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The initial training of mathematics teachers: a case study from the perspective of the documentational approach to didactics

La formación inicial del profesor de matemáticas: un estudio de caso desde la perspectiva del enfoque documental del didáctico.

La formation initiale de l'enseignant de mathématiques : une étude de cas dans la perspective de l'approche documentaire du didactique

A formação inicial do professor de matemática: um estudo de caso na perspectiva da abordagem documental do didático

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Abstract

Initial training is one of the conditions that determine teaching professional development. Therefore, it is essential to analyze the didactic and pedagogical processes underlying it in order to provide adequate training that enables the teacher's future professional experiences. The present study aims to explore the development of the professional knowledge of a Mathematics degree student from the perspective of the Documentary Didactic Approach. The nature of the research is qualitative, in the form of a case study, based on the Reflective Research Methodology with articulations between analytical results from lexical analysis software and the participant's academic activities. The student's teaching documents are unstable, with advances and setbacks resulting from the training process. However, the results indicate that it is possible

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to extract operational invariants and action rules, arising from their interaction with resources from their teacher trainers and their educational process as a whole.

Keywords: Mathematics education, Documentational approach to didactics, Initial training, Case study.

Resumen

La formación inicial es una de las condiciones que determinan el desarrollo profesional docente. Por lo tanto, es fundamental analizar los procesos didácticos y pedagógicos que subyacen en él con el fin de brindar una formación adecuada que posibilite las futuras experiencias profesionales del docente. El presente estudio tiene como objetivo explorar el desarrollo del conocimiento profesional de un estudiante de Licenciatura en Matemáticas desde la perspectiva del Enfoque Didáctico Documental. La naturaleza de la investigación es cualitativa, en forma de estudio de caso, basada en la Metodología de Investigación Reflexiva con articulaciones entre los resultados analíticos de un software de análisis léxico y las actividades académicas de los participantes. Los documentos docentes del estudiante son inestables, con avances y retrocesos derivados del proceso formativo. Sin embargo, los resultados indican que es posible extraer invariantes operacionales y reglas de acción, derivadas de su interacción con los recursos de sus formadores de docentes y de su proceso educativo en su conjunto.

Palabras clave: Licenciatura en matemáticas, Enfoque documental de la enseñanza, Formación inicial, Estudio de casos.

Résumé

La formation initiale est l'une des conditions qui déterminent le développement professionnel de l'enseignement. Il est donc essentiel d'analyser les processus didactiques et pédagogiques qui la sous-tendent afin de fournir une formation adéquate qui permette aux futures expériences professionnelles de l'enseignant. La présente étude vise à explorer le développement des connaissances professionnelles d'un étudiant en licence de mathématiques dans la perspective de l'approche didactique documentaire. La nature de la recherche est qualitative, sous forme d'étude de cas, basée sur la Méthodologie de Recherche Réflexive avec des articulations entre les résultats analytiques des logiciels d'analyse lexicale et les activités académiques du participant. Les documents pédagogiques de l'étudiant sont instables, avec des avancées et des reculs résultant du processus de formation. Cependant, les résultats indiquent qu'il est possible d'extraire des invariants opérationnels et des règles d'action, découlant de leur interaction avec

les ressources de leurs formateurs d'enseignants et de leur processus éducatif dans son ensemble.

Mots-clés : Licence de mathématiques, Approche Documentaire du Didactique, Formation initiale, Étude de cas.

Resumo

A formação inicial é uma das condicionantes que determinam o desenvolvimento profissional docente. Portanto, é imprescindível analisar os processos didáticos e pedagógicos subjacentes a ela de modo a oportunizar formação adequada que capacite às futuras vivências profissionais do professor. O presente estudo objetiva explorar o desenvolvimento dos saberes profissionais de uma estudante de Licenciatura em Matemática sob a perspectiva da Abordagem Documental do Didático. A natureza da pesquisa é a qualitativa, na forma de um estudo de caso, com base na Metodologia de Investigação Reflexiva com articulações entre resultados analíticos de um software de análise léxica e de atividades acadêmicas da participante. Os documentos didáticos da licencianda se apresentam em instabilidade, com avanços e retrocessos decorrentes do processo formativo. Contudo, os resultados apontam que é possível extrair invariantes operatórios e regras de ações, provenientes de sua interação com recursos de seus professores formadores e de seu processo educacional como um todo.

Palavras-chave: Licenciatura em matemática, Abordagem documental do didático, Formação inicial, Estudo de caso.

The knowing of teachers holds a plurality of conceptions that make it heterogeneous from its roots, making the understanding of the constitution of the teaching profession difficult and complex, once its components are debatable and multifaceted. On the other hand, understanding such knowing is essential to identify the domains of professional development and the underlying challenges.

In this context, theorists such as Shulman (1986; 1987) outline the essential knowing that permeates the teaching profession, while Ball, Hill and Bass (2005) specifically detail what refers to the mathematics teacher. Additionally, Remillard (2000) and Adler (2000) focus on clarifying tangible aspects in the training of the teacher and the repercussion of the employed resources on the teacher's professional knowing. Likewise, in this study, the definition of teacher knowing is covered by Fiorentini, Nacarato and Pinto's (1999) approach, which synthesizes it as being reflective, plural, complex, historic, provisional, contextual, affective and cultural, forming a web of scientific knowings stemming from the educational sciences, the knowings of the courses and from the curricula and knowings gathered from experience.

Considering this scenario, the Documentational Approach to Didactics - DAD (Trouche *et al.,* 2020) emerges as a theoretical option which allows for a deeper understanding of a teacher's professional development through the analysis of their interactions with resources, starting from the initial training and considering, in a reflective manner, the experience contexts under which the teaching instrumentation and instrumentalization, concepts that will be explained in the theoretical foundation of this work, were processed.

As pointed out by Assis and Trouche (2021), the initial training holds moments of status change, from the student to the teacher, which are easier to capture. Hence, the DAD applied to the understanding of the future teacher's resources allows to understand the aspects determining the possible professional development paths considering their pedagogical beliefs, didactic actions and the materiality analysis of their academic activities.

In this sense, the initial training is considered one of the conditioning aspects of a teacher's professional development, since this is the starting point of their didactic documents' genesis - which are fundamental in the construction of future teaching practices, as presented by Assis and Trouche (2021). Such practices have, as a determining factor, the observation time for the understanding of the phenomenon and the identification of usage schemes for the formulation of conjectures about the documentational genesis, according to a study by Xavier

Neto, Silva and Trouche (2021) about the academic production related to documentational genesis between years 2012 and 2020.

In the same sense, Silva and Lima (2021) analyze studies showing that the Reflective Investigation Method applied to the DAD enables the studied teacher to analyze and criticize their own choices, favoring a better understanding of the teacher's interactions with the resources in the documentational genesis, in a longitudinal supervision of the teacher's work.

