

**A study of the documentational geneses of teachers for the teaching of piecewise defined function in basic education**

**Un estudio de la génesis documental de docentes para introducir la enseñanza de la función en educación básica considerando la función definida por tramos**

**Une étude des genèses documentaires des enseignants pour l'enseignement de fonctions par morceaux dans l'éducation de base**

**Um estudo da gênese documental de professores para introduzir o ensino de função na educação básica considerando função de uma variável real com várias sentenças matemáticas**

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**Abstract**

The present study is a part of a sequence of situations elaborated with the objective of investigating the documentational geneses of teachers for the teaching piecewise defined function in basic education. The theoretical framework used was the Documentational Approach to Didactics and the methodology was based on the principles of reflective investigation methodology. The documentational geneses can be observed through the mobilization and/or creation of resources and documentational schemes during the appropriation and application of one of the resources that integrated the sequence of professional situations.

**Keywords:** Documentational genesis, Piecewise defined function, Instrumental orchestration.

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## Resumen

El presente estudio es un extracto de una secuencia de situaciones elaboradas con el objetivo de estudiar la génesis documental de docentes de la función en educación básica a partir de la función definida por tramos. El marco teórico utilizado fue el Enfoque Documental de la Didáctica y la metodología se basó en los principios de la metodología de investigación reflexiva. La génesis documental se puede observar a través de la movilización y/o creación de recursos y esquemas documentales durante la apropiación y aplicación de uno de los recursos que integraron la secuencia de situaciones profesionales.

**Palabras clave:** Documental génesis, Función definida por Tramos, Orquestración instrumental.

## Résumé

La présente étude est un extrait d'une séquence de situations créées dans le but d'enquêter sur la genèse documentaire des enseignants pour introduire le concept de fonction à partir de situations mettant en jeu la fonction par morceaux. Le cadre théorique utilisé était l'approche documentaire du didactique et la méthodologie était basée sur les principes de la méthodologie de recherche réflexive. La genèse documentaire pourrait être observée à travers la mobilisation et/ou la création de ressources et de dispositifs documentaires lors de l'appropriation et de l'application d'une des ressources qui intégraient l'enchaînement des situations professionnelles.

**Mots-clés :** Genèse documentaire, Fonction par morceaux, Orchestration instrumentale.

## Resumo

O presente estudo é um recorte de uma sequência de situações elaboradas com o objetivo de investigar a gênese documental de professores para introduzir o conceito de função utilizando situações que envolvem função de uma variável real com várias sentenças matemáticas. O referencial teórico utilizado foi a Abordagem Documental do Didático e a metodologia foi embasada nos princípios da metodologia de investigação reflexiva. A gênese documental pôde ser observada por meio da mobilização e/ou criação de recursos e esquemas documentais durante a apropriação e aplicação de um dos recursos que integrou a sequência de situações profissionais.

**Palavras-chave:** Gênese documental, Função de uma variável real com várias sentenças matemáticas, Orquestração instrumental.

## **A study of the documentational geneses of teachers for the teaching of piecewise defined function in basic education**

This article presents an excerpt from the first author's Ph.D. thesis, defended in the frame of the Program of Mathematics Education at PUC/SP in 2021, which studied, in an environment of teacher training, the phenomenon of teachers' documentational genesis for a Piecewise Defined Function – PDF, as a resource to introduce the notion of function to students of the 1<sup>st</sup> grade of High School – HS.

The choice of PDF was supported by didactic studies and a historical-epistemological narrative that clarified the development of the formal definition of function, the understanding of PDF and the intertwining between them. It also clarified that the understanding of PDF was epistemologically hampered by the idea of representing function by through of a single expression with continuous and regular behavior.

According to Xavier Neto (2021), such ingredients lead teachers to reproduce for students the idea that every function must have this orientation, leading to a didactic obstacle in the learning process.

This makes us infer that the *raison d'être* of the notion of the construction of the notion of function would be related to the PDF and that it could be introduced, in basic education, by a problem involving PDF. These arguments served as a basis for structuring a teacher training, aiming to propose an introduction to the notion based on the PDF, our research objective being to study the documentational geneses of its participants.

In this article, we will focus on the study of the evolution of the documentational geneses of two teachers participating in the training - Gabriel and Lucas -, by monitoring the evolution of their documentation throughout a process of appropriation and application of one of the resources that integrated a didactic sequence – DS, to start function teaching based on PDF.

To this end, a methodological approach was developed inspired by the principles of the reflective investigation methodology, proposed by the Documentational Approach to Didactics - DAD, which constituted the theoretical framework of the present study, the details of which are presented below.

The methodology, the principles of reflective methodology and the methodological path are then discussed. Next, the analysis of the results obtained are presented and finally, the final remarks.

## Documentational approach to didactics - DAD

The DAD was introduced by Gueudet and Trouche (2008, 2009) and later received contributions from Gueudet, Pepin and Trouche (2012) and Gueudet (2019). It proposes to analyze teachers' work from the perspective of their interaction with resources, considering this relationship as eminently creative since, to engender their daily work, the teachers relate to a set of resources that, in turn, are adapted, revised or even reorganized to enable the planning and implementation of their professional activity.

For the DAD, the notion of resource is broad, including "everything that can be a source again the work of teachers" (Trouche, Gueudet and Pepin, 2020, p. 3). Resources are categorized as curricular when designed for teaching purposes in the frame of a given institutional prescription, such as a textbook, although there may be others without this purpose, such as a journal article.

Another fundamental element to understand the teacher's relationship with resources is related to the concept of scheme, which is central to grasping the processes that make up the DAD.

