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Mathematics for teaching the concept of polynomials from recontextualizations in textbooks

Matemáticas para la enseñanza del concepto de polinomios a partir de recontextualizaciones en libros de texto

Mathématiques pour l'enseignement du concept de polynômes à partir de recontextualisations dans les manuels scolaires

Matemática para o ensino do conceito de polinômios a partir de recontextualizações em livros didáticos

Alana Santiago Oliveira[1]
Secretaria de Educação do Estado da Bahia (SEC-BA)
Mestrado em Educação
https://orcid.org/0000-0003-4556-0565

Jaqueline de Souza Pereira Grilo[2]
Universidade Estadual de Feira de Santana (UEFS)
Doutorado em Educação
https://orcid.org/0000-0002-0408-047X

Abstract

The research was developed with the objective of systematizing Mathematics for Teaching the concept of Polynomials based on achievements present in textbooks. For this, we had Bernsteinian theory and the Study of the Concept as theoretical and methodological foundations. The results obtained from the analysis of the concept of Polynomials in ten textbooks point to a variability of achievements, grouped into six panoramas, namely: generalization, problem situations, algebraic structure, geometric, manipulable materials and illustrative. These panoramas indicate that this concept tends to maintain a weak degree of isolation between the concept of Polynomials and other mathematical concepts, as well as with concepts outside Mathematics, revealing the establishment of links between them. Furthermore, we identified that the achievements recognized in the generalization, algebraic structure and geometric panoramas, establish combinations with Abstract Algebra and allow deeper interpretations.

Keywords: Teaching, Polynomials, Basic education.

[1] alanasantiagooliveira@hotmail.com
[2] jspgrilo@uefs.br

Resumen

La investigación se desarrolló con el objetivo de sistematizar la Matemática para la Enseñanza del concepto de Polinomios a partir de logros presentes en los libros de texto. Para ello tuvimos como fundamentos teóricos y metodológicos la teoría bernsteiniana y el Estudio del Concepto. Los resultados obtenidos del análisis del concepto de Polinomios en diez libros de texto apuntan a una variabilidad de logros, agrupados en seis panoramas, a saber: generalización, situaciones problemáticas, estructura algebraica, geométrica, materiales manipulables e ilustrativa. Estos panoramas indican que este concepto tiende a mantener un débil grado de aislamiento entre el concepto de Polinomios y otros conceptos matemáticos, así como con conceptos ajenos a las Matemáticas, revelando el establecimiento de vínculos entre ellos. Además, identificamos que los logros reconocidos en la generalización, estructura algebraica y panoramas geométricos establecen combinaciones con el Álgebra abstracta y permiten interpretaciones más profundas.

Palabras clave: Enseñanza, Polinomios, Educación básica.

Résumé

La recherche a été développée dans le but de systématiser les mathématiques pour l'enseignement du concept de polynômes sur la base des acquis présents dans les manuels scolaires. Pour cela, nous avions la théorie bersteinienne et l'Etude du Concept comme fondements théoriques et méthodologiques. Les résultats obtenus à partir de l'analyse du concept de Polynômes dans dix manuels pointent vers une variabilité des acquis, regroupés en six panoramas, à savoir : généralisation, situations-problèmes, structure algébrique, géométrique, matériaux manipulables et illustratif. Ces panoramas indiquent que ce concept tend à maintenir un faible degré d'isolement entre le concept de Polynômes et d'autres concepts mathématiques, ainsi qu'avec des concepts extérieurs aux Mathématiques, révélant l'établissement de liens entre eux. De plus, nous avons identifié que les acquis reconnus dans la généralisation, la structure algébrique et les panoramas géométriques, établissent des combinaisons avec l'algèbre abstraite et permettent des interprétations plus profondes.

Mots-clés : Enseignement, Polynômes, Education de base.

Resumo

A pesquisa foi desenvolvida com o objetivo de sistematizar uma Matemática para o Ensino do conceito de Polinômios a partir de realizações presentes em livros didáticos. Para isso, tivemos a teoria bernsteiniana e o Estudo do Conceito como fundamentações teóricas e metodológicas.

Os resultados obtidos com a análise do conceito de Polinômios em dez livros didáticos, apontam para uma variabilidade de *realizações,* agrupadas em seis *panoramas,* a saber: generalização, situações problemas, estrutura algébrica, geométrico, materiais manipuláveis e ilustrativo. Esses panoramas apontam que tal conceito tende a manter um fraco grau de isolamento entre o conceito de Polinômios e outros conceitos matemáticos, bem como com conceitos fora da Matemática, revelando o estabelecimento de *vínculos* entre eles. Além disso, identificamos que as realizações reconhecidas nos panoramas generalização, estrutura algébrica e geométrico, estabelecem *combinações* com a Álgebra Abstrata e permitem interpretações mais profundas.

Palavras-chave: Ensino, Polinômios, Educação básica.

Mathematics for teaching the concept of polynomials from recontextualizations in textbooks

Several studies in the field of Maths Education are inspired by the classic Shulman (1987) and focus on the understanding of the disciplinary and pedagogical knowledge needed by Maths teachers for teaching purposes (Ball et al., 2008; Ribeiro, 2020). These authors discuss a particular aspect of the teacher, which differs from the Mathemathics practised in other areas, such as physics, engineering, etc. According to Davis and Renert (2014) this particularity of the teacher, who must know how to use specific maths for teaching purposes, consists of a network of understandings about mathematical concepts and pedagogical skills that are not easily acquired by teachers. Although content knowledge is indispensable, teaching mathemathics requires essential pedagogical aspects so that specific concepts are understood by students (Menduni-Bortoloti, 2016; Rangel et al., 2015; Santos & Barbosa, 2016).

Regarding the specific mathematics that teachers should know for teaching purposes, the authors Adler (2005), Davis and Simmt (2006) and Davis and Renert (2014) discuss it in terms of Mathematics for Teaching (M4T), which we translate as "*Matemática para o Ensino*". Our understanding of M4T stems from our Bernsteinian theoretical understanding, which will be better presented in the next section.

From an M4T perspective, several authors have developed studies interested in investigating how teachers communicate certain mathematical concepts (Campos, 2021; Coutinho, 2015; Gomez, 2016; Menduni-Bortoloti, 2016; Santos, 2017). Like Davis and Renert (2014), we believe that for teaching purposes, a mathematical concept can be understood as an organization of procedures of communicating it, which can be formal definitions, applications, drawings, metaphors, among others. Furthermore, we understand that effective knowledge for teaching "is much more than a readily catalogued or objectively tested set of concepts" (Davis & Renert, 2014, p.3).

Given that mathematical communication in teaching contexts is based on specific mathematical concepts, the concept chosen to be investigated was polynomials. The pretext for choosing this concept lies in the fact that it was excluded from the list of secondary school content after the approval of the "National Common Curriculum Base" (BNCC - *Base Nacional Comum Curricular*) and was restricted to primary school, the stage at which contact with algebraic language and the structuring of algebraic thinking formally begins (Brasil, 2017). Our intention is to identify the achievements of the concept proposed for this stage of Basic Education in order to fulfill the research objective: to systematise a Mathematics for Teaching the concept of Polynomials based on realisations present in textbooks. Here we understand realisations as the different ways that could be used to communicate a concept: a formal definition, applications, drawings, metaphors, among others.

