl (Personal collection)

The researcher Mathematics teacher, the deaf student and the SLI participated in this moment. The researcher could interact with the SLI and with each student, throughout the activities, when considered convenient, asking: "Why this result?" or "How did you think?".

To identify the schemes and their content, reviewing the videos several times was necessary. Then, each interactive dialogue was transcribed into Portuguese, preserving the structure of Libras, a task that involved the researcher and two interpreters. To assist in the analysis of records of deaf students' actions during problem solving, Peixoto (2015a) developed a model presented in Table 3.

[4] AEE is a Special Education service (in comprehensive perspective) which identifies, develops and organizes pedagogical and accessibility resources that eliminate barriers to the full participation of students, considering their specific needs. It complements and/or supplements the student's training aiming autonomy and independence at school and outside of it (Brasil, 2011).
[5] According to the approval of the protocol by the Ethics Committee of UESC (CAAE: 25500713.5.0000.5526)

Table 3.

Analysis model designed to record the actions of deaf students in each problem situation (Prepared by the authors)

Student		**Actions recorded**		
Action scheme	**Mobilized knowledge**	**Libras**	**Gestures**	**Written productions**
...	...	...	...	...

This model was created for each student, in which it was possible to analyze the resolution of the problem situation according to the action scheme and the type of registration in Libras, gestures and written productions. These records express the organization of the actions that student uses to resolve a given situation, in which it is possible to observe the concepts in action.

It is worth noting that Libras meets all the linguistic criteria of a language, with lexicon, syntax and ability to generate sentences. This means that it does not correspond to a set of gestures or mimics, on the contrary, gestures are identified in signed languages (Correa, 2007; Goldin-Meadow et al., 2012). However, it should be noted that such identification is not an easy task.

Thereunto, we consider that Libras, gestures and signs are mobilized by the same visual-spatial modality, then the signs were investigated with dictionaries, glossaries of that language, with the interpreter and the deaf themselves. Articulation of hands or the body that were not included in the dictionary was called gesture, unless it was a regional sign, being investigated with other SLI and with the deaf themselves. To classify gestures, we use the typology of gestures by David McNeill (1992) as a starting point.

The next section presents the profile and analysis of students' schemes in combinatorics situation.

Analysis of deaf students' schemes

This section presents the analysis of schemes of each student based on the dialogue between the students and the researcher, mediated by the SLI. Considering that the scheme is constructed by the individual himself and is shaped by the set of concepts acquired throughout formal and informal experiences (Muniz, 2009), we chose to present the profile of each student, recognizing the importance of their life trajectory, both personally and at school.

Fábia and her schemes

Fábia is 18 years old, her parents are listeners, she was diagnosed with profound bilateral congenital deafness. She started studying at the age four at regular, private school in her neighborhood, and at six years old, she went to the public school in the same local neighborhood. She learned Libras in the Specialized Educational Service at her school and with people from a church near her home. Within the family context, only her mother and a cousin know a little Libras. At the beginning of her schooling, she didn't like going to school because she didn't understand anything, but she started to like it after she had SLI in the classroom. She can read and write Portuguese and enjoys the subject, her biggest difficulty is in Chemistry. Regarding Mathematics subject, the student states that she likes it and considers that the SLI is an important help in the classroom, although there are some signs that she does not know. Whenever possible, she has a cousin who helps her with her studies at home. Although she learned 2nd degree equation easily, she has difficulty with a system of equations. She didn't know how to identify which Mathematics subjects she would like to learn more about.

Next, the records of the dialogue between Fábia and SLI in the situation of Measurements Product (Combination) are presented, which are analyzed in this paper: "*At a ball it was possible forming 12 different couples to dance. As there were 4 boys and everyone danced, how many girls were there*?"

1 **SLI**: *At a ball there were 12 different pairs...*

2 **Fábia**: *Oh! Is pair, pair* [signal "pair" alternating in space] 6?

3 **SLI**: *12 different pairs, different, man and woman, changed, 12 different, again, different, different.*

4 **Fábia**: *12 people a pair, a pair* [alternating the signal in space].

