

**Didactic suitability in the initial training of mathematics teachers: reflections on aspects of interaction**

**Idoneidad didáctica en la formación inicial de profesores de matemáticas: reflexiones sobre aspectos de interacción**

**Aptitude didactique dans la formation initiale des enseignants de mathématiques : réflexions sur les aspects de l'interaction**

**Idoneidade didática na formação inicial de professores de matemática: reflexões sobre aspectos de interação**

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**Abstract**

In this study, we seek to investigate how the interactions experienced in the context of the initial training of mathematics teachers, taking into account the planning, execution and evaluation of pre-professional practices, can influence the development of teaching autonomy. This is qualitative research in which participated three future teachers of Mathematics and a teacher of Basic Education, residents and preceptor, respectively, participants of the program of initiation to teaching Pedagogical Residence. The production of data was achieved from three instruments: questionnaire, interview and observation of training practices. For data analysis, three categories were elaborated which emerged from the dialogue between the notion of Didactic Adequacy and the production of data, in particular, with the component autonomy of interaction adequacy, namely: autonomy in planning, autonomy in execution and autonomy in

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evaluation. The results show that collective and collaborative work, especially with the participation of an experienced teacher working in the professional field becomes a necessary element for future teachers to develop autonomy in front of educational practices. In addition, it is a fact that the teaching of mathematics today requires, among so many variables, reflective teachers and capable of making decisions regarding curricular prescriptions, the demands of students and social, cultural and institutional contexts. By way of conclusion, it is necessary to recognize the importance of teacher training courses articulate Mathematics training with pedagogical didactic training.

**Keywords:** Didactic adequacy, Autonomy, Training of mathematics teachers, Pedagogical residence.

### Resumen

En este estudio, buscamos investigar cómo las interacciones vividas en el contexto de la formación inicial de profesores de Matemáticas, teniendo en cuenta la planificación, la ejecución y la evaluación de las prácticas pre-profesionales, pueden influenciar el desarrollo de la autonomía docente. Se trata de una investigación cualitativa en la que participaron tres futuros profesores de Matemáticas y un profesor de Educación Básica, residentes y preceptor, respectivamente, participantes del programa de iniciación a la docencia Residencia Pedagógica. La producción de datos fue concretada a partir de tres instrumentos: cuestionario, entrevista y observación de prácticas formativas. Para el análisis de los datos fueron elaboradas tres categorías las cuales emergieron del diálogo entre la noción de Idoneidad Didáctica y la producción de los datos, en particular, con el componente autonomía de la adecuación de la interacción, a saber: autonomía en la planificación, autonomía en la ejecución y autonomía en la evaluación. Los resultados evidencian que el trabajo colectivo y colaborativo, en especial, con la participación de un profesor experimentado actuante en el campo profesional se convierte en elemento necesario para que los futuros profesores desarrollen la autonomía frente a las prácticas educativas. Además, es hecho que la enseñanza de la Matemática en la actualidad requiere, entre tantas variables, profesores reflexivos y capaces de realizar toma de decisiones frente a las prescripciones curriculares, las demandas de los estudiantes y los contextos sociales, culturales e institucionales. A modo de conclusión, hay que reconocer la importancia de que los cursos de formación de profesores articulen la formación Matemática con la formación didáctica pedagógica.

**Palabras clave:** Idoneidad Didáctica, Autonomía, Formación de profesores de matemáticas, Residencia pedagógica.

## Résumé

Dans cette étude, nous cherchons à étudier comment les interactions vécues dans le contexte de la formation initiale des professeurs de mathématiques, en tenant compte de la planification, de l'exécution et de l'évaluation des pratiques pré-professionnelles, peuvent influencer le développement de l'autonomie des enseignants. Il s'agit d'une recherche qualitative à laquelle ont participé trois futurs professeurs de mathématiques et un professeur de l'éducation de base, résidents et précepteur, respectivement, participants au programme d'initiation à l'enseignement de la résidence pédagogique. La production de données a été réalisée à partir de trois instruments : questionnaire, entretien et observation des pratiques de formation. Pour l'analyse des données, trois catégories ont été élaborées qui ont émergé du dialogue entre la notion d'adéquation didactique et la production des données, en particulier avec la composante autonomie de l'adéquation des interactions, à savoir : autonomie dans la planification, autonomie dans l'exécution et autonomie dans l'évaluation. Les résultats montrent que le travail collectif et collaboratif, en particulier avec la participation d'un enseignant expérimenté travaillant dans le domaine professionnel, devient un élément nécessaire pour que les futurs enseignants développent l'autonomie face aux pratiques éducatives. En outre, il est un fait que l'enseignement des mathématiques de nos jours exige, parmi tant de variables, des professeurs réfléchis et capables de prendre des décisions face aux prescriptions du programme, aux exigences des étudiants et aux contextes sociaux, culturels et institutionnels. En guise de conclusion, il faut reconnaître l'importance que les cours de formation des enseignants articulent la formation mathématique avec la formation didactique pédagogique.

**Mots-clés** : Aptitude didactique, Autonomie, Formation des enseignants de mathématiques, Résidence pédagogique.

## Resumo

Neste estudo, buscamos investigar como as interações vivenciadas no contexto da formação inicial de professores de Matemática, levando em conta o planejamento, a execução e a avaliação das práticas pré-profissionais, podem influenciar o desenvolvimento da autonomia docente. Trata-se de uma pesquisa qualitativa da qual participaram três futuros professores de Matemática e um professor da Educação Básica, residentes e preceptor, respectivamente, participantes do programa de iniciação à docência Residência Pedagógica. A produção de dados foi concretizada a partir de três instrumentos: questionário, entrevista e observação de práticas formativas. Para análise dos dados foram elaboradas três categorias as quais emergiram do

diálogo entre a noção de Idoneidade Didática e a produção dos dados, em particular, com o componente autonomia da idoneidade interação, a saber: autonomia no planejamento, autonomia na execução e autonomia na avaliação. Os resultados evidenciam que o trabalho coletivo e colaborativo, em especial, com a participação de um professor experiente atuante no campo profissional torna-se elemento necessário para os futuros professores desenvolvam a autonomia frente às práticas educativas. Além disso, é fato que o ensino da Matemática na atualidade requer, entre tantas variáveis, professores reflexivos e capazes de realizar tomada de decisões frente às prescrições curriculares, as demandas dos estudantes e os contextos sociais, culturais e institucionais. À guisa de conclusão, há que se reconhecer a importância de os cursos de formação de professores articularem a formação Matemática com a formação didático pedagógica.