According to the authors, a scheme has four components:

- The aim of the activity (the aim characterizing the class of situations).
- Rules of action, of retrieving information and of control.
- Operational invariants, which are knowledge components of two (associated) kinds: theorem-in-action, proposition considered as true and concepts-in-action, a concept considered as relevant and
- Possibilities of inferences, of adaptation to the variety of situations. (Trouche, Gueudet and Pepin, 2020, p. 241).

During their professional activity, teachers enrich their schemes, including the new action rules or develop new schemes. Proposed by Vergnaud (1998), the concept of scheme "is strongly linked to another, the class of situations, which, in this theoretical context, designates a set of professional situations corresponding to the same objective of the activity" (Trouche, Gueudet and Pepin, 2020, p. 5). Messaoui (2020) calls documentational schemes all those that directly refer to documentational work.

The DAD calls the result of the interaction between resources and schemes of usage a document, that, however, is not so simple a process and requires a long development period. According to Hammoud (2012, p. 46), "it is the observable regularities of the teacher's activity or its uses, that condition the construction of a document," thus, not always a specific resource

used for an action will result in a document, as the development of a document continues throughout its cycle, making it difficult to determine at which stage it will actually be formed.

To describe the stages of document formation in DAD, three notions have been proposed: mother resource, daughter resource, and intermediate resource.

According to Hammoud (2012, p. 50), "a mother resource is a set of initial resources that the teacher mobilizes to prepare a given course", a daughter resource is the resource that the teacher has prepared and will implement in his or her class and is a product of the mother resource, while an intermediate resource is a resource that is considered an unfinished version of the daughter resource, i.e., a resource that has not yet been fully completed.

The documents prepared by the teachers are organized in a "structured system in which the documents are correlated" (Bastürk-Şahin, Tapan-Broutin and Trouche, 2021, p. 106), while the resources are organized in another system, created from the those that teachers use.

An aspect that accompanies the professional activity of teachers and is related to resources is collective work. According to Trouche, Gueudet and Pepin (2020, p. 241), "when teachers share documentation work, for example, in a group preparing lessons collectively", it is possible that they also develop a shared resource system, even if the members of this collective refer to a certain resource and develop different schemes.

According to Trouche, Gueudet and Pepin (2020, p.4), the process of creating a document "developed by a particular teacher for a particular purpose", known as documentational genesis, consists of two ways: "instrumentation and instrumentalization". The former occurs when resources instrumentalize the teacher's didactic action, while the process of appropriation and modification of resources characterize instrumentalization.

However, documentational genesis is not a process that happens naturally. To guide it and to enhance the development of the teacher's resource system, we found support in the theory of Instrumental Orchestration - IO, which has been incorporated, in a sense, into the DAD theoretical framework. An IO is characterized as "the systematic and intentional arrangement of an environment, carried out by an agent, to effect a given situation" (Trouche, 2005, p. 126), which, in the teaching context, refers to a set of professional situations corresponding to an activity objective. According to Trouche (2005), planning an IO from this perspective means creating a fruitful environment with an adequate structure to support the actions that will be developed in it in a didactical execution scenario.

According to Drijvers (2012), three elements can be distinguished in an IO: a didactical configuration, an exploitation mode and a didactical performance. The didactical configuration is responsible for managing the artifacts, subjects, time, and technologies in which the planned

situation takes place; an exploitation mode will put into practice the arrangement designed in the didactical configuration, including details of how the situation can be worked on in such a way as to encourage the mobilization of schemes and class of situations, while the didactical performance refers to verifying the performance achieved by planning the situation, when it will be possible to reflect on the success of the IO.

The principles of the reflexive investigation methodology and the methodological path used in the study are presented below.

## **Methodology**

The research carried out in the sphere of DAD uses its own methodology, the reflective investigation methodology, which was developed specifically for the purpose of studying teachers' documentation. As we have used it in the study presented here, we will detail its principles below.

### **The principles of the reflexive investigation methodology**

The reflexive investigation methodology is part of the DAD theoretical framework and was initially proposed by Gueudet and Trouche (2008). It then received further contributions from Trouche, Gueudet and Pepin (2012; 2020) until it reached its current state with its five main principles:

The principle of a broad collection of the material resources used and produced in the documentation work throughout the follow-up.

The principle of long-term follow-up. Geneses are ongoing processes and schemes develop over long periods of time,

The principle of in-and out of-class follow-up. The classroom is an important place where the teaching elaborated is implemented, bringing adaptations, revisions, and improvisations. However, an important part of the teacher's work takes place beyond the students' presence – at school, at home, in teacher development programs, etc.

The principle of reflective follow-up of the documentation work.

The principle of permanent confronting the teacher's views on her documentation work, and the materiality of this work (materiality coming, for example, from the collection of material resources; from the teacher's practices in her classrooms)". (Trouche, Gueudet and Pepin, 2020, p.243-244).

Considering that the preparation of documents is intertwined with the development of geneses and that its evolution requires time to observe, we had methodological choices to follow up the development of Gabriel and Lucas' geneses, which will be detailed in the next section dedicated to the methodological path of the investigation.

## **The methodological path**

To follow up the evolution of Gabriel and Lucas' documentation, we initially decided to find out how their resource system was organized. For this, as suggested in Gueudet and Trouche (s/n), we decided to organize a guided tour into their resources, accompanied by a semi-structured interview and the completion of a logbook.

During the visit, we planned to propose to the teachers the elaboration of the outline their Schematic Representation of a teacher's Resource System - SRRS, also used by Gueudet and Trouche (2010), to understand the structure and hierarchy of the existing resources and to make them reflect on how they are used to introducing function teaching.