Given these aspects, this study aims to explore the development of the professional knowings of a mathematics undergraduate student from the perspective of the Documentational Approach to Didactics through a qualitative research study in the form of a case study, based on the Reflective Investigation Method - underlying the DAD, which combined interviews and the analysis of the student's academic activities with the researchers' empirical observations.

In addition to this introduction section, this study presents the employed theoretical references, the methodology, the analysis and discussion of the results and the final considerations. The next section provides the DAD's conceptual basis and its relation to the initial training.

Theoretical concepts of the documentational approach to didactics

The DAD is based on the instrumental conception (Rabardel, 1995; Artigue, 2002; Guin, Ruthven & Trouche, 2005), initially developed to study how students learn mathematics with technology. In the DAD, a teacher's professional development is considered through their interactions with the teaching-learning resources (Gueudet & Trouche, 2009; Bellemain & Trouche, 2019), which may be books, pieces of software, videos, curricular guidelines or even conversations with their peers. In addition to those, the DAD is supported by the usage schemes, the instrumentation, the instrumentalization, the documentational genesis and the didactic documents, which are in the heart of understanding the teaching practice.

The usage schemes are seen as an interaction process in which the teachers develop specific forms of using a resource for a given didactic objective. These schemes may differ from teacher to teacher, even when related to the same resources and goals. Hence, for a given class of situations, a teacher develops a stable activity organization - a scheme, which consists of the activity's goal, action rules, operational invariants and inference possibility (Trouche *et al.,* 2020).

Two other processes of interest for the DAD are the instrumentation and the instrumentalization. The former refers to the teacher's contact with resources that may present the potential to change the teaching practice, while the latter are the dispositions and knowledge

that guide the teacher's preference for a given resource. The instrumentation and the instrumentalization emphasize the dialectical nature of the DAD, as they refer to teacher-resource interactions in recombination processes (Trouche, Gueudet & Pepin, 2020).

In the basis of the DAD, this creation process is subjected to the operational invariants, which may be one of two types: the theorems-in-action and the concepts-in-action. The former are propositions seen as true - pedagogical ideas valued and absorbed by the teacher - their didactic beliefs on the teaching needs, developed during their initial training process and during work. Concepts-in-action, on the other hand, are representations seen as relevant in the sphere of the contents studied. Furthermore, in the process of class conception, there are still rules of action, which are seen as the ways to substantiate the invariants (Trouche *et al.* 2020; Gueudet & Trouche, 2015).

In terms of processes during the interaction with a certain resource or set of resources, teachers develop a didactic document, which is, in short, a resource adapted for a pedagogical objective. The conception of these documents is called documentational genesis (Trouche, Gueudet & Pepin, 2020).

The professional teacher development, then, involves a process in which resources influence the teacher (instrumentation) who transforms those resources by adapting them to their practice (instrumentalization). Over time, this movement peaks at the documentational genesis, determined by the usage schemes, which, in turn, are linked to the operational invariants and action rules. Thus, a teaching document is the result of a resource that is subjected to the usage schemes of a given teacher.

The time variable is interesting for the process, as the teacher's knowings are cumulative constructions during their professional life. However, this research requires understanding the initial movements of teaching. Hence, the next section offers an approach related to the initial training.

The DAD in the initial training

The DAD provides a structure for the analysis of a teacher's work based on the study of their interactions with the resources they use and develop to produce their teaching. However, during initial training, a moment in which the future teacher develops the formal knowledge needed for the profession, Trouche (2021) raises the question about the transitivity of the operational invariants and the resource system, as the individual is halfway between the conditions of teacher and student, and their actions are yet to become stable.

In this sense, Assis and Trouche (2021) present a research work aiming to illustrate the documentational genesis, considering a situation of planning and implementation of a class by a mathematics undergraduate student, from a perspective that highlights the support resources that she uses and the adaptations that are made. In the study, the authors identified reasons for choices and the learning found in the training process, highlighting the future teacher's beliefs and learning.

Two situations are evident for the future teacher: studying and teaching. Studying situations include experiences related to the study of any topic during the program, while teaching situations consider experiences aiming at teaching something, be it in the university, with colleagues as the audience, or at a school, with students during pedagogical practices and internships (Assis, Gitirana & Trouche, 2019).

The category shift from student to teacher is understood by Assis, Gitirana and Trouche (2019) as a metamorphosis. Resources and their uses influence initial training and, therefore, it is plausible to understand that the resources of the training professors and even the institution in which the undergraduate students apply their professional learning activities interfere in their training. After discussing the theoretical concepts that structure this research work, the methods used will be presented in the next section.

Methods

This research uses a qualitative approach, characterized by a case study involving a student - named "Aline" in the context of this work - on the fifth semester of the undergraduate program in Mathematics from a public university in southern Brazil.

This study is part of a doctoral research project, approved by the research ethics committee (CAAE: 558483622.6.0000.0118), with the scope of analyzing the conception process of statistical knowledge of the future mathematics educator. Thus, Aline was invited to take part in the study, as she was enrolled and studying Statistics and Probability, and she was involved with other academic activities related to statistics. Thus, the future teacher's didactic documents analyzed here have statistics as their theme.

In this context, this research work considered Aline's participation in the Statistics and Probability classes, observed by the researcher (Author 01) during the first semester of 2022, the preparation of simulated classes in the Mathematics Teaching Laboratory IV course, two interviews lasting 35 min one and 36 min the other, conversations using instant messaging applications and e-mail records. Moreover, the student carried out her final thesis research under the supervision of the researchers involved in this study during the following semester.

All these aspects provided the opportunity to apply the Reflective Investigation Method, which is part of the DAD's theoretical construction and is based on the analysis of the teacher's documentational work considering elements such as: the diversity of resources supporting such work and their results; the variety of interactions (collective, institutional and social) influencing this work; the required time for the development of the documentational geneses (Trouche *et al.*, 2020).

In this sense, data referring to the reflective investigation were collected in semi-structured interviews (found in appendix 1 of this study), informal communication via text messages and in the student's didactic documents, such as activities in teaching laboratories, internship plans and articles written for the courses. In the analysis, the following were considered as reflective domains: training, experiences outside the program, degree in mathematics, remote teaching, learning of contents and resources.

The analyses were carried out with the help of the *Iramuteq* software (Ratinaud & Marchand, 2012), using the Descending Hierarchical Classification (DHC) lexical analysis and similarity graphs searching for categories, similarities and linguistic representation patterns. The filtered terms were verbs and nouns, which represent names and actions in the speech of the student participating in the research.

The Reinert method (Reinert, 1987) was used for the DHC, using the double configuration on text clustering (double on RST), as there were short text segments. According to Salviati (2017), double clustering is performed on two tables, in which the rows are a cluster of the text segments. The same treatment is, therefore, performed twice, changing the number of active forms for the RST. This allows for a deeper lexical analysis of the text segments. The participant's statements are shown in italics - those longer than three lines are offset from the text by 4 cm from the margin.

