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Micropaths in the study of symmetry: conditions and restrictions in continuing education with mathematics teachers

Microvías en el estudio de la simetría: condiciones y restricciones en la formación continua de profesores de matemáticas

Les micropathes dans l'étude de la symétrie : conditions et restrictions en formation continue auprès des professeurs de mathématiques

Micropercursos no estudo de simetria: condições e restrições em uma formação continuada com professores de matemática

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Abstract

This article is an excerpt from a doctoral research that investigated possibilities for developing micro Study and Research Paths (PEP) through a praxeological study developed in continuing education for mathematics teachers. The training was carried out with a group of twelve teachers working in basic education, in which we worked with didactic systems intrinsic to the studies of geometric contents supported by the SRP methodology. All sessions were recorded in audio and video and later transcribed for data production analysis. For the study and development of the SRP, we sought to understand the Anthropological Theory of Didactics (ATD), with emphasis on the didactic paradigm questioning the world and the levels of codetermination, identifying conditions and restrictions that permeate the pedagogical practices of the teachers participating in the research. By analyzing the didactic systems, we could see

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that teachers are dependent on textbooks and that this resource has directed their pedagogical practices. We also saw how much the conditions imposed in their academic training end up being reflected in the way they conduct their practices in the classroom, reproducing a teaching directed by the paradigm of the visit to the works with regard to the teaching of Geometry. In addition, we observed the teachers' need to study and seek out more information about the concepts covered and how much their praxeologies are in the doing block, with few theoretical and technological justifications. In this scenario, we saw that the SRP caused a praxeological destabilization of their geometric knowledge in teachers.

Keywords: Micropathways, Continuing education, Pedagogical practices, Geometric contents.

Resumen

Este artículo es un extracto de una investigación doctoral que investigó posibilidades para el desarrollo de micro Recorridos de Estudio e Investigación (REI) a través de un estudio praxeológico desarrollado en la educación continua de profesores de matemáticas. La capacitación se realizó con un grupo de doce docentes que laboran en educación básica, en los cuales se trabajó con sistemas de enseñanza intrínsecos al estudio de contenidos geométricos apoyados en la metodología REI. Todas las sesiones fueron grabadas en audio y video y posteriormente transcritas para el análisis de la producción de datos. Para el estudio y desarrollo del REI, buscamos comprensiones de la Teoría Antropológica de la Didáctica (TAD), con énfasis en el paradigma didáctico, cuestionamiento el mundo y los niveles de codeterminación, identificando condiciones y restricciones que permean las prácticas pedagógicas de docentes participantes en la investigación. Al analizar los sistemas de enseñanza, pudimos ver que los docentes dependen de los libros didácticos y que este recurso ha guiado sus prácticas pedagógicas. También vimos cómo las condiciones impuestas en su formación académica terminan reflejándose en la forma en que realizan sus prácticas en el aula, reproduciendo una enseñanza guiada por el paradigma de la visita a obras en lo que respecta a la enseñanza de la Geometría. Además, observamos la necesidad de que los docentes estudien y busquen más información en torno a los conceptos tratados y en qué medida sus praxeologías están en el "los profesores están en el modo hacer que en el modo saber hacer ", o sea, con pocas justificaciones teóricas y tecnológicas. En este escenario, vimos que el REI provocó en los docentes una desestabilización praxeológica de sus conocimientos geométricos.

Palabras clave: Micro recorridos, Educación continua, Prácticas pedagógicas, Contenidos geométricos.

Résumé

Cet article est un extrait d'une recherche doctorale qui a étudié les possibilités de développement de micro Parcours d'Etudes et de Recherche (PER) à travers une étude praxéologique développée en formation continue pour les professeurs de mathématiques. La formation a été réalisée avec un groupe de douze enseignants travaillant dans l’éducation de base, dans lequel nous avons travaillé avec des systèmes didactiques intrinsèques aux études de contenus géométriques soutenus par la méthodologie PER. Toutes les sessions ont été enregistrées en audio et en vidéo, puis transcrites pour l’analyse de la production de données. Pour l’étude et le développement du PER, nous avons cherché à comprendre la théorie anthropologique de la didactique (TAD), en mettant l’accent sur le paradigme didactique, en interrogeant le monde et les niveaux de codétermination, en identifiant les conditions et les restrictions qui imprègnent les pratiques pédagogiques des enseignants participant à la recherche. En analysant les systèmes didactiques, nous avons pu vérifier que les enseignants sont dépendants des manuels scolaires et que cette ressource a orienté leurs pratiques pédagogiques. Nous avons également vu à quel point les conditions imposées dans leur formation académique finissent par se refléter dans la manière dont ils mènent leurs pratiques en classe, reproduisant un enseignement dirigé par le paradigme de la visite des œuvres en ce qui concerne l’enseignement de la géométrie. De plus, nous avons observé la nécessité pour les enseignants d’étudier et de chercher plus d’informations sur les concepts abordés et sur la place de leurs praxéologies dans le bloc, avec peu de justifications théoriques.

Mots-clés : Micro-parcours, Formation continue, Pratiques pédagogiques, Contenus géométriques.

Resumo

O presente artigo é um recorte de uma pesquisa de doutorado que investigou possibilidades do desenvolvimento de micro Percursos de Estudo e Pesquisas (PEP) por meio de um estudo praxeológico desenvolvido em uma formação continuada de professores de matemática. A formação foi realizada com um grupo de doze professores atuantes na educação básica, na qual trabalhamos com sistemas didáticos intrínsecos aos estudos dos conteúdos geométricos amparados pela metodologia do PEP. Todas as sessões foram gravadas em áudio e vídeo e, posteriormente, transcritas para a análise da produção dos dados. Para o estudo e desenvolvimento do PEP, buscamos compreensões da Teoria Antropológica do Didático (TAD), com ênfase no paradigma didático, questionamento do mundo e dos níveis de codeterminação, identificando práticas pedagógicas dos professores participantes da pesquisa. Ao analisar as condições e restrições que permeiam sistemas didáticos, pudemos perceber que os professores são dependentes dos livros didáticos e que esse recurso tem direcionado suas práticas pedagógicas. Vimos também o quanto as condições impostas nas suas formações acadêmicas acabam sendo refletidas no modo como conduzem suas práticas em sala de aula, reproduzindo um ensino direcionado pelo paradigma de visita às obras no que diz respeito ao ensino de Geometria. Além disso, observamos a necessidade dos professores de estudar e buscar mais informações em torno dos conceitos abordados e o quanto suas praxeologias estão no bloco fazer, com poucas justificativas teóricas e tecnológicas. Nesse cenário, vimos que o PEP provocou nos professores uma desestabilização praxeológica dos seus conhecimentos geométricos.

Palavras-chave: Micropercursos, Formação continuada, Práticas pedagógicas, Conteúdos geométricos.

Micropaths in the study of symmetry: conditions and restrictions in continuing education with mathematics teachers

As scientists in the field of Mathematics Education and members of the Didactics of Mathematics Research Group (DDMat), we are aware of the importance of research and studies on the continuing education of teachers (Santos; Freitas, 2013). Based on these training contexts and through educational, social and political changes in current times, we understand, in agreement with Pais (2002), that Mathematics Didactics (DM) meets the training aspirations of mathematics teachers, since this is, in the words of author:

> One of the trends in the broad area of mathematics education, whose object of study is the elaboration of concepts and theories that are compatible with the educational specificity of mathematical school knowledge, seeking to maintain strong links with the formation of mathematical concepts, both at the experimental level of pedagogical practice and in the theoretical territory of academic research (Pais, 2002, p. 11).

