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Specialized knowledge in teaching exponential function via problem solving in the context of PIBID

Conocimiento especializado en la enseñanza de la función exponencial via Resolución de Problemas en el contexto del PIBID

Connaissance spécialisée dans l'enseignement de la fonction exponentielle par la résolution de problèmes dans le contexte du PIBID

Conhecimento especializado no ensino de função exponencial via resolução de problemas no contexto do PIBID

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Abstract

This article aimed to identify and describe the specialized knowledge mobilized in the teaching of exponential function via problem-solving by two mathematics PIBID fellows. We adopted a qualitative research methodology, following the interpretative paradigm, which took place in the context of PIBID, focused on the elaboration and implementation of a teaching sequence in the Teaching-Learning of Mathematics via Problem Solving (EAMvRP) approach, in high school. Based on the Mathematics Teacher's Specialized Knowledge (MTSK), the results show the potential of EAMvRP as a teaching approach that, addressed in PIBID in the theory-practice relationship, provided the development of MTSK. We conclude that the specialized knowledge built by the two fellows on content, teaching organization, and their classroom experience can contribute to a more successful future teaching practice.

Keywords: Mathematical knowledge, Pedagogical knowledge, Teaching sequence, Teacher training.

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Resumen

Este artículo tuvo el objetivo de identificar y describir el conocimiento especializado movilizado en la enseñanza de la función exponencial mediante la resolución de problemas por dos participantes del PIBID en matemáticas. Adoptamos una metodología de investigación cualitativa, siguiendo el paradigma interpretativo, que se llevó a cabo en el contexto del PIBID, enfocada en la elaboración e implementación de una secuencia de enseñanza en el enfoque de Enseñanza-Aprendizaje de Matemáticas via la Resolución de Problemas (EAMvRP) en la Educación Secundaria. Basados en el Conocimiento Especializado del Profesor de Matemáticas (MTSK), los resultados evidencian el potencial del EAMvRP como un enfoque de enseñanza que, tratado en el PIBID en la relación teoría y práctica, proporcionó el desarrollo del MTSK. Concluimos que los conocimientos especializados construidos por las dos participantes del PIBID sobre el contenido, la organización de la enseñanza y su experiencia en el aula pueden contribuir a una práctica docente futura más exitosa.

Palabras clave: Conocimiento matemático, Conocimiento pedagógico, Secuencia de enseñanza, Formación docente.

Résumé

Cet article avait pour objectif d'identifier et de décrire les connaissances spécialisées mobilisées dans l'enseignement de la fonction exponentielle par la résolution de problèmes par deux participantes du PIBID en mathématiques. Nous avons adopté une méthodologie de recherche qualitative, suivant le paradigme interprétatif, qui s'est déroulée dans le contexte du PIBID, axée sur l'élaboration et la mise en œuvre d'une séquence d'enseignement dans l'approche Enseignement-Apprentissage des Mathématiques par la Résolution de Problèmes (EAMvRP) au lycée. Basés sur les Connaissances Spécialisées de l'Enseignant de Mathématiques (MTSK), les résultats montrent le potentiel de l'EAMvRP comme une approche d'enseignement qui, traitée dans le PIBID dans la relation théorie et pratique, a permis le développement du MTSK. Nous concluons que les connaissances spécialisées acquises par les deux participantes du PIBID sur le contenu, l'organisation de l'enseignement et leur expérience en classe peuvent contribuer à une pratique pédagogique future plus réussie.

Mots-clés : Connaissance mathématique, Connaissance pédagogique, Séquence d'enseignement, Formation des enseignants.

Resumo

Este artigo teve o objetivo de identificar e descrever o conhecimento especializado mobilizado no ensino de função exponencial via resolução de problemas de duas pibidianas de matemática. Adotamos uma metodologia de pesquisa qualitativa, seguindo o paradigma interpretativo, que ocorreu no contexto do PIBID, voltado à elaboração e implementação de uma sequência de ensino na abordagem do Ensino-Aprendizagem de Matemática via Resolução de Problemas (EAMvRP), no Ensino Médio. Pautados no Conhecimento Especializado do Professor de Matemática (MTSK), os resultados evidenciam o potencial do EAMvRP como uma abordagem de ensino que, tratada no PIBID na relação teoria e prática, proporcionou o desenvolvimento do MTSK. Concluímos que os conhecimentos especializados construídos pelas duas pibidianas sobre o conteúdo, a organização de ensino e sua vivência em sala de aula podem contribuir para uma prática docente futura mais exitosa.

Palavras-chave: Conhecimento matemático, Conhecimento pedagógico, Sequência de ensino, Formação docente.