The choice of textbooks was based on the study by Macêdo et al. (2019). The authors state that teaching work is still intertwined with the way mathematics appears in textbooks. Thus, each textbook we analyzed allowed us to see mathematics for teaching the concept of polynomials. The contribution of this study is the systematization of mathematics for teaching the concept of polynomials that is scattered in different textbooks.

Based on Davis and Renert (2014), the concept of Polynomials is taken as the grouping of different realisations that can be used to communicate and associate with the word Polynomials. For example, the formal definition below, expressed by Hefez and Villela (2012, p. 74), is an example of a realisation of the concept.

> Seja *A* um anel. Um símbolo *x* não pertencente ao anel *A* será chamado uma *indeterminada* sobre *A*. Utilizaremos um símbolo x^j, para cada número natural $j \geq 0$, e escreveremos x^0=1 e $x^1 = x$. Um Polinômio $f(x)$ com coeficientes em *A* é uma expressão formal do tipo:
>
> $$f(x) = a_0 + a_1 x + \cdots + a_n x^n = \sum_{j=0}^{n} a_j x^j,$$
>
> onde $n \in N$, $a^j \in A$, para $0 \leq j \leq n$.
> Para $0 \leq j \leq n$, os elementos a_j são chamados de coeficientes do Polinômio $f(x)$, as parcelas $a_j x^j$ de termos e os termos $a_j x^j$ tais que $a_j \neq 0$ de monômios de grau *j* do Polinômio $f(x)$. O coeficiente a_0 é chamado de termo constante.

Historically, the development of algebraic concepts has been associated with the generalization of arithmetic. According to Eves (2002, p. 546), "at the beginning of the 19th century, algebra was considered simply as symbolic arithmetic". According to the author, this is still the way algebra is presented at school and also in higher education courses. In this context, Moura and Santos (2021) discuss some problems from before the 16th century that sought to interconnect Arithmetic, Algebra and Geometry and refer to Book II of Euclid's Elements, specifically on the possible relations between Algebra and Geometry, which "presents algebraic traits, being known as the 'geometric algebra' of the Greeks" (Moura & Santos, 2021, p. 307).

According to Ibrahim (2015), understanding the concept of polynomials is essential as it enables the understanding of other related mathematical concepts and, consequently, the development of algebraic skills. Studies also point to difficulties in the process of teaching and learning polynomials that may be associated with the way this concept is approached in schools (Ibrahim & Resende, 2018; Lauteschlager & Ribeiro, 2017). Other studies present suggestions for activities aimed at contributing to the teaching and learning of Polynomials (Bressan, 2021; Coutinho, 2019; Massarioli, 2022; Pereira, 2020; Silva, 2021), as well as proposals aimed at a more in-depth study that establish relationships with Rings of Polynomials and how they could be worked on in Basic Education (Dias, 2021; Santos, 2021; Sousa, 2020).

Next, we present a brief discussion of concepts from Bernstein's theory (2000; 2003), specifically the concept of Pedagogical Recontextualization, which will underpin our interpretation of Mathematics for Teaching.

Paths of pedagogical recontextualization and the constitution of an M4T

According to Bernstein (2000; 2003), pedagogical practices are not just restricted to the relationships established in the school context but consist of social relationships where cultural production and reproduction take place. To deal with the different forms of communication in the context of pedagogical practices, Bernstein (2000; 2003) uses the word text as any pedagogical, gestural, spoken, written or visual representation that reveals the subjects' positions. Following the meaning expressed, the interest in this research is in the texts that circulate in the pedagogical practices established in the school context, materialised in textbooks that communicate the concept of Polynomials.

In order to explain the path of constitution of the texts that are reproduced within pedagogical practices, Bernstein (2000; 2003) has structured three fields. The field of production, where new ideas are created and original and specific texts are developed. Grilo, Barbosa and Luna (2016) point to the Institute of Pure and Applied Mathematics (IMPA) as the main centre for the production of texts related to mathematics in Brazil. This is where, for example, the development of the theories that make up Abstract Algebra takes place, in other words, where the original texts on the concept of Polynomials are produced.

The recontextualization field, where transformations and selections of original texts take place, is classified into two types: the Official Recontextualizing Field (ORF) and the Pedagogic Recontextualizing Field (PRF) (In portuguese, the "*Campo de Recontextualização Oficial (CRO)*" and the "*Campo de Recontextualização Pedagógica (CRP)"*, respectively). The Pedagogic Recontextualizing Field is composed of universities, colleges of education, foundations, schools, book publishers, among others. The Official Recontextualizing Field is represented by state sub-agencies and specialised departments, such as the secretariats of the Ministry of Education (MEC). The main objective of the recontextualizing fields is to regulate and transform texts from the field of production into pedagogical discourses to be reproduced in the fields of reproduction. In the context of our research, textbooks present recontextualized texts on polynomials under the influence of the recontextualization processes operated by the official and pedagogical departments.

The field of reproduction, in turn, is where texts are selectively reproduced that were produced at the original level of production and have undergone transformations in the recontextualizing fields until their final communication in this field. It is common for research that deals with pedagogical relationships developed in school contexts to consider this field to be the classrooms of Basic Education (Grilo, Barbosa & Luna, 2016; Lima & Oliveira, 2019; Prado, Oliveira & Barbosa, 2020). However, this association of classrooms with the Field of Reproduction does not mean that in these contexts, or even outside them, there is no production of new texts (Grilo, Barbosa & Luna, 2017).

The fields structured by Bernstein (2000; 2003) can have specific meanings and legitimate communications, according to the boundaries that are established in the relationships between and within these fields. The specific character is only created and preserved when there is isolation in these relationships (Bernstein, 2003). This isolation can be strong, increasing the boundaries and delimiting the specificities of each field, or weak, if the texts that circulate in the context of pedagogical practices show greater closeness, with no limited or absent relationship between the texts.

The principle that regulates the degree of isolation is called "classification" by Bernstein (2003), and can vary between strong and weak. Classification refers to the ways in which texts can be communicated between/within fields and the distinction of "which" texts are considered specific and legitimate, through the rules of recognition. If we consider the concepts Polynomials and Functions, a weaker classification will make it possible to establish connections and associations between the texts specific to these concepts. On the other hand, a stronger classification tends to keep these texts increasingly specialised and unrelated to the two concepts.

The principle that regulates communication within the fields is called "framing" and also varies between strong and weak. It is the rules of realisation that allow control over "how" texts should be transmitted and acquired within any pedagogical practice, by establishing criteria for selecting and producing legitimate texts. For example, it is the rules of realisation that determine that the legitimate way to perform an addition of polynomials consists of grouping similar terms and then reducing them. Analysing the rules of recognition and realisation, and consequently the principles of classification and framing, makes it possible to investigate "which" and "how" texts are communicated within any pedagogical practice.