In the field of DM, the theoretical contribution of the Anthropological Theory of Didactics (TAD) (Chevallard, 1992, 1994, 1998, 2001, 2002, 2003, 2007, 2009a, 2009b, 2009c, 2009d, 2009e, 2009f, 2009g, 2012) has made it possible to analyze, investigate and reflect on the teaching of mathematics through the study of situations present in the classroom and its surroundings - school, pedagogy, society - as well as the analysis of pedagogical practices. In view of the above, the reflections lead to the question: how to develop continuing education for mathematics teachers based on the development of micropaths in order to provoke changes in pedagogical practice?

Based on individual reports from some teachers about students' difficulties in learning and working with geometry content, as well as on research on geometric praxeologies in textbooks (Andrade, 2012; Correia and Lobo, 2011), we made this choice to somehow affect teachers' practice, without it being a training that "indicates recipes for the classroom" or something unrelated to the real situation of school life, such as a mechanical and merely theoretical training. On the contrary, it is necessary that, based on the dialogue and concerns of the participating teachers themselves, the training develops, enabling studies and reflections on their practices through the exchange of experiences. Thus, developing continuing education with mathematics teachers who are interested in discussions on Geometry topics became the object of study of the research to be reported.

According to TAD, all human activity can be described through praxeology, meaning the inseparability between know-how and knowledge. Following this principle, in a classroom, through this theory, we can investigate the mathematics taught – Mathematical Organization (MO) – and how this mathematics is taught – Didactic Organization (DO).

To this end, we based ourselves on studies developed by Chevallard (2012), in which the author instigates a paradigm shift. He highlights two paradigms called visits to the works and questioning the world. According to Chevallard (2012), visits to the works is the paradigm in which you observe the situation with little protagonism, and in the new paradigm of questioning the world, you observe, analyze and question the situation, being an active person in the process of teaching and learning mathematics.

Based on this last paradigm, Chevallard (2009a) has developed the methodology called Percurso de Estudos e Pesquisas (PEP), with the intention of triggering pedagogical practices that lead students and teachers to question, conjecture, autonomy, to appear as researchers, and to be, in fact, constructors of their own knowledge. Therefore, we hypothesize that we can promote changes in the praxeologies of the Basic Education mathematics teacher related to geometric content in the execution of PEP.

In the PEP proposal, when forming a study group around a specific type of problem, a didactic relationship is established between those involved in the study. This relationship, considered "open," consists of the development of a formative proposal, in which students will not have prior knowledge of the problems to be answered throughout the study and the supervisor of this study will not be able to foresee all the discussions or difficulties that may arise in the study process, as the authors state: "Teaching, as a means of the didactic process, should not seek to absolutely control the development of this process. The didactic relationship is an 'open' relationship" (Chevallard; Bosch; Gascón, 2001, p. 201).

Considering these foundations, this research was developed through several micro-PEP, in which the study supervisor, a role developed by the researcher, together with mathematics teachers who were in the position of students, carried out studies of questions that contained Geometry content, guided by questioning of the world paradigm.

In developing the micro-PEP, we conducted studies of different praxeologies with mathematics teachers related to geometric content, permeated by the exchange of experiences, and sought to understand possibilities in producing PEP in teacher training, since, for Chevallard (2009a), working collectively on praxeologies allows teachers to discuss how the teaching and learning process of a given concept occurs and is the essential tool to combat routine teaching in the classroom. For this article, we carried out an excerpt from the doctoral research, in which we will present an analysis of the micro-pathways developed around the content of Symmetry.

Theoretical and methodological contribution

In the research carried out by Santos and Freitas (2013), we identified, when analyzing the pedagogical practice *in loco* , that there is a diversity of factors that are not only present in the classroom or at school, but in a broader environment, such as in society, and that directly interfere in the classroom context and, mainly, in pedagogical practice. In this perspective, SDT, through the levels of codetermination, makes it possible to understand these factors (whether didactic or not) that permeate the school environment.

The conceptions of praxeology in SDT allow us to affirm that “didactics is dedicated to studying the conditions and restrictions under which praxeologies begin to live, migrate, change, operate, perish, disappear, be reborn, etc. within human groups”, that is, in social institutions” (Chevallard, 2007, p. 14). To establish a praxeology related to mathematical knowledge, this knowledge must be associated with a hierarchical scale, in which each level corresponds to a reality and determines the habitats [4]and niches [5]of the Mathematical Organization (MO) and the Didactic Organization (DO), as pointed out by Chevallard (2009a, p. 12):

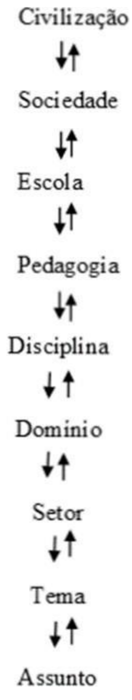

Figure 1.

Levels of Codetermination, Chevallard (2009a, p. 12)

[4]For Chevallard (1994): The habitat of a mathematical object refers to the institution in which the knowledge is found, its address.

[5]The niche corresponds to the relationship between knowledge and the object of study, determining the function of knowledge, that is, its niche. (CHEVALLARD, 1994)

In this context, mathematical knowledge is related to the hierarchical scale (figure 1), which refers to didactics, which according to Chevallard (2009b, p. 1): “Didactics is the science of the conditions and restrictions of social diffusion of praxeologies”. It is therefore necessary to understand the meaning of the words “conditions and restrictions”. When teaching something to someone, there are some conditions that, as Chevallard himself mentions, “in the beginning everything is a condition” (2009a, p. 12), the moment that this condition cannot be modified, by the person or institution, the condition becomes a restriction. Understanding the above directly: the teacher should not assign his students independent studies of the contents without considering the reality that his student experiences.

In this way, we can understand the levels of codetermination subdivided into two sublevels, according to Chevallard (2002): higher levels (Civilization, Society, School and Pedagogy) and lower levels (Discipline, Domain, Sector, Theme and Object), in which, in this scale, the double arrow represents that the creation or modification of a condition in a certain level can make a difference in the other levels. For the higher levels, we have the following scale:

Civilização¶

↓↑

Sociedade·

↓↑

Escola·

↓↑

Pedagogia¶

↓↑

Figure 2.

Higher levels of codetermination, Chevallard (2009a, p. 12)

The higher levels correspond to the conditions and restrictions imposed by situations outside the classroom that were not created by the teacher but directly affect the execution of his/her praxeologies. In this scenario, we can exemplify different situations. Let’s imagine, for example, a school in which the pedagogy is based on preparing students for entrance exams at the largest Brazilian universities and for the National High School Exam (ENEM). For such an institution, the pedagogy implies that the teacher's practice must be guided by the system of handouts (provided by the school) and by the activities presented in previous ENEM exams and entrance exams provided by universities.

This model directly imposes restrictions on the teacher's practice, since he or she cannot fail to follow these methodological procedures because, following the handout, he or she must develop all the activities in the handouts, as well as the execution of different entrance exam activities. These are situations that present themselves as restrictions that cannot be modified.

This being said, we can observe that, in the school environment, there are different situations correlated to higher levels that directly imply the teacher's practice in the classroom, and it is up to researchers in didactics to understand that these are situations that must undoubtedly be analyzed, even if they cannot be modified, as stated by Chevallard (2009a):

> [...] It is up to educational researchers to take into account as much as possible at a given time all the conditions that they suspect weigh on the diffusion studied praxeologically , although they do not have the capacity to have these conditions modified in a way that is eventually desired (Chevallard, 2009a, p. 13).

The levels of codetermination represent, at each level, the subjection of X in relation to the Institutions Ii. We understand that the TAD allows us to understand the personal relationships R(X, O) and the institutional relationships RI(O). Furthermore, we can, through this theory, analyze the relations between subject X and institution I, which involves the subjection of X to I and the conditions and restrictions of I to X. In this aspect, we can think of the relations R(X, R(I, O); R(I, R(X, O) and, further, analyze each relation depending on institution I. In this case, the relations are modified or altered when we assume I = School, or I = Society, I = Class, or even, I = Y (study supervisor), as well as when we assume the different subjects X involved in these relations.