**Palavras-chave:** Idoneidade didática, Autonomia, Formação de professores de matemática, Residência pedagógica.

## **Didactic suitability in the initial training of mathematics teachers: reflections on interaction aspects**

The teaching and learning process of Mathematics increasingly demands that teachers develop the ability to reflect and exercise autonomy in their practice (Font et al., 2023). Frequent curriculum reforms in basic education require decision-making regarding the content to be addressed and its possible interrelations with other areas of knowledge. In this context, it is crucial to reflect on teacher training for contemporary times and how programs organize their practices, especially with regard to bringing future teachers closer to the professional field (Silva & Tinti, 2021).

It is undeniable that teacher education institutions face the challenge of creating curricular arrangements that promote reflective processes and autonomy within pedagogical practices. However, it is important to emphasize that there are different theoretical constructs addressing this topic. In particular, notable approaches include the reflective practitioner (Schön, 2000; Perrenoud, 2004), Lesson Study (Fernández & Yoshida, 2004), the development of teaching competence for "professional noticing" in the teaching and learning of Mathematics (Llinares, 2013), and the discussion of Didactic Suitability Criteria within the Ontosemiotic<sup>4</sup> Approach.<sup>2</sup>

At times, Mathematics teacher education programs lack theoretical foundations that can support the various stages of their educational practices, especially in planning, execution, and assessment (Silva & Tinti, 2021; Horta & Silva, 2023). One potential contribution to this context in Mathematics teacher education is Didactic Suitability (DS), which offers components and indicators for the analysis of practices (Godino, 2013, 2016; Godino et al., 2013; Breda et al., 2018).

Given the above, this study seeks to analyze how interactions experienced in the context of the initial training of Mathematics teachers—taking into account the planning, execution, and evaluation of practices—can influence the development of teaching autonomy. This text is structured as follows: this introduction is followed by the theoretical foundations, in which we present the construct of DS, the methodological approaches, results, and discussions, concluding with final considerations.

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<sup>4</sup> In the original "Ontosemiotic Focus". When we use the acronym EOS, we refer to this theoretical construct.

<sup>2</sup>Since we do not yet have a consensus on the best translation into Portuguese for the term "Didactic Suitability", we will use its spelling as it appears in the original language (Spanish). When we refer to the expression Didactic Suitability Criteria, we will use the acronym DSC and when it is simply Didactic Suitability DS.

## **Theoretical assumptions**

One of the fundamental issues that teacher education (both initial and continuing) must address is what we will refer to here as the problem of instructional design: what criteria can be provided to and/or developed alongside teachers to help them plan, implement, and re-plan increasingly effective sequences of tasks.

A first response to this problem could be what we will call here a positivist approach (objectivist, scientific, etc.), which can be formulated as follows: scientific research conducted in the field of Mathematics Education (ME) will tell us which causes need to be modified in order to achieve the desired effects or objectives. This approach involves using empirical research to inform and improve pedagogical practices in the classroom. It emphasizes the importance of applying scientific methods to evaluate the effectiveness of various teaching and learning strategies. This entails providing teachers with practices based on scientific evidence, such as expert reports, case studies, controlled experimental studies (e.g., studies where participants are randomly assigned to an experimental group receiving an intervention and a control group that does not), meta-analyses (studies that combine the results of multiple research projects on a specific topic to derive an overarching conclusion), and even meta-analyses of meta-analyses.

This perspective assumes that research conducted in MT and related fields will identify the criteria (causes) necessary to achieve the desired effects (objectives) in the teaching and learning process (effective learning). In other words, evidence-based scientific research will offer teachers "practices that work" because they are supported by studies conducted, for instance, with experimental and control groups, through meta-research on many of these studies, and through meta-research conducted on these meta-analyses. These are practices that can be claimed to work in nearly all contexts because there is objective data to support them (evidence). These are practices that, according to researchers, are objectively validated empirically by quantitative methods that ensure control and verification of the results.

From this perspective, the strategy to improve Mathematics teaching and learning processes should be vertical in nature, that is, it should be implemented top-down. This implies the production of curricular materials developed by specialists who apply scientific knowledge and evidence-based practices, or alternatively, the creation of these materials by teachers themselves, following such practices proven to be effective.

A second response to the problem of instructional design is institutional administration. From this perspective, it does not matter how a particular orientation, criterion, or practice was

introduced into the curriculum; what matters is that, once it becomes part of the curriculum, it becomes an obligation for the teacher.

These two responses position the teacher as an executor with little room for maneuver, either as someone who applies "practices that work" (as there is scientific evidence to support them) or as someone who fulfills curricular obligations.

A third response to the instructional problem, which serves as an intermediate approach between the two previous ones, is the idea that the solution partly involves fostering teachers' reflection and autonomy. In this regard, various studies have analyzed teachers' practices in their different dimensions, particularly focusing on how they reflect on the teaching and learning process of Mathematics (Beltrán-Pellicer; Godino; Giacomone, 2018; Breda; Font; Pino-Fan, 2018; Seckel; Font, 2020; Silva; Manrique, 2021; Horta; Silva, 2023). In this sense, it is important to emphasize that teachers—or future teachers—should have access to means and tools that support their ability to critically analyze the process of planning and/or designing, executing and/or implementing, and evaluating the teaching trajectory (Godino; Burgos, 2020). However, broader and more general guidelines may not sufficiently promote reflection on Mathematics teaching practices, which often involve the handling of complex concepts.