All data were analyzed in convergence with the theoretical approaches that support this research work, such as Gitirana and Trouche (2019), Trouche *et al.* (2020) and Assis, Trouche, Gueudet and Pepin (2020), among others. The next section provides evidence of the genesis of a resource system - still as protoresources and protodocuments[4], which will be the basis for the professional development of Aline as a teacher.

[4] Terms coined by Author 01 that mean the resources and documents that are in an instability process and will result in the first resources and documents of the future teacher.

Analysis of the case study

Aline attended Basic Education entirely in public schools. She always liked studying and always wanted to be a teacher. She wanted to be an "educator" and, later, a "mathematics teacher". She states that when she started the undergraduate program, she had a difficult time because she lacked the knowledge about some contents, a gap she had to fill by herself.

> [...]I look back and remember this strongly because, in the first classes, when I saw symbols on the chalkboard, I didn't even know what those were. An equation, for example, an inequality, I didn't even know it was a thing, that that kind of equation existed. (Aline, Research Data, 2022).

Thus, she had to make a huge effort to overcome the difficulty she had regarding the contents and be able to follow the program. According to her, the realization that she needed to study came after a test in the second semester, in which her performance made it clear that she needed to overcome the gaps arising from Basic Education.

> And so, it took me a while... it's an obvious thing, if you don't know something, you try to learn it, but it took me a while before this dawned on me. It was in the second semester, I know when it happened, it was in Analytical Geometry, that I did really poorly on the first test and that's when I realized 'man, this is not going to work, if I just keep attending the classes and taking the tests... it's not going to work. Now I understand, but I need a foundation, I need to chase this and study". (Aline, Research Data, 2022).

Like that, she reckons, she learned how to go after what she needed, to do some research and to study. By the time she gave us the interview, she realized she had succeeded in her learning endeavor, and it was "*a dream*" to be at the end of the fifth semester of the program, with plans for her graduate studies.

One of the experiences that can be highlighted in Aline's training is the optional internship, which she completed at a school that offers personalized training for the development of mathematical skills. With this experience, the undergraduate student had contact with the teaching of mathematical content, as she needed to monitor the activities carried out by the students. Aline said that her internship did not help in the construction of her knowledge of contents, but she learned to dialog with students and deal with different moments in class and, therefore, she developed general pedagogical knowledge, which is based on understanding student characteristics, cognitive processes and how students learn (Schulman, 1986).

> [...]I supposed that would enable me to explain every topic very well. But then I realized I couldn't. [...]. I learned to create a rapport with the students, to spot their qualities and

> to talk to them. I learned to share this moment, now we are going to study, now we are going to talk, now things are slower, now we have to go a little faster. To really have this teacher attitude. To tell, you are the student, I am the teacher, but we do get along, but, at the same time, we are here with a goal: to study, to learn mathematics. (Aline, Research Data, 2022).

One of Aline's concerns during her internship was working with students with disabilities. She stated, at the time, that she had no knowledge in this area and that she had plans to study the subject. She helped three students with disabilities during her internship and made it clear that she needed to delve deeper into the topic, as she considered herself "lost" in her actions.

> I find it difficult; I don't know how to deal with it, I don't know if I'm being rude or too nice, or if I'm treating people differently, or in the same way and that's a problem, I feel really lost. It's something I want to do, but in the future, now I can't. (Aline, Research Data, 2022).

The future mathematics teacher needs to be prepared to work with heterogeneity, understanding differences and valuing the potential of each student so that teaching favors everyone's learning (Rosa, 2020). When a student is transitioning from the status of learner to that of a teacher, it is essential to have contact with resources that will allow them to deal with differences. This contributes to the expansion of equitable relationships at school.

Aline also held a scholarship by PIBID (Institutional Teaching Initiation Scholarship Program). She was, however, frustrated for not joining projects and not living the school dynamics, since her group focused closely on the reading and discussion of papers in detriment of their presence in the school. This may have been caused by the sanitary measures necessary to contain COVID-19, which occurred at the same time.

Regarding the Pandemic, which should not be overlooked as a variable when analyzing the training of future teachers who went through this moment of health crisis, Aline stated that, at the beginning, she had problems such as not having a personal computer or a suitable place to study at home. Later on, she was unable to pay attention to the remote video classes, often having to rewatch them, as the amount of information around her or on-line made her lose focus. When she gave the interview, she stated that she was going through a process of reeducation for the face-to-face classes, since now she could not "rewatch them, as they are not recorded".

The Pandemic posed new challenges to the use of teaching and learning resources in response to the emergency (Huang, Huang & Trouche, 2023). These aspects must be studied, evaluating their impact on the documents of teachers who experienced this period, whether as

learners or working in the classroom, exploring potentialities for the mathematics education as a whole and understanding systemic weaknesses arising from this period.

The graph in Figure 01 shows the ideas emerging from Aline's interview, considering only verbs and nouns, to capture ideas related to her actions in the movement of her initial training. Three centers may be observed: "knowing", "class" and "teacher".