Regarding the lower levels of codetermination (Discipline, Domain, Sector, Theme and Object), we have:

Disciplina·

↓↑

Domínio·

↓↑

Setor·

↓↑

Tema·

↓↑

Assunto¶

Figure 3.

Lower levels of codetermination, Chevallard (2002, p. 9)

In this context, we can understand that in this research the lower levels are: Discipline (Mathematics); Domain (Geometry); Sector (Plane Geometry); Theme: (Isometry); Subject (Symmetry). Chevallard (2002) mentions that these correspond to the study of praxeologies, but not necessarily a study in a fragmented way, located at each level. On the contrary, the studies of the lower levels should provide motivation for the study of different tasks, as he points out:

> The main deficit created by the current state of affairs prevailing today in high schools and lycées concerns first of all the mathematical organizations actually implemented in classes: this deficit is felt in the absence of motivation for the T types of tasks studied. Very generally, the 'motivating' tasks are missing and, at the limit, no one even knows where to look for them anymore! [...] the types of motivating tasks are found at higher levels of determination of mathematical organizations - sectors and domains (Chevallard, 2002, p. 4).

In this sense, the levels of codetermination aim to break away from the mathematics of *visiting works* , working “silently to replace a secular tradition, in which the student waits without blinking, according to the immemorial problem of copying works and cultural mimicry, for the teacher to teach - that is, to show what it is, how to do it and why to do it that way” (Chevallard, 2002, p. 1). For the author, the lack of connection (study in a fragmented way) between the Subject and Theme levels with the Sectors and Domain levels, and also the Discipline level, results in the impossibility of thinking about motivating tasks; in addition, the organization of the study considering all levels is no longer mathematically viable. In general, teachers only go through the Theme and Subject levels.

The lower levels allow us to understand how mathematical knowledge exists in a given institution, since in each of them there are conditions and restrictions that must be considered so that knowledge can exist there. An example of this situation can be clarified by Bittar (2017):

> Let's take the concept of area as an example, present in all levels of basic education and in many university courses in exact sciences. In the 4th year of elementary school, for example, the calculation of the area of flat figures is explored through the idea of paving. This idea appears implicitly in the study of multiplication, in the 1st or 2nd year. In the final years of elementary school, this same concept is studied in a slightly more formal way with the presentation of formulas for calculating the area of polygons that can almost always be justified with the congruence of triangles. If in the initial years the idea of area is explored intuitively, in the final years a more formal idea of the mathematical object area of flat figures begins to have space and, in a course of differential and integral calculus, the concept is expanded to the calculation of any flat figures with the presentation of the Riemann integral. In each of these institutions, it is necessary to make adaptations so that the idea of area can exist; such adaptations are a consequence of the conditions and restrictions imposed by the institution itself (Bittar, 2017, p. 367).

In this example, we have the conditions and restrictions of mathematical knowledge according to the institutions (níveis de ensino) in which it is inserted. It is clear that a pedagogical practice developed in the classroom is not based only on the execution of a study carried out by teachers or a group of teachers; didactic and mathematical issues permeate conditions and restrictions determined by the levels of codetermination.

According to Chevallard (2009a), the teacher, as an education professional, must first understand that didactics evolve. However, the evolution of didactics alone is not enough. In addition to political and administrative impositions, studies on the subject are necessary in the school system to meet the particularities of each discipline. The teacher must seek this study, identifying the useful and dispensable praxeologies for the exercise of the profession, praxeologies resulting from the conditions and restrictions, in the case of the discipline of mathematics, which can be imposed by higher levels (school, pedagogy, society and civilization) and lower levels (theme, subject, sector, domain and discipline).

In this context, understanding that all levels of codetermination directly interfere in school practice, we must ask ourselves: how to teach Mathematics in the classroom? Therefore, supported by the *Questioning of the World Paradigm,* Chevallard (2009g, 2012) aims to break with practices that the author calls the *Visiting the Works Paradigm* , with the purpose of encouraging teachers to develop practices that aim at a curriculum generated through questions that instigate both the student and the teacher to adopt a researcher's stance in relation to the knowledge taught in the classroom. To this end, Chevallard (2009a) has elucidated a Study and Research Path (PEP), which includes teaching through a Q question, which generates other questions, permeating a practice that stimulates an investigative and questioning stance.

In this context, the PEP is based on the paradigm of questioning the world, providing a broader view of teaching, in order to consider conditions and restrictions that are external to the classroom and that directly interfere with pedagogical practice. For Bosch (2018), we can visualize a PEP structure as follows:

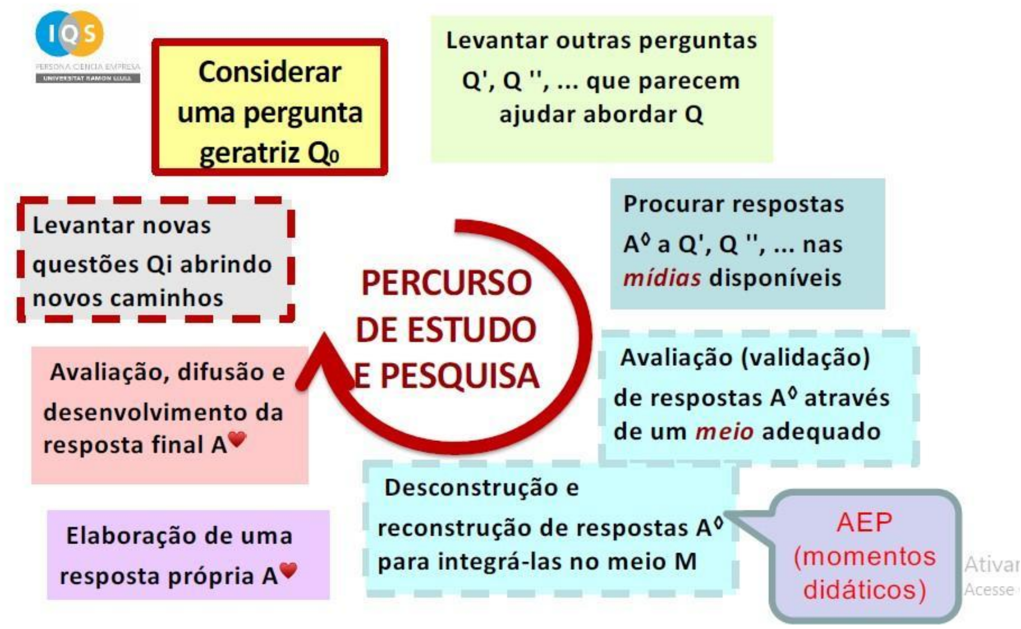

Figure 4

PEP description. Source: Bosch lecture slides (2018)

According to Bosch (2018), during the course, we will have situations in which it will be necessary to work with didactic moments (AEP). Among them, work with technique, which, directs the study towards a more technical development (Gascón, 2003). Certainly, such studies are necessary to build a praxeology of the desired response and allow us to understand that we may have situations during the course that will not necessarily be guided by the *questioning of the world paradigm* .

For Ruiz (2015), the structure of a PEP is always open and indeterminate at its beginning, since it is the process itself that will outline the possible forms for the path (with many setbacks, detours and shortcuts, as necessary) and, furthermore, throughout the PEP, the generating question Q evolves and transforms into one or more new questions. For this author, the notion of PEP arises from the need for educational organizations, such as schools or any other institution, to have a truly functional epistemology, which appears as machines for producing useful knowledge for creating answers R to questions Q.

For Andrade (2012, p. 36), the PEP:

> [...] it constitutes a Teacher Training Path, in initial and continuing training, as it constitutes study processes to face the praxeological problem of the teaching institution, of issues that emanate from and in teaching practices, of the conditions and restrictions imposed on these problems, eliminating the risks of wanting to train teachers based on an immutable EP that would be left under the responsibility of the teacher to mobilize him/her in concrete situations.