Understanding that the textbook used by teachers and students communicates a version of mathematical concepts in written form (Menduni-Bortoloti, 2016), investigating it allows us to identify the recontextualized texts that teachers generally use when planning and developing lessons and, consequently, in the constitution of Mathematics for Teaching. Like Davis and Renert (2014), we understand Math for Teaching a given concept as a set of different strategies and forms used to communicate it. As the fruit of a process of successive recontextualizations, M4T is not something fixed and it's possible to be determined. From the perspective of Davis and Renert (2014), it is constituted in practice, in the interactions and sharing of teachers with their peers and students, which is close to the Bernsteinian understanding of pedagogical recontextualization as a movement of transformations of texts not only of an instructional order, but also of a moral order, with the aim of complying with the orders that regulate the pedagogical practice established in school contexts.

According to Davis and Renert (2014), Maths for Teaching consists of a network of understandings about mathematical concepts and competences that are not easily acquired by teachers. Although content knowledge is indispensable, teaching maths requires essential pedagogical aspects so that certain concepts are understood by students (Rangel et al., 2015; Menduni-Bortoloti, 2016; Santos & Barbosa, 2016).

Methodological approach

This study was developed on the basis of qualitative research and therefore presents an interpretation of the object under investigation based on the experiences of the researcher (Creswell, 2010), without seeking to quantify the ways in which the concept of Polynomials is communicated in the selected textbooks. In this sense, it is a documentary study, which gives the researcher access to first-hand data, i.e. data that has not yet been analysed by other authors (Gil, 2010).

As data sources, we selected the eleven collections of mathematics textbooks for the final years of primary school that were approved by the National Textbook Programme (PNLD) in 2020, as shown in Table 1. The PNLD takes place every three years and recommends, after careful pedagogical evaluation, textbook collections to be chosen by school teachers for use in the following three-year period, through the National Textbook Guide (GNLD).

We chose these collections as our data sources, since up until the year this article was written they were the approved collections and could therefore reflect the pedagogical practices of maths teachers working in the final years of primary school. We would like to point out that we are not interested in analysing the selected textbooks in their entirety, but rather specifically the topics on polynomials.

Table 1.

Math Textbook Collections approved by PNLD 2020 (Brasil, 2019)

Collection	Authors
A Conquista da Matemática	José Ruy Giovanni Júnior, Benedicto Castrucci
Apoema Matemática	Adilson Longen
Araribá Mais - Matemática	Obra coletiva. Editores Responsáveis: Mara Regina Garcia Gay e Willian Raphael Silva
Convergências Matemática	Eduardo Chavante
Geração Alpha Matemática	Carlos de Oliveira, Felipe Fugita
Matemática Bianchini	Edwaldo Bianchini
Matemática - Compreensão e Prática	Ênio Silveira
Matemática Essencial	Patrícia Moreno Pataro, Rodrigo Balestri
Matemática Realidade & Tecnologia	Joamir Roberto de Souza
Teláris Matemática	Luiz Roberto Dante
Trilhas da Matemática	Fausto Arnaud Sampaio

Each collection contains four volumes (6th to 9th grade). At first, we consulted the summaries of all the volumes of the collections to see which books covered the concept of polynomials. We found that in the collection "*Apoema - Matemática*" the concept under study did not appear in any of the volumes. In the collections "*Matemática - Compreensão e Prática*" and "*Matemática Realidade & Tecnologia*", polynomials are presented in volume four, corresponding to 9th grade. In the other collections selected, this content is covered in volume three (8th grade).

After identifying the ten textbooks, eight of which are from the 8th grade and two from the 9th grade of the final years of primary school, which deal with the concept of Polynomials, we went to the units and specifically to the chapters dealing with the content to identify the ways in which the concept of Polynomials was communicated by the authors and possibly by the teachers who adopt these books in their classrooms. Once we had the printed textbooks and some in digital format available on the publishers' own websites, we proceeded to search for topics related to the concept of Polynomials in all the copies of the collections. When these topics were identified, the textbook was selected for detailed analysis of the corresponding pages.

We followed the perspective of the language of description defined by Bernstein (2000), which consists of looking at empirical data (external language) and relating it strictly to the theoretical concepts studied (internal language), and from there, interpreting and producing specific texts. In addition to concepts from Basil Bernstein's theory (2000; 2003), we used the Concept Study (CS) methodological framework proposed by Davis and Renert (2014). CS consists of a collaborative investigation where teachers can question and devise new ways of communicating a mathematical concept for teaching purposes. Even though our study was not carried out with a group of teachers in situ, we believe that this is an appropriate methodological strategy given that the textbooks were written by a group of authors/teachers who express ways of communicating a concept and from these it is possible to devise new ways of communicating it. In addition, as Menduni-Bortoloti (2016) and Santos (2017) point out, these books can reflect the pedagogical practices in which teachers participate.

Based on the results of different CSs, Davis and Renert (2014) systematised four emphases: realizations, landscapes, entailments and blends, which we translate to portuguese as: *realizações, panoramas, vínculos* and *combinações,* respectively. For the authors, it is through realisations that students interpret and make sense of the concept. According to Santos and Barbosa (2017, p. 320), "concepts exist only as attributes of their realisations, in other words, it is in the realisations and through the realisations that concepts are constituted". Panoramas are the result of groupings of similar realisations. The links establish how the realisations of a given concept are capable of shaping the understanding of other concepts and the combinations indicate how, from the realisations of a given concept, it is possible to establish more comprehensive and complex connections.

In this sense, given the aim of our study, we initially identified the realisations of the concept of Polynomials and grouped them into panoramas. In order to group them, we considered the principles of classification and framing (Bernstein, 2003) and, consequently, the rules of recognition and realisation, to the extent that we recognised which realisations could be brought together in each panorama and how these realisations are legitimized in the face of the variety of recontextualised textual forms on the concept of Polynomials present in textbooks. In addition, it was possible to identify links and combinations that were established in observance of the strength of the classification of the realisations of the concept of Polynomials with the understanding of other related concepts.

Data presentation

In order to systematize a “Mathematics for Teaching the concept of Polynomials”, we rely on the <u>realisations</u> identified in the selected textbooks and present them in <u>panoramas</u>. Six panoramas were identified, namely: generalisation, problem situations, algebraic structure, geometry, manipulable materials and illustration.

Generalization Panorama

The panorama we have named generalization groups together the realizations of the concept of Polynomials recognized as ways of generalizing mathematical patterns. The realizations linked to this panorama were essentially presented at the opening of the chapters and in proposed exercises. Most of the authors suggest in their guidelines for teachers that students discuss the meaning and importance of a specific language that allows for generalizations in mathematics. Figure 1 shows examples of what is recognized as a generalization.

Figure 1.

Realisations examples of generalisation panorama

(A)

• Qual é a expressão geral de um número natural (n) par? E de um número natural ímpar?

Source: Giovanni e Castrucci (2018c, p. 99)

(B)

4. A sequência $(xy, x^3y^2, x^5y^3, \ldots, A)$ tem 6 termos. Descubra o padrão de montagem dessa sequência e escreva o monômio representado por A.