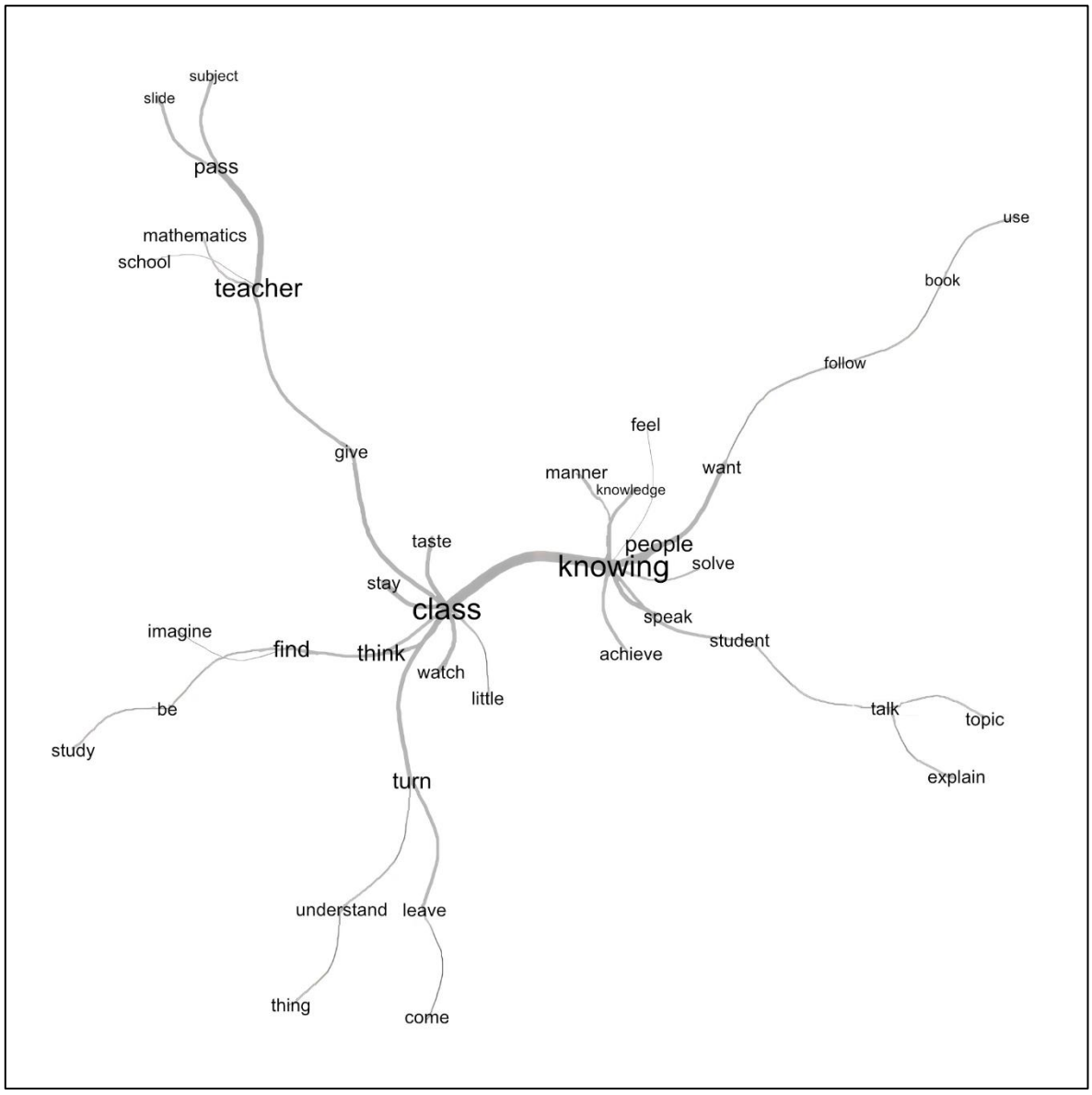


Figure 01.

Some ideas emerging from Aline's statements.

Ideas expressing Aline's concern about the knowings of a teacher emerge from the cluster centered on "knowing". The branch containing "people" stands out, which, throughout the interview, substantiates Aline's view on the relationship with her colleagues and the

relationship of trust built with some of her professors, the one from the Mathematics Teaching Laboratory IV course being one of them.

> He has this lightness and he knows how to win us over, as he talks. This way, we build a friendship. I can't really explain it, because we don't talk to him outside of the class, or about off-class topics. But, in there, we feel connected, confident. (Aline, Research Data, 2022).

The associations between "knowledge", "manner" and "feel" also stand out in the "knowing" branch, evoking the thought that the professors inspired Aline with their knowledge: *"It's just that he inspires me in a certain manner... how he knows so much. [...] And he shows so much knowledge, we ask anything, and he knows the answer. In the exercises, he can solve everything. So, he inspires me in knowledge."* (Aline, Research Data, 2022).

Another context of the "knowing" branch is in the attention to the student while in classroom, with associations to the terms "speak", "student", "talk", "topic" and "explain", from which the undergraduate student's insecurities about the context and the dynamics of the classroom emerge, as illustrated in the following excerpt from the interview.

> But I have a personality trait that is really part of me, that I can't get them to stay... I can't scold them... I can't ask them to stop talking so I can explain something. So, I would prefer to split them in pairs, in small groups, to make some islands in the classroom. I believe that is how I would organize the class. I think it would be more a chalkboard-and-chalk manner. (Aline, Research Data, 2022).

Gravitating towards the center "class", we can see the various meanings and perceptions the student attributed to this moment, such as "taste" (expressing her preference for certain teaching styles, in her case, the traditional); "stay" (regarding the state of the students in class - distracted, disciplined); "give" (always connected to the act of teaching), "watch" (participate in the class) and "think" (the act of planning a class). The center "teacher", in turn, evokes two conceptions marking the transience through which Aline went as a student, with her concerns about "passing", "subject" and "study". The student's preparation to enter the teaching profession also emerges as "mathematics" and "school".

It is clear from Aline's statements that the undergraduate program in mathematics brought about changes in her way of studying and her perspective on the development of knowledge and skills. When she states that she had some difficulties and she had to search for alternatives, this change becomes evident. Also, when she explains that she did not know how to search for information, to look into books and that, normally, she chose to acquire knowledge from the chalkboard only, she indicates that those are teaching situations experienced by her

in the largely traditional Basic Education, which did not allow the development of her cognitive autonomy.

As for the resources she employs when studying, Aline referred to the material provided by the professor and exercise lists, not including new materials, even those suggested by the professor: *"He recommended a book and I even took it from the library, but I didn't use it. I never got to open it, and I didn't watch any recorded supporting classes, I only used the professor's materials"* (Aline, Research Data, 2022). Figure 02 provides a record of the studying resources representation, made by the student.

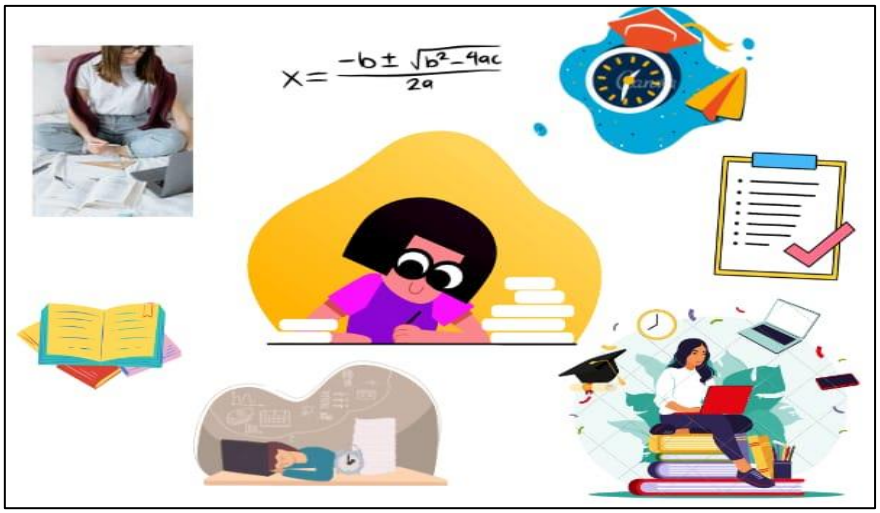


Figure 02.

Record of resources representation.

Aline also explained, on a conversation using an instant messaging application, the resources she employs to study (Figure 03). She showed the professor's slides, the exercise lists and her notes from the classes, situations that were also highlighted in the analysis of the interviews.

