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An organization of teaching-learning systems of linear equations in high school, from a developmental perspective: a didactic-training experiment

Una organización de sistema de enseñanza-aprendizaje de ecuaciones lineales en una escuela secundaria, perspectiva del desarrollo: un experimento didáctico-formativo.

Une organisation de l'enseignement-apprentissage des systèmes d'équations linéaires au lycée, dans une perspective développementale : une expérience didactique-formation

Uma organização do ensino-aprendizagem de sistema de equações lineares no Ensino Médio, na perspectiva desenvolvimental: um experimento didático-formativo

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Abstract

When considering the scenario of Brazilian education, at the beginning of the 21st Century, marked by the utilitarian value of school content, and, taking into account the need to define a specific mathematics of teacher action, the research question presented is: How to organize the teaching-learning of Systems of Linear Equations in High School, from a developmental perspective, based on principles of the Historical-Cultural Theory (Vygotsky, 1896-1934) and Study Activity Theory (Davydov, 1930-1968), so as to create conditions for the development of students' theoretical thinking? To this end, this article presents discussions on the conception and execution of a Task, which comprised a didactic proposal, in the context of a didactic-training experiment, with the objective of proposing an organization of the teaching-learning process of Systems of Linear equations, with 2nd year high school students, aiming to develop

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theoretical thinking. Three Study Tasks were created, which constitute the basic unit (cell) of the Study Activity. It is concluded that: 1) the organization of teaching required consistent knowledge of the theoretical assumptions of the Study Activity Theory and the logical and historical aspects of Systems of Linear Equations, fertile theorizations to discuss mathematical knowledge for teaching; 2) the teacher plays a fundamental role in the process as an organizer of teaching and as being responsible for mediation, working in the Zone of Proximal Development – ZDP, which requires teaching-specific mathematical knowledge ; 3) concern with the development of theoretical thinking can lead to the appropriation of concepts, in essence.

Keywords: Theory of study activity, Didactic-formative experiment, Historical-cultural theory, System of linear equations, Teacher's mathematical knowledge.

Resumen

Al considerar el escenario de la educación brasileña, en el inicio del siglo XXI, marcado por el valor utilitario de los contenidos escolares, y teniendo en cuenta la necesidad de definir una Matemática específica de la acción docente, la pregunta de investigación que se presenta es: ¿Cómo organizar la enseñanza-aprendizaje de Sistema de Ecuaciones Lineales en la Escuela Secundaria, desde una perspectiva evolutiva, basada en principios de la Teoría Histórico-Cultural (Vygotsky, 1896-1934) y la Teoría de la Actividad de Estudio (Davidov, 1930-1968), apuntando a crear condiciones para el desarrollo del pensamiento teórico del estudiante? Para ello, este artículo presenta discusiones sobre la concepción y ejecución de una Tarea, que constituyó una propuesta didáctica, en el contexto de un experimento didáctico-formativo, con el objetivo de proponer una organización del proceso de enseñanza-aprendizaje de Sistema de Información. Ecuaciones Lineales, con estudiantes de 2do año de secundaria, con el objetivo de desarrollar el pensamiento teórico. Se crearon tres Tareas de Estudio, que constituyen la unidad básica (célula) de la Actividad de Estudio. Se concluye que: 1) la organización de la enseñanza requirió el conocimiento consistente de los supuestos teóricos de la Teoría de la Actividad de Estudio y los aspectos lógicos e históricos de los Sistema de Ecuaciones Lineales, teorizaciones fértiles para discutir el conocimiento matemático para la enseñanza; 2) el docente juega un papel fundamental en el proceso, como organizador de la enseñanza y como responsable de la mediación, actuando en la Zona de Desarrollo Próximo – ZDP, lo que requiere conocimientos matemáticos propios de la enseñanza; 3) la preocupación por el desarrollo del pensamiento teórico puede conducir a la apropiación de conceptos, en esencia.

Palabras clave: Actividad teórica de estudio, Experimento didáctico-formativo, Teoría histórico-cultural, Sistema de ecuaciones lineales, Conocimientos matemáticos del profesor.