This last response to the instructional problem, which serves as an intermediate approach between the first two, considers that the criteria guiding teachers' reflection and autonomy should be of a consensual nature. The aim is to establish consensus on criteria to guide practice (considered high-quality) that are useful in two key moments of mathematical instruction processes. A priori, consensual criteria serve as principles that guide "how things should be done." A posteriori, these criteria are used to evaluate the instructional process that has been effectively implemented. The construct of the Didactic Suitability Criteria (DSC) forms the basis of this response, as its inclusion in teacher training allows teachers to develop the competency to evaluate DS, which in turn enables them to organize their reflection and fosters autonomy. The DSC is a multidimensional construct that enables teachers to reflect on their practice and guide its improvement within the context in which the teaching and learning process takes place (Godino, 2013; Godino; Batanero; Rivas; Arteaga, 2013; Breda; Font; Pino-Fan, 2018; Silva; Tinti, 2021; Godino; Batanero; Burgos, 2023).

This third response, although it considers scientific research and teachers' obligations, does not limit itself to their mechanical application. Instead, it empowers teachers to make more informed decisions about how to develop specific practices, taking into account the context and their own judgment regarding the importance of various criteria (Godino, 2013; Godino;

Batanero; Rivas; Arteaga, 2013; Breda; Font; Pino-Fan, 2018; Silva; Tinti, 2021; Godino; Batanero; Burgos, 2023).

Various authors provide definitions of the term "Didactic Suitability" (Breda et al., 2018; Beltrán-Pellicer et al., 2018; Castillo et al., 2020), and overall, there is evident consensus on the following points: within the Onto-semiotic Approach (OSA), according to Breda et al. (2018), the Didactic Suitability (DS) of a teaching process is understood as the degree to which it (or a part of it) exhibits certain characteristics that qualify it as optimal or appropriate for achieving alignment between the personal meanings achieved by students (learning) and the institutional meanings intended or implemented (teaching), considering the circumstances and resources available (environment). See Figure 1.

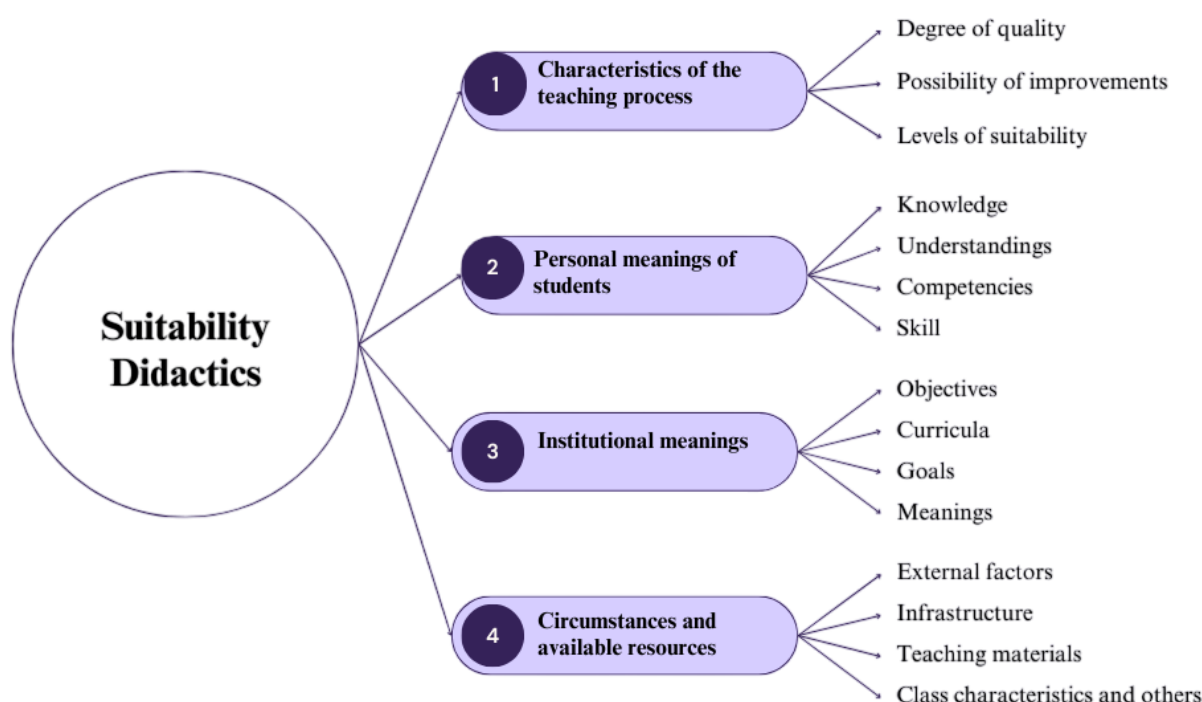


Figure 1.

*Didactic Suitability. Adapted from Font, Planas and Godino (2010).*

The foundations that support the construct of Didactic Suitability (DS) are organized into decisions, which aim to clarify its continuous, dialogical, and inclusive development. In Figure 2, these decisions are outlined:

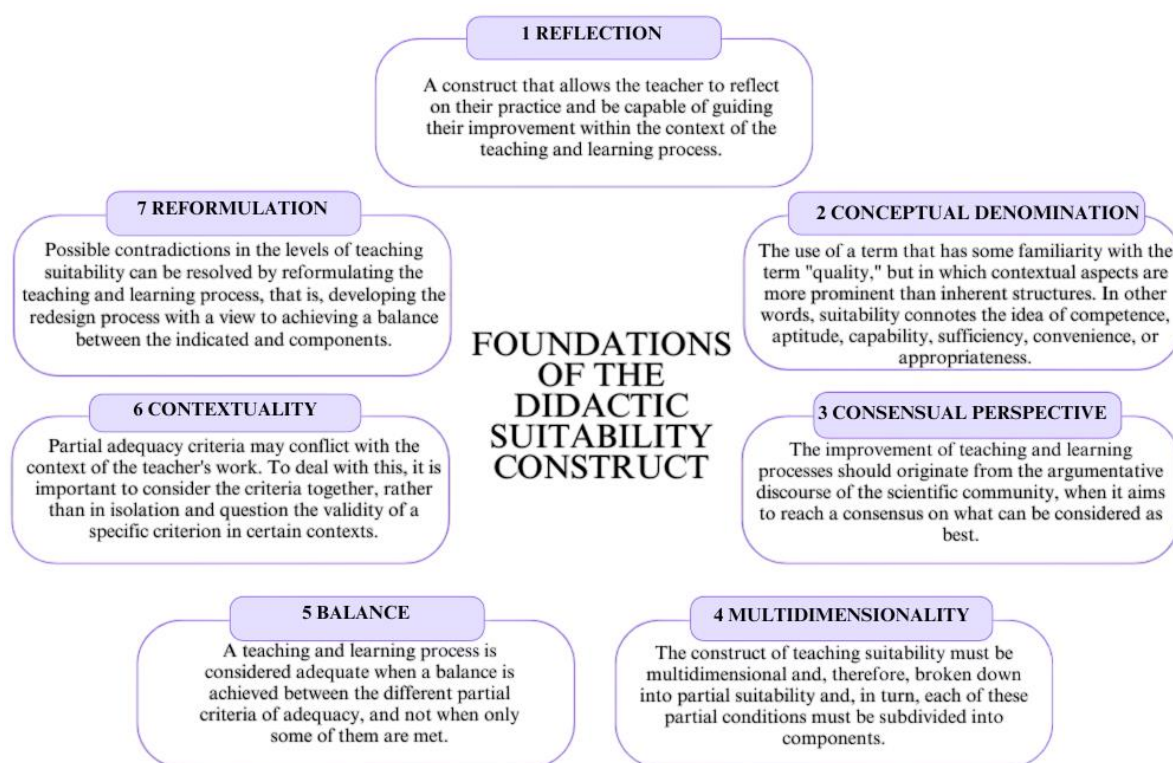


Figure 2.

*Foundations of the Didactic Suitability Construct*

*Adapted from Breda et al. (2018)*

In the context of Didactic Suitability (DS), we identify six dimensions, each containing specific components and indicators. Each of these dimensions can be analyzed independently, and within them, different levels of improvement potential can be identified. The didactic analysis through the DS Criteria (DSC) can be applied in various situations, such as lessons, curricular proposals, planning, among others (Godino, 2011). Complementing this, Font et al. (2023, p. 382) highlight.:

In the theoretical system that underpins the Onto-semiotic Approach (OSA), the notion of Didactic Suitability (DS) has been incorporated as a systemic criterion for optimizing a teaching and learning process in Mathematics. It is defined as the degree to which this process (or part of it) exhibits certain characteristics that qualify it as appropriate for achieving alignment between the personal meanings attained by students (learning) and the institutional meanings intended or implemented (teaching), taking into account the circumstances and resources available (environment).

To assist in the analysis of each dimension, Godino (2011, 2013) developed a description that outlines key points for each dimension of analysis, as presented in Table 1.

Table 1.

*Indicators of didactic suitability (Godino, 2011,2013 - Adapted)*

<b>Epistemic Suitability</b>		
<b>Components</b>	Problems	A representative and coherent sample of contextualization, exercise, and application situations is presented, proposing problem-generating situations (problematization).
	Languages	The use of different modes of mathematical expression (verbal, graphical, symbolic, etc.), translations and conversions between them, and the level of language appropriate for the children being addressed are proposed in situations of mathematical expression and interpretation.
	Rules	<b>Indicators</b> The definitions and procedures are clear and correct, and are adapted to the educational level they are aimed at. They present fundamental statements and procedures of the topic for the given educational level, and propose situations where students have to generate or negotiate definitions, propositions, or procedures.
	Arguments	The explanations, verifications, and demonstrations are appropriate for the educational level they are directed at, and situations are promoted where students are required to argue.
	Relations	Mathematical objects (problems, definitions, propositions, etc.) are related and connected to each other. The various meanings of the objects involved in the practices are identified and articulated.
<b>Cognitive Suitability</b>		
<b>Components</b>	Pre-existing knowledge	Students have the necessary prior knowledge for studying the topic, and the intended content can be achieved (it has manageable difficulty) in its various components.
	Curricular adaptations	Students have the necessary prior knowledge for studying the topic, and the intended content can be achieved (it has manageable difficulty) in its various components.
	Learning	<b>Indicators</b> The various modes of evaluation indicate that students achieve the intended knowledge acquisition (including understanding and competence): conceptual and propositional understanding; communicative and argumentative competence; procedural fluency; understanding of the situation; metacognitive competence; the assessment takes into account different levels of understanding and competence, and the results of the evaluations are shared and used to make decisions.
<b>Affective Suitability</b>		
<b>Components</b>	Interests and needs	The tasks are of interest to the students, and situations are proposed that highlight the utility of mathematics in everyday and professional life.
	Attitudes	<b>Indicators</b> Participation in activities, perseverance, responsibility, etc., is promoted. Argumentation is encouraged in situations of equality; the argument is evaluated by its merit, not by who presents it.
	Emotions	Self-esteem is promoted, avoiding rejection, phobia, or fear of mathematics. The aesthetic and precision qualities of mathematics are emphasized.
<b>Interactional Suitability</b>		
<b>Components</b>	Teacher-student interaction	<b>Indicators</b> The teacher makes an appropriate presentation of the topic (clear and well-organized presentation, does not speak too quickly, emphasizes the key concepts of the topic, etc.), recognizes and resolves students' conflicts (asks and answers questions appropriately, etc.). Efforts are made to reach consensus based on the best argument, using various rhetorical and argumentative resources to engage and capture students' attention, and student inclusion in the classroom dynamics is facilitated.