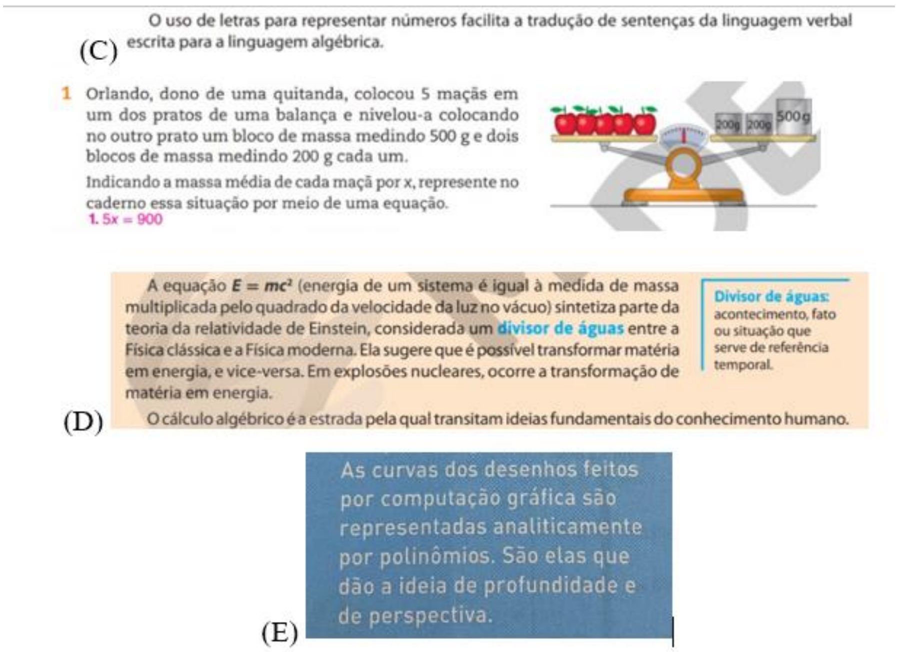
Source: Giovanni e Castrucci (2018c, p. 114)

Example (A) is a question that appears in the teaching guidelines. The authors suggest that teachers discuss with students the general expression of an even natural number and an odd natural number. Example (B) consists of a question on sequences. Students are expected to determine the assembly pattern of the given finite sequence and the monomial that represents the last term. This is an attempt to present algebra to primary school students as a generalisation of arithmetic.

The realisations of the concept recognised in the generalisation panorama reveal a weak classification between texts on Polynomials and other mathematical concepts, when establishing links with the concepts of Natural Numbers and Sequences, for example. In terms of combinations, we observed a weak degree of isolation between the texts in this panorama and Abstract Algebra. This tendency towards a weak classification can be seen in the way algebra is approached - as a generalization of arithmetic - which goes back to the beginning of the process of building algebra.

Problem Situations Panorama

Grouped in this panorama, which we call problem situations, are the realisations of the concept of Polynomials recognised as ways of simplifying and representing mathematical sentences by means of algebraic expressions. Polynomials appear in this case as a way of shaping problems and manipulating structures through algebraic language in order to synthesize information, simplify writing and represent problem situations through algebra. Figure 2 contains examples of the realizations recognized as problem situations.


(C) O uso de letras para representar números facilita a tradução de sentenças da linguagem verbal escrita para a linguagem algébrica.

1 Orlando, dono de uma quitanda, colocou 5 maçãs em um dos pratos de uma balança e nivelou-a colocando no outro prato um bloco de massa medindo 500 g e dois blocos de massa medindo 200 g cada um.
Indicando a massa média de cada maçã por x, represente no caderno essa situação por meio de uma equação.
1. $5x = 900$

(D) A equação $E = mc^2$ (energia de um sistema é igual à medida de massa multiplicada pelo quadrado da velocidade da luz no vácuo) sintetiza parte da teoria da relatividade de Einstein, considerada um divisor de águas entre a Física clássica e a Física moderna. Ela sugere que é possível transformar matéria em energia, e vice-versa. Em explosões nucleares, ocorre a transformação de matéria em energia.
O cálculo algébrico é a estrada pela qual transitam ideias fundamentais do conhecimento humano.

Divisor de águas: acontecimento, fato ou situação que serve de referência temporal.

(E) As curvas dos desenhos feitos por computação gráfica são representadas analiticamente por polinômios. São elas que dão a ideia de profundidade e de perspectiva.

Figure 2.

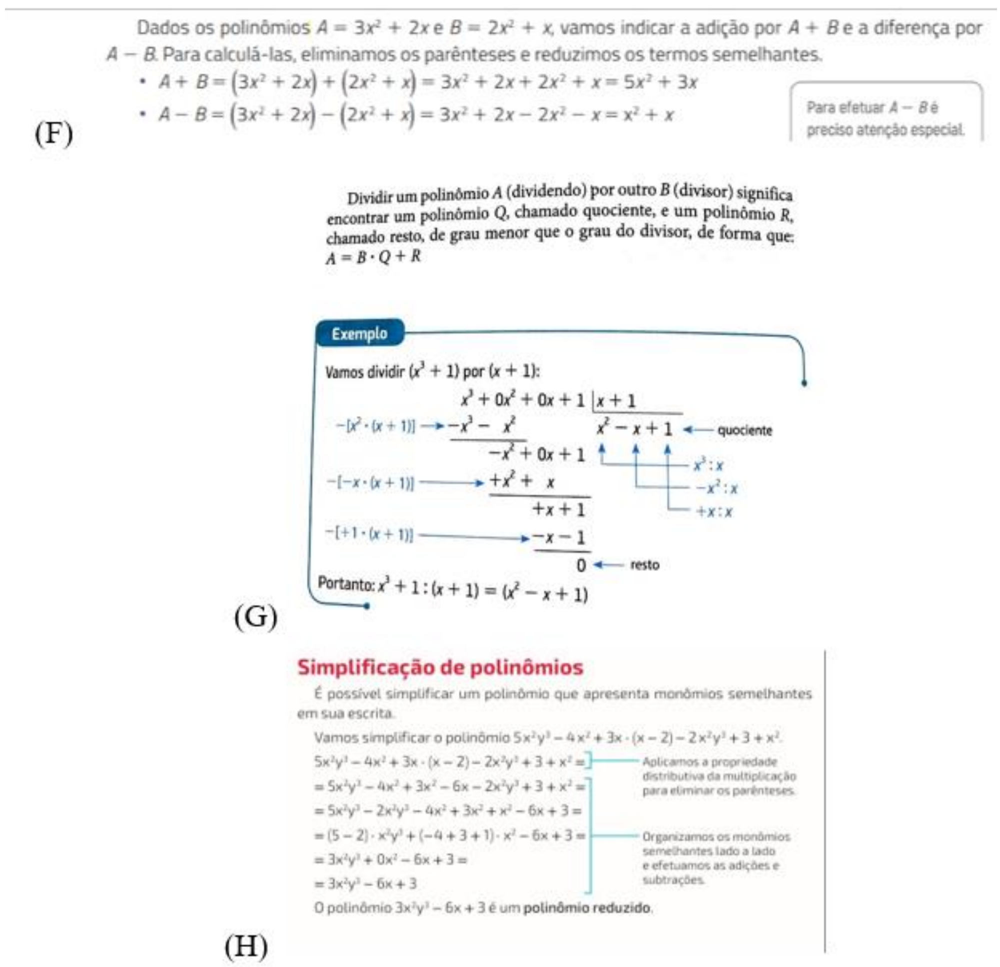
Exemplos de realizações do panorama situações problemas (Bianchini (2018, p. 91, 93), Oliveira e Fugita (2018c, p. 33)

The examples shown above are the result of discussions at the beginning of the chapters. In examples (C) and (D) we can see that the authors are trying to convince students of the importance of studying algebraic expressions and, consequently, polynomials for modelling mathematical problems that can be applied to various areas. In all the books we analyzed, we identified exercises that allow students to recognize that the transition from expressions written in the mother tongue to mathematical language, via problem solving, is a legitimate realisation of the concept of Polynomials.