5 **SLI**: *For example, there are men and women together, different, there are 12 different. There are 4 boys, how many women?*

6 **Fábia**: *There are 12 people in pairs dancing. There are 4 boys, how many women are missing?*

7 **SLI**: *How many women?*

8 **Fábia**: [opened both hands, held 5 in her left hand and 3 in her right hand (*metaphorical gesture*)] *It seems to be 8.*

9 **SLI**: *No, it is too much!* [She pointed to the Figure 2] *here, they are dancing and exchanging [pointing out] there are 12 different ones, different, different. There are 4 men [pointing out], how many women?*

10 **Fábia**: *3.*

11 **SLI**: *Why?*

12 **Fábia**: [Pointing the signal 4 for boys and with right hand she pointed to the woman in the Figure 2] *are missing for the three men.*

13 **Researcher:** [explained the solution on the board]

To express this situation to Fábia, SLI mobilized deictic gestures (Alibali et al., 2014) to support the interpretation in Libras, according to paragraphs 9 and 12. We assessed that in this problem, SLI had difficulty interpreting it, as the interpreter often encounters barriers in organizing mathematical discourse in Libras, perhaps due to a lack of experience with concepts of the subject and with mastering Libras itself in context (Peixoto, 2015a; Muniz 2018). Such difficulty was expressing the meaning of words "pair" and "different" (paragraphs 1 to 5 and 9). We may infer that Fábia associated the word pair with the set of natural numbers, since from 1 to 12 there are 6 of them which are pair (2, 4, 6, 8, 10, 12).

Regarding the word "different", for Fábia, it is associated with the class of situations in the Additive Conceptual Field, precisely with the subtraction operation (Vergnaud, 2011). This inference is based on Fábia's response in paragraph 6, when signaling "missing" and "it seems to be 8", when using gestures considered metaphorical ones, which are reflections of abstraction (McNeill, 2006). In this case, the linguistic aspect influenced the student's performance (Magina, Santos & Merlini, 2011) because the signals "different" and "missing" are common terms for subtraction, whether in Libras or in Portuguese. Therefore, it is reasonable to assume that by showing the eight fingers of her hands, Fábia brings the abstract idea of the subtraction operation (12 – 4 = 8), although this scheme did not lead to success.

Next, SLI showed Figure 2, and apparently Fábia got it right by answering 3, but this result came from incorrect reasoning, which may have been caused by the figure itself that separately presents "four men" and "one woman". Then, according to her justification in paragraph 12 "it is missing women for the three men", it is possible that the student performed the subtraction operation 4 – 1 = 3 and not the division 12 : 4 = 3.

Table 4 presents a summary of Fábia's knowledge mobilized in the combinatorial situation.

Figure 4.

Summary of knowledge mobilized by Fábia in resolving the combinatorial situation (Prepared by the authors)

Fábia		**Actions recorded**		
Action scheme	**Mobilized knowledge**	**Libras**	**Gestures**	**Written production**
She visually performed the one-to-one correspondence in Figure 2: man (4) and woman (1) and subtracted 4 - 1 = 3	Subtraction operation on the set of natural numbers two-way correspondence	Paragraph 6: *There are 12 people in pairs dancing.* *There are 4 boys, how many women are missing?* Paragraph 12: *[...]are missing for the three men.*	Paragraph 8: she represented 5 and 3 with hands, perhaps to get 8 = 12 - 4 by *Metaphorical gesture*	No records.

In short, we observed the predominance of reasoning expressed in gestures and Libras, since written records were never used. Faced with the figure of four boys and a girl and the words “missing” and “different”, it is possible that Fábia’s reasoning was centered on these specific words, distancing the scheme from the conceptual meaning present in the situation.

Regarding the gestures, we called them metaphorical, as they sought to relate the problem data (paragraph 8) expressing the mutual influence between linguistic and gestural elements in Libras (McCleary & Viotti, 2011).

Another important fact, which we did not realize at first, was that the illustrative figure of situation (Figure 2) used unfortunately led to mistaken reasoning. It is worth noting that deaf students are prone to paying attention to details, especially visual ones, and operating with these aspects (A. A. Queiroz, 2022).

Schemes by Frank

In a similar way, to analyze Frank’s schemes in dialogues referring to situation of combinatorics, we first brought his profile, as we understand that his schemes and performance are linked to his life story, whether private or school.

Frank, whose parents are hearing, was diagnosed with bilateral profound deafness at two months age. He began attending school at age three, in regular education at a public school.