Résumé

En considérant le scénario de l'éducation brésilienne, au début du XXIe siècle, marqué par la valeur utilitaire du contenu scolaire, et en tenant compte de la nécessité de définir une mathématique spécifique de l'action de l'enseignant, la question de recherche présentée est la suivante : Comment organiser l'enseignement-apprentissage des systèmes d'équations linéaires au lycée, dans une perspective développementale, basé sur les principes de la théorie historique et culturelle (Vygotsky, 1896-1934) et de la théorie de l'activité d'étude (Davidov, 1930-1968), visant à créer les conditions d'un le développement de la pensée théorique de l'étudiant ? À cette fin, cet article présente des discussions sur la conception et l'exécution d'une tâche, qui comprenait une proposition didactique, dans le contexte d'une expérience didactique-formation, dans le but de proposer une organisation du processus d'enseignement-apprentissage des systèmes d'information. Equations Linéaires, auprès de lycéens de 2ème année, visant à développer la réflexion théorique. Trois tâches d'étude ont été créées, qui constituent l'unité de base (cellule) de l'activité d'étude. On conclut que : 1) l'organisation de l'enseignement exigeait une connaissance cohérente des hypothèses théoriques de la théorie des activités d'étude et des aspects logiques et historiques des systèmes d'équations linéaires, des théorisations fertiles pour discuter des connaissances mathématiques pour l'enseignement ; 2) l'enseignant joue un rôle fondamental dans le processus, en tant qu'organisateur de l'enseignement et en tant que responsable de la médiation, travaillant dans la Zone de Développement Proximal – ZDP, qui nécessite des connaissances mathématiques spécifiques à l'enseignement ; 3) le souci du développement de la pensée théorique peut conduire, pour l'essentiel, à l'appropriation de concepts.

Mots-clés : Théorie de l'activité d'étude, Expérience didactique et formative, Théorie historico-culturelle, Système d'équations linéaires, Connaissances mathématiques de l'enseignant.

Resumo

Ao considerar o cenário da educação brasileira, no início do Século XXI, marcado pelo valor utilitário dos conteúdos escolares, e, levando-se em conta a necessidade de definir uma Matemática específica da ação do professor, apresenta-se a questão de pesquisa realizada: Como organizar o ensino-aprendizagem de Sistema de Equações Lineares no Ensino Médio, na perspectiva desenvolvimental, a partir de princípios da Teoria Histórico-Cultural (Vigotski, 1896-1934) e da Teoria de Atividade de Estudo (Davidov,1930-1968), visando criar condições para o desenvolvimento do pensamento teórico do aluno? Para tanto, apresenta-se neste artigo discussões sobre a concepção e a realização de uma Tarefa, que compôs uma proposta didática, no contexto de um experimento didático-formativo, com o objetivo de propor uma organização do processo de ensino-aprendizagem de Sistema de Equações Lineares, com alunos da 2ª série do Ensino Médio, visando ao desenvolvimento do pensamento teórico. Foram elaboradas três Tarefas de Estudo, que se constituem na unidade básica (célula) da Atividade de Estudo. Conclui-se que: 1) a organização do ensino exigiu do professor conhecimentos consistentes dos pressupostos teóricos da Teoria da Atividade de Estudo e dos aspectos lógicos e históricos de Sistema de Equações Lineares, teorizações férteis para discutir o conhecimento matemático para o ensino; 2) o professor tem papel fundamental no processo, como organizador do ensino e como responsável pela mediação, atuando na Zona de Desenvolvimento Próximo – ZDP, o que exige conhecimentos matemáticos próprios para o ensino; 3) a preocupação com o desenvolvimento do pensamento teórico pode conduzir à apropriação de conceitos, em sua essência.

palavras-chave: Teoria da atividade de estudo, Experimento didático-formativo, Teoria histórico-cultural, Sistema de equações lineares, Conhecimento matemático do professor.

An organization of teaching-learning systems of linear equations in high school, from a developmental perspective: a didactic-training experiment

Introduction

This article is the result of a doctoral dissertation, whose research is part of the project titled *Conteúdos algébricos no ensino médio: discussões e propostas na perspectiva da teoria histórico cultural* (*Algebraic content in high school: discussions and proposals from the perspective of historical-cultural theory*), approved by FAPEMIG in 2017, APQ-01914-17. The project is coordinated by Professor Dr. Marilene Ribeiro Resende and aims to develop, within high school educational practice, an organization of the teaching-learning process of Systems of Linear Equations, to enhance the development of theoretical thinking, which is essential for human development.

Considering that the organization of teaching-learning requires mathematical knowledge and understanding specifically geared toward this purpose—knowledge that is developed, applied, and discussed in light of consistent theoretical frameworks—this article presents the contributions of the Historical-Cultural Theory, proposed by L. S. Vygotsky and his collaborators in the former Soviet Union, and of Theories derived from it, such as V. V. Davydov's Study Activity Theory, within the perspective of what has come to be known as Developmental Didactics. We present some aspects of these theories and a portion of a didactic-formative experiment carried out with high school students, focusing on the topic of Systems of Linear Equations, which is typically covered at this level of education.