The realizations of the concept of polynomials through problem situations identified in the textbooks establish links with other mathematical concepts such as equations, functions and interest, and also with other areas such as economics, physics, medicine, cryptography, computer graphics, etc., as can be seen in (D) and (E). Therefore, the results of this panorama reveal a weak classification of polynomial texts with other mathematical concepts or not. In terms of combinations, the realizations in this panorama reveal a strong classification between texts on polynomials and abstract algebra.

Algebraic Structure Panorama

This panorama includes the realisations of the concept of Polynomials recognised only as algebraic expressions made up of letters, numbers and symbols that satisfy the formalization of a set of elements with structured operations and properties, i.e. Polynomials. In all the books, the realisations of the algebraic structure type predominate. Because of their specific nature and because they present well-synthesized sentences, these realisations appear as: reduced polynomial, degree and numerical value of a polynomial, equality between polynomials and operations with polynomials. Figure 3 shows three examples of realisations recognized as algebraic structure overview.



(F)

Dados os polinômios $A = 3x^2 + 2x$ e $B = 2x^2 + x$, vamos indicar a adição por $A + B$ e a diferença por $A - B$. Para calculá-las, eliminamos os parênteses e reduzimos os termos semelhantes.

- $A + B = (3x^2 + 2x) + (2x^2 + x) = 3x^2 + 2x + 2x^2 + x = 5x^2 + 3x$
- $A - B = (3x^2 + 2x) - (2x^2 + x) = 3x^2 + 2x - 2x^2 - x = x^2 + x$

Para efetuar $A - B$ é preciso atenção especial.

(G)

Dividir um polinômio A (dividendo) por outro B (divisor) significa encontrar um polinômio Q, chamado quociente, e um polinômio R, chamado resto, de grau menor que o grau do divisor, de forma que: $A = B \cdot Q + R$

Exemplo

Vamos dividir $(x^3 + 1)$ por $(x + 1)$:

$x^3 + 0x^2 + 0x + 1$ | $x + 1$

$-[x^2 \cdot (x + 1)] \rightarrow -x^3 - x^2$; $x^2 - x + 1$ ← quociente

$-x^2 + 0x + 1$; $x^3 : x$; $-x^2 : x$; $+x : x$

$-[-x \cdot (x + 1)] \rightarrow +x^2 + x$

$+x + 1$

$-[+1 \cdot (x + 1)] \rightarrow -x - 1$

0 ← resto

Portanto: $x^3 + 1 : (x + 1) = (x^2 - x + 1)$

(H)

Simplificação de polinômios

É possível simplificar um polinômio que apresenta monômios semelhantes em sua escrita.

Vamos simplificar o polinômio $5x^2y^3 - 4x^2 + 3x \cdot (x - 2) - 2x^2y^3 + 3 + x^2$.

$5x^2y^3 - 4x^2 + 3x \cdot (x - 2) - 2x^2y^3 + 3 + x^2 =$ — Aplicamos a propriedade distributiva da multiplicação para eliminar os parênteses.

$= 5x^2y^3 - 4x^2 + 3x^2 - 6x - 2x^2y^3 + 3 + x^2 =$

$= 5x^2y^3 - 2x^2y^3 - 4x^2 + 3x^2 + x^2 - 6x + 3 =$

$= (5 - 2) \cdot x^2y^3 + (-4 + 3 + 1) \cdot x^2 - 6x + 3 =$ — Organizamos os monômios semelhantes lado a lado e efetuamos as adições e subtrações.

$= 3x^2y^3 + 0x^2 - 6x + 3 =$

$= 3x^2y^3 - 6x + 3$

O polinômio $3x^2y^3 - 6x + 3$ é um **polinômio reduzido**.

Figure 3.

Exemplos de realizações do panorama estrutura algébrica (Dante (2018c, p.83), Araribá Mais Matemática – obra coletiva (2018c, p. 202), Pataro e Balestri (2018c, p. 82))

In example (F), the author presents the algebraic manipulation needed to perform the operations of addition and subtraction of polynomials. In (G), the conditions for carrying out the division operation between polynomials are established using the key method. In example (H), the authors present in detail the step-by-step process and the properties that must be applied to obtain the polynomial in its reduced form.

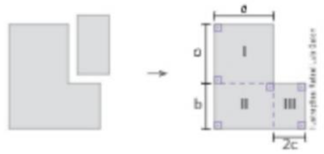
We recognise these examples as a panorama of algebraic structure in that they consider purely algebraic procedures and techniques in the manipulation of mathematical sentences. Recognising these ways of communicating the concept of Polynomials is very much required in exercises in all the selected books, when the authors suggest algebraic manipulation, the application of properties to simplify and, above all, questions involving operations with Polynomials.

Realisations of the concept of Polynomials grouped in the algebraic structure panorama maintain links with specific topics of the content itself, such as operations, simplifications and properties of Polynomials, thus revealing a strong classification between their texts and other concepts. In terms of combinations, these realisations and links maintain a weak classification between their texts and Abstract Algebra, since they reveal possibilities for connections and associations with more specific Abstract Algebra texts. Examples of this are the division of polynomials using the key method, which we can associate with the Euclidean division theorem, and the operations carried out with polynomials that are strictly linked to the basic properties of Rings.

Geometry Panorama

This panorama is made up of the realisations of the concept of Polynomials recognised as expressions that represent perimeter, area or volume of figures and distances between points, associated with a geometric figure or drawing. Figure 4 shows examples of the realisations identified in the textbooks analysed.

De uma chapa metálica com formato retangular foi retirada uma peça, também com formato retangular, e o restante foi divido em três partes, como mostra o esquema.



De acordo com as medidas indicadas no esquema, podemos representar a medida da área restante da chapa metálica calculando a medida da área de cada uma das três partes e adicionando-as em seguida. Desse modo, temos:

Medida da área I	Medida da área II	Medida da área III	Medida da área restante
$a \cdot a = a^2$	$a \cdot b = ab$	$b \cdot 2c = 2bc$	$a^2 + ab + 2bc$

A expressão algébrica que representa a medida da área restante da chapa metálica é chamada **polinômio**.