Specialized knowledge in teaching of exponential function via problem solving in the context of PIBID

The Mathematics teacher training is a subject which has been investigated by studies which have assumed as focus the specialized knowledge of the teachers (Moriel-Junior & Carrillo, 2014; Avila, 2015; Vasco & Climent, 2018; Zakaryan & Ribeiro, 2018; Carreño & Climent, 2019; Advíncula-Clemente et al., 2022; Martín-Díaz & Montes, 2022; Vieira, Ponte & Mata-Pereira, 2022), searching to establish some understanding on the importance of knowing how to solve a problem and knowing Mathematics in manners which allow its usage in teaching activities (Ponte & Chapman, 2016). Therefore, the studies on the teacher's knowledge have evidenced the need that the teacher knows the subject in specialized manner in relation to teaching (Ball, Thames & Phelps, 2008; Carrillo et al. 2014; Carrilo-Yañez et al. 2018) and knows aspects related to teaching and learning (Shulman, 1986, 1987).

The authors Bromme and Tillema (1995) associated the development of professional knowledge to the reflection of the teacher's action. The authors highlighted that for the development of such knowledge, the teacher must not only accumulate theoretical knowledge, he needs to develop professional practice. In this scope, the *Programa Institucional de Bolsas de Iniciação à Docência (PIBID)* might favor this professional knowledge, one it aims to establish the comprehension of the teaching-learning process through the theory and practice relation.

The present study follows PIBID's context, in a manner the subject of exponential function was proposed, which must be worked at school (Brasil, 2018). However, some studies demonstrated difficulties in teaching which generated difficulties in the learning of students of such content (Silva, 2012; Keeling, 2015; Oliveira, 2015; Mendonça & Pires, 2016). Such difficulties are related with factors such as the poor understanding of the content, the way the subject is taught, demanding only the reproduction of ideas or even as the didactic books present the work with the exponential function.

In this sense, it is important that in the teacher training it occurs the development of teaching professional knowledge as those indicated in the model of Mathematics Teacher's Specialized Knowledge (MTSK) by Carrilo-Yañez et al. (2018), having as reference the subject of exponential function. With this, the objective of our study is identifying and describing the specialized knowledge mobilized in the teaching of exponential function via problem solving of two Mathematics PIBID participants. Searching to reach such objective, we approach the understanding by Carrilo-Yañez et al. (2018), analyzing moments of planning and implementation of a teaching sequence. This article is organized in sections which present the teaching via problem solving, approaching the Mathematics teacher specialized knowledge, describe the methodological procedures, discuss the obtained results and, lastly, present the conclusions.

The teaching via problem solving

The approach of teaching based on Problem Solving is widely recognized, both in regulatory documents and in research, as one of the privileged manners to promote mathematical activity (Brasil, 2018; Proença, 2021; Proença, Campelo & Santos, 2022). In the international scenario, studies as those by Lester and Cai (2016), Schoenfeld (2020), Liljedahl and Cai (2021), and Olivares, Lupiáñez and Segovia (2021) highlight the importance of the Problem Solving in the teaching of Mathematics, evidencing how it contributes to the development of mathematical teaching and learning by the students. In a similar manner, in the national context, research by Andreatta & Allevato (2020); Proença (2022); Marcatto (2022); Silveira & Andrade (2022); Santos, Campelo & Proença (2023); Travassos (2023) and Mendes (2023) also stress the relevance of Problem Solving, also defending its usage favors not only the understanding of mathematical concepts, but also the development of cognitive and mathematical skills of teachers, future teachers and students.

About the Problem Solving treated in teaching, the authors Schroeder and Lester Junior (1989) highlight approaches, which imply an understanding on how Problem Solving was approached in the decade of 1980. According to the authors, there were three approaches, to be known: teaching *about* Problem Solving, teaching *for* Problem Solving and teaching via Problem Solving.

The teaching *about* Problem Solving refers to the student learning about the stages of how to solve a problem, as the example of the four stages defined by Polya (1994) - comprehension of the problem, plan elaboration, plan execution and retrospect. The teaching *for* Problem Solving consists in thinking paths for the resolution, which consists in an utilitarian approach of Mathematics, because what is learned must be applied in problems and exercises only with the purpose of obtaining an answer. The teaching *via* Problem Solving comprehends the usage of a problem as a starting point of mathematical activity, which provides a privileged moment for the learning of mathematical content.

In the understanding by Schroeder and Lester Junior (1989), the teaching *via* Problem Solving is pointed as the most indicated for the teacher's job, because it allows the students to, beyond mobilizing previous knowledge, understand mathematical concepts, processes and techniques. About the teaching *via* Problem Solving, Proença (2018) proposed an organized teaching sequence in five actions, as it is illustrated in Figure 1, which he denominated Teaching-Learning of Mathematics via Problem Solving (EAMvRP).