Comparing the data obtained in the semi-structured interview with those from the SRRS outline provided us with information on the way in which function these teachers proposed teaching, while the logbook provided data on the nature of the activities they developed, both inside and outside the classroom.

Inspired by the teacher training model proposed by Lucena (2019), we subsequently studied the Gabriel and Lucas actions in a teacher training with eight basic education teachers during 30-hours in 2019. We followed the two teachers, the subjects of this study, during a process of appropriation and application of one of the resources that was part of a DS to introduce the teaching of functions based on PDF to students in the 1<sup>st</sup> grade of HS, in which we analyzed the evolution of their documentation during the experience of a professional situation entitled "appropriation" supported by an IO. Finally, we carried out a final visit to both teachers to evaluate the genesis process through the elaboration of the Inferred Mapping Resource System - IMRS, which was based on contributions proposed by Wang (2019).

## **Analysis of the results**

Next, we present professional information about teachers Gabriel and Lucas, followed by the analysis of the guided tour through the resources and the semi-structured interview. Then, we analyze the experience of the situation appropriation and application of the resource to introduce function teaching based on PDF and, finally, we discuss the information obtained during the final visit of both teachers.

## **Teachers Gabriel and Lucas**

Gabriel holds a degree in mechanical engineering and has two years of experience teaching high school mathematics. In 2019, he worked for two private schools, one of them

with Lucas, with a total workload a total of 30 hours per week. Lucas, in turn, has a degree in mathematics and one-and-a-half experience as a high school mathematics teacher in a single private school and, in 2019, worked 25 hours a week.

### Guided visits (tours) into Gabriel and Lucas' resources

Initially, we intended to conduct guided tours and semi-structured interviews at the teachers' homes, as we assumed that it was where they kept the resources, they used to prepare the documentation work. However, when trying to implement the idea, we realized some resistance on their part to conduct the interviews in their homes, so we decided to conduct them in the schools where they worked. For convenience, we called School A the institution where Gabriel and Lucas worked together, and School B, where only Gabriel taught.

### Guided visit through resources and interview with Gabriel

The visit, followed by an interview with Gabriel, took place in the library of School A in August 2019 and was scheduled in advance through an email exchange. The teacher was asked to bring to the meeting detailed information about the resources he used to prepare and implement his lessons on functions. We requested access to the logbook notes, but the teacher claimed that he didn't have time to do this. Then, as planned, he was invited to reflect on his documentation work and outline the SRRS (Figure 1) with the resources he usually used to start function teaching in the 1<sup>st</sup> grade of HS.

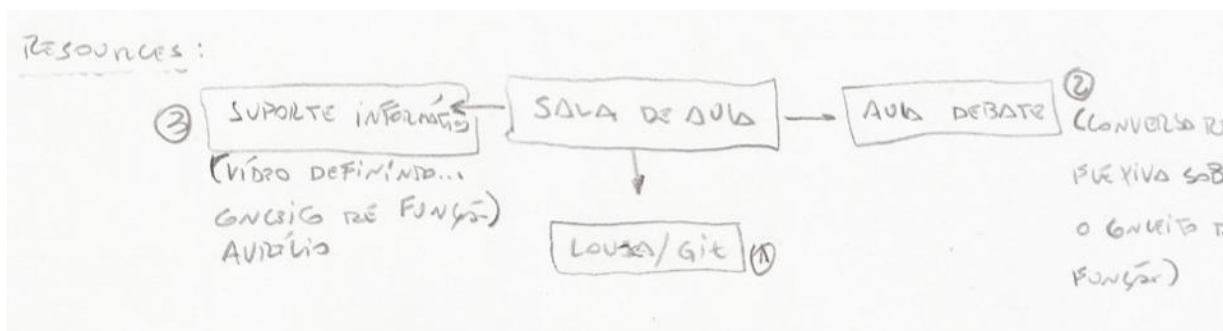


Figure 1.

*SRRS prepared by Gabriel (Xavier Neto, 2021, p.168)*

He explained that the numbers in the sketch were related to a sequence of events, and his explanation showed a hierarchy among the resources he used to teach function. According to the teacher, it was based on the doubts raised by the students in class (number 2) that he usually looked for exercises (in video lessons and in a notebook with solved exercises from the



main entrance exams - number 3) to plan his lessons, and for this he mostly used blackboard and chalk (number 3).

In view of this, we hypothesized that "students doubts raised in class", "video classes", and "notebook with solved exercises from the main entrance exams" represent a group of mother resources to start teaching function. To prove it, we asked the teacher to tell us how he used them to prepare his classes, and he told us that he used to start the teaching of function by emphasizing the need to use a notation, because he had noticed in previous years that a lack of this approach led to gaps in the subject learning. From this, we deduced that Gabriel developed a documentational scheme throughout his teaching activity to introduce the content and that the operational invariant "it is important for students to understand function notation" was one of its characteristics.

The teacher also gave us access to a form he had recently prepared, containing instructions for the students to fill in a table with values for "y" in the expression  $y = x + 1$ , asking them to complete it to explore the notation and the construction of a graph with the help of Geogebra. He told us that his experience of collective work at School A involved participation in the planning of the school year and creating resources for students who had learning difficulties, and that in School C, only bureaucratic meetings. The observations made during the interview led us speculated that Gabriel's mother resources to start teaching function were composed of doubts produced by students about notation, video classes, and exercises from entrance exams, and the script mentioned by the teacher probably represented a daughter resource because of its recent use.