	Interaction between students	Dialogue and communication among students are promoted, and they try to convince themselves and others of the validity of their statements, conjectures, and answers, relying on mathematical arguments. Inclusion in the group is encouraged, and exclusion is avoided.
	Autonomy	There are moments when students take responsibility for their studies (they raise questions and present solutions; explore examples and counterexamples to investigate and conjecture; use a variety of tools to reason, make connections, solve problems, and communicate problems).
	Formative evaluation	Systematic observation of students' cognitive progress.
Mediational Suitability		
Components	Material resources	Indicators Manipulative and computer-based materials are used, which allow for the introduction of good situations, languages, procedures, and arguments adapted to the intended content. Definitions and properties are contextualized and motivated using concrete situations, models, and visualizations.  The number and distribution of students allow for the intended teaching to be carried out, the course schedule is appropriate (e.g., not all sessions are held at the last minute), and the classroom and student arrangement are suitable for the development of the intended instructional process.  The time (both in-class and out-of-class) is sufficient for the intended teaching, with adequate time dedicated to the most important content of the topic and those that present greater difficulty in comprehension.
	Classroom conditions	
	Time	
Ecological Suitability		
Components	Adaptation to the curriculum	The content, its implementation, and evaluation align with the curricular guidelines.
	Didactic innovation	Innovation based on research and reflective practice. Integration of new technologies (calculators, computers, ICT, etc.) into the educational project.
	Socio-professional and cultural adaptation	Indicators The content contributes to the socio-professional development of students.
	Education in values	
	Intra and interdisciplinary connections	The content is related to other intra- and interdisciplinary content.

With the DS Criteria (DSC), it is possible to reflect on the process of planning, execution, and evaluation of the teaching and learning process; however, it is necessary to consider that teachers, in their initial and/or continued education, develop practices that encourage recognizing the limits and possibilities of educational action in mathematics. In this sense, we turn to Freire (1996, p. 39), who warns us:

(...) In the ongoing professional development of teachers, the fundamental moment is that of critical reflection on practice. It is by critically thinking about yesterday's practice that the next practice can be improved. The theoretical discourse itself,

necessary for critical reflection, must be concrete enough that it almost blends with practice.

Thus, it is essential to understand and analyze how teacher education programs articulate the different dimensions of reflection on practices. Therefore, we base ourselves on Godino et al. (2013), who transpose the notion of DSC (Didactic Suitability Criteria) to the context of mathematics teacher education programs. Specifically, for this research, we focus on the adaptation made in the context of interactional suitability, with an emphasis on the autonomy component, to analyze the reflections that emerge from the discourse of future teachers, both regarding mathematical content and didactic content.

We want to emphasize that in this document we are considering two different uses of the term autonomy. The first use is the one we have discussed earlier, where autonomy is understood as the ability of the teacher to assign more or less weight to a didactic suitability criterion depending on the context and their judgment. On the other hand, now autonomy is a component of the interactive criterion that the educator (in our case, the educator of future teachers) should seek to develop in their students (future teachers).

### **Methodology**

Based on the above, this study is a qualitative research (Godoy, 2005; Creswell, 2007; Gatti, 2012), in which three future teachers – residents – of Mathematics and one Basic Education teacher – mentor – participated in the teaching initiation program, called the Pedagogical Residency Program at the Federal Institute of Education, Science, and Technology of Minas Gerais – São João Evangelista Campus (IFMG/SJE).

The Pedagogical Residency Program is a public policy aimed at teacher education, which began in 2018. This initiative has a hierarchical structure that extends from the Ministry of Education, through the Coordination for the Improvement of Higher Education Personnel (Capes), passing through teacher education institutions, and reaching Basic Education schools, known as field schools.

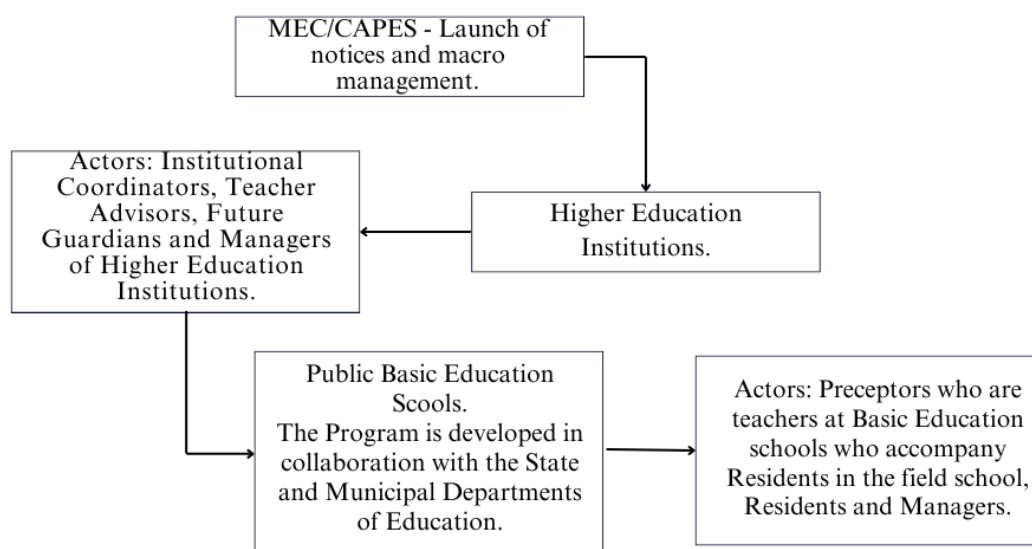


Figure 3.