The teaching-learning of Systems of Linear Equations, like other school content, encompasses knowledge that has been historically and socially produced, arising from a range of social and cultural contexts, needs, progressions, contradictions, and setbacks. Thus, when this content is taught based on its logical and historical development, it can foster the development of theoretical thinking and the holistic formation of the individual.

Bringing this contribution to the context of Brazilian high schools—which are increasingly oriented toward technical and utilitarian training—is important for researchers, teachers, and students. The teaching of algebra, and specifically Systems of Linear Equations at this level, cannot be limited to simply finding the "value of x," fulfilling a curriculum component just to pass the academic year or to prepare for an exam or university entrance test. The teaching-learning process must consider that high school students, as they are shaping their personalities, need a humanistic education—which can occur through school subjects such as algebra, becoming significant mediating elements.

From this perspective, this article aims to support studies on the teaching and learning of algebra, grounded in Historical-Cultural Theory and related theories, by presenting the theoretical foundations of these frameworks and part of a didactic-formative experiment.

The text is thus organized into five sections. In the first, we present some contributions from the theories on which the study is based; next, considerations regarding the didactic-formative experiment; in the third, a brief overview of the logical-historical development of Systems of Linear Equations; in the fourth, the description and analysis of Task 3 from the experiment; and finally, the concluding remarks.

Contributions of the Historical-Cultural Theory and the Study Activity Theory

This study is grounded in the dialectical-materialist conception of educational research. It reflects a worldview in which the production of knowledge occurs as a process within the historical movement of society and is realized when it meaningfully transforms concrete and historical reality (Souza, Magalhães & Silveira, 2014).

The teaching of mathematics, guided by the assumptions of Historical-Cultural Theory and its derived theories, such as the Study Activity Theory, leads to the historical appropriation of concepts and recognizes that movement is inherent in the construction of knowledge. It considers that the proposed teaching stems from human necessity and human beings' relationship with the world. Theory and practice form a unified whole, giving meaning/significance to the knowledge being learned (Moura, 2000).

Basing the teaching of algebra on this perspective can provide teachers with a new quality of teaching, as asserted by Sousa, Panossian & Cedro (2014). According to these authors, when teachers understand the logical and historical development of algebra, they can work not only with its end products but also with its processes. They also emphasize that teaching the essence of algebraic concepts, considering the totality of the object, contributes to the development of humanization through knowledge—humanization that emerges from human activity and the advancement of theoretical thinking.

The research conducted focused on the development of students' theoretical thinking, as proposed by authors such as V. V. Davydov, who emphasizes the importance of acquiring theoretical knowledge for intellectual and psychological development. In Davydov's framework (1988), the Study Activity focuses on developing theoretical knowledge (a unified combination of substantive abstraction, generalization, and theoretical concepts) and has a specific structure that contributes to mental and personality development in children. For him, school plays a crucial role in individual formation, as it is through engagement in this environment that children assimilate the most developed forms of *social consciousness*, since the appropriation of school knowledge stems from the activity of study.

Theoretical thinking is formed through the dialectic between concrete reality and concrete thought, in an abstraction process where the particular generates the general, while concrete thought reconstructs concrete reality. "Theoretical thinking reflects the object in terms of its internal relations and laws of movement, knowable through rational processing of empirical knowledge" (Kopnin, 1978, p. 152).

We understand concept formation as an "*authentic and complex act of thinking*" that requires embracing intricate forms of thought. Concept formation is not a mechanical or automatic mental activity; it does not refer merely to a set of associative connections assimilated by memory (Vygotsky, 1934).

According to Davydov (2001), a concept behaves like a mental activity that reproduces the idealized object and its system of relations. The concept acts simultaneously as a reflection of the material object and as a means of mentally reproducing and structuring it—that is, as a special mental action "To have a concept of an object means to be able to mentally reproduce its content, to construct it; the mental action of constructing and transforming the object constitutes the act of its understanding and explanation, the discovery of its essence" (Davydov, 1988, p. 128). In the process of concept formation, mental processes of abstraction and generalization are key. These begin from a concrete given and lead to a concrete thought synthesis of generalizations. This process is dynamic, as each approach to the given object allows for the production of new syntheses.