### **Guided visit through resources and interview with Lucas**

Like Gabriel, the interview with teacher Lucas was scheduled in advance by email exchange and took place in the library of school A. At the beginning of the interview, it emerged that the teacher had not even started filling in the logbook. After a specific request, Lucas drafted the SRRS to start teaching functions (Figure 2), with the support of a text editor installed in his private laptop.

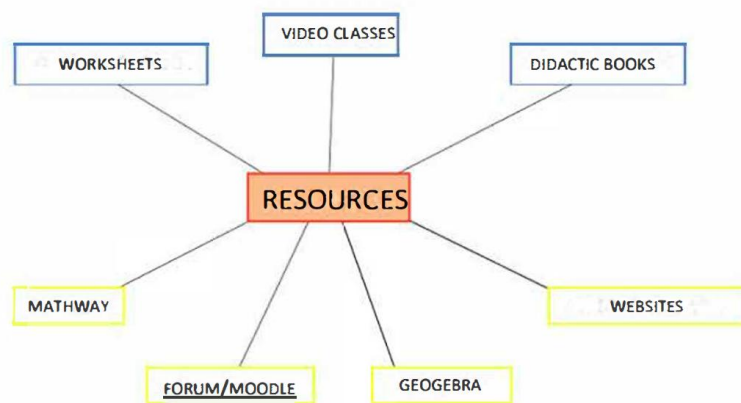


Figure 2.

*SRRS prepared by Lucas (Xavier Neto, 2021, p.173, our own translation)*

From the outline of the SRRS, it was assumed that the resources marked in blue represent those used in the preparation of his lessons - mother resources - and those marked in yellow represent those used in their execution. However, we identified that the suggestions of more experienced colleagues at School A were important for the teacher, that part of the worksheets were prepared with the help of PowerPoint, and that in a Moodle environment at the school, Lucas made available several files containing graphs drawn up with the help of GeoGebra, to improve the learning process.

The influence of his colleagues led Lucas to develop his own strategy for teaching function, which he says has been repeated without significant change. Convinced that it should facilitate the students' content understanding, he chose to start teaching function by positing an exercise from everyday life, related to taxi rides. These statements allowed us to assume that the invariant "facilitate content understanding through a simple a simple exercise" was a vestige of the existence of a documental scheme developed by the teacher throughout his teaching activity for the class of situations "start function teaching".

Due to the confrontation of the information obtained during the semi-structured interview and the outline of the SRRS, it was possible to understand that Lucas favored the introduction to function teaching by essentially using the affine function with a single sentence, and the fact that this strategy was repeated in his teaching activity could indicate the presence of a document. We deduced that the collective work developed with colleagues in the area, the problem sheets, the textbooks, and the video classes constituted his mother resources, while the

Geogebra files made available in Moodle environment to improve the learning of function were their daughter resources.

We believe that the guided visits to the two teachers and the semi-structured interviews carried out in the schools where they taught allowed us to understand that the choices they made to introduce the teaching of function were based on the use of affine functions with a single sentence. We understand that the use of the logbook as a tool to follow up the documentation inside and outside the classroom did not prove effective, possibly for both teachers' cultural reasons, who were not used to producing detailed reports on their professional practice. The simple production of the SRRS was not enough to understand the structure and hierarchy of the teachers' resource systems, and it was necessary to compare their draft with the data obtained during the guided tour and the interview. Finally, regarding the types of curricular resources used by the teachers to introduce function teaching, we realized that there was a combination of several of them rather than a predominance of one type over others.

Next, we will analyze the data obtained from a professional situation called appropriation.

### **The situation of appropriation**

The appropriation situation consisted of intentionally proposing the resolution of an activity to introduce the teaching of function from PDF who integrated the class of situations "solve and evaluating whether an activity is appropriate to introduce the function teaching".

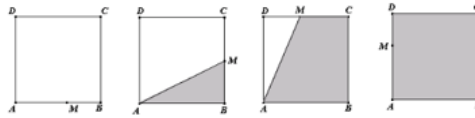
To implement the situation, we created an IO whose didactic configuration was planned, considering that two face-to-face meetings of three hours each would be sufficient for the process of appropriating the resource. In the first one, we thought that the time could be used to discuss the activity, and in the second one, to plan its application in the classes of 1<sup>st</sup> grade of HS students where Gabriel and Lucas were teaching.

The other element of the didactic configuration, the resource (Figure 3) to introduce function teaching via the PDFs was composed of activities originally designed by Almouloud (2016), "inserted in a geometric framework well known to the students and composed of affine functions" (Xavier Neto, 2021, p. 200).

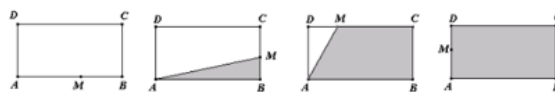
1. A point  $M$  moves on the side of a square  $ABCD$  whose sides measure 4 u.m. (fig. below). We will call  $x$  the measurement in centimeters of the length of the path from  $A$  to  $M$ .

a. Estimate the area  $a(x)$  of the colored part, according to the position of point  $M$ .

b. Represent the corresponding application graphically.



2. Return to the same tasks given in the first question, where  $ABCD$  is a rectangle of length 4 and width 2 (fig. below).



3. Return to the same tasks given in the first question,  $ABCD$  now being a rhombus whose sides measure 4 u. m. and angle  $\widehat{C}$  measuring  $60^\circ$ .

Figure 3.