(I)

2 O desenho a seguir representa o esboço de uma rodovia que passa pelas cidades *A*, *B*, *C* e *D*. A distância de *A* a *B* é igual à distância de *B* a *C*, e ambas podem ser representadas por *x* quilômetros. Sabendo que a distância de *A* a *D* é de *y* quilômetros, qual é a expressão algébrica que representa a distância de *C* a *D*?

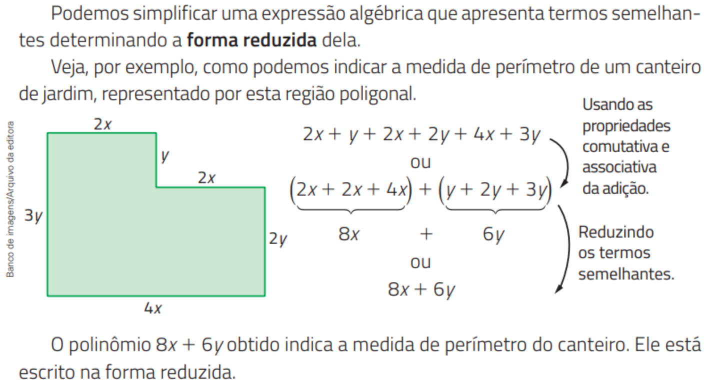
Observando o esboço, podemos concluir que a distância de C a D é dada pela diferença entre as distâncias de A a D e de A a C:

$y - 2x$ ⟶ A expressão algébrica $\mathbf{y - 2x}$ representa a distância entre as cidades *C* e *D*.

As situações que acabamos de apresentar nos mostram expressões algébricas que indicam, respectivamente, uma adição ou uma subtração de monômios, ou seja, indicam uma **adição algébrica de monômios**.

Qualquer adição algébrica de monômios denomina-se **polinômio**.

(J)

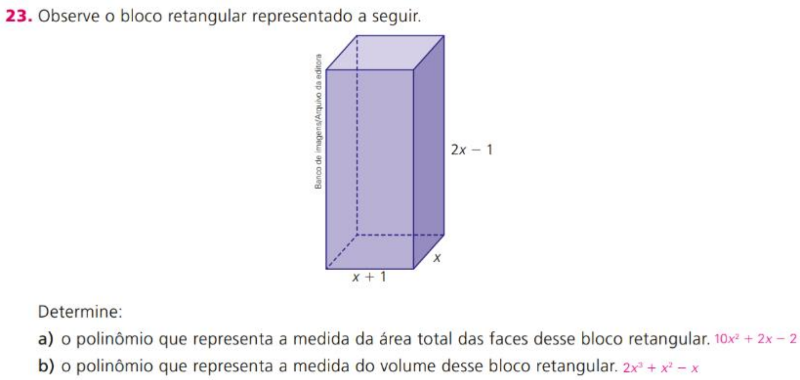


Podemos simplificar uma expressão algébrica que apresenta termos semelhantes determinando a **forma reduzida** dela.

Veja, por exemplo, como podemos indicar a medida de perímetro de um canteiro de jardim, representado por esta região poligonal.

O polinômio $8x + 6y$ obtido indica a medida de perímetro do canteiro. Ele está escrito na forma reduzida.

(K)



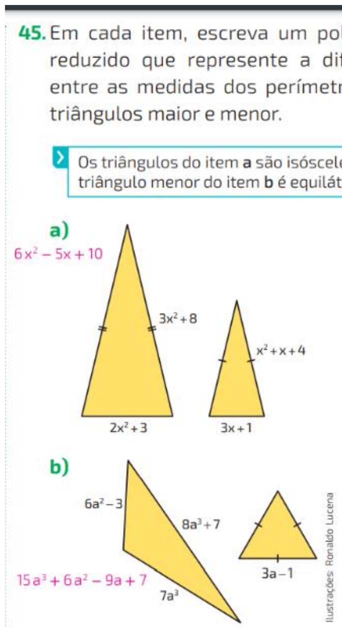
23. Observe o bloco retangular representado a seguir.

Determine:

a) o polinômio que representa a medida da área total das faces desse bloco retangular. $10x^2 + 2x - 2$

b) o polinômio que representa a medida do volume desse bloco retangular. $2x^3 + x^2 - x$

(L)



45. Em cada item, escreva um polinômio reduzido que represente a diferença entre as medidas dos perímetros dos triângulos maior e menor.

Os triângulos do item **a** são isósceles e o triângulo menor do item **b** é equilátero.

(M)

Fonte:

Figura 4.

Exemplos de realizações do panorama geométrico (Chavante (2018c, p. 159), Giovanni e Castrucci (2018c, p. 118), Dante (2018c, p.81), Sampaio (2018c, p. 71), Pataro e Balestri (2018c, p. 86))

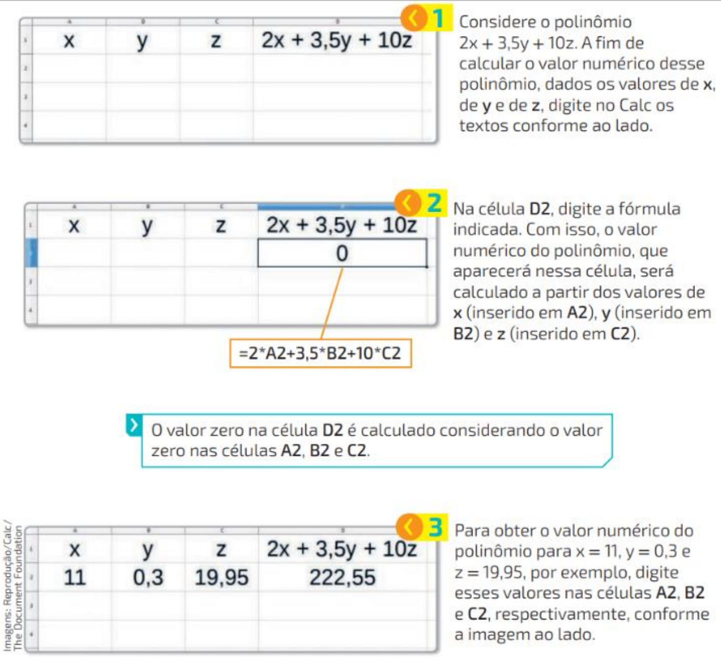
Example (I) shows a realisation of the concept of polynomials associated with the area of a rectangular sheet of metal. Example (J) deals with a hypothetical situation that problematizes the distance between cities and determines the algebraic expression that represents this distance. The authors then emphasize that situations of this type reveal algebraic expressions that express the addition or subtraction of monomials, i.e. polynomials.

Example (K) uses the perimeter of a polygonal region to determine the algebraic procedure for writing a polynomial in its reduced form. Examples (L) and (M), on the other hand, deal with questions proposed by the authors on Polynomials which refer to total area, the volume of a rectangular block and the difference between the perimeters of triangles. As in the previous overviews, questions of this type were identified in all the books and are suggested in activities such that students should recognise that polynomials can be used to represent areas, perimeters, volumes and distances.