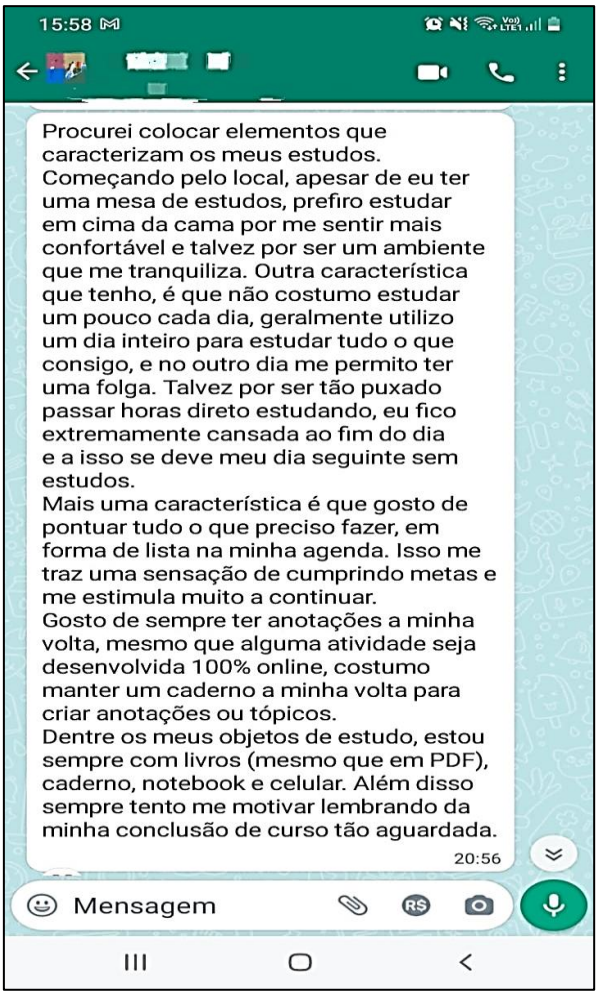


Figure 03[5].

Excerpts from a conversation with Aline via an instant messaging application.

The next section presents the analysis of an activity carried out in the Mathematics Teaching Laboratory IV course, in which Aline developed two simulated classes: one in the traditional format and the other, which was called "differentiated".

[5] [I tried to include elements that define my studies. Starting from the place, although I have a desk, I prefer to study on the bed, as I feel more comfortable and maybe because it's a soothing environment. One of my traits is that I don't usually study a little every day. I normally use a whole day to study everything I can, then I allow myself to take the next day off. Maybe, because studying long hours is such hard work, I feel extremely tired by the end of the day, and that's why I don't study the next day. Another thing about me is that I like to list everything I need to get done, as a to-do list on my calendar. This makes me feel I'm keeping up with my goals and drives me to continue. I like to always have notes around me, even when a certain activity is carried out completely on-line, I usually have a notepad nearby to take notes. Among the objects I use to study, I always have books (even if in PDF), a notebook, a laptop computer and a cell phone. Besides that, I always try to stay motivated by thinking about my highly-expected graduation.]

Analysis of Aline's simulated activities.

Aline's simulated activity was about probability. The goal of the Mathematics Teaching Laboratory IV course was, at first, working the contents using a traditional conception and, later, using a differentiated method.

In the statements and activities developed by the student, the use of the terms "differentiated class" and "traditional class" was noticed, with opposite meanings. These terms are found in the Mathematics Teaching Laboratory IV syllabus, under the program content: "Introduction of examples of traditional and differentiated mathematics classes" (Research Data, 2022). Thus, in order to coherently conceptualize the terms in relation to what was presented in the observations, the professor who taught the course was asked to explain the concepts, which were elucidated from his reflections in a 14'40" audio, as well as a text, spreadsheets with tabulated research data on the topic and slides. The professor's understanding clarifies that the traditional class and the differentiated class are contrasted by the idealization of the intellectual field, once they are not distinct sets, since they have similar characteristics. In a differentiated class, the learning process is centered on the student, who is the protagonist and will learn mathematics by making conjectures, creating from investigative practices. In this type of class, the teacher guides the process. In a traditional class, in turn, the student is a passive agent in their learning process, which has the teacher in the center, with contents being exposed and examples being repeated through exercises. He exemplifies both types with classes developed by his own students, and observes that the concepts are very volatile, since an expository class, in which the students do an exercise, may be organized as an investigative practice (Training professor - Mathematics Teaching Laboratory IV course, textualization of research data, 2022).

Aline's simulated class had, in its educational plan, the following goals: understanding the concept of probability; learning to interpret random experiments using probability to calculate the odds of certain events; calculating the probability of a random event and express it as a fraction; learning the rules of probability and using the ideas of sets to facilitate the development of the calculations. In this sense, it has an essentially content-based scope, giving indications of Aline's perspective on planning a class with this characteristic.

The didactic organization of the class was based on coin-tossing-related problems (heads and tails) and the random picking (drawing) of colored balls to explain probability as the odds of an event taking place, directing to the equation $P = \frac{Number\ of\ positive\ cases}{Number\ of\ possible\ cases}$. These examples were explained through an expository class using the chalkboard, as defined by the

educational plan: “Next, I will draw on the board a box with eight balls: three white, three blue and two orange ones, and I will ask the question: What is the probability of drawing: Example 1: a blue ball? Example 2: A blue ball or an orange ball?” (Educational Plan - Aline, 2022). Figure 04 shows one of the records from the chalkboard as used by Aline showing the proposed drawing for the explanation of the proposed problem situation from the educational plan.

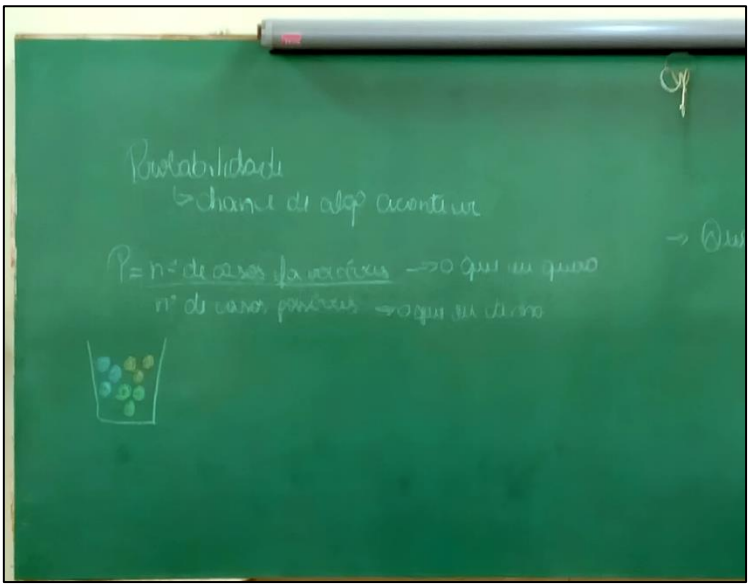
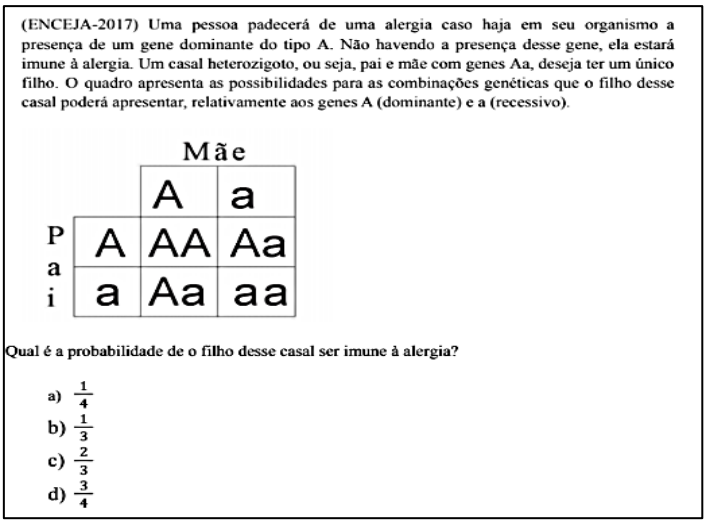


Figure 04.