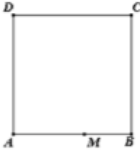
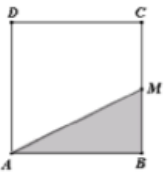
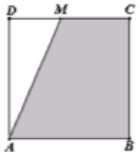
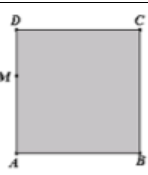
*Resource for introduce function through the PDF, Almouloud (2016, p. 135, our own translation)*

During the process of appropriating and planning of the activities planned in the resource, we expect that Lucas and Gabriel would be able to mobilize documental schemes and mathematical knowledge, allowing us follow up on their documentation. We also understand that the activities in the resource could provoke reflection about the students' role in the learning process and, therefore, the follow up of documentation work, since the activities "raise questions such as: What will students do? Will the work be carried in a group? What instructions to provide? What knowledge can be socialized?" (Almouloud, 2016, p. 129, our own translation).

Some mathematical solutions could be proposed for the activities, but we expected that the teachers would evaluate possible difficulties that the students of the 1<sup>st</sup> grade of HS could face when displacing point  $M$  on the figure of the square, as shown in Table 1.

Table 1.

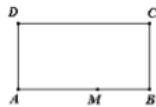
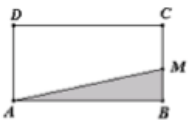
Expected solution for activity 1 - question a, adapted from (Xavier Neto, 2016, our own translation)

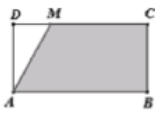

Displacement of M point	Expected action
	Identify that there will be no area formed while M moves along the side $\overline{AB}$ . Therefore, the measure of the area in relation to x will be: $a(x) = 0$ , if $0 \leq x \leq 4$ .
	Understand that when point M belongs to side $\overline{BC}$ , $x = AB + BM$ and that the area sought is that of the right triangle ABM, that is $A_{\Delta ABM} = \frac{AB \times BM}{2}$ . Therefore, the measure of this area in relation to x will be: $a(x) = 2x - 8$ , if $4 \leq x \leq 8$ .
	Understand that when point M runs along side $\overline{CD}$ , observe that the figure of the trapezoid ABCM is identified, whose area is determined by $A_T = \left(\frac{AB+CM}{2}\right) \times BC$ . Therefore, as $CM = x - 8$ , the measurement of this area in relation to x will be: $a(x) = 2x - 8$ , if $8 \leq x \leq 12$ .
	When the point M runs along the side $\overline{DA}$ , we expected that they to realize that the area of the square has been swept. So the last area in relation to x will be: $a(x) = 16$ se $12 \leq x \leq 16$ .

Regarding Activity 2, it was thought to be close to the one presented in Table 2.

Table 2.

Expected solution for activity 2 - question a, adapted from (Xavier Neto, 2016, our own translation)

Displacement of point M	Expected action
	Identify that there will be no area formed while M moves along the side $\overline{AB}$ . Therefore, the measurement of the area in relation to x will be: $a(x) = 0$ , if $0 \leq x \leq 4$ .
	Understand that while point M runs along side $\overline{BC}$ , the area considered is that of the right triangle ABM, whose measurement is given by: $A_{\Delta ABM} = \frac{AB \times BM}{2}$ . Therefore, the measurement of this area in relation to x will be: $2x - 8$ , if $4 \leq x \leq 6$ .


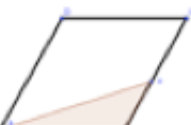
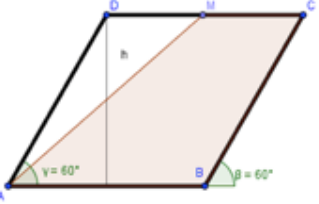
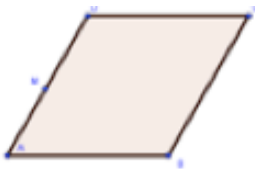
	<p>We expect that when the point M runs along the side <math>\overline{CD}</math>, they will see the trapezoid <math>ABCM</math>, whose area can be expressed by: <math>A_T = \left(\frac{AB+CM}{2}\right) \times BC</math>.</p> <p>Therefore, we expect then to observe that the measurement of this area in relation to <math>x</math> will be: <math>a(x) = x - 6, \text{ if } 6 \leq x \leq 10</math>.</p>
	<p>When point M runs alongside, <math>\overline{DA}</math>, we expected then to notice that the area of the square has already been swept. Therefore, the last area measurement will be: <math>a(x) = 8, \text{ if } 10 \leq x \leq 12</math>.</p>

In the first two activities, the teachers should identify students' need to mobilize knowledge about line segments; magnitude and proportionality, perimeter, and area of plane figures; additive and multiplicative structures of real numbers and algebraic expressions; and to discuss strategies to overcome any gaps to prepare the required formation law and graph.

Regarding the question of activity 3, the mathematical solution found in Table 3 was the one that we considered most likely.

Table 3.

Expected solution for activity 3 - question a, adapted from (Xavier Neto, 2016, our own translation)

Displacement of point M	Expected action
	<p>If point M belongs to side <math>\overline{AB}</math>, there is no area, which occurs when: <math>a(x) = 0</math>, if <math>0 \leq x \leq 4</math>.</p>
	<p>When M runs alongside <math>\overline{BC}</math>, the total area will be represented by a triangle with one of the angles equal to <math>120^\circ</math>, whose area will be equal to a <math>\frac{1}{2} \overline{AB} \overline{BM} \sin 120</math>. In this sense, it is possible to find the following relationship can be found: <math>a(x) = \sqrt{3}(x - 4)</math>, if <math>4 &lt; x \leq 8</math>.</p>
	<p>While M displaces along the side <math>\overline{CD}</math>, the segment <math>\overline{AM}</math> sweeps an area (which can be represented by a trapezoid), which can be calculated by the difference between the measurement of the rhombus and the area measurement of the triangle ADM. Thus, the expression for the area on this path is given by the function: <math>a(x) = \sqrt{3}(x - 4)</math>, if <math>8 &lt; x &lt; 12</math>.</p>
	<p>In the last step, in the section <math>\overline{AD}</math>, it can be seen that the segment <math>\overline{AM}</math> sweeps the total area of the rhombus and thus it is possible to determined it by the expression <math>2\left(\frac{\overline{AB} \overline{BC} \sin 120}{2}\right)</math>. In it, inside the parentheses, the area of half the rhombus is calculated; for this reason, it is multiplied by two, which guarantees, an area equal to the function: <math>a(x) = 8\sqrt{3}</math>, if <math>12 \leq x \leq 16</math>.</p>