The realisations identified in the geometric panorama reveal a weak classification between the texts on Polynomials and other mathematical concepts, i.e. they establish links with geometric concepts such as: Polygons, Perimeter, Area, Volume of geometric figures and Distance between Points. Furthermore, the realisations of this panorama identified in the selected textbooks reveal a weak classification between the texts on Polynomials and Abstract Algebra, to the extent that they associate the resolution of algebraic problems with Geometry.

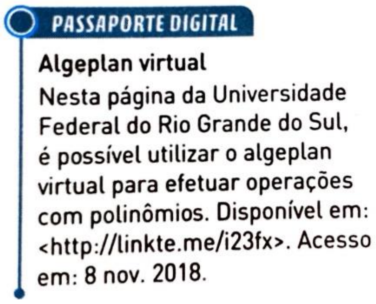
Manipulable Materials Panorama

The overview that we call manipulable materials is composed of realisations of the concept of Polynomials that suggest the use of digital or non-digital tools to approach and explore topics of Polynomials. The realisations of this overview were identified only in Bianchini (2018c), Pataro and Balestri (2018c) and Oliveira and Fugita (2018c). Picture 5 presents examples of realisations of this panorama.



(N)

Fonte:

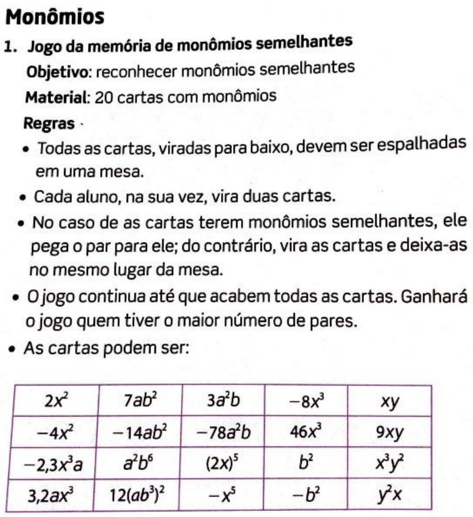


PASSAPORTE DIGITAL

Algeplan virtual

Nesta página da Universidade Federal do Rio Grande do Sul, é possível utilizar o algeplan virtual para efetuar operações com polinômios. Disponível em: <http://linkte.me/i23fx>. Acesso em: 8 nov. 2018.

(O)



Monômios

1. **Jogo da memória de monômios semelhantes**

Objetivo: reconhecer monômios semelhantes

Material: 20 cartas com monômios

Regras

- Todas as cartas, viradas para baixo, devem ser espalhadas em uma mesa.
- Cada aluno, na sua vez, vira duas cartas.
- No caso de as cartas terem monômios semelhantes, ele pega o par para ele; do contrário, vira as cartas e deixa-as no mesmo lugar da mesa.
- O jogo continua até que acabem todas as cartas. Ganhará o jogo quem tiver o maior número de pares.
- As cartas podem ser:

$2x^2$	$7ab^2$	$3a^2b$	$-8x^3$	xy
$-4x^2$	$-14ab^2$	$-78a^2b$	$46x^3$	$9xy$
$-2{,}3x^3a$	a^2b^6	$(2x)^5$	b^2	x^3y^2
$3{,}2ax^3$	$12(ab^3)^2$	$-x^5$	$-b^2$	y^2x

(P)

Figura 5.

Exemplos de realizações do panorama materiais manipuláveis (Pataro e Balestri (2018c, p. 293), Oliveira e Fugita (2018c, p. 55), Bianchini (2018c, p. L))

In example (N), the authors suggest the use of the spreadsheet tool to explore the numerical value of Polynomials. The spreadsheets consist of tables that allow the organization of texts, numerical data and formulas, facilitating the performance of algebraic calculations. Example (O) deals with an access link to a virtual Algeplan that allows the exploration of operations with Polynomials, through the *GeoGebra* software. Algeplan consists of teaching material composed of rectangular pieces of six different sizes and their respective geometric representations. In this case, Oliveira and Fugita (2018c) suggest using the virtual version of Algeplan. Other tools used in the implementation of this panorama involve the use of games and the construction of materials to explore the concept of Polynomials. In example (P), Bianchini (2018c) suggests, in the instructions to the teacher, the use of a memory game to explore similar monomials. The author presents the rules and a suggestion of possible cards for developing it. The implementations of the concept of Polynomials identified in the textbooks recognized as manipulable materials establish a weak classification, that is, links with other concepts, such as: operations, area, equation, function, games and mathematical software. In terms of combinations with Abstract Algebra, they reveal a strong classification since the use of manipulable materials, whether virtual or physical, are not part of the repertoire of implementations of the Production Field where Abstract Algebra is located, so that these texts remain isolated.

Illustration Panorama

This panorama, which we call illustration, includes the realisations of the concept of Polynomials through comic strips. The comic strip consists of a textual genre that associates images and humor. Communicating the concept of Polynomials using comic strips involves students in a relaxed environment, leading them to think and interpret the message by associating it with the mathematical concept. Realizations of this overview were identified in "*Araribá Mais Matemática – obra coletiva"* (2018c) and *"Oliveira e Fugita*" (2018c). Figure 6 presents examples of realisations of this panorama.

(Q) 

(R) 

Figura 6.

Exemplos de realizações do panorama ilustrativo (Oliveira e Fugita (2018c, p. 40, Araribá Mais Matemática – obra coletiva (2018c, p. 193)))

In example (Q), the authors suggest that students recognize that the problem in the comic strip can be written in the form of a Polynomial and that they calculate the numerical value assuming that the rat had thought of the number five. In example (R), the humor of the comic strip lies in the ambiguity of the crossword answer, since “algebraic expression formed by the product of a number by one or more variables with natural exponents” with seven letters is a monomial, and the bird's answer was an example of a monomial, $xyzw^2abc$.

The achievements recognized in the illustration panorama reveal a weak classification of texts about Polynomials with other concepts outside the mathematical context, such as textual genre and text interpretation. In terms of combinations with Abstract Algebra, achievements of this type maintain a strong classification, as they do not establish relationships between their texts.

Cross-Sectional Analysis and Discussion of Data

After presenting the realizations that we grouped into panoramas, it is noted that the selected textbooks move through different ways of communicating the concept of Polynomials. However, the focus on realizations recognized in problem situations, algebraic structure and geometric panoramas stands out in all the books. On the other hand, little emphasis is placed on the achievements recognized in the generalization, manipulative materials and illustration panoramas.

The realizations of the concept observed in the textbooks reveal that they are not limited to the context of the Polynomials content, as they establish connections with other concepts and related areas, for example, Economics, Physics, Cryptography and Computer Graphics. Like Davis and Renert (2014), we analyzed these connections using the emphases of the CS, links and combinations, as explained at the end of the presentation of each panorama. These panoramas reveal that the concept of Polynomials tends to maintain weak isolation, as it establishes links with mathematical concepts, such as: Equation, Function, Polygons, Natural Numbers and Sequences, and other concepts outside the mathematical context, such as textual genre and text interpretation, for example. In addition, we identified that the realizations, especially, of the panoramas, generalization, algebraic structure and geometric make it possible to establish combinations with Abstract Algebra, by maintaining a tendency for a weaker classification among its texts.