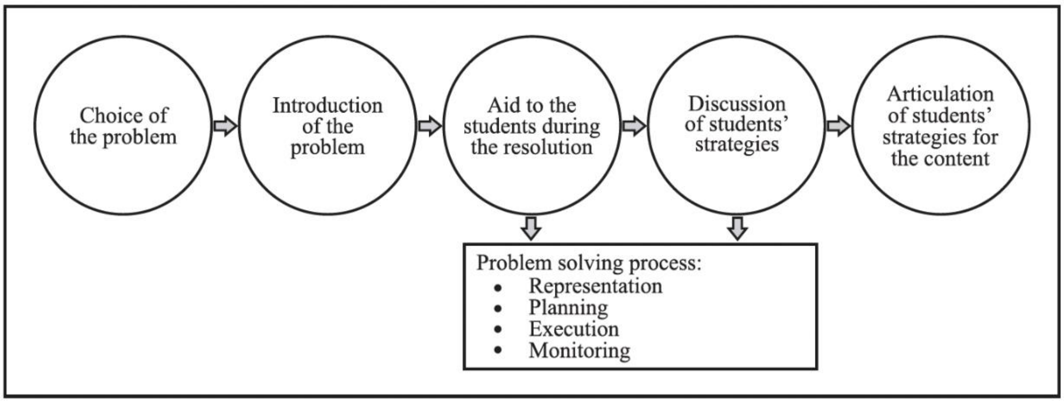


Figure 1.
Work scheme by mean of sequence of actions (Proença, 2018, p.46)

In the action of *choice of the problem*, the teacher should create, rework, or choose the mathematical situation as a whole. Proença (2018) highlights that the mathematical situation chosen by the teacher must favor the mobilization of previous knowledge, the usage of different strategies to obtain an answer, in a manner to contribute to the construction of new knowledge. The action of *introduction of the problem* is considered by the author as a moment of teacher-student contact, in which it will be presented the mathematical situation, the teacher will define a classroom organization in groups, favoring collaborative work and making it easier for the teacher's work in following the students.

In the *aid to the students during the resolution*, the teacher assumes a learning observer, directive and encouraging role. The action of *discussion of the students' strategies* is what Proença (2018) calls socialization, the moment in which the students are going to present their strategies to everyone, and the teacher must explore the possible difficulties or mistakes made in the process of resolution. In the *articulation of the students' strategies to the content*, the teacher must utilize strategies, paths and reasoning imagined by the students in a manner to articulate the forms utilized by the students to think what is wanted to be taught.

Proença (2018) highlights some necessary knowledge for the stages of *representation, planning, execution* and *monitoring* in the process of problem-solving. The stage of *representation* involves the comprehension of the problem and, such comprehension, depends on the mobilization of three knowledges: *linguistic*, which corresponds to words or expressions which belong to the mother language of the chosen mathematical situation which regards to the actions involved; *semantic*, which corresponds to mathematical terms or words/expressions present in the mathematical situation and which demands mathematical knowledge; *schematic*, which corresponds to the nature of the mathematical situation, what is, the involved content.

The stage of *planning* involves proposing a strategy and includes the *strategic* knowledge, that is, the paths of resolution (as the usage of charts, Tables, draws, trial and error, among others). The stage of *execution* is the moment in which is/are put in practice the strategy(ies) chosen, such action involves the *procedural* knowledge, which implies in the execution of calculations/representations. The stage of *monitoring*, besides not involving a specific knowledge, from the cognitive point of view, demands an evaluation of the obtained answer(s) according to the context of the problem.

The specialized knowledge of the mathematics teacher

The *Mathematics Teacher's Specialised Knowledge* - MTSK is a theoretical model which describes the specific and professional knowledge which the Mathematics teacher must have (Carrillo et al., 2014; Carrilo-Yañez et al., 2018) and appears from the studies by Shulman (1986, 1987) and Ball, Thames and Phelps (2008). O MTSK is organized in two domains, *Mathematical Knowledge* - MK and *Pedagogical Content Knowledge* - PCK and six subdomains, as illustrated in Figure 2. In the center of the hexagon, we have the teachers' beliefs about Mathematics, its teaching and learning. Below, we describe each of the six subdomains.

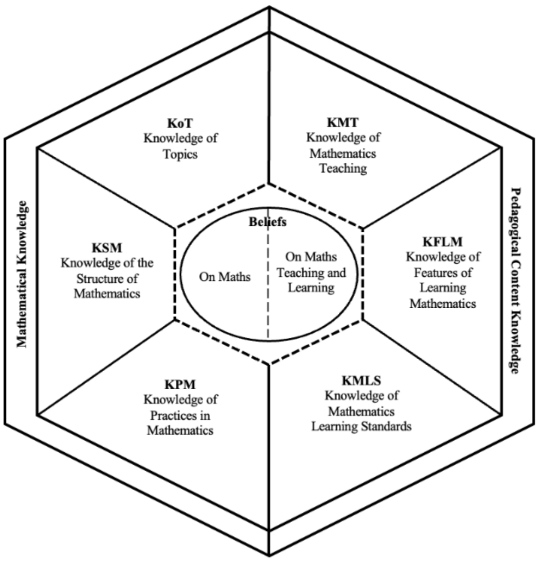


Figure 2.

Based on Carrillo-Yañez et al. (2018, p. 241), our own translation

Mathematical Knowledge (MK) includes three subdomains: *Knowledge of Topics* - KoT, *Knowledge of the Structure of Mathematics* - KSM and the *Knowledge of the Practices in Mathematics* - KPM.