> In concept formation, analysis plays an enormous role as a movement that starts from the concrete given in sensations and proceeds to the abstract, while synthesis is also essential as a movement from the abstract to a new concrete, which is the set of definitions. The analytical process is inconceivable without induction and deduction. Once formed, the concept implicitly contains, in an original form, all the judgments and deductions that occurred during its formation process. **The concept is the confluence, the synthesis of the most diverse ideas, the result of a long process of knowledge** (Kopnin, 1978, p.191, author's emphasis)**.**

In the appropriation of scientific concepts, Vygotsky (2001) emphasizes the cooperation between teacher and student, where the adult enables the child to develop higher psychological functions. These are a genetically distinct system, composed of heterogeneous structures formed on the basis of historical and social development.

Higher psychological functions, from their most elementary forms to objective reality and mediating activity - through the use of words, are linked to the formation of concepts, which does not happen linearly. Thought processes vary in different phases in a dialectical movement that enables individual development and concept appropriation. Bases on this movement, school-aged children develop voluntary attention and logical memory – higher psychological functions, alongside abstraction, generalization and theoretical thinking. The student's awareness of these higher functions intellectualizes them, promoting their development.

Collaboration with others is the source of individual development. To Vygotsky (2001, p. 270), "[...] when we apply the principle of collaboration to establish the zone of proximal development, we gain the possibility of directly investigating the most determining factor of intellectual maturation, which will culminate in the next and subsequent stages of development".

Teaching and pedagogical practice have the potential to foster student development. When the teacher teaches and/or proposes a collaborative group activity involving immature processes—those not yet fully mastered but nearing maturation—they are working in the Zone of Proximal Development (ZPD), which is:

> [...] the distance between the actual level of development, which is usually determined by independent problem-solving, and the level of potential development, determined by problem-solving under adult guidance or in collaboration with more capable peers (Vygotsky, 2001, p.97).

Regarding collaboration, Vygotsky (2008) views mediation as a contributing factor to rational and intentional transmission of human experience and thought. Mediation allows for understanding and connection between the parts that make up a whole – supporting process maturation. Its absence results in a fragmented whole.

From Vygotsky's work (2001, 2008), it can be inferred that mediating activity relates to the teacher's role in school, particularly regarding the planning, preparation, organization, and development of a lesson; the attributes of teaching; the formation of concepts; the means of instruction; the artifacts of material and spiritual culture; technologies; and other essential aspects aimed at promoting the student's development.

The teacher, then, serves as the organizer of mediating elements during student learning, organizing content and guiding the teaching-learning process. They act, intentionally and consciously, in the ZDP, enhancing student development. Since teaching occurs jointly with learning, mediating activity contributes to the development of student learning. In this organization process, the teacher needs to understand the essence of mathematical concepts, beyond representations and external features.

Research on teaching and learning requires a unit of analysis that expresses or encompasses an individual’s sociocultural experiences. Students and teachers, upon entering a classroom, do so as human beings, carrying with them desires, doubts, expectations, and cultural experiences from the social group to which they belong. They also hold a particular worldview. Additionally, the classroom itself is already socially typified as a specific space.

Socialization allows children to appropriate empirical manifestations, to understand rules and behaviors. Gradually, they observe the concrete, analyze the material, and interpret it logically to begin the process of abstraction. The relationship between mediating activity, communication, and an individual’s sociocultural experiences does not occur solely at the biological or cognitive level. The objective reality present in society influences communication and, consequently, also the teaching-learning processes.

Thus, this study is based on discussions concerning the assumptions of the Study Activity Theory, which establishes a set of principles and actions that guide the process of concept formation based on substantive abstractions and generalizations. It considers that both content and its forms influence the learning process and the development of theoretical thinking. By appropriating content, the student tends to develop theoretical thought, promoting reflection, analysis, and mental experimentation, as stated by Davydov (1988).

The concept of "activity" is fundamental in Soviet psychology and has its origins in dialectical materialism and the discovery of universal schemes of human activity. Therefore, it must be understood in a broader sense, since “[...] dialectical logic studies and describes historically significant and universal forms of practical and mental activity of people, forms that underlie the development of all material and spiritual culture in society” (Davydov, 1988, p. 22, our translation). This author states that activity can manifest in different forms: emotional communication, object manipulation, play, study, and socially useful activities—such as work. The Study Activity Theory, developed in the 20th century by Soviet pedagogues and psychologists as a continuation of the ideas of Vygotsky, Leontiev, Luria, and other collaborators, highlights the transformation of students through study activity. This involves not only mental development, which encompasses the appropriation of knowledge and skills, but also moral and personal development—what we refer to as integral development.