Figure 7 presents a summary of "Mathematics for Teaching the concept of Polynomials" carried out in Basic Education textbooks. The identified panoramas are represented in diagrams with rounded corners, containing a brief description of the realizations that were grouped in these panoramas. In addition, associated with each panorama we have another rectangle containing the links that were identified in compliance with the strength of the classification between the texts on Polynomials and other mathematical concepts or not. The combination and links between the texts grouped in each panorama with other concepts and with Abstract Algebra is described around the strength of the classification and represented using continuous and dashed lines.

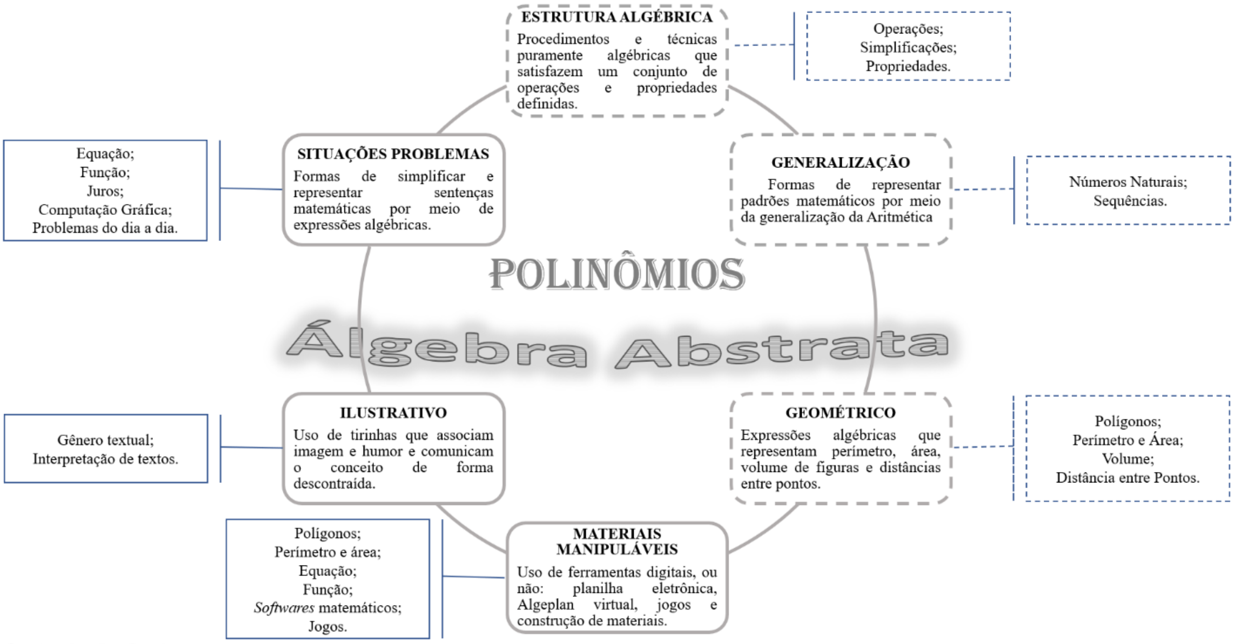


Figura 7.

Uma Matemática para o Ensino do conceito de Polinômios recontextualizado nos livros didáticos

The variability of the levels of approximation between the described panoramas and Abstract Algebra is represented by solid and dashed lines, as shown in the figure. The solid lines indicate a strong degree of classification and, consequently, less approximation between the texts of these panoramas and Abstract Algebra. The dashed lines, on the other hand, represent weak classification and greater approximation between their texts. Thus, it is observed that panoramas problem situations, manipulable materials and illustrations expose a tendency towards strong classification in relation to Abstract Algebra, that is, the texts on Polynomials recognized in these panoramas do not allow for the establishment of deep connections with legitimate Abstract Algebra texts. The panoramas, generalization, algebraic structure and geometry, which are delimited by dashed lines, reveal a weaker classification, that is, greater approximation between the texts on Polynomials and Abstract Algebra texts.

The possibilities of deeper understandings related to the concept of Polynomials, based on the realizations and links of the algebraic structure panorama, can be exemplified when presenting the division of Polynomials relating it to the Euclidean Division, where dividing a complex Polynomial D by another d, not identically zero, consists of determining complex Polynomials, quotient q and remainder r, such that they fulfill: *degree (r)* < *degree (d)* and $D = dq + r$.

Specific characteristics of the study of algebraic structures that are generally addressed through theorems, definitions and properties of Abstract Algebra, appear in realizations identified in textbooks in a more simplified and implicit way among their topics. An example of this are the properties and rules of operations with Polynomials that obey which are consequences of the definition of Rings and Fields. These connections are possible through the successive recontextualizations that Abstract Algebra texts undergo until their legitimate communication in the context of the Basic Education textbook.

Final considerations

This paper presents the results of a study that aimed to systematize “Mathematics for Teaching the concept of Polynomials” based on realisations present in textbooks. With the theoretical and methodological foundations of Bernstein and the Study of the Concept by Davis and Renert (2014), we identified a variability of realisations of the concept of Polynomials in ten textbooks and grouped them into six panoramas: generalization, algebraic language, algebraic structure, geometry, manipulative materials and illustration.

The study corroborated some results already pointed out in the literature that deals with the presentation of Algebra as a generalization of Arithmetic and its relationship with Geometry, especially in the Generalization and Geometric panoramas. The realisations that make up the Problem Situations panorama, to convince students of the importance of Algebra, reinforce the idea present in the literature that the study of Polynomials is important for the development of algebraic skills.

We infer that the predominance of realizations of the Algebraic Structures panorama in all the books analyzed may contribute to the difficulties related to learning this concept already pointed out in the literature, by presenting it as operations and properties structured by well-summarized sentences. As a counterpoint to the realizations that integrate this panorama, the books analyzed did not present a formal definition for the concept of Polynomial, since they are intended for the final years of Elementary School.

We consider that the variety of realizations of the concept of Polynomials and their connections play an important role in understanding the concept, since it reaffirms the learning of other concepts already studied, enhancing the development of language and algebraic thinking. However, we consider that the fact that this concept was restricted to the final years of Elementary School demands new ways of approaching it in teacher training courses. Such courses, as a Field of Pedagogical Recontextualization, can share with teachers different realizations of the concept of Polynomials in order to contribute to teaching and learning. It is expected that this study will help teachers gain insight into the different ways of communicating the concept of Polynomials, possible links with other concepts, and combinations with Abstract Algebra.

Finally, we emphasize that the option to analyze only the chapter that dealt with the concept of Polynomials was a limitation of the realizations identified in the books analyzed. However, we believe that the realizations documented were capable of synthesizing *a* Mathematics for teaching this concept that, while not intended to be unique or complete, can be used as a starting point for pedagogical practices established between teachers and students when addressing the concept of Polynomials.

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