KoT describes the deepened knowledge of the teacher of mathematical contents to be taught, which involves knowing procedures, definitions and properties, representations and models, as well as contexts, problems and meanings, and, in this sense, acknowledges the complexity of the mathematical objects which may arise in the classroom. KSM involves the mathematical knowledge on advanced and elementary concepts, previous and futures, which allows the teacher to understand the school Mathematics under a wider perspective, from the point of view of the many contents and their organization (Carrillo-Yañez et al., 2018). KPM includes the manners of doing and proceeding in Mathematics, which involves the forms of creating or producing, selecting representations, defining, making deductions and inductions, giving examples and understanding the role of counterexamples, knowing how to explore and generate new knowledge, managing the mathematical reasoning mobilized by the students, knowing different strategies to solve a problem (Carrillo-Yañez et al., 2018).

The Pedagogical Content Knowledge (PCK) includes three subdomains: *Knowledge of Mathematics Teaching* – KMT, *Knowledge of Features of Learning Mathematics* – KFLM, and the *Knowledge of Mathematics Learning Standards* – KMLS.

The Pedagogical Content Knowledge includes the theoretical knowledge (formal or personal) of the teacher on the Mathematics teaching, which involves knowing approaches (problem solving, exploratory teaching, modeling among others, having knowledge of resources and teaching materials (potentials and limitations), as in the example of books, manipulative materials, softwares among others (Carrillo-Yañez et al., 2018). KFLM involves the teacher's knowledge in interpreting the procedures and reasoning of the students (possible mistakes, difficulties and strong points, articulating contexts which might motivate and influence the interest and the learning, identify the modes (interest and expectation) how students deal with the content (Carrillo-Yañez et al., 2018). The KMLS, according to Carrillo-Yañez et al. (2018, p. 13), involves the "teacher's knowledge about everything the student must have or is capable of reaching in a specific level, in combination with what the student studied previously and the specifications for the subsequent levels".

Methodological procedures

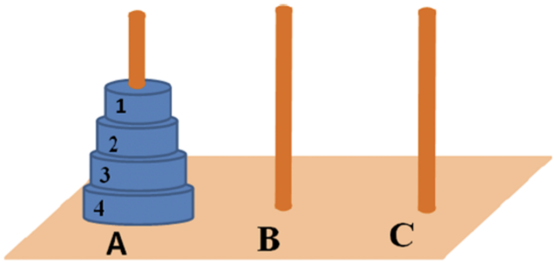
The research was developed from a qualitative perspective, because it values the processes in its natural environment, assuming a paradigm of descriptive and interpretative character (Bogdan & Biklen, 2010). In this study, the university and the school were the natural environment, because the training activities were of theoretical-practical character in the context of PIBID. In order to realize our study, there was the approval of the Permanent Comitee of Ethics in Research with Human Beings (*Comitê Permanente de Ética em Pesquisa com Seres Humanos* - COPEP) under the Certification of Ethical Presentation and Appreciation (Certificado de Apresentação de Apreciação Ética- CAAE) nº 71695523.0.0000.0104.

Two female prospectives mathematics teachers participated in the study from a public state university from Paraná, who were scholarship holders in PIBID, a subproject of Mathematics, in the period of October from 2022 to March 2024. Both PIBID participants experienced training activities, referent to the professional teacher learning to approach the Teaching-Learning of Mathematics via Problem Solving (EAMvRP), in which according to the author of this article was the coordinator of the subproject and the first author was the researcher.

The training activities were organized in two large moments, in a theoretical-practical movement. In the first moment, it was realized as a theoretical approach on the Problem Solving and the stages involved in this process. The students who hold PIBID scholarships also solved and discussed strategies of problem solving as part of a training moment. Based on this study and discussions, they created a mathematical situation which was applied to students from High School in a partner school. The students' resolutions were tabulated and analyzed at the university, and the resulting material was shared with the teachers and the management team of the school. In the second moment, there was the work about the five actions of EAMvRP in which the PIBID participants produced a teaching sequence, which was debated collectively and then was implemented in High School classrooms. Then, there was the socialization with the Mathematics teachers and the school principal.

In this context, the focus of our study is about this second moment, in which the two participants were a team who chose the subject of exponential function for the sequence of teaching they created in the perspective of EAMvRP. The scholarship holders implemented the teaching sequence in three classes of the third year of High School from a partner school, which comprehended three hours/class in each class, a period occurring in October 2023. Figure 3 below demonstrates the problem which was proposed by the PIBID scholarship recipients as a starting point, based in the didactic material known as Hanoi Tower, to introduce the subject of exponential function.

The Hanoi Tower is a mathematical game based on an ancient Indian legend. According to it, Brahma, the creator god, created three diamond poles in a platter of bronze in the temple in Benares, on which is located the dome which marks the center of the world. In one of the poles, Brahma put sixty four discs of solid gold, in decreasing sizes, being the biggest one exactly on top of the platter and the smallest on the top of the tower and ordered his priests to transfer them to the other pole, following two rules: they could move only one disc at a time and they could never put a larger disc on top of a smaller one. When the task got complete, the world would end, and all would disappear with a thunder. Ever since this, the priests work tirelessly, day and night moving the discs.