*Structure of the Pedagogical Residency Program. Silva e Tinti (2021)*

The residents are students enrolled in undergraduate courses who have completed at least 50% of the course or are in their 5<sup>th</sup> semester or beyond. They engage in planning and teaching activities under the direct supervision of a teacher from the Basic Education school, known as the Mentor. In addition to these participants, there is the university supervisor, who provides support from a broader perspective. The program has a total duration of 18 months, but for this research, we will focus on the last six months. The research was conducted in three stages, which involved the following instruments:

- I. **Questionnaire:** The questionnaire was applied online to a group of nineteen (19) future teachers, using *Google Forms* for its creation and distribution. In this phase, volunteers were requested for the interview.
- II. **Interview:** For the interviews, residents and mentors working in schools of the State Education Network and participants in the PRP were selected. To characterize them within the context of the research, participants were identified by fictitious names. The mentor was referred to as Mariana, and the residents were given the names Davi, Cristina, and Kelly. These interviews followed the guidelines of Yin (2016), who emphasizes the need for them to take place within the specific reality, not adhering to a rigid model, allowing for a bidirectional relationship.
- III. **Observation:** Observation of the environment where the activities proposed by

the PRP take place, aiming to understand the actions carried out.

Based on the theoretical support, the autonomy component was adopted as the category of analysis, considering the need for a thematic focus, since discussing all the components would not be feasible within the scope of this article. The aim was to analyze the interactions between the mentor and the residents that foster the development of autonomy in the context of the professional development of the PRP participants within teacher education. In this regard, Contreras (2002) emphasizes that:

Autonomy (...) does not consist of isolation, nor the abandonment of schools and teachers to their own fate. It starts, more precisely, from conceiving the relationship between teachers and society on different grounds (those of community constitution), so that the bonds are not bureaucratic or market-driven, but political and personal. (Contreras, 2002, p. 269).

Thus, the descriptor Autonomy was grounded in the theoretical framework of the research with support from the assumptions of Godino (2011), who describes autonomy as one of the components of the Interactional Facet, signaling its occurrence when the presence of its indicators, as presented in Table 2, is observed.

Table 2.

*Autonomy component and its indicators (Godino, 2011; 2013)*

Component	Indicators
Autonomy	- It includes moments in which students take responsibility for their studies (ask questions and present solutions; explore examples and counterexamples to investigate and conjecture; use a variety of tools to reason, make connections, solve problems and communicate them).

The organization and adaptation of the indicators were guided by the dialogical interaction between data production, analysis, and the theoretical framework. Throughout this process, three indicators were developed, thus becoming the subcategories of analysis, as shown in Table 3.

Table 3.

*Autonomy component and its indicators adapted based on (Godino, 2011; 2013)*

Category	Indicators (Subcategories)
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Autonomy	<ul style="list-style-type: none"> <li>- Autonomy in planning: Includes moments of responsibility for the actions planned with the group (they ask questions and present solutions; explore examples and counterexamples to investigate and conjecture; use a variety of tools to reason, make connections, solve problems and communicate them);</li> <li>- Autonomy in execution: Includes moments of responsibility for the actions carried out in the context of schools (they ask questions and present solutions for the reality experienced; choose different resources for teaching, make connections, solve problems, get to know the students, ...)</li> <li>- Autonomy in evaluation: Includes the evaluation of the path of the practices carried out throughout the teaching and learning process (they analyze the practices, list the difficulties, the challenges, ...)</li> </ul>
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## **Results and discussion**

In this section, we will address the data starting with the subcategory Autonomy in Planning, which deals with the organizational actions and responsibilities of residents and preceptors. Next, we will discuss the perspective of Autonomy in the Execution of planned practices and, finally, Autonomy in Evaluation, which refers to the ability to reflect on practices.

### **Autonomy in planning**

Autonomy in Planning aims to discuss the idealization and organization of formative activities within the Pedagogical Residency Program (PRP), considering the context of the actions of future Mathematics teachers. It is important to highlight that, according to the Coordination for the Improvement of Higher Education Personnel (2018a, p. 2), the Pedagogical Residency Program's objective is “to improve the training of residents, students from teacher education courses, with a focus on the development of projects that strengthen the professional practice of the future teachers.” In this way, it was found that the interactions between the program participants grant the resident autonomy in planning activities, as resident Cristina mentioned, “The preceptor entrusted the residents with the task of structuring the activities to be carried out with the students in the field school” (Excerpt from the interview conducted with Cristina on March 28, 2022).

According to the preceptor, her responsibility is to guide the residents, mediating and articulating the different paths that lead to learning. According to her:

I try, in the same way that I work with my students (basic education students), to guide my residents. I show them that there are different paths to achieving a certain goal in the classroom. And in the same way in my guidance... I demonstrate this to them because our resident is also one of our students who is learning to become a teacher. (Excerpt from the interview conducted with Mariana on March 28, 2022)

The preceptor's account highlights a work with the residents that reflects the engagement of their actions in constructing a process, the product of which are interactions that promote knowledge resulting from the motivation and experiences lived within the context of the PRP. In other words, for the "construction of didactic-mathematical knowledge and competencies, it is necessary to design and implement formative processes that contribute to the learning of future teachers" (Godino et al., 2017, p. 103), enabling professional growth and the opening to innovations in Mathematics teaching practices. In line with the aforementioned, the teacher states: "(...) it is necessary to encourage, seek new ways to capture attention, point out new paths and new learning resources, diversify the way of guiding the residents, showing that a new perspective is needed for pedagogical practice" (Excerpt from the interview conducted with Mariana on March 28, 2022).

Given the above, the relevance of the knowledge built during the planning of the activities to be carried out by the resident is evident. This was clear in the words of the resident Kelly, when she points out that:

The time to plan the activities was very important for our learning as a resident. We were guided to work with a specific content. However, planning the activities and the path to follow was a responsibility assigned to us. The tutor guided us, but gave us autonomy to plan the activities. And that was very important. It made me think about my responsibility in planning my classes in the future. (Excerpt from the interview conducted with Kelly on March 28, 2022)

These results point to a proposal for professional development focused on both theory and teaching practice, considering that the residents, by having autonomy during the planning of the activities proposed within the PRP, perceive themselves as fitting into the reality of their intended field of action, which, according to Tinti and Silva (2020), favors the training of these future teachers.