We therefore adopted short or micro-PEPs [6]with the purpose of investigating and analyzing their contributions and limitations as a methodological proposal for the continuing education of mathematics teachers. We worked with a group of twelve teachers, all of whom work in public education in the final years of elementary school, in order to encourage these teachers to study and rethink the way they conduct their classes on geometric content, based on the paradigm of questioning the world. It is important to emphasize that developing a PEP is not basically about bringing teachers together and presenting an activity. This implies, first, a moment of planning the activities, with a lot of study, so that they can be characterized as PEPs, as we will report in the course of the analyses.

Thus, the methodological procedures of the Study and Research Path (PEP) seem viable in working with continuing education, since the development of teaching systems, guided by the world questioning paradigm, enables a broader study of conditions and restrictions that arise from the higher levels of codetermination, directing the entire school environment, be it school practices, coordination, curriculum, among others.

Analysis of micropaths

All training sessions were guided by several Didactic Systems (DS), represented by $S_i = \{x_i, Y, Q_i\}$, in which the elements x_i represent the teachers participating in the training (which varies according to their availability), Y the teacher supervising the study (researcher) and Q_i the generating questions that led the study of the mathematical object in the continuing education. For this article, we will present the analysis of two didactic systems guided by the question Q_4 = How to teach symmetry? Called PEP - S_7 and PEP – S_9. In the Micro PEP - $S_{7\ session}$, we had the following didactic system: $S_7 = \{(x_1, x_2, x_{12}), Y, Q_4\}$, in which teacher x_{12} was participating for the first time. Supported by the *questioning the world paradigm* , the study supervisor (Y) began to dialogue focusing on question Q_4 for the beginning of the study:

> [...] when we work on geometry, we can use some geometric construction instruments in the classroom, that's why I brought a ruler and compass, because it can also be an opportunity when

[6]We assumed that these were short or micro-courses, since in the ongoing training we carried out, we did not have a continuous flow of teacher participation due to different conditions and restrictions. In this sense, we assumed that each training session would be a moment of study, a micro-course.

working on this symmetry content, to work a little on constructions, to work with a ruler, set square (Y)
They have difficulty working (students) (x_2)
That would be an opportunity, if they have difficulty even starting to handle it (Y)
They show, like, teacher, what is this called? What is this one here? What is this for? They have a lot of difficulty. Honestly, I don't give much value to this content here, I just go through it superficially (x_2)
It's what I said, we just go through this content [...] then we think about this symmetry sequence (x_1)
You know what happens, the questions that the textbook brings, they do not promote or demonstrate any difficulty for the student and, then when the student does it, they question the teacher, is this for us? This here can't be made difficult, can it? (x_2)
It depends, right? Yes, it does (Y)
Why are the ones in the book of a very low difficulty level (x_2)?
So let's solve it here, and we can discuss about (Y)
If I know you well, this one will raise doubts, it will cause problems for me again (x_2)

At the beginning of the study, teacher x_2 had already expressed her OD regarding the symmetry content and answered $R_{4.1}$: *"I don't give much value to this content here, I just skim through it* (x_2)", and she also added that the fact that the textbook only presents trivial activities makes even the students question whether the activities are for them. Certainly, the symmetry content is taught by the teacher following the didactic organization presented in the textbook and, at the beginning of the study, she commented that she had no difficulty with the content. However, the teachers have chosen this content because they would work with it in the classroom, and they saw in the continuing education an opportunity to prepare these classes, through the PEP methodology.

During all the sessions, we tried to get the teachers to participate actively and for the study to contribute to their classroom practice. As the teacher has been participating assiduously in the training sessions, the paths developed have destabilized her certainties. She says: "If I know you well, this one will raise doubts, it will cause me problems again (x_2)". Therefore, when they start participating in a new micro-PEP session, the teachers already expect situations that will allow them to rethink their OD and OM around geometric content. For all these reasons, we began our study by revisiting the meaning of geometric transformations:

[...] this word is now in the sixth grade book, I don't know about yours? Homothety, the textbook is more or less there [...] in the ninth grade, in the seventh grade, there is something about congruence, isn't there? Oh, no! In the eighth grade, there are cases of congruence [...] And it's pure rote learning, I'm going to talk about pure rote learning that the student does [...] Which, in the OBMEP, there is a lot, a lot of context about this, right, you have to master this part in the OBMEP, then you get this part [...] That's why you have to go through the ABCs themselves or vice-versa and then relate, but then you also have that availability of time too, because until the student assimilates it, or you start there with the context, until you get there you have to talk about the technical part too, you have to point out several things within the content too, you can't just stick to the story, right? You have to show the technical part of the thing, right, the

theory, I think the content is very extensive and I would like there to be only one, two or three contents, then maybe we could open the student's mind to other things (x_2)
Because, in truth, whether we like it or not, we end up making some choices, we end up prioritizing some content over others (Y)
Or there are teachers who end up skimming everything and going through everything. When I say skimming, you see, but you don't see the ideal, the ideal would be for you to stop and study in depth, as you are saying, then maybe even the symmetry itself that I am considering, an easier content that we go through, has something for us to see more (x_2).

In this fragment, the teacher once again emphasizes her OD, which teaches the content with a focus on technique, since she does not have the conditions or time to work on the entire OM in its complexity. These conditions, in having to comply with the syllabus according to the state's curricular reference, were not created by the teachers, but they directly affect their praxeologies in the classroom and end up becoming restrictions for their practice. The teacher ends up making some choices, because, in a classroom with 40 students (heterogeneous) , with different learning times, it is difficult, according to the teacher, to work with a technological-theoretical block. After these reflections, the teachers began to solve the following activity:

1) In each case, indicate the number of axes of symmetry for each figure:

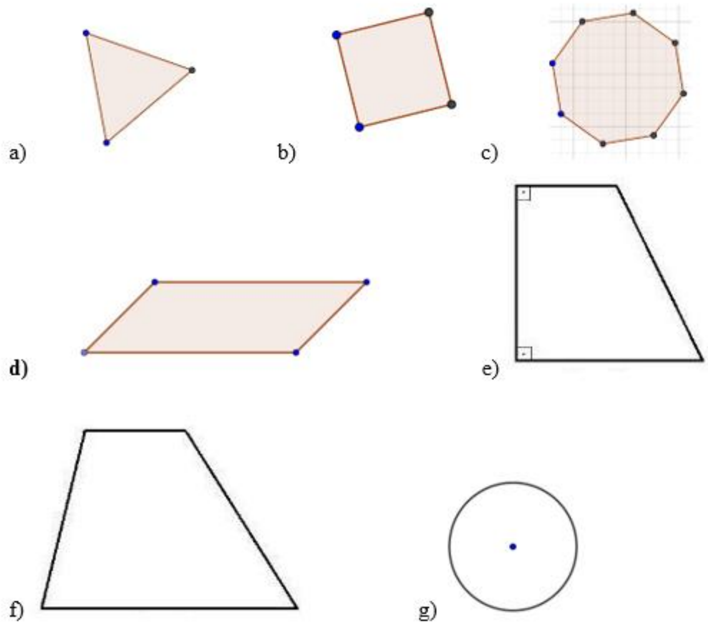

Figure 5.

Activities adapted from Silva's thesis (2015)

a) What did you do to find the axes of symmetry of the figures?
b) Can the direction of the axis of symmetry (horizontal, vertical, oblique) be a variable that intervenes in its identification?
c) Can figures arranged on blank paper or graph paper make it easier or harder to identify the axes of symmetry?

d) What is symmetry?