Use of the chalkboard.

After this initial explanation, Aline understood the need to formalize the contents, presenting the concepts of an event and sample space in class, from examples related to a deck of playing cards and a set of dice. The examples used in the first conceptual approach and for formalization were taken from the textbook listed in the educational plan - Mathematics: Context and Applications (Dante, 2011). After the expository class, Aline moved to the exercise lists. She observed that the context of the exercises is not the same as the examples, using contextualized themes, as shown in Figure 05.



(ENCEJA-2017) Uma pessoa padecerá de uma alergia caso haja em seu organismo a presença de um gene dominante do tipo A. Não havendo a presença desse gene, ela estará imune à alergia. Um casal heterozigoto, ou seja, pai e mãe com genes Aa, deseja ter um único filho. O quadro apresenta as possibilidades para as combinações genéticas que o filho desse casal poderá apresentar, relativamente aos genes A (dominante) e a (recessivo).

		Mãe	
		A	a
Pai	A	AA	Aa
	a	Aa	aa

Qual é a probabilidade de o filho desse casal ser imune à alergia?

a) $\frac{1}{4}$
b) $\frac{1}{3}$
c) $\frac{2}{3}$
d) $\frac{3}{4}$

Figure 05[6].

Example of activity from the exercise list.

Regarding the activity related to the differentiated class, Aline went with a playful proposal, using the "Game of Maximum" adapted from Vianna and Silva (2021), whose article inspired the activity. The educational plan had, as the goal: "to learn how to interpret random experiments using probability to calculate the odds of a given event to take place; to calculate the probability of a random event and express it as a fraction; to interpret data to draw a conclusion". The first difference that stands out is the presence of the verb "to interpret", which was not employed by Aline in the traditional class.

For the class, Aline proposed dividing the students into groups of two or three to play the game, which uses two fair dice and works as follows: After an initial draw to find who starts the game, each student rolled their dice. If the highest score from the dice were between 1 and 4, the first player would win and, if it were between 5 and 6, the second player would be successful.

The students also enter data into a table and built graphs during the game. Written records during games are important for the combined study of other contents, for the understanding of the reasoning employed by the students, and they can even be considered part of the assessment (Moreira & Souza, 2020). Figure 06 shows these extra materials proposed by Aline.

[6] [A person will develop an allergy if their organism has a dominant type-A gene. Without this gene, they will be immune to the allergy. A heterozygous couple, that is, father and mother with Aa genes, wants to have only one child. The table shows the possible genetic combinations for a child of this couple, regarding genes A (dominant) and a (recessive). What is the probability of the child being immune to the allergy?]

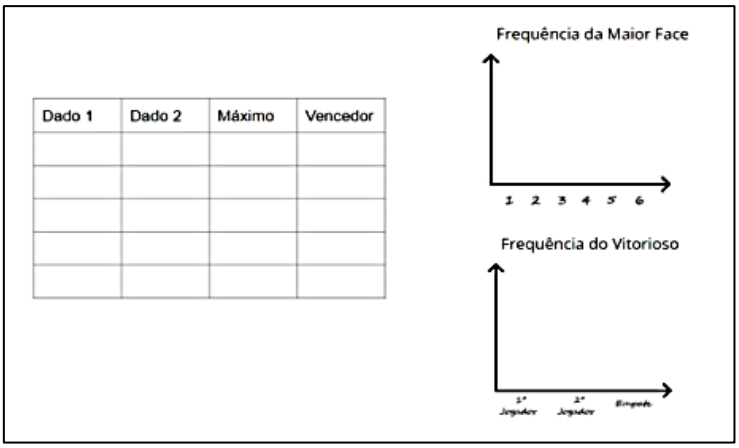


Figure 06[7].

Table and graphs used during the game.

The educational plan defined that, while the students were playing, Aline would write a few questions (Figure 07) on the chalkboard, which would be discussed and answered on a separate sheet of paper the students would hand in later. "It's important that you answer these questions on a sheet of paper to be handed in." (Educational plan - Aline, 2022).

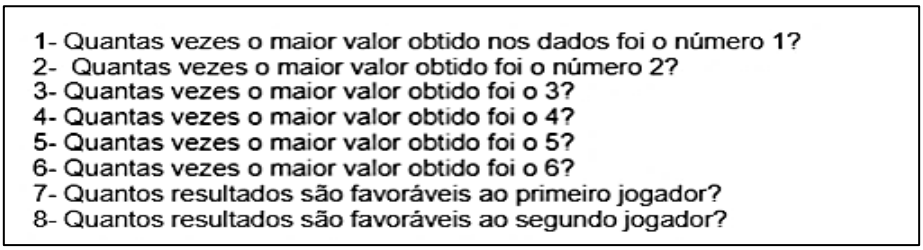


Figure 07[8].

Few questions on the chalkboard.

After the game, a new table was provided to the students with more space to expand the activity. At the end, students were asked who the winner was, and if the game was fair. Like so, Aline encouraged the students to realize the game was advantageous for Player 2.

For the second part of the class, the game continued with a new table, in which students had to input the highest number to come up on the upper face of the dice at the intersection between players 1 and 2. For example, if the first die were a 4 and the second a 6, the highest

[7] [Die 1, Die 2, Maximum, Winner / Frequency of highest number / Frequency of winner]

[8] Questions proposed by Aline. [1- How many times was the highest number 1? / 2- How many times was the highest number 2? / 3- How many times was the highest number 3? / 4- How many times was the highest number 4? / 5- How many times was the highest number 5? / 6- How many times was the highest number 6? / 7- How many results favor the first player? / 8- How many results favor the second player?]

number (6, in this case) should be written at the intersection between row 4 and column 6, as Figure 08 shows.