At the same school unit, he began attending the Specialized Educational Service to learn Libras, and at the same time began acquiring reading/writing in Portuguese; according to the Service teacher's statement, with good performance in reading and writing in that language. From the age of 10 to 12, he studied at a private, integral special school that did not have an SLI. After this period, he returned to regular public school, with a SLI in the classroom. Frank states that he currently enjoys school, but when he was younger, he was lost because of how messy his classmates were in class. His favorite subjects are Arts and Physical Education, and he has difficulty in Physics and History, he really likes Mathematics, but finds it difficult. He has difficulty with 2nd degree equations and considers the SLI in his classroom to be very "weak", because she helps little. He tries to study the subjects covered in the classroom at home, but he doesn't remember the ones he learned easily. He would like to learn more about the subjects of the high school 1st grade.

The dialogue between Frank and ILS is below.

1 ***SLI**: Dancing at a ball there are 12 pairs, they are together and changed again, again and again. There are 4 boys, how many girls?*

2 ***Frank**: Again.*

3 ***SLI**:* [Repeated the same way].

4 **Frank**: *4?*

5 ***SLI***: *Yes.*

6 **Frank**: [Registered on the board 12 12 12 12 vertically].

7 ***SLI***: *No. For instance, I dance with you, then I change, I dance with someone else, I change again, there are 12. There are 4 boys dancing and exchanging, different, how many are the girls?*

8 **Frank**: [Touched the index finger of the left hand to 4 fingers of the right hand (*deictic gesture*)] *3.*

9 ***SLI***: *Right, why?*

10 **Frank**: *Because there are 4 boys, one with three, one with three, one with three, one with three* [Signaled 4 (with the right hand) and pointed the sign 3 (with the left hand) for each finger on the right hand (*deictic/metaphorical gesture*), according to Figure 3)] *adding it, it is 12.*

Figure 3.

Frank signaling 4 (right hand) and pointing the sign 3 (left hand) for each finger on the right hand (Research data)

Initially, it is clear in paragraphs 1 to 6 that Frank did not understand the SLI's interpretation of situation, since he thought of adding four times the number of couples (12), in other words, making a repeated addition (12+12+12+12). However, in the third interpretation, SLI staged the problem "[...] *I dance with you, then I change, I dance with someone else, I change again, there are 12"* (paragraph 7).

This translation choice was decisive for resolving the situation, this way of interpreting is similar to a "Constructed Action", a linguistic element commonly identified in speech in Libras and oral languages, in which the narrator expresses his/her perception of the event; this aspect is "essential to understand utterances by deaf people, as it expresses a wealth of details essential for recognizing the referent" (Bernadino et al., 2020, p. 17).

From that moment on, the student quickly answered "three", explaining his reasoning in Libras "*one with three, one with three, one with three, one with three*". Thereunto, he used *deictic/metaphorical gestures*, expressing two-way correspondence "1man-3girls", he added 3girls + 3 girls + 3 girls + 3 girls = 12, value that does not correspond to 12 girls, but to 12 couples. Probably, the student was explaining in Libras/gestures (paragraph 10) the combinations {(r_1,m_1), (r_1,m_2), (r_1,m_3), (r_2,m_1), (r_2,m_2), (r_2,m_3), (r_3,m_1), (r_3,m_2), (r_3,m_3), (r_4,m_1), (r_4,m_2), (r_4,m_3)}, according to the scheme represented in Figure 4.

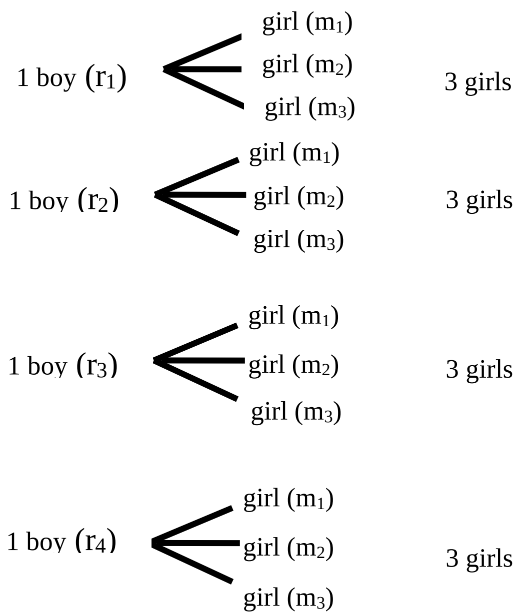

Figure 4.