When presenting the activity, we begin the study:

> Let's indicate the axes of symmetry (Y)
> In G, are there infinities? (x_2)
> Why? (Y)
> In G, are there infinites? Because every time I cross it out there will be, here passing through the center, right? Or am I mistaken, guys, please help me, you end up forgetting, you know what happens when you are a teacher, and you are around other teachers in the area, right, you are afraid to say something, to speak and, suddenly, the other one will say something like that, then she will call me stupid. (x_2)
> No problem, you can talk. (x_1)
> [ ] Our training is not enough. Sometimes you don't know because you didn't see it, or sometimes also because you don't remember, right? (x_2)
> For example, if you haven't worked with the ninth grade for so long, five years, you have to go back and take a look at the content, plan. That's how it works to go back. I used to work at a private school, and it was like that at the private school, you would take the ninth grade, and you would only teach the ninth grade. (x_1)

Teacher x_2 points out how difficult it is to express herself in a group of teachers and, mainly, to admit the difficulties she has in relation to the content taught. We have seen that her more active participation in the courses allows her to have more freedom and trust in the study advisor (Y) to expose her difficulties. This is an important characteristic in the Study and Research Course – Teacher Training (PEP–FP), since teachers must assume themselves as 'students of the issue' and it becomes a challenge for the study advisor to direct the courses so that teachers have this responsibility and expose their praxeologies developed in the classroom.

Therefore, one of the limitations of PEP-FP programs composed of mathematics teachers who already have a degree in the area is the difficulty in assuming to other professionals that they do not understand a certain content, which is completely different from dealing with teachers in initial training who are still in the process of training. We are not saying that the teacher is the professional who already understands everything about the school environment. On the contrary, we believe that, according to Gatti (2016), continuing education is precisely to continue the training process. The important thing is that teachers seek, in training, an opportunity to study the concepts that they will work on in the classroom in the future. We continue the discussion around the activity:

> The axis doesn't necessarily have to be from vertex to vertex, right? You just have to cross it out, right, start it the same. (x_2)
> It's the mirror, just like the mirror. (x_1)
> A technique that we can think of by folding the figure if I can fold it. (Y)
> And stay the same. (x_2)
> One part overlaps the other equally, so I can find it. (Y)

I wanted to say the following, right, to make it clear. Here are two, you can divide them here, look, then you can, it has to be exactly the same size, understand? So, for example, if I divide the triangle in half, fitting two parts is the axis, you have to fit two parts, right? (x_1)
This (Y)
So fold here, not here, it didn't work, it didn't work (x_2)
It has to be symmetrical like a mirror, right? (x_1)
It doesn't work here, I'm doubling it here, it doesn't work, so it's wrong (x_2)
So let's start here (Y)
Now here it gives (x_2)
Let's start at the letter, the letter 'a' (Y)
In the letter 'a' (x_1)
How many axes of symmetry does it have? (Y)
There are two (x_1)
There are three (x_2)
[...] in the letter 'a' how many did you find? How many axes of symmetry? (Y)
If you asked more than once it's because mine is wrong (x_2)
So to find three, you're considering this triangle what? (Y)
Equilateral (x_1)
To me, it's equilateral, isn't it? (x_2)
Because then, in case (Y)
If it were isosceles, we wouldn't find this, because isosceles is just one axis (x_1)
It wouldn't work, then there would be fewer axes, right? (x_2)
How many axes could we think of, if it were isosceles how many axes of symmetry could we think of? (Y)
Just one, just the one in the middle, right? Right? It would only fold here, it wouldn't fold here, it wouldn't fold there, now this one, because it's equilateral, it has three. (x_2)
That's why I'm measuring here, I have to measure it exactly, right? There's no scalene, right? If it is, if I'm going to measure it, right? (x_1)
Scalene does not have, none. (x_1)

The teacher mentioned at the beginning of PEP – S_7 that it was an easy subject. However, when she started to solve the activity, she raised the question: $Q_{4.1}$: *When tracing the axis, doesn't it necessarily have to be from vertex to vertex?* As the studies developed, we realized that this understanding of it being an 'easy' subject was changing. When solving the activity with the figures of the rectangle, square, and parallelogram, the teachers had no difficulty in solving it, but when trying to solve the situation for the eight-sided polygon, they had doubts about their certainties:

In the letter C? (Y)
I'm not seeing much of the letter C. (x_1)
Eight sides. (Y)
No, no, in the letter c, the letter c is this one, right, it's an octagon, I got four too, wasn't that it? (x_2)
Let me see 2, 3, 4, 5, 6, 7 and 8. (x_1)
1, 2, 3, 4 is four, isn't it? (x_2)
Are there four? (Y)
Ah, I found more, am I wrong? Let me make a bigger one here, one... (x_1)
One, two... there are only four. (x_2)
Two, three and four, which I made here, look (x_1)

No, Y! I'm wrong, look here, Y, there's a pile, which has 4, 5 in mine there will be infinite ones too. (x_2)
Infinite? (Y)
Look here, look, if you cross it out next to this other one, no, no, no, no, no, wait a minute, if here it gave 5, 6, 7, 8, it gave eight, out of eight, then that means the number of sides is the number of axes? Is that so? (x_2)
Guys, I'm thinking, 4, 5, 6, eight. (x_1)
But you didn't cross it out here. (x_2)
I'm thinking to cross it out, wait. (x_1)
When we sit down to solve these exercises, do we have the same difficulty that the student would have? After we study and pass it on to them, we want them to learn quickly, right, and we are great students, right? (x_2)
That's what I said to another teacher, we demand too much from our students. (x_1)
Yeah, poor things, I didn't find this one (x_2)
[...] I can then say that, in a regular polygon, the number of axes of symmetry is equal to the number of sides? (Y)
With these three examples, I'll already say it's true. (x_2)
I was going to say yes, Y. (x_1)
Can I say? No? (x_1)
I don't know (Y)
Calm down, let's analyze (x_2)

When analyzing the activity solutions, the teachers realized that the number of axes of symmetry of the figures was equal to the number of sides of the regular polygons. Therefore, another question asked by the teachers arose. $Q_{4.2}$: *Is the number of sides of the regular polygon equal to the number of axes of symmetry?* In raising these questions, we continued the study in order to find the answers and we saw that the teachers traced the axes of symmetry, but were unable to justify their solutions:

But then, in this case, when you draw the axis of symmetry in the figure, are you going from vertex to vertex? (Y)
That was my initial question. (x_2)
No. (x_1)
Let's look here, for example, let's look here, look, in the case of the octagon. (Y)
When it is regular, you go from vertex to vertex. (x_2)
What happened in the case of the square? (Y)
Here you did not pass all the vertices, here is not a vertex. (x_2)
In this case it is a case that was the vertex. (Y)
True, Y. (x_2)
So, in these three cases here, did you draw a single line that passes through the vertices? (Y)
No. (x_2)
You guys drew it too. (Y)
Side by side, half, vertex to vertex, half, midpoint, by the midpoint of each side, each side, exact. (x_2)
So, in this case, you have two options, in this case, is it the same rule in the triangle? (Y)
Vertex, the vertex was. (x_2)
Triangle went from vertex to vertex? Look at the quadrilateral, the square octagon, you started, which is from vertex to vertex. (Y)
It's not here, at the midpoint of the triangle (x_1)
So, look, this was it, here it was from the midpoint to the midpoint of the opposite side. (Y)
Here it was from vertex to midpoint. (x_1)

Here it was from vertex to midpoint. (Y)
It's different here, don't you agree? (Y)
I agree. (x_1)
Here it is vertex to vertex, here midpoint to midpoint. (Y)
Same thing here. (x_2)
Where do you want to go? (x_1)
Are you seeing vertex by vertex? So, I ask you this: if I consider a regular polygon, can I say that they will all have a number of axes of symmetry with the same number of sides? (Y)
Yes? (x_2)
That was clear. (x_1)

We guided the course so that the teachers could answer question $Q_{4.2}$ and arrive at the answers $R_{4.2}$: *In a regular polygon, the number of axes of symmetry is equal to the number of sides of the polygon* . During the study, the teachers began to notice some differences in the way the axes of symmetry are found in polygons with even and odd sides. The advisor asked:

But what about the axes of symmetry to find, let's say, the even-sided polygons, is it the same for you to find the axes of symmetry of the odd-sided polygons? (Y)
Send us a polygon with odd sides so we can see. (x_1)
So let's draw, for example, a pentagon, a regular pentagon. (Y)
It will be difficult for me to draw a pentagon now. (x_1)
So let's think about the axis of symmetry. (Y)
[...] It's not regular, it's difficult to draw freehand. (x_1)
If I double it, it won't work either, Y. (x_2)
Isn't it possible? Let's think here, look, from this vertex, it's the drawing that isn't there. (Y)
Let's make the correct drawing. (x_1)
No way, Y (x_2)
A regular pentagon. (x_1)
So, it's been a while since I drew a pentagon, but we can try it like this, let's suppose here, side five, side five, there. (Y)
So, let's think like this, let's make another format for it. (x_2)
Side five. (Y)
Let's try another format for it then, Y. (x_2)
So, here we can draw five, here too, right (Y), it won't be a pentagon. (x_1)
It will give a monstrous figure. (x_2)
Regular teacher, I can't understand a regular there. (laughs) (x_1)
It won't work out. (x_2)

The teachers were unable to draw a pentagon. When they tried to draw the polygon, they only considered the sides, but not the angles. It is worth noting that teacher x_1 had participated in the course on tiling and we discussed the regular pentagon. However, at that moment, she did not score anything. We then continued the discussion:

No way, shall we use the cell phone? (x_2)
Teacher, can I use the cell phone? (laughs) (x_1) (takes the cell phone)
[...] could the problem be in our design? (Y)
But this is a regular pentagon. (x_2)
It's a pentagon, but to be regular, what do they have to have? (Y)
Ah, from the vertex to the center it must have the same measurement as the side. (x_2)
When we talk about regular polygons here, what are we just considering? (Y)

Only side. (x_2).
Only side? (Y)
It's wrong, our drawing. (x_2)
What's wrong here? Is this a regular polygon? Looking here, I'm just considering the sides, but the regular polygon, it has equal sides and angles, so here, for example, here is an angle of 90... here it will be a little bigger than 90 here, this figure is not regular. (Y)
But can the pentagon be regular? (x_2)
Yes. (Y)
Let's put it on the internet. (x_2)
There is a regular pentagon, isn't there? Or not? (Y)
I have my doubts. (x_2) (At this moment, the teacher searches on the internet with the help of her cell phone.)
Yes! Do you have it? (Y)
I thought I had it, but now I don't know anymore, based on this drawing here I don't know anymore, teacher, until now I thought I had it. (x_1)

In this excerpt, it is clear that the teachers do not know what a regular polygon is, and the question $Q_{4.3 \text{ is raised}}$: *But can a pentagon be regular?* As we go along, we realize that we assume that the teachers already know some geometric concepts, which are necessary conditions for developing the activities. However, as we carry out the studies, we have strong evidence that the basic concepts of Geometry are not understood by the teachers, and they need to resort to the internet (media) to understand the content. Teacher x_2 expressed herself as follows:

Oh my God! It does. It's not in the shape of our little house and it will have angles and sides, look here. (x_2)
The angles here must all be equal, and here what happens is that we are only considering the side, in a regular polygon with five sides, their internal angles are 108°. (Y)
The sum is 540, right? (x_2)
[...] And if it resembles a triangle, will it resemble a heptagon too? I'll look it up on the internet (x_1). The teacher starts to look it up on the internet. The heptagon has seven sides, it's a polygon with odd sides, right? From the drawing on the internet, she starts to count the axes of symmetry. (x_1)
One, two, three, four, five, six and seven, always at the vertex to the midpoint and, then, the characteristics always remain for regular polygons, the characteristics also remain for polygons with even sides and odd sides as well (x_2)
What happens, in regular polygons, we can say that the number of sides is the number of axes of symmetry, however, the way I find the axes of symmetry of an even-sided polygon is different from the way I find the axes of symmetry of an odd-sided polygon? (Y)
In pairs, it's vertex to vertex and midpoint to midpoint, is that it? (x_2)
Vertex to vertex and midpoint to midpoint. What about the odd-sided polygon? (Y)
Only from vertex to midpoint. (x_2)
From vertex to midpoint. (Y)

In this excerpt, after the teachers researched on the internet (media), we were able to institutionalize some particularities of symmetry that were recurrent in the activities developed, having obtained the following answers: $R_{4.3}$: *In regular polygons with even sides, the axes of* *symmetry are drawn from vertex to vertex and midpoint to midpoint* and $R_{4.4}$: *In regular polygons with odd sides, the axes of symmetry are drawn from vertex to midpoint.*

Despite the difficulties, it is importante to note that all the time the teacher x_1 and x_2 are always writing everything down. As they themselves expressed, it is a moment of study, but teacher x_{12} just watches, perhaps because it is the first time she has participated. The study continues, when the advisor presents some concepts of orthogonality on the bisector as a geometric locus, because the teachers, when asked about these concepts, did not remember. After understanding the content, the teachers evaluated the possibilities of developing these activities in the classroom (OD) and resumed the discussion about the study:

> This activity in the lab would be cool. You can distribute it to the group, you can distribute the questionable ones to the group, right, for example, a polygon with even sides, one that does not have an axis of symmetry. (x_2)
> Cases that do not have axes, cases that do, such as even and odd polygons. (Y)
> And from there, start questioning, asking questions so they can answer. It could be a cool lab class, right? (x_2)
> OY, that figure I mentioned at the beginning, which has infinite axes of symmetry, I still keep harping on about it and it has finite axes of symmetry, there will be 'n' lines passing through the center, and it will be divided. (x_2)
> It will be divided equally, so in the case of the circumference, it has infinite axes of symmetry, when it passes through the center and touches two points of the circumference. In fact, I'm working with the (Y)
> Radius, the diameter. (x_2)
> That, with the diameter. (Y)
> You can also pull the content, all about circumference in some year there, I think it's in the ninth, won't it be? In the eighth too, last year I remember, and then it's cool to also pull the hook for that. (x_2)
> So, you can start working, and I even brought some questions in addition to these, so we can even think of questions to ask the students, or sometimes we don't even need to. I will necessarily apply the definition, axis of symmetry is this, this, this, then you can talk about these figures, let's double, the concept of superimposing on the other, then they will discover that these are the axes of symmetry. (Y)
> They will form their own theory, how do you want to write it? What is the axis of symmetry? Each one will say something different. (x_2)
> There, in fact, is what I can divide the figure into. (Y)
> In two overlapping parts. (x_2)
> Equal, when you superimpose, it is exactly the same, so, like this, you can even do an activity from this and ask the students, what did you do to find the axes of symmetry of the figures? (Y)
> You can't say that it's dividing the figure into two equal parts, otherwise the parallelogram comes in, right, I have to say something else, right, equal parts that overlap. (x_2)
> In two symmetrical parts. (Y)
> You can put something else instead of symmetrical, maybe they will understand better, I can't talk about two equal parts, otherwise I will lead to error. (x_2)
> In the case of the parallelogram, it is precisely this that leads to this error, so there would be two equal parts that overlap. (Y)
> Likewise. (x_2)
> Likewise. (Y)

Certainly, in the teachers' speech, the concept they expressed that it was "easy" has already changed. During the study, we found that the teachers understood the content much better and began to think about a DO to be developed in the classroom. As we continued studying the content, we proposed a discussion about some conceptual errors presented in textbooks related to symmetry. One of the most serious mistakes mentioned in the activity is the concept of axis of symmetry in three-dimensional beings. Thus, we tried to study that in three-dimensional objects there are planes of symmetry and not axes of symmetry. Therefore, we point out:

> [...] what he puts with more emphasis than what I'm talking about here, axes of symmetry, what are we working with, with figures. (Y)
> Flat, he places with three-dimensional figures. (x_2)
> That's it. So, for example, the human body, it's not flat, so I can't talk about an axis. (Y)
> But his shadow usually comes from the shadow (x_2)
> If it is the shadow on the plane, I can talk about symmetry, but when I talk about a three-dimensional figure, what do I talk about? (Y)
> It doesn't have that name anymore, it's reflection symmetry, is that the name? (x_2)
> [...] Reflection symmetry and axial symmetry are the same thing, but what happens when I work with three dimensions is that there is no axis of symmetry, there is a plane of symmetry. I will talk about the planes of reflection symmetry, that's why he says that the book is misleading, because when I work with, for example, a building. The building is a three-dimensional figure, so there will be no axis, there will be no axis of symmetry, there will be a plane of symmetry. (Y)
> I think the book wanted to contextualize and ended up getting it wrong (x_2)
> In fact, there are no axes in the case of three-dimensional figures, and think about the three-dimensional object, let's suppose there, if I think about dividing this object. (Y)
> I would have a plan, I would pass a sheet of paper in the middle, not a straight line. (x_2)
> That's it. (Y)
> I pass a sheet in the middle which is a plane. (x_2)
> This is the plan that will divide it into two equal and overlapping parts. (Y)
> As if it were a wall. (x_2)

With the presentation of the activity on conceptual errors in the textbook regarding the study of symmetry, we concluded PEP-S_7 and, at this point, we have noticed that due to difficulties in relation to the content, the teachers hardly ask questions. Bosch and Gascón (2010) argue that the paths should lead to an expansion of praxeologies. We observed that, during the study of this session, the teachers' praxeologies regarding the concept of symmetry expanded, since, according to reports, they only taught how to draw the axes of symmetry. Certainly, at the end of this session, they no longer found it simple to teach the content and their praxeologies became more complete after the study. The route was composed of the following system: $[S_7 = \{(x_1, x_2, x_{12}), Y, Q_4\} \rightarrow M = \{R_{4.1}, R_{4.2}, R_{4.3}, R_{4.4}; Q_{4.1}, Q_{4.2}, Q_{4.3}\}]$.

At the end of these activities, the teachers requested that the study of Symmetry be continued. Thus, it was proposed to do so in the next session. In short, we understand that there would be a lot to do for the study of Geometry with these teachers, but their relationships with geometric content will not be the same after the micro-paths.

Since some teachers who requested the study of question Q_4 were not present at PEP–S_7, we resumed the discussion of the study of symmetry in PEP–S_9. The study began with the activity: In each situation, trace the symmetrical figure in relation to the given line and explain the procedures used, as shown in figure 6.

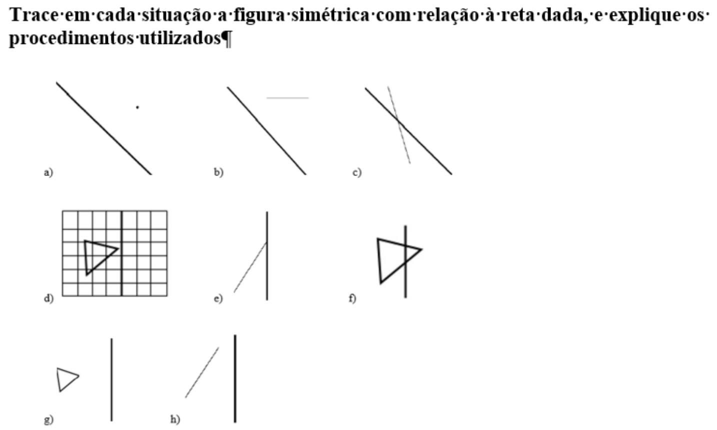

Figure 6

Symmetry about a given line (Activities adapted from Silva's thesis, 2015)

The teachers initially solved the activity using the technique they know, τ: *folding the figures in half so that one part overlaps the other.*

Are you going to bend over the axis? (x_1)
I fold it here, I put the shadow and that's it, I get the same figure [...] an easier and more accurate way, that's it, right? You have to fold and overlap (x_2)
Is there only this way? What does this axis mean when I fold the figure, what does this axis become in the figure? (Y)
The base, when I folded it, it stayed here, right? [...] it is the midpoint (x_2)
[...] what is the segment that passes through the midpoint here, it has a name, do you remember? (Y)
It is the bisector (x_2)
So, the axis of symmetry becomes the mediator of all figures (x_2)
That, he is the mediatrix of the figures (Y)
Since the teachers had already participated in PEP-S_7 on the study of symmetry, they adopted the $\tau_{1.9\ technique}$. However, in the next activities, we began to encourage them to think of another technique to solve the activity: How am I going to construct this point on the other side without overlapping? Without using the folding technique? (Y)
And without measuring too? With the ruler? (x_2)
Even when measuring, you can deviate here and there, and it doesn't necessarily overlap. (Y)
And it has to be symmetrical. (x_2)
How to do this? (Y)
I don't know how to do it. (x_2)

The teachers would have to reproduce a symmetrical point in relation to the given axis and, to develop this activity, a technique that is done through drawing, reproducing the point with the help of a compass. The teachers really did not understand how to do it and the study ended up being directed by (Y), teaching some drawing procedures to reproduce the symmetrical figures in relation to the axis. In view of this, teacher x_2 also expressed the students' difficulties in working with the compass and the protractor:

> [...] The simple fact that they handle this thing here (compass) is already an improvement. They have extreme difficulty with it. When I worked on the sector chart with them, they understood it better, but the protractor was chaos for them [...] the one they sent from the government is very bad (x_2)

The teacher expressed the difficulty in working with drawing instruments with students and that they are used in other mathematics content (sector graphs) and not for geometric content. Perhaps the teacher's own difficulty in remembering the procedures for geometric drawings is an obstacle to working with her students. Given the findings and reported here, we explained to the teachers how to construct the symmetrical point in relation to the given axis with the help of the compass and ruler and, subsequently, we solved situations that would require reproducing the symmetrical figure of the triangle, which they had no difficulty in doing, since it was enough to reproduce the three vertices. As the activities continued, we asked the teachers about the figures that intersected the axis of symmetry, a different case from those they had solved until then:

> [...] and in these cases where the axis of symmetry intercepts the figure, what happens? (Y)
> You're going to have to send it the other way around, right? This one you're going to send there and this one you're going to send here. You take this point, send it there, this one you're going to send there and this one you're going to send here, is that it? (x_2)
> That's right. (Y)
> But it's hard work to do with this (compass). (x_2)
> One of the focuses of teaching Geometry is for you to work with geometric constructions. Therefore, work with the instruments compass and ruler. (Y)
> If it's by folding, I move there, draw here, move there, draw here. (x_2)
> In the case of the folding technique, I use the axis as a reference [...] but in this case, where does the axis cut the figure? [...] In this case, is the folding or construction technique more convenient? (Y)
> I think that what comes closest to the ideal, to the exact, is this one, right, from what I'm seeing, but I think it's laborious and if I don't study everything will get messed up here, is it possible to do this one? (x_2)
> Shall we try? (Y)

The teachers try to solve it and cannot reproduce it using the bisector. They know the technique of reproducing the three points of the triangle and then drawing its segments by connecting the points, but when drawing using the compass they cannot do it, even though the tutor did the procedures minutes before:

> Why don't we build the right point? Wow, complicated, huh? The student won't like it, no. (x_2)

The teachers build the figure as the advisor (Y) explains the procedures for finding symmetrical points with the help of a compass, and they carry out the activity. However, even with the guidance, the teachers were unable to reproduce the figure. One of them expressed that this type of activity is not feasible to work on in the classroom, as it takes up a lot of class time.