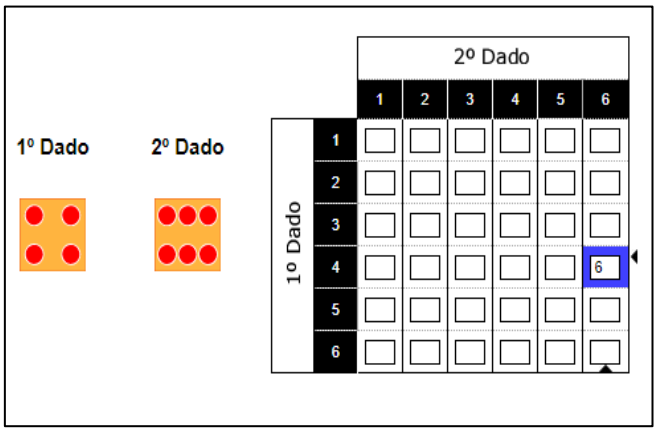


Figure 08.

Example of the Game of Maximum during the second part. (Source: IME/USP)

Since this was a simulated class, Aline's colleagues - mathematics teachers in training, answered the activity sheets (Figure 09) and reasoned about the dice game, trying to find patterns in relation to the game's result. In this sense, the undergraduate students also performed a deep analysis of a pedagogical activity, which may become a resource in their future professional experiences (Trouche *et al.*, 2020).

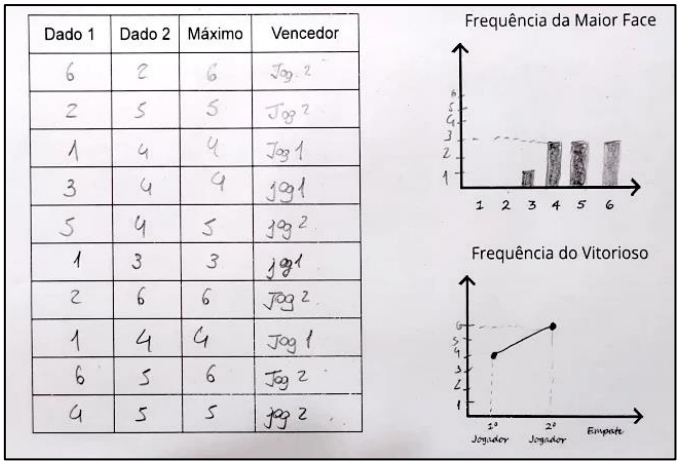


Figure 09[9].

Answered activity proposed in the simulated class.

After the game, Aline proposed some questions to figure out the reason why Player 2 had an advantage in the game: "What is the probability of Player 1 winning? What is the

[9] [Die 1 / Die 2 / Maximum / Winner / Frequency of highest number / Frequency of winner]

probability of Player 2 winning?" As a challenge, Aline proposed that ways to make the game fair should be analyzed. In terms of assessment, the undergraduate student Aline suggested observing the students while they play, in addition to grading the sheet they handed in during the first part of the class.

This activity was adapted from the paper by Vianna and Silva (2021) and is characterized by the transformation of a digital activity into a physical one. Like so, a resource was adapted according to usage schemes developed by the undergraduate student, that is, the genesis of the didactic document (Trouche *et al.,* 2020). It is worth noting that, according to Aline's statements during the interview, she had a preference for resources that did not use digital technologies.

> [...]So, about technology, actually, I'm always thinking that I needed to bring more of it. Because, when I imagine my classes in the future, I picture them being differentiated, of course, but not with technology. [...] I would have to find inspiration in a class, but I would try not to move towards technology, because I don't like it very much. (Aline, Research Data, 2022).

A few months after the first interview, an extra conversation was held with the undergraduate student so that she could present her perceptions about the traditional and differentiated activities and the process of class development regarding resources and inspirations. Aline said she did not use to search for practices in scientific papers, this was the first time, because she had only had contact with that kind of resource when she needed to write a text: "This was something that changed, because I didn't use to search scientific papers for some class inspiration, I only used it for texts I had to write. Never to search for something in practice." (Aline, Research Data, 2022*).*

Regarding the practice, the undergraduate student was concerned about her performance and about how the students would view her class. Thus, she saw in the games a chance to bring lightness to the teaching-learning moments. This lightness Aline refers to is a topic that emerged during the first interview, and its meaning is centered on the students' perception about the class not being boring, dull or unattractive.

> I don't want to walk into the classroom and hear students saying: 'My God, it's her class again'. Math again... I believe that mathematics is the course that gets the most judgement from everyone. It needs to be approached with lightness, tranquility, to provide tranquility, not despair: 'I am learning a new topic, math is nice. It has a bright side.' And the game provided me that. It provides such lightness. It tells me that mathematics can be fun, we can play, we can have a playful activity. (Aline, Research Data, 2022).

Some differences between the simulated traditional and differentiated classes (in Aline's conception) are organized in Table 1. Although both had been planned to be antagonistic, they hold common aspects regarding the content-oriented proposal and the assessment style.

Table 1.

Aspects of the traditional and differentiated classes from Aline's perspective.

Traditional	**Differentiated**
Expository class to present the content.	Expository class with dialog and questions as a way to build reasoning about a concept/content.
Students organized individually.	Students working in teams.
Use of the chalkboard as a resource to present the contents and to do the exercises.	Chalkboard still in use, representing students' reasoning and for questions.
Resolution of exercises.	Playful proposal.
Assessment by participation.	Assessment by participation and written.

From Table 01 and the reports on the differentiated activity handed to the professor of the MTL-IV course, in addition to the undergraduate student's statements during the interviews, some theorems-in-action and operational invariants that provide a momentary basis for Aline's practices can be evaluated. (Table 2).

Table 2.

Analysis of Aline's activities

Action rules	**Theorems-in-action**	**Operational invariants**
Conduct a simulated traditional class about the contents of probability.	Use of textbooks and video classes as didactic references.	The textbook helps to define the list of contents that must be provided at each level of education.
	Use of the expository class and use of the chalkboard.	The teacher, in a traditional class, exposes knowledge (contents) for the students orally and with written information on the chalkboard.
	Provide students with an exercise list.	The use of exercise lists reinforces and systematizes content memorization.
Conduct a simulated differentiated class about the contents of probability.	Enable interaction with the contents.	Students and teachers must interact to reach the common goal of learning.
	Provide playfulness through the game.	The game is a resource for learning mathematics which can awaken the interest in learning and break away from traditionalism.

Regarding traditional classes, Aline expressed in her report that "when designing this class, I could picture myself giving a speech where the students were spectators and I, the one who conveys the knowledge, was the main actor in the process, leaving the students in a passive

position." (Aline, Research data, 2022). The undergraduate student also pointed out that this type of class may be complemented with other teaching methods.

Regarding the differentiated class, however, she evoked the word "*interaction*", thinking of the learning process as a relationship between the students and the teacher. Aline also brought a context of "playfulness" into the differentiated class, in which the students could play and learn, considering the game as a didactic resource that provides moments of relaxation, making learning mathematics fun. As a future teacher, the undergraduate student constructed an idea in terms of using games in the learning context: *"I see it as an extra option, I already see it as an option to offer a given content. Sometimes I wonder about some contents, how I would do it, what game I would bring in"* (Aline, Research data, 2022).