In the same way, we thought that the teachers, when discussing the probable difficulties that the students would encounter in solving the problem, would identify the same knowledge as in the first two activities, as well as the knowledge of trigonometric relations and the use of technological resources. For the execution of the situation, we planned to set aside 120 minutes for the face-to-face meeting; for the mathematical resolution of the activity, we allocated 90 minutes. The rest of the time was programmed for the exchange of impressions of any difficulties that the students could find. For the second meeting, we set aside all the time to prepare the application with Gabriel's and Lucas's students.

The data collection through video and audio was organized by a communication professional and an observer who filled out an observation protocol, which should help to

identify the documentational schemes mobilized by the teachers, as well as the resources mobilized and/or produced by them. We believe that the comparison of the results obtained by these devices would be sufficient to identify documentational schemes throughout the process of appropriating the resource to introduce function teaching using PDF.

Regarding the didactic performance achieved by OI-2, we observed that the teachers did not have any difficulties with the mathematical solution of the activity and that the main problems arose because of the difficulties that the students might have in solving it. In one discussion, Gabriel claimed that students would have doubts about their understanding because he thought the wording was confusing. We don't think this was due to the original wording of the activity, but rather because the activities encouraged students to take a leadership role, something the teachers had not experienced before.

Regarding the issue that involved the construction of the subdomains of function sentences, we observed that the discussion provided evidence of an appropriation of the activity as a means of introducing the teaching of function from the PDF. Such appropriation was revealed to us by the actions, speeches, and notes existing in the observation protocol by Lucas's and Gabriel's, when they referred to the displacement of the M point on the side  $\overline{CD}$  of the flat square figure and the measurement of the trapezoid area. The transcript of the speech that follows seemed to highlight the arguments presented by Gabriel:

In interval III, the area required is that of a trapezium, when M is between points C and D, but neither at C nor at D. The smaller base, which varies, can be seen here (*Gabriel shows his solution*)! If M moves one unit from C to D, we would have  $x = 9$ , so the smallest base would be 1. In other words,  $9 - 8 = 1$ . If M moves three units from C to D, we would have  $x = 11$ , so the smallest base would be 3. In other words,  $11 - 8$ . That is,  $11 - 8$ . So the smallest base is  $x - 8$ , with  $8 < x < 12$ . We need to keep this in mind, because this doubt will clearly arise... To explain it better, I even think it's really cool to use the 9<sup>th</sup> grade textbook and even take one of these exercises and solve it on the blackboard to put it in context, you know... (Xavier Neto, 2021, p. 223, our own translation)

We noticed that when trying to anticipate the appearance of an error in the students' elaboration of the subdomain range, Gabriel shows signs of appropriation of the resource pertinent to this part of the activity. We believe that the transcript shows the mobilization of a



documentational scheme, whose operative invariant contains the statement 'clearly this doubt will arise'.

The teacher's ability to express his arguments showed us that his knowledge was in an operative form, which corroborates the observations made by Vergnaud (2013) that a subject is capable of not only solving a given problem and therefore being competent, but also of being able to explain his solving process to his colleagues. The differentiation between the forms of knowledge mentioned by Vergnaud “are also suitable for describing documentational practices” (Messaoui, 2020, p. 64, our own translation) which reinforces the fact that when “teachers share documentation work, for example in a group preparing lessons collectively, they may also develop a shared resource system” (Trouche, Gueudet and Pepin, 2020, p.241).

Understanding that the teachers were making progress towards appropriating the resource to introduce PDF-based function teaching and that they could mobilize it together with other resources from their resource systems, we planned and organized its application with students attending schools in which Gabriel and Lucas taught in 2019. During this process, the teachers led by Lucas developed a tool in the GeoGebra program (Figure 4) to simulate the displacement of point M in the flat figure of the square.