Tow-way Correspondence "1 boy – 3 girls" (Prepared by the authors)

The strategy of bringing the enunciation to the deaf context may help understanding the problem statement, since these students tend to get involved in the context of situations presented. Study by Segadas-Vianna et al. (2016, p. 30) identified initial difficulties for deaf people in "interpreting and understanding the statement in Portuguese, in addition to difficult to understand that the question was not personal in nature". Corroborating this, study by Queiroz (2022, p. 139) involving situations of propositional logic showed that deaf people tend to bring their ethical and moral principles into the context of questions, "which influenced their responses and justifications through these contexts and experiences". Table 5, next, presents a summary of knowledge mobilized by Frank in combinatorics situation.

Figure 5

Summary of knowledge mobilized by Frank in resolving combinatorial situation (Prepared by the authors)

Frank		**Actions recorded**		
Action schemes	**Mobilized knowledge**	**Libras**	**Gestures**	**Written productions**

Mentally search (trial/error) for the number of girls from a reference value of 12 (number of possible combinations) and the number of boys (4)	Two-way correspondence (by finger sign) Addition Operation	Paragraph 10: *Because there are 4 boys, one with three, one with three, one with three, one with three* 3-1,3-1,3-1,3-1	Paragraph 10 - Addition: 3 + 3 + 3 + 3 = 12 Paragraphs 8 and 10: pointing sign 3 for each finger of configuration 4 : 3-1,3-1,3-1,3-1 Mental addition: 3+3 + 3 + 3 = 12 (Finger-sign correspondence by deictic/metaphorical gestures)	Paragraph 6: Addition N 12 + 12 + 12

Then, it is possible to observe how the work of the SLI is fundamental, and the importance of her mathematical competence in appropriation and communication of situation. Perhaps this is one of the factors that led Frank's performance to be satisfactory, getting the situation right with few interactions from the SLI and none from the researcher, but his reasoning was based on the Additive Conceptual Field; however, it is necessary for it to make a qualitative leap.

Conclusions

The analysis of action records in Libras, gestures and written productions of two deaf students in a combinatorial situation involving division, allowed an approximation of circumstantial conceptualization stages of each of them. It is worth remembering that only a small portion of our knowledge may be explained (Vergnaud, 2009), then our interpretation was an "approximation", a constant formulation of hypotheses about the real meanings that student was attributing to situations.

In general, these students' schemes were based on additive reasoning or perhaps in a transition phase from additive to multiplicative reasoning. Frank solved the combinatorial situation in space, using gestures and signs in Libras, and the linguistic aspect influenced this student's choice of resolution strategies. For instance, when SLI staged the problem "[...] *I dance with you, then I change, I dance with someone else, I change again, there are 12"* (paragraph 7), using a linguistic resource placing himself in the context of situation when interpreting the situation for Frank, it is possible that this reflected in his better performance regarding to Fábia. However, the knowledge repertoire presented by Frank is richer than that of Fábia, as he responded more promptly and performed the mental calculation.

In mathematical activity, the gestures occurred both in interpretation in Libras by SLI, who inserts her own fundamental mathematical understanding into her mediation in the appropriation process by the student, and during the student's cognitive action. Furthermore, appeared combined, for instance, "*deictic/metaphorical*" expressed concepts of addition and one-to-one or finger-to-sign correspondence, suggesting that gestures may be used to teach these concepts in situations.

The results obtained in this study highlight the importance of reflecting, in translated linguistic environments, on the elaboration of problem situation statements in both Portuguese and Libras, considering the linguistic competence of each deaf student. In Portuguese language, understanding the proposed situation may be compromised, due to the possibility of terms unknown to deaf students, since their first language is Libras. Therefore, it is important that presenting the statement in Portuguese include figures that explore the situation, considering the visual experience of the deaf person. Furthermore, there is the possibility of preparing utterances in video-libras using the students' schemes and the linguistic resources of sign language, for example, gestures and Constructed Action.

The method developed here may be a tool for evaluating mathematical activity of deaf signers to understand their potentialities, errors, incompleteness, aiming to seek teaching situations to expand this conceptual field. Thereby, detailed description of schemes mobilized by deaf students may benefit mathematics teacher in teaching this concept within the comprehensive educational context.

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