The essence of the Study Activity concept "[...] consists of the desire to analyze the transition from activity to its 'subjective product'—in the analysis of new formations, qualitative changes in the child's psyche, and their intellectual and moral development" (Davydov, 2019, p. 194). This involves appropriation—meaning the processes of assimilation that move from the interpsychological to the intrapsychological, as proposed by Vygotsky.

Study activity includes a conceptual apparatus shared with Soviet pedagogical psychology, according to Davydov & Márkova (1987), but this theory advanced as experiments were conducted. Thus, to clarify the fundamental assumptions of Study Activity, it is important to understand the concepts of "assimilation," "development," and "learning" in this theory.

The concept of *assimilation* or *appropriation* is fundamental because, according to Davydov, the student subjectively reproduces the historical-social experience of humanity. According to Davydov (2019, p. 196):

> Assimilation is the process of reproducing the modes of action formed by the individual during the historical process of transforming objects and surrounding reality, their types of relationships, and the process of converting these social patterns into forms of individual subjectivity.

It is not, therefore, a passive reproduction, but rather the student's active engagement in appropriating and making this experience—present in culture and science—personal and meaningful. This assimilation does not occur only in the school setting but also in other types of activity. However, "it seems that only in study does the specific objective of assimilation appear, whereas in other types of activity, assimilation is a derived product" (Davydov & Márkova, 1987, p. 323), because in study activity, there is intentionality. The school proposes suitable (though not identical) activities to those found in scientific fields, aiming to assimilate the human historical-social experience, in which generic human activity is embodied. In this sense, it is necessary to organize teaching in a way that requires teachers to have specific knowledge—not only about the subject they teach but also about how to make it teachable.

In correlating assimilation and development, Davydov (2019, p.196) states: "[...] development occurs through the assimilation (appropriation) of the individual of historical-social experience". Ressalta, ainda, que esses processos não são independentes, no entanto, a assimilação pode não promover, necessariamente, o desenvolvimento. He further emphasizes that these processes are not independent, although assimilation does not necessarily promote development. In this context, Davydov (2019) presents developmental learning as the way of organizing assimilation under the historical and concrete conditions defined by society. Furthermore, he stresses that it is in Study Activity that the assimilation of socially elaborated experience is developed.

Regarding study activity and its structure, Elkonin (1961), one of the forerunners of this theory, affirms that the Study Task is its basic unit. According to him, the Study Task modifies the subject of the activity, transforming and developing them. This transformation involves new modes of action with scientific knowledge.

In the research conducted, which involved teaching and learning algebraic knowledge, we sought to provide a study activity that went beyond the appropriation of mathematical techniques and procedures, aiming also at the self-transformation of the individual in their human actions. According to Davydov (2019, p. 199): "The main content of Study Activity is the assimilation of generalized modes of action in the sphere of scientific concepts and the qualitative changes in psychic development [...]".

Study Activity has a structure, which is presented by Davydov (2019, pp. 199–200) as follows:

> 1. *The student's understanding of the Study Tasks*. The Study Task is closely linked to substantive (theoretical) generalization. Its goal is to lead the student to master generalized relationships in the knowledge field and stimulate them to adopt new modes of action. The acceptance of the Study Task by the student "for themselves" and their autonomous formulation is closely related to their motivation to learn, and to the transformation of the child into the subject of the activity.
> 2. *Execution of study actions by the student* – with proper organization of the learning process, aiming to reveal general relationships, guiding principles, key ideas in a given field of knowledge, to model these relationships, to master the transitions from universal relationships to their concreteness and vice versa, and to transition between the model and the object and vice versa.
> 3. Execution, by the student themselves, of *control and evaluation actions*. (author's emphasis)

Thus, study activity is structured around Study Tasks that require specific actions from the student, which are carried out through operations. During a school task, students must identify the essence of the object, based on abstraction, to appropriate theoretical knowledge. The tasks consider the analysis of particular situations, allowing the general characteristic to emerge within specific contexts. They must offer autonomy to students, enabling them to develop actions toward concept formation.

One significant component of Study Activity is the system of particular actions required to solve the tasks. Learning, therefore, is produced through the execution of the following actions: 1) transformation of task data to discover the universal relationship of the object, which should be reflected in the corresponding theoretical concept; 2) modeling the universal relationship in object form, graphically or with letters; 3) transforming the model to study the property of the universal relationship identified in the object; 4) constructing a system of particular Tasks that can be solved using a general procedure; 5) Controlling the execution of the previous actions; 6) evaluating the assimilation of the general procedure as a result of solving the given learning Task. (Davydov, 1988, p. 174).