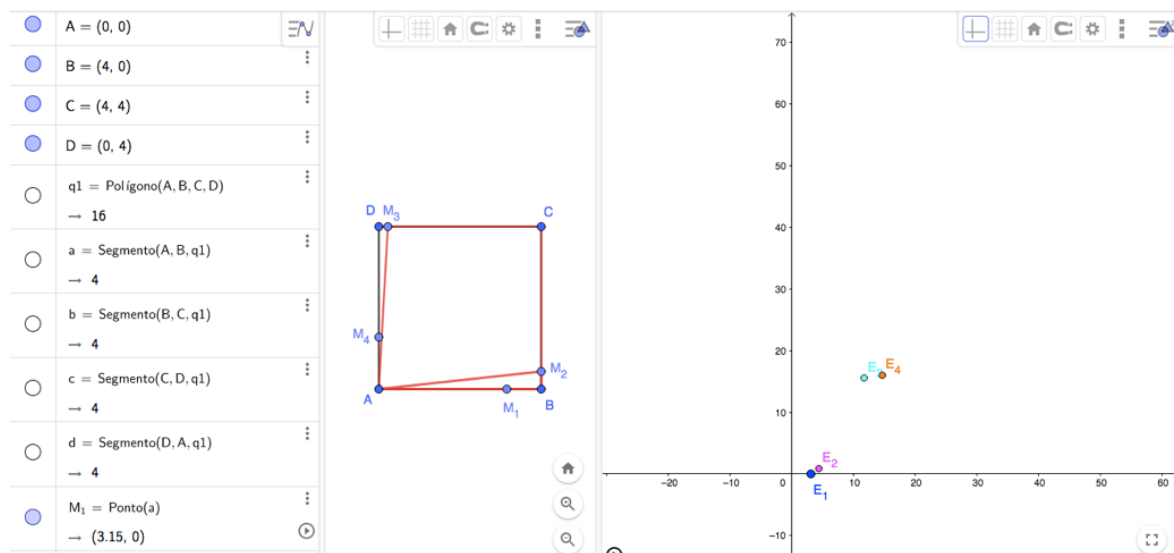


Figure 4.

*Tool developed with the help of the slider (GeoGebra), Xavier Neto (2021, p. 251)*

Initially, the teachers felt that the tool could be useful for students to understand the formation of the area and its relationship to the PDF, but since there was no consensus on its use, it was up to the teacher using the activity to make that decision. On the other hand, they raised the importance of providing basic training for students to use the program.

We believe that the actions taken by Lucas point to the development of a documentational scheme whose invariant "could be useful for students to understand the formation of the area and its relationship with the PDF formed" was one of the components promoted by the articulation between the resources of the GeoGebra application and the exercises in the activities. Regarding the time and the way in which the activities could be carried out, it was decided to do it during the after-school hours and in the form of workshops, to provide the students with a less formal learning environment and more time to carry out the activities.

### **Application of activities**

The activities were applied at the end of October and the beginning of November 2019, and the process was documented through audio and video, with the first author carrying out this task. We sought to analyze the data obtained to continue identifying in the teachers' actions, signs of appropriation of the resource to introduce function teaching based on PDF through the identification of resources and schemes mobilized and/or developed during the process of applying the activities.

Lucas was the first to apply the activities, and Gabriel acted as an assistant teacher. Twelve students, organized into three groups, participated in the application and, according to Lucas, had basic knowledge of functions, affine function, and set theory. The classroom where the workshops took place had a computer with Internet access, a conventional blackboard, a data show, and if necessary, Lucas could request individual laptops. During the application, we initially observed that Lucas was uncomfortable with the role of mediator suggested by the activity, which seemed evident to us when he didn't understand when to provide (or not) guidance to the students and when one of the groups (Figure 5) had difficulties to the displacement of the point M on side  $\overline{BC}$  of the flat figure of the square.

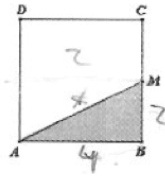


Figure 5.

Activity 1, Group 2 solution - School A (Lucas). Lucas's production, (Xavier Neto, 2021, p. 255)

At this point, instead of trying to resolve the students' doubts, Lucas chose to lead them to the correct answer. We understand that he may not have remembered the discussions with the other teachers about the fact that the problem involved the notions of magnitude and proportionality. Specifically in relation to this group of students, we observed that Lucas's attitude increased the loss of meaning in learning, which confirms the fact that if "students are not invited to think but are simply led to carry out the procedure suggested by the teacher" (Almouloud, 2016, p. 111), this increases the misrepresentation of the knowledge to be taught.

Two other facts stood out in our observation: the way Lucas proceeded when one of the groups of students solved Activity 3 (point M moving along the sides of the representation of a rhombus), and in relation to outline of the PDF graphs made by the students. Regarding the displacement of point M along the flat figure of the rhombus, Lucas noticed that the students had difficulty in identifying the measure of the area of the triangle ABM (Figure 6) and the measurement of the angle  $\hat{B}$  as  $120^\circ$ .

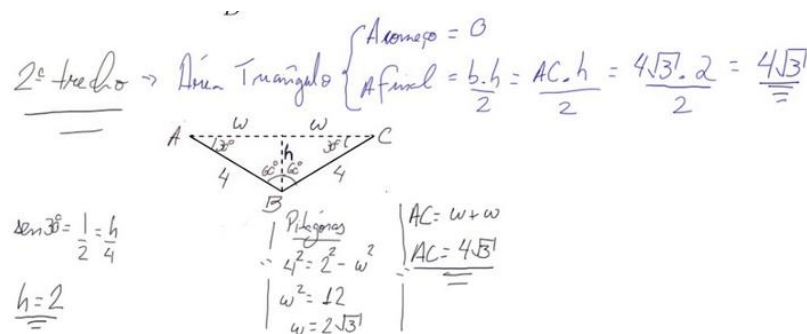


Figure 6.

Activity 3a, Group 2 solution - School A (Lucas). Lucas's production, (Xavier Neto, 2021, p. 257)

In view of this finding, he decided to share with the students a sheet that presented trigonometric ratios in the same context as what was proposed at that time, which he had

prepared in previous years, and which helped the members of this group to build the PDF properly. We believe that the use of this resource enabled the students to remember the notion of sine from an angle and evidenced a process of instrumentalization, since the mobilization of a resource like the one in Activity 3 ended up influencing their teaching.