A. According to the rules of the legend, what is the minimum of movements needed to assemble a tower with 1, 2, 3, 4, 5, 6 and 7 discs?
B. (Orally/On the board) What if we wanted to know the minimum of movements for a tower with 10 discs? And of 100 discs? Or for any amount of discs? Can we get to a formula/expression for it?

Figure 3.
Created Mathematical Situation (Research Data, 2023)

In order to compose the research data, we had as initial sources the teaching sequence and the registers in the field journal to help in the collection (Roese et al., 2006), in which the researcher registered the data, directly in the classroom, the implementation by the PIBID scholarship holders in the three High School classes. Thus, we conducted an interview which used these initial sources in order to support our questions to both PIBID scholarship recipients, aiming to understand their explanations on elaboration and implementation of the EAMvRP to the subject exponential function. We elaborated a semi-structured interview, realized a week after implementing the teaching sequence, having in sight that "[...] it is characterized by the flexibility and exploring the maximum of a determined subject, demanding from the source some dynamic subordination to the interviewed" (Duarte, 2005, p. 3).

To analyze the data, we realized an organization of Tables which correspond to the five actions of EAMvRP to situate them and reveal in the training context the dimensions of MTSK which were mobilized. In an specific manner, the data analysis occurred by mean of the Content Analysis (Bardin, 2011), organized in three moments: 1) Pre-analysis, in which the transcribed data were organized in a way to constitute the *corpus* of the research; 2) Material exploration, which constituted in a meticulous reading aiming to establish the categories (aspects of the five actions of EAMvRP) and subcategories (mobilized specialized knowledge), which were obtained *a posteriori*, based on the registry units; and 3) Results treatment, which focused on the discussion and interpretation of the data, starting from studies about the MTSK and studies about the teaching *via* Problem Solving.

Results and discussions

The charts ahead demonstrate the results and discussions about the process of creating and implementing the teaching sequence to introduce exponential function and reveal the mobilized specialized knowledge of the MTSK. Table 1 below demonstrates categories (aspects of the choice of the problem), and subcategories (mobilized specialized knowledge) based on the explaining of both participants (registry units) about the action of *choice of the problem*, occurring in the classroom.

Table 1.

Choice of the problem

Aspects of the choice of the problem	Mobilized specialized knowledge	Registry Units
Criteria for choosing the subject	KMLS	We received the subjects [from the school] and (...) about the date each one would be implemented, then we tried to choose one that had a good time for us to prepare [the teaching sequence], (...) then we chose the subject of Exponential Function from the Thematic Unit of Algebra.
	KMT	We thought (...) a mathematical situation in which we could utilize manipulable material, with the objective of making [the subject] more palpable for the students.
	KMT	Since we had already worked with the Hanoi Tower, which could involve the subject of exponential function, it happened that (...) we decided to use the Hanoi Tower.
	KFLM	We already knew the class had difficulties paying attention to the mathematical subject, so we wanted to bring something more palpable,more attractive to them, to catch attention.
	KPM	(...) we searched for something (...) that also brought subtle extra knowledge, right.
Kind of problem (created, reworked, or chosen mathematical situation as a whole)	KMT	(...) we researched in website and articles, where we found the Hanoi Tower. We reworked (...) with the end of presenting only the key points and summarizing it. (...) The mathematical situation we created fits in a historical context, in an adapted manner.
Strategies/Paths to solve the problem	KoT	We started thinking [strategies] in the most obvious thing and which was where we wanted to get and we went on thinking in other mathematical forms which could get to that [the answer], but what the students could think, not with our view, but with theirs.

	KPM	(...) We already knew they liked a lot, for example, chart, we could also put something as a diagram, draw, (...) that they could think for being something more palpable, more visual, also trial and error and formulas, and we went on thinking in something like this to get to our strategies.

In the action of *Choice of the Problem*, three main categories were identified: Criteria to choose the content, Kind of problem (created, reworked, chosen as a whole) and Strategies/Paths to solve the problem. These categories evidenced the mobilization of different subdomains of specialized knowledge, inducing two subdomains of Mathematical Knowledge (KoT, KPM) and three of Pedagogical Content Knowledge (KMT, KFLM, KMLS). The Knowledge of Mathematics Teaching (KMT) was mobilized in a predominant manner by the PIBID participants, as demonstrated by the choice of the Hanoi Tower, a pedagogical resource acknowledged by its capacity of exploring the concept of exponential function. This choice reflects a conscious planning which articulates mathematical content and didactical strategies, aligning to the findings of Martín-Díaz and Montes (2022), which highlight the central role of the Pedagogical Knowledge of Content in the creation of problems. Furthermore, the Knowledge in Practices of Mathematics (KPM) of the PIBID participants emerges by prioritizing the introduction of new knowledge and the management of mathematical reasoning possibilities of the students. This approach evidences an effort in diversifying representations and promoting the learning through strategies which dialogue with the interests and needs of the students.