> [...] I think the following, put a very elementary one and one of these is enough to learn the technique [...], because otherwise, you don't have much time in class, then you can put situations, which way you can reproduce on the other side, then they will talk about the folding technique, they will never talk about these. (x_2)

Certainly, during PEP-S_9, a continuation of PEP-S_7, it became clear that the teachers teach the content of 'superficial' symmetry, work only with the folding technique and close the content, as if the teaching of symmetry was limited to that. Furthermore, they do not work with other types of symmetry, such as rotation and translation. In fact, as teacher x_2 said at the beginning of PEP-S_7: “[...] I do not give much value to this content here, I go through it superficially (x_2)”, considering that there was no difficulty in teaching this content and nor for the students in learning it, but, at the end of the micro-paths, she pointed out:

> [...] you also noticed that for all the content it seems to me that we stay at the elementary level, that if you think about it, as I told you in the first meeting, I said that I didn't give much importance to the symmetry content, that it seemed very easy to me and I just skimmed through it and look how much I didn't know, do you understand? So it seems that the book, at the same time that it helps you, also hinders you, you had to stop to study, and if you think about the class, well-designed for each content you enter, you had to study, you had to work hard [...] because we open a page of the book and read it, it happens to everyone, you go to talk to the students and you didn't prepare that class accordingly, you accept what is there, it's true, you agree with everything and if you look, behind the scenes the questions are long. (x_2)

In this speech, we can see that the teacher assumes that her teaching organization is subject to the textbook and, when preparing her classes according to the content presented in the textbook, she does not have any difficulty, since she reproduces it as it is on paper. However, the journey made her realize the importance of studying and how much she does not study enough to teach a class with all the mathematical organization necessary for the student's understanding. Teachers teach geometric content based only on techniques, which are conditions that end up restricting their practices.

In fact, the micro-course developed around the $S_{9\ system}$ was based on a more local study of the symmetry content; the development of the $S_9 = \{x_1$ and $x_2, Y, Q4\}$ system did not allow for any further questions to be raised. During this course, the teachers expressed themselves quite a bit. However, we saw that the questions arise from some knowledge that the person has about the content to be studied. In this case, we realized that the teachers had a superficial knowledge of the symmetry content and could not ask about something they did not know. Therefore, the PEP – S_7 and S_9 had the following directions:

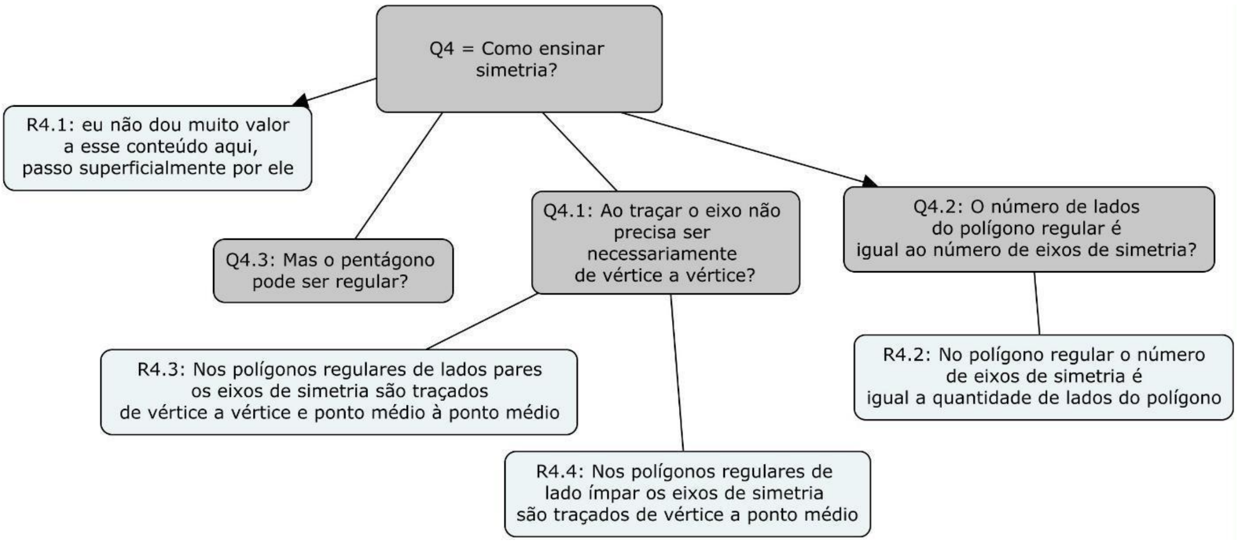

Figure 6

Questions and answers about how symmetry (Paths S_7 and S_9, prepared by the researcher)

In the research by Barquero, Bosch and Gáscon (2011), we can see that, in the development of the PEP, in order to overcome some limitations of the pathways (students' passivity, direction of the pathway completely guided by the advisor), the teachers proposed that, at the end of each session, each student would bring a question to be studied and, at the beginning of the next session, everyone would discuss it together. We believe that, during the development of pathways, there was a lack of time for the teachers to study and research the content studied. Perhaps if we had provided more guidance, for a time dedicated to individual study, the pathways would have had other directions.

Certainly, during the development of the micro-PEP–FP, we found that the praxeological study did not develop as we had hoped. Firstly, we did not have continuous and active participation from the teachers. As previously mentioned, the PEP–FP requires the teacher to assume the 'role of student of the question', to expose his/her doubts and certainties. However, the discontinuity in the participation of the micro-paths meant that the teachers did not build trust in the group, which would allow a more detailed discussion of their praxeologies with their peers. On the contrary, the teachers, such as x_1 and x_2, who had a more continuous participation in the micro-paths, allowed themselves to question and expose their praxeologies around the geometric contents.

During the planning of the micropaths, these have been guided by the paradigm of questioning the world, however, in their execution, we identified that there are some characteristics of this paradigm, but, in general, the didactic systems, mainly the studies of geometric content in which the teachers had more difficulty, focused on paths totally guided by the study advisor, which does not match the proposed paradigm.

During the development of the courses, moments of reflection have been provided on the initial training of teachers, and how much the fully directed teaching they received during their undergraduate studies has an impact on their school practice, which are therefore conditions that reflect on the exercise of the profession. It was also assessed how difficult it is to work in the paradigm of questioning the world, since we are rooted in practices in which the teacher directs all the study developed.

Final considerations

In the study of geometric content, we saw how necessary it is to develop more training sessions with mathematics teachers around these concepts and how the field of numbers and operations prevails in mathematics teaching, restricting the teaching of Geometry. Certainly, given the different teaching systems, we noticed the difficulty teachers have in presenting their OM, which, for the most part, are based on reproductions of techniques without theoretical justification. In general, they present the 'doing' block that is dissociable from the 'know-how' block.

We have also seen how many textbooks are present in teachers' OD. In general, the didactic organizations of the teachers in the research regarding geometric content are around the OD of the textbooks, which reproduce exactly as the book presents it.

In the development of micro-courses, some teachers are used to participating in training courses in which they only 'listen' and, at the end, receive a certificate. These are courses that do not address specific areas, as the teacher mentions about the need to "promote a greater number of specific continuing education courses per area of knowledge" (x_1). The implementation of micro-courses made it possible to break with these types of training courses;

teachers were encouraged to participate all the time and, more than that, they were the protagonists of the training courses that directed which content to study and how to teach.

We can infer that the micro-paths destabilized the participants' OM and OD and, in addition, made it possible to reflect on the school system and the conditions and restrictions imposed on teachers, such as an extensive list of content to be completed, the lack of dialogue between teachers in the same area of knowledge, the lack of support from the coordination for classroom work, limitations of initial training and the daily challenges of school practice. The micro-paths allowed teachers to dialogue not only about geometric content, but also about their concerns about the school environment.

We observed, however, that in the development of the research micro-paths there were approximations with the theoretical/methodological contribution and that the constitution of the subjects involved made it possible to dialogue with the theory in the face of a reality of Brazilian continuing education.

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