Thus, it is important to note that the resident's detachment in planning pedagogical practices, based on the interactions established within the PRP, evokes the inherent knowledge of teaching, as indicated by the words of resident Davi:

There is no way to enter the Pedagogical Residency Program without being involved with the reality we will face. The program aims to integrate us into the daily life of education, and my preceptor does this masterfully. She gives us the autonomy and responsibility to plan our activities to be carried out at school. Of course, with her supervision. And we know that it will be like this when we start working in the school. (Excerpt from the interview with Davi on March 23, 2022).

Therefore, regarding the autonomy of residents in planning, anchored in the interactions

they establish in the PRP space, the data point to an adequacy of the category's indicators. Thus, the actions instituted through the program consider the moments in which residents assume responsibility and idealization of the activities developed with the group.

### **Autonomy in execution**

The descriptor of Autonomy in execution addresses the actions of the resident during the implementation of activities with students from the Basic Education school associated to the PRP. Thus, understanding autonomy as a fundamental element for the resident, as a future teacher, to implement pedagogical procedures in the teaching and learning processes, it is considered that, according to Campos (2007), the autonomy of the teacher involves the decision-making moment regarding content and didactic methods aimed at educational objectives, in the classroom practice and in the interaction with participants in the educational process. Therefore, it is understood that autonomy is not built individually. In the context of the PRP, this autonomy is achieved in collaboration with the Preceptor, which is supported by the statement of resident Kelly, when she points out: “The preceptor encouraged us to take the lead in the activities. We had the autonomy for execution, including in regard to the evaluation of student participation” (Excerpt from the interview with Kelly on March 28, 2022).

This movement of support and assistance, in the process of implementing classes, is fundamental in the training process of future teachers. Reflections in this context have the role of providing future teachers with awareness about the profession (Silva & Tinti, 2021). In this sense, the resident points out:

I learned a lot from the preceptor. I learned how to argue with students, I learned how to engage students in classroom discussions. She did this even with us (the residents) when she had us participate in choosing the topics to be worked on in the workshops." (Excerpt from the interview with Kelly on March 28, 2022)

It is worth noting that this relationship based on autonomy is triggered in the process of teaching identity, as the same resident states: “The tutor taught me to be a teacher, to have an outlook focused on the collective. So you have to look for methods of working without excluding anyone” (Excerpt from the interview conducted with Kelly on March 28, 2022).

Therefore, these results indicate that the interactions established with the PRP participants include moments in which the residents take responsibility for the actions carried out in the field school. As observed, resident Cristina emphasizes that: “(...) the preceptor always gave us an indication of how to build the path to carry out the proposed tasks, but made it clear that we were the ones who would follow the path” (Excerpt from the interview conducted with

Cristina on March 29, 2022). Thus, it is highlighted that, from the planning to the execution of the activities, the residents had the freedom to make suggestions that could contribute to the execution of the proposals.

That said, Freire (1996) emphasizes that teaching is not about transmitting knowledge and that students learn when they become deeply involved in the situation. In this context, the data related to the Autonomy in execution descriptor indicate a successful practice in the context of interactions established between PRP participants, given that they are consistent with the teaching knowledge that arises from the autonomy that the Preceptor grants to the resident during the execution of activities, culminating in knowledge relevant to the teacher.

### **Autonomy in evaluation**

It is worth mentioning that part of the training activities took place during the pandemic, which highlights important challenges in their implementation. However, the practices took place in a virtual format, as reflected by the preceptor Mariana:

When the program went online, the interactions were different. However, it was beautiful to see the unity and participation of most of the residents. Some faced many difficulties, both technological and emotional. It was not an easy period, but it was a period of great learning for me and for them. Experiencing their professional and personal growth in front of the students. Seeing them becoming confident in teaching content, dealing with students, and arguing with authority in their speech was very enriching."(Excerpt from the interview with Mariana on March 28, 2022).

From the above, challenges are evident in the training context and in the implementation of practices, however, it should be emphasized that interactions constituted important elements in the context of PRP actions.

When asked about how he dealt with the challenges experienced, resident Davi replied:

We did not have problems with the school or the university... And with the students, the only problematic issue was their participation in the activities. In this regard, the preceptor did what she could. She would talk to the students to try to understand why they were absent, and then, with us (the residents), she would look for solutions to this problem, which was their participation in the activities."(Excerpt from the interview with Davi, on March 23, 2022).

In this regard, the preceptor develops an interaction process that fosters the training of future teachers. Davi's reflections highlight issues related to his formative process and the challenges faced when teaching Mathematics in the context of basic education.

Among the challenges mentioned, those related to the use of technology stand out, especially those dependent on an internet connection, as stated by resident Cristina.:

I saw that the pandemic really hampered the performance of activities. There were days when we didn't have a good internet signal, or sometimes even the preceptor didn't have one. On rainy days, for example, it was a struggle. The internet didn't work at all. (Excerpt from the interview with Cristina on March 29, 2022).