The Knowledge of Topics (KoT) was identified in the PIBID participants' skills in dominating the mathematical content and their representations, demonstrating security in the planning and application of the mathematical situation. However, it is worthy stressing the literature, as in the study by Vieira, Ponte and Mata-Pereira (2022), points out the mobilization of KoT could be challenging in the initial training, especially in complex tasks which involve the identification of mathematical patterns, as figure sequences. The other two knowledges mobilized were the Knowledge of the Features of Learning Mathematics (KFLM), by thinking in a context (situation) with potential to encourage and influence the students' interest and the Knowledge of Mathematics Learning Standards (KMLS) which involves the PIBID participants' knowledge about what the student must learn in a specific level of school education.

Table 2 below demonstrates categories (aspects of the introduction of the problem) and subcategories (mobilized specialized knowledge) based on the explanations of both the participants (registry units) about the action of introduction of the problem, which occurred in the classroom.

Table 2.

Introduction of the Problem

Aspects of the introduction of the problem	**Mobilized specialized knowledge**	**Registry Units**
Organization/Contact of/with the class for collaborative work	KMT	(...) We thought about splitting the class in groups of 3 or 4 people, no more than this (...) because we thought it was important having the discussion with other people (...) one observes a manner, the other observes another manner, and we did not want big groups.
	KMT	(...) We had already done the prediction of the groups and also the prediction of reading with them and delivering the Hanoi Towers for them to start situating.
	KMT	(...) We delivered the sheet with the problem and the Hanoi Towers and did the reading of the problem with them and then we asked if all of them understood, and everyone said “yes”
Stimulus to search for answer(s)	KMT	(...) After presenting the situation to the groups, we asked them to use the knowledge they had and attempted to solve the situation in different manners.
	KMT	(...) We walked around explaining again the rules and asking how they would use the 4 discs made available for them (...) to see if they would think about strategies or see some pattern.
	KFLM	A student asked if she could get more discs because the situation asked for the movements with 1, 2, 3, 4, 5, 6 and 7 discs, then we asked her and her group to think what occurred when they used the 4 discs available and asked them to represent forms of representing what they were doing.

In the action of *Introduction of the Problem* we identified two categories: Organization/Contact of/with the class for collaborative work and Stimulus to search for answer(s). In this action, the PIBID participants mobilized two subdomains of Pedagogical Content Knowledge (KMT, KFLM). The Knowledge of Mathematical Teaching (KMT) was the most frequent. In the study by Advíncula-Clemente et al. (2022) which involved the specialized knowledge about polygons of Mathematics teachers in formation, KMT was the second most frequent subdomain, which the authors highlight as an important data to understand the participant teacher’s manners. In this action, the mobilization of KMT is shown when the PIBID participants utilized some referrals from EAMvRP, as an example the division of the students in small groups, aiming the sharing of knowledge, the presentation of the mathematical situation to the students and the stimulus to the students in the search for different paths to obtain an answer.

The Knowledge of the Features of Learning Mathematics (KFLM) is evidenced when the PIBID participants aim to identify possible difficulties or the good development of the students by dealing with the proposed problem.

Table 3 below demonstrates categories (aid in the classroom), and subcategories (mobilized specialized knowledge) based on the explanations of both participants (registry units) when the action of *aid to the students during the resolution,* which occurred in the classroom.

Table 3.

Aid to the students during the resolution

Aid in the classroom	**Mobilized specialized knowledge**	**Registry Units**
Observer teacher	KFLM	We walked by in the first moment and saw the groups with difficulty in knowing how to find a pattern, because they were only used for 4 [discs].
	KMLS	(...) I was observing if they would be able to get to the formula of exponential function. We looked and then,at most only one group did not get it, like, everyone got to the formula and they have so much potential.
	KFLM	(...) they have so much potential even with Mathematics. I saw they like it, calculating something without properly knowing how they are discovering it, they are scared of hard Mathematics, right. But I found it really cool.
	KFLM	(...) when we were looking at them, it was like, very interesting seeing their thoughts(...) a very very quick thought, really good.
Encouraging teacher	KPM	(...) we went on giving suggestions like, ‘why do you not show the numbers for 1, for 2, for 3, for 4 discs to check if you see something.
	KoT	And that is where we found ourselves (...) with the exponentiation of two minus one. And it was where letter b fit. Then we (...) asked: how can you do it without needing 99 discs?
	KFLM	(...) we were not prepared for them reaching the formula so soon (...) and then we also improvised a little bit by asking them to write a script of how to get to the answer.
Directive teacher	KPM	a student (...) said, how can you keep talking about 5 discs if you gave me only 4, then we explained what the intention was.
	KoT	(...) in a second moment, after they had found this result, we kept trying to take them to see what pattern existed in this result. How this result was increasing.

	KPM	(...) a group looked at me and said, “no, we are going to do the 99 results. It was it, we kept trying to guide them to the train of thought.