The class Aline called differentiated, using the nomenclature provided by the course's professor, is conceptually classified as an active method, which consists of specific strategies to drive the protagonism of the students, who become responsible for their own learning process. In turn, in the traditional approach, students are passive agents when learning the proposed contents. This difference impacts when choosing a project which integrates the main contents, using research, interviews, narratives and games as a part of the teaching-learning process (Carvalho, 2021) and, consequently, it impacts the resources the teacher uses. These pedagogical options converge to the operational invariants of the teachers, assuming teaching-learning beliefs constructed during their professional development and initiated during the initial training courses.

Other aspects worth mentioning from our work with Aline are the impact of the training professors' practices in the construction of the undergraduate student's knowings, which can be summarized in the language aspects, the connection between courses and the guidance itself.

Regarding the language, Aline states that she usually replicates speeches, examples and contexts from her professors' classes, which became evident in her simulated classes, regarding the training professor who teaches, in the mathematics program, the curricular component Probability and Statistics, as can be extracted from the following excerpt from the second interview.

> I ended up catching some lines. [...]I clearly remember I started talking the same way he would explain to us. [...]. And I noticed that in my speech. When I was explaining something: 'Oh..., but why is it like this? Why does this behavior take place.' I ended up stealing some ideas, some manners, I'm not sure how to explain... (Aline, Research data, 2022).

Another aspect is the activity connection between the courses of the undergraduate program, which is fundamental to suppress the dichotomy between theory and practice. In this

context, Aline says she adapted a game that was initially developed for the practical activities of the MTL-III course to the Practices of Mathematics Teaching (PMT) course.

> It ended up like this, when we considered it for Laboratory III. And, when we had to come up with an activity for PMT, I immediately remembered that game and said: 'Let's apply it!' So we weighed our ideas and made a few changes, we made an actual board, we didn't use the students as pawns. We had the little pieces to place on the game board, and they went to the chalkboard to solve the questions. Those were all ideas we had in a different course. (Aline, Research data, 2022).

Aline remembers that, during her differentiated class, the training professor (responsible for the MTL-IV course) had her reflect on her actions, on what she was offering her colleagues and how that would affect a real High School class, as the dialog shows.

> There was a moment, in the final part, when they had to fill in that table. Then I, once again, left them with the data. [...]Meanwhile, I was writing some questions on the chalkboard, when the professor came up to me and said:
> - Aline, do they really need to keep rolling the dice?
> Because there was a logic to build that table... there was a pattern. Then I told him:
> - I want them to realize that this is a thing. I want them to try until they realize it',
> But what he said kept me thinking, and it made me realize that, maybe, I had left things much... like: 'Construct the table, fill it in'. And I didn't give it enough thought. So, when I asked them:
> - Guys, aren't you tired of rolling the dice?
> And they said yes, they were. Then I thought that, if they were High School students, they would get tired much before the end of the activity. Then that would be when I could say: Then, talk to the other teams, see what you can get, try to find a pattern. (Aline, Research data, 2022).

Thus, the activities proposed by the training professors turned into resources for the future teacher Aline, who acted upon them, adapted them using her own usage schemes, interrelating pieces of knowledge, integrating them and developing her didactic documents, which, through her professional experiences, will become her operational invariants. In this context, Assis, Gitirana and Trouche (2019) consider that the metamorphosis which takes place during the initial training must be supervised by the training professors, who need to reflect on how to motivate the development of new usage schemes and operational invariants, and how a repertoire of resources and usages are being offered to the undergraduate students in the courses and projects that make up the training.

Thus, Aline's instrumentation was based on the numerous resources she had available in her academic life, considered in her path, just like her instrumentalization is also based on these pieces of knowledge. Her influences were also subjected to the forge of her experiences as a student - her usage schemes had this peculiarity.

With that in mind, Assis, Gitirana and Trouche (2019) reflect that the initial training is a metamorphosis through which students transform studying resources into teaching resources. In this sense, analyzing Aline's statements during the interviews and her resources shows that she watches the knowledge and the posture of her professors and her peers, denying them or mirroring them. Her resources were considered protoresources, which yielded protodocuments, which are fundamental for the professional development of the future teacher, meaning that they are the means through which the future teacher starts to develop their own knowing, integrating theory and practice.

Final considerations

From the perspective of the initial training incompleteness, the future teacher Aline's resources were still unstable, since her instrumentation/instrumentalization process was subjected to the conditioning factors of her student cognitive processes - she was constructing her teaching knowings. These selected and combined resources, along with pedagogical goals and the future professional experiences will constitute what the DAD defines as the teaching professional development, which is dialectical and, thus, will be in motion throughout the professional life.

Thus, it is possible to see that the professional development process is a creative one in which the teacher (here, future teacher) creates new things or adapts them with the aim of achieving a didactic objective. This construction takes place collectively, as Aline's statements show, integrating a community of practice, a definition that is in the foundations of the DAD. Furthermore, her academic experiences yield the constitution of a corpus of invariants associated with specific rules of action, which will outline the genesis of a teacher's didactic documents.

The initial training is a laboratory of formative processes which lead to the construction of a teacher's knowings from a dialectical perspective. Therefore, future longitudinal research is coherent with this approach, once it will provide an opportunity to study the impact of such initial conceptions in future professional experiences.

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Appendix 1

Script of the semi-structured interview conducted with the undergraduate student

Dimension 1: Conception of training

1. What was your high school education like?
2. Do you have any other higher education degrees?
3. Why did you choose the undergraduate program in Mathematics?
4. Do you intend to take other programs after you graduate in Mathematics? Which ones?
5. Are you taking any other courses simultaneously to the undergraduate program in Mathematics? Which ones?
6. Do you intend to work as a teacher?
7. What hurdles did you find in the undergraduate program in Mathematics?
8. Did you have any trouble keeping up with the emergency remote classes? Which ones?

Dimension 2: Learning statistics

9. How do you go about studying statistics? Which instruments or means do you use?
10. Do you usually ask your professors for extra-class explanations?
11. What resources/methodology your professors use for teaching statistics contents in class that usually have a positive effect on your learning?
12. Do you often ask/talk to your colleagues to improve your understanding of the contents?
13. Do you take part in any study groups or something similar?
14. Do you look into other sources but those suggested by the professor to find information and explanations to understand topics of statistics?
15. Do you read your professor's educational plan? What information do you look for in this document?
16. Do you usually read documents such as the program's pedagogical project or other curricular guidelines for the undergraduate program in Mathematics? Why?
17. What didactic resources did you learn about in the undergraduate program in Mathematics that you would like to employ in the statistics classes you will teach in your future professional experience?
18. Would you say that teaching a class on statistics for Basic Education is easy, considering the knowledge acquired during the undergraduate program in Mathematics?

19. Could you create an illustration that shows how your statistics studying moment takes place? It should contain what you use to study, be it materials, books, articles, technologies or anything that is part of your studying moment.

20. Using the following link, could you write down the first ten words that represent your studying and learning in statistics?