As for the graph required in the activity, when he observed that one of the groups was drawing non-objective scales related to the distance of point M from the sides of the representation of a square, Lucas decided to use the GeoGebra, available on the classroom computer to emphasize its importance in the processes of constructing and understanding a graph. Although Lucas's activity aroused great enthusiasm among the students, when he realized the technical difficulties in using the software, he just pointed out differences in the students' production instead of offering training, as had been planned during the organization of the activity. We believe that Lucas' choice may have inhibited the visualization of different ways of representing functions, whose potential in the digital environment was observed in study by Geiger and Redmond's (2013).

Gabriel's activity application took place a few weeks after the experience with Lucas at School A and had the participation of 8 students from the first grade of HS who had contact with the idea of function, linear and affine function in the 9<sup>th</sup> grade of a secondary school and set theory during that school year. However, before starting the application, Gabriel informed the students that they would need an introductory lesson to review the notions of function, trigonometry, and plane geometry. This decision was motivated by the fact that he observed gaps in these contents during the application of the activity with the group of students at School A. We believe that the teacher's action reinforces the findings made by Messaoui (2020) about the importance of collective documentation work for the development of the teacher's professional activity, as well as indicating a process of instrumentalization.

After the development of the content review, Gabriel carried out the activities with the students as planned, and in relation to the evolution of his documentation, two events seemed relevant to us. The first one related to his attitude towards the developing the application of the activity by proposing mathematical reflections to students who had doubts, identified when a

student had difficulties in solving the third movement (Figure 7) of point M (side  $\overline{CD}$  f activity 1a).

$$\begin{array}{l}
 \text{L. } \overline{AB} = 0 \leq x \leq 4 \rightarrow A = 0 \\
 \overline{BC} = 4 < x \leq 8 \rightarrow A = b \cdot h = 2(x-4) = 2x-8 \\
 \overline{CD} = 8 < x \leq 12 \rightarrow A = \frac{(b+b) \cdot h}{2} = \frac{(4+x-8) \cdot 2}{2} = x-4 \\
 \overline{DA} = 12 < x \leq 16 \rightarrow A = 16
 \end{array}$$

Figure 7.

*Activity 1a, Gabriel. Gabriel's production, (Xavier Neto, 2021, p. 265)*

Gabriel asked the student to reflect on the construction, suggesting that he consulted the notes taken during the preparatory lesson, especially about the trapezoidal figure, as the intervals might not be correct. After this intervention, the student corrected the sentence corresponding to the movement of point M in the trapezoid but continued to have doubts about the formulation of the sentence intervals. To resolve the doubt, the teacher asked another group of students who had completed the exercise correctly to present their solution on the blackboard to their colleagues who had doubts about the function sentences, while he acted as a mediator.

Corroborating Trouche, Gueudet and Pepin (2020), we understand that Gabriel developed a new class of situations with the aim of applying the activities and his interactivity with the resources mentioned in the action were decisive for his professional development, leading him to develop design skills in the face of the didactic scenarios presented to him.

The second event took place when the teacher formalized the solution of each of the activities proposed in the resource, emphasizing the idea of correspondence between variables without referring to the idea of a set. We infer from this that there was a change in the way Gabriel introducing function teaching since he didn't refer to the notion of notation, as he did during the interview at the school where he taught.

### The final visit

The final visit took place a few weeks after the application of the activities at PUC-SP and lasted about 60 minutes. We sought to confront teachers' opinions about their

documentation work to understand the impact of appropriation and application process of the activity on their resource systems so that the evolution of the documentation could be clearly understood.

When they were asked about how they would introduce function teaching in the 1<sup>st</sup> grade of HS, Lucas said that he intended to adapt his courses to avoid teaching only single sentence functions, but it was not yet clear whether he would do so using the activities he had applied with his students at School A. With this, we understand that his IMRS could possibly include some of the resources mentioned during the interview, and it is not possible to say whether they would have the status of mother resources.

Gabriel, in turn, expressed that his experience certainly made him reflect on the way he had been introduced function teaching, stating that he did not intend to do it through the exclusive emphasis of affine functions with a single function sentence and that he would try to integrate the activities into his didactic strategies to teach function in the 1<sup>st</sup> grade of HS, which leads us to believe that in the future his IMRS could include the activities as a new resource.

### **Final considerations**

This investigation consisted of a study of how documentational genesis occurs in two teachers from basic education during the process of appropriating and application of a resource to start a function teaching from the PDF.

To achieve this, a methodological approach based on the five principles proposed by the reflexive investigation methodology was developed to follow up the evolution of the teachers' documentation.

The data that was collected during the visits to the schools where the teachers worked allowed us to understand the constitution and the hierarchy of their resource systems, providing details of the mother resources, daughter resources and documents they used to introduce function teaching in the 1<sup>st</sup> grade of HS.

The follow up of the evolution of the teachers' documentation showed that Gabriel apparently changed his emphasis on the notion of notation to introduce the teaching function, developing documentational schemes throughout the experience of situations and application

of activities, leading us to infer that he had appropriated the resource and changed the way of teaching the content. Gabriel's follow up the implementation of the activities at the school where Lucas worked, before implementing then himself at his institution, offered elements about the importance of collective work, even though it was not done in a planned way.

Lucas, in turn, brought elements that made it possible to deduce that he was appropriating the way of introducing function teaching using the PDF. This may have occurred when he developed a scheme to articulate existing tools in GeoGebra for creating graphs. The fact that the notion of a document is linked to its use, combined with the idea that documental genesis is a phenomenon whose follow up takes place over long periods of time, encourages us not only to continue to follow up the teachers' documentation as a strategy for investigate in more depth the phenomenon of documental genesis, but also to develop more evidence for the correctness (or incorrectness) of the decision to begin function teaching in the 1<sup>st</sup> grade of HS using PDF.

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