In the action of *Aid to the students during the resolution* three categories were identified in the teaching action: Observer teacher, Encouraging teacher and Directing teacher. In these interactions, the PIBID participants mobilized specialized knowledge which belong to two subdomains of Mathematical Knowledge (KoT, KPM) and two subdomains of Pedagogical Content Knowledge (KFLM, KMLS). Between the mobilized subdomains, it is highlighted the Knowledge of the Features of Learning Mathematics (KFLM), evidenced in the identification of the difficulties faced by some groups when searching for patterns, and in the acknowledgement of interest and success of other students in problem solving. A notable example of this subdomain is the observation that some students demonstrated “fear of hard Mathematics”, but engaged positively when exploring calculations without the complete domain of what they were discovering. This ability of perceiving and interpreting the behavior of students was aligned to what Meléndez and Grueso (2021) highlight as essential for the teacher, allowing the offer of directed feedback and the anticipation of mistakes. Studies by Carreño and Climent (2019) reinforced the relevance of KFLM, by showing the teachers in training mobilized this knowledge by understanding how the students learn and progress.

The Knowledge of Mathematical Practices (KPM) was mobilized by the PIBID participants by exploring and managing the mathematical reasoning of the students. This competence manifested, as an example, in the suggestion of representing numbers for different quantities of discs (1, 2, 3, 4) as an early strategy to detect patterns, beyond encouraging the students to explain their discoveries. These actions evidenced the PIBID participants’ skill in guiding the students in the construction of new mathematical knowledge. The Knowledge of Topics (KoT) emerged especially in connecting the strategies of the students with the concept of exponential function and by introducing questions which lead to the discovery of a mathematical pattern, as an item (b). The acknowledgement of appropriated moments to deepen the subject and guiding the students in direction to generalizing reflect the domain over the mathematical content needed for the teaching practice. This domain, according to Carreño and Climent (2019), was observed in teachers in training, although difficulties in the usage of KoT in teaching situations have also been reported.

The Knowledge of Mathematics Learning Standards (KMLS) was evidenced in the capacity of the PIBID participants of identifying the students’ potential in combining previous knowledge and exponentiation in advancing to the generalization of the expression which represented the number of movements. This identification is crucial to promote the learning and is compatible with what previous studies indicate about the importance of understanding how the students progress their mathematical reasoning.

Table 4 below demonstrates categories (socialization) and subcategories (mobilized specialized knowledge) based on the explanations of both participants (registry units) about the action of e *discussion of the students' strategies*, occurring in the classroom.

Table 4.

Discussion of the students' strategies

Socialization	**Mobilized Specialized Knowledge**	**Registry Units**
Socialization of the groups' strategies on the blackTable	KPM	We ordered the groups through strategies which complemented each other (...) so the students could observe how one strategy is directly connected to another.
	KFLM	(...) we thought (...) calling first the students who did strategies different from our articulation (...) we saw most made a Table, we planned the articulation thinking about it. But there were some who made paper discs, thought about the cap, did a graphic, who drew. We tried to take them, one of each at least.
Evaluation of the problem solving process	KFLM	After we analyzed the strategies chosen by the students, we invited to the Table the groups who made any mistakes, to encourage the students to aid this group and identify what occurred (...) if necessary with our intervention, avoiding any embarrassment.
	KPM	(...) We called the groups which obtained significant resolutions and which would engage our articulation.
	KPM	(...) explaining in front of the others, many got nervous, lá stuttering, not being able to explain correctly, but we helped

In the action of *Discussion of the students' strategies*, two categories were identified: Socialization of the groups' strategies on the blackTable and Evaluation of the problem-solving process. During this action, the PIBID participants mobilized a subdomain of Mathematical Knowledge (KPM) and a subdomain of Pedagogical Content Knowledge (KFLM). The KPM - Knowledge of Practices in Mathematics was evidenced when the PIBID participants selected representations and strategies of the groups to manage the mathematical reasoning developed by different groups. This process included a deliberate choice of the strategies to be socialized on the blackTable, with the object of allowing the students to perceive how an approach connects directly to another. The KPM mobilization reflects the competence of the PIBID participants in facilitating the articulation between the different forms of resolution, stimulating the reflection about the mathematical approaches adopted by the groups.

The care in the choice of the strategies to be presented aimed not only to show the diversity in solutions, but also evidencing the connections between them, promoting more integrated and contextualized learning. The Knowledge of the Features of Learning Mathematics (KFLM) manifested mainly when the PIBID participants interpreted the problem-solving strategies of the different groups, identifying mistakes and difficulties. From this analysis, they were capable of acting in an strategies manner, inviting the groups with mistakes to explain their solutions so the other students could contribute with corrections, if needed, mediated by the intervention of the PIBID participants. This teaching posture is aligned to the view of Vasco and Climent (2018) and Advíncula-Clemente et al. (2022), who highlight the importance of the teacher understand the common difficulties of the students by interacting with mathematical contents. By dealing with these mistakes, the PIBID participants not only offered corrections, but also promoted a collaborative environment and mutual reflection, which favors collective learning.