The comment made by the resident reflects some of the difficulties discussed in Breda, Farsani, and Miarka (2020), related to the political, technical, and pedagogical limitations for conducting Mathematics teaching at different educational levels during the COVID-19 pandemic. However, based on the statement from participant Cristina, it is important to highlight that future Mathematics teachers demonstrate autonomy in reflecting on the context they experienced, pointing out challenging elements of the teaching and learning process. According to evidence from the data, this ability comes from interactive experiences that resulted from consensus between residents and the preceptor. This is corroborated by resident Davi's statement when he describes, in an interview, that:

Our relationship (the residents) with the preceptor is a relationship of knowledge exchange. The preceptor, with her experience and her arguments, can make us think: isn't she right? Doesn't it really work that way? (Excerpt from the interview conducted with Davi on March 23, 2022)

The above leads us to reflect on the practice that takes into account dialogical, consensual, and democratic aspects, such aspects coinciding, for example, with those considered in the construction of the CAD tool (Breda, Font, and Pino-Fan, 2018). In this sense, "It is precisely the reflective dialogue that leads to reflective thinking and prepares for methodological reflective practice" (Domingo, 2021, p. 18). Regarding decision-making, another important aspect in the construction of autonomy, Davi reports: "The preceptor always involved us in the process of thinking about the proposed activities. She always asked: Do you think it will work? Which path do you consider to be the best for us in order to reach the maximum number of students possible?" (Excerpt from the interview conducted with Davi on March 23, 2022).

In view of the above, it is considered that the attitude of co-responsibility in the elaboration of decisions contributes to future teachers developing the ability to act more confidently when they finally enter the career. This perspective leads us to agree with Pinzón and Gómez (2021, p. 2) when they state that "the teacher's decision-making process in the classroom is considered one of the key elements of teaching." According to these authors, it is necessary to recognize "that teaching is a task of high social interaction (Schoenfeld, 2015), that classroom conditions change unpredictably and, as a reflection of these complexities, that teachers make hundreds of non-trivial decisions every day (Clough et al., 2009)." In short, the

analysis reveals that interactions are fundamental for the development of autonomy in the training of future teachers, that is, this shows us the importance of the social, historical, and cultural aspects of the teaching profession:

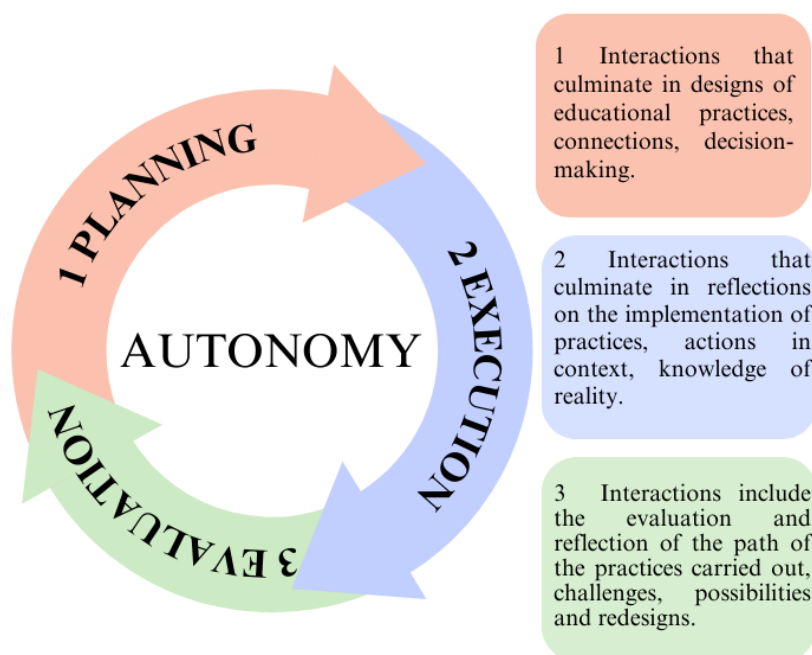


Figure 4.

*Autonomy in teacher training. (Elaborated by the authors, 2024)*

As revealed by Morreti and Moura (2010, p. 358), the process of work and training within the collective allows us to question the historical conception of the "individual competence" of people in teacher training and shows us the importance of creating collaborative settings, not only for initial training but also for continuing education.

### Final considerations

In this study, we aimed to analyze how the interactions experienced within the context of Mathematics teachers' initial training, including the planning, execution, and evaluation of practices, can influence the development of teaching autonomy. We consider that the curricular arrangements of teacher education programs require the support of theoretical constructs that provide elements favoring the reflective process, both of future teachers and their educators. These elements, in practice, should translate into indicators that enable individuals to organize the teaching of content, taking into account the different dimensions that make up the educational scenario, ranging from epistemological to social, political, cultural, and economic contexts. In this perspective, the DS approach contributes to the understanding and evaluation

of the Mathematics teacher education scenario because, by enumerating its components and indicators, it allows us to look at formative processes in their multiple facets.

In particular, the context of the interactions throughout the practices carried out in the training of future teachers stands out, as they are fundamental to consolidating the training. Collective and collaborative work, especially with the participation of an experienced teacher in the professional field, becomes a necessary element for future teachers to develop autonomy in relation to educational practices (Morales-López et al., 2023).

The teaching profession is mainly social, and in this sense, it needs to be dialogical, as decision-making that is consensual tends to be more solid and produce better outcomes (Breda, Font, and Pino-Fan, 2018). This perspective is crucial for teaching and learning processes in Mathematics in light of current challenges, among them, the challenging results shown by external and internal evaluations conducted in our country.

It is a fact that teaching Mathematics today requires, among many variables, reflective and autonomous teachers to make decisions in the face of curricular prescriptions, student demands, and social, cultural, and institutional contexts. In this regard, we must recognize the importance of teacher training programs linking Mathematical education with didactic-pedagogical training (Godino et al., 2013).

Although in this study we have addressed aspects related to the dimension of interaction, we are aware that the teacher's learning process is complex and encompasses elements from other dimensions because, as Godino et al. (2008) suggest, when planning teaching, it is necessary to consider the institutional meanings that one intends to study, adopting a broad view, not reduced to discursive aspects (epistemic suitability). Additionally, these authors emphasize the need to conceive and implement a didactic pathway that takes into account students' prior knowledge (cognitive suitability), identifies and resolves conflicts, mobilizes interactions (interactional suitability), and chooses and uses the necessary material and technological resources (mediational suitability). This context also includes the recognition of beliefs, values, conceptions, and emotions (affective suitability) and the importance of dealing with the evaluative processes that support learning (cognitive suitability).

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