Table 5 below demonstrates categories (articulation to the subject) and subcategories (mobilized specialized knowledge) based on the explanations of both participants (registry units) about the action of *Articulation of the students' strategies to the subject*, occurring in the classroom.

Table 5.

Articulation of the students' strategies to the subject

Articulation to the subject	**Mobilized Specialized Knowledge**	**Registry Units**
Resumption of the strategy for using the table	KMT	(...) we retraced with them every strategy to see if they remembered. They remembered it easily.
	KPM	(...) Since all of them used the Table, we were not afraid of articulating using the table and other groups feel upset (...) we started talking about the possibility of assembling a table with the data of the number of discs and the number of movements. Then we worked easily through the Table we built with them.
Use of the central points of the table strategy	KPM	We were questioned on how to write the numbers. And it was nice seeing them participating. And then we developed until reaching our formula, which was 2 elevated to n, everything minus one.
	KFLM	(...) we built our table on the Table with their participation. We asked if they remembered what they have done, how they wrote it and they answered, then, their participation was really nice. We asked if they remembered other forms of writing the numbers, if they remembered what they had observed there.

	KoT	Then we used everything to make a graphic with them, and then we also utilized a strategy from the class, right. Of a group which had done it through a graphic.

In the action of *Articulation of the students' strategies to the subject*, were identified two main categories: Resumption of the strategy for using the table and *Use of the central points of the table strategy*. During this action, the PIBID participants mobilized two subdomains of Mathematical Knowledge (KoT, KPM) and two subdomains of Pedagogical Content Knowledge (KMT, KFLM). The Knowledge of Practices in Mathematics (KPM) was the subdomain most frequently mobilized. The KPM was evidenced when the PIBID participants demonstrated domain over different solving strategies, especially in the construction and management of the table with the students. By proposing the collaborative construction of the chart, the PIBID participants not only ensured the articulation of different forms of representation. but also made easier the comprehension of the mathematical relations involved, allowing the students to appropriate from this tool to develop the necessary reasoning to solve this problem. The mobilization of KPM reflects the PIBID participants competence in guiding the process of construction of mathematical representations which make easier the understanding and the problem solving. Studies such as those by Avila (2015) and Vieira, Ponte and Mata-Pereira (2022) stress the importance of convincing and using different records of representation which allow the teachers in formation to propose diversified strategies and adapt to the needs of the students.

The Knowledge of Topics (KoT) is mobilized by the PIBID Participants and demonstrates mathematical knowledge about the procedures, definitions, properties and representations which involved the subject to be taught, the Exponential function. The authors Vasco and Climent (2018) also identified in the mobilization of KoT procedures and register of representations. Zakaryan and Ribeiro (2018) in a specific study about the subcategories of the subdomain KoT, identify and differ specific categories of the teachers' knowledge. The Knowledge of Mathematics Teaching (KMT) was mobilized when the PIBID participants encouraged the resumption of the students' ideas, rescuing what was produced by them all. Lastly, the Knowledge of the Features of Learning Mathematics (KFLM) appeared when the PIBID participants articulated contexts aiming to motivate and influence the students' interests and searched to understand the manners and how the students handled the subject during the problem solving.

Conclusion

The objective of this article was identifying and describing the specialized knowledge mobilized in the teaching of exponential function via problem solving of problems of two PIBID participants of Mathematics. In order to do it, we analyzed the planning and implementation of a teaching sequence, elaborated in the five actions of EAMvRP, of two PIBID participants of Mathematics in three classes of the third year in a High School school.

The results demonstrated that the Mathematics Teacher's Specialized Knowledge (MTSK) allowed us to deepen the understanding of knowledge mobilized by the PIBID participants of Mathematics by teaching the subject of exponential function in the approach of EAMvRP. The categories and subcategories of analysis helped revealing the mobilization of knowledge of the subdomains of *Mathematical Knowledge* – MK and of *Pedagogical Content Knowledge* – PCK, in the elaboration of the teaching sequence and in the work in the classroom when implemented, with exception of the *Knowledge of the Structure of Mathematics* - KSM. The study reveals the mobilization of expected knowledge in each one of the actions of Teaching-Learning of Mathematics via Problem Solving (EAMvRP).

We believe our study contributes for the formation of future Mathematics teachers in the field of problem solving and of teacher's specialized knowledge, by treating about forms of organizing the Mathematics teaching and of the knowledge the teacher must have in order to plan and implement a class about exponential function (and other subjects from the school curriculum), besides helping the teachers in early training to reflect on the need of having mathematical and didactical knowledge on the content to be taught.

From the point of view of scientific research, our study also contributes to the development of future research, in a diversity of contexts and aspects, which advance in the identification and understanding of the mobilization of the teacher's specialized knowledge, having the example of *Knowledge of the Structure of Mathematics* – KSM, which we believe needs more time to observe the teacher's work. Therefore, future research might be done to conduct the elaboration of new proposals which contribute to teacher training and in reflections about knowledge and teaching practices.

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