The organization of content—how it is taught—is of fundamental importance in the student's learning development. If school tasks proposed by teachers deal only with content and exercises that students already know and lack proper organization, what new concepts will these students actually form? Teachers must challenge their students through well-structured content, creating motives that trigger the need to learn new concepts, acting within their zone of proximal development.

To organize the assimilation of scientific content, one must understand its essence, that is, its history. Vygotsky (1934/2001) emphasized that concept development cannot occur through the assimilation of something finished, static, and devoid of meaning. It is therefore essential to know and master the content to be taught, understanding its logical-historical development, since its essence should guide the organization of teaching.

Teaching and learning actions pave the way for concept formation. They start with the first contact with the object in question, identifying its general characteristics, highlighting similarities and differences. Then comes modeling, an action that enables the understanding of the object beyond appearances, analyzing internal features and establishing relationships among its parts. The model allows investigation of the object in its "pure" form, through substantive abstraction, deduction, and generalization. Identifying the general concept enables the solution of the Task.

For example, in the study of Systems of Linear Equations, solving the equations (unknowns) is considered the main general relationship of this topic. When a student learns the methods to solve a system of equations, they are developing their substantive abstraction; by identifying that the solution to the system is also valid for each of the individual equations that make up the system, substantive generalization is established. In other words, to understand a system of equations—as a whole—it is necessary to generalize the result to the equations, which are the parts, and this stems from substantive abstraction. This expresses the essence of a system: the simultaneity of the solution of the equations that comprise it. The solution to the system is also a solution to each of the equations, if such a solution exists. Thus, we perceive the fluid movement from the general to the particular—the system to the equations—and from the particular to the general—the equations to the system. The internal links in this movement are the presence of unknowns that must satisfy certain conditions present in the equations, whose solution must simultaneously satisfy all of them.

In this process, the teacher must monitor students' actions, mediating in order to enhance learning and address difficulties. Assessment plays a crucial role in the success of the proposed task. It shows not only whether the concept was assimilated but also reveals the process as a whole—errors and successes alike. It enables the teacher to reflect on the task, its organization, and their interventions. Assessment helps the teacher rethink the teaching-learning processes and improve their planning and practices.

Didactic-Formative Experiment: A Proposal for Research and Teaching

The *Study Activity* uses as its main method the formative experiment, which is considered the most appropriate in developmental psychology and pedagogy, according to Davydov & Márkova (1987). According to these authors, this method has its origins in the ideas of L. Vygotsky's genetic-experimental method and was expanded through the work of A. Leontiev, A. Luria, P. Galperin, A. Zaporozhets, and D. Elkonin. The essence of this method lies in creating conditions for psychological transformations and development in the learner. It was applied in experimental school programs in the former Soviet Union during the second half of the last century.

In Brazil, research in the field of developmental didactics has employed the didactic-formative experiment, although not in the same format as the long-term, multidisciplinary team projects conducted by Soviet pioneers. Here, experiments are proposed within the realistic conditions of our educational context, aiming to introduce new methodologies and resources. Most importantly, they seek to assess to what extent a teaching organization based on the assumptions of these theories can lead to the appropriation of concepts and action procedures within a specific school subject, with a view to the student's integral development. This is a method of both didactic and psychological investigation. Thus, we can affirm that carrying out a didactic-formative experiment, based on the assumptions of the Study Activity Theory, requires teachers to have knowledge not only of the logical and historical aspects of the content they teach but also of the pedagogical and didactic elements involved in the organization of teaching. This approach is based on the internal connections of concepts, not merely their external relations.

Aquino (2017) emphasizes that, in current Brazilian research, the didactic-formative experiment is aimed at enhancing students' learning and intellectual, physical, and emotional development, and proposes the following stages for its implementation: 1st) Literature review and diagnosis of reality 2nd) Development of the experimental didactic system; 3rd) Development of the didactic-formative experiment; 4th) Data analysis and report writing.

The research[3] we conducted, a didactic-formative experiment on the teaching of Systems of Linear Equations in High School, based on the assumptions of Davydov's Study Activity Theory—was carried out in a private school in Uberaba, Minas Gerais, during after-school hours for students in the second year of high school, since this is the year in which Systems of Linear Equations are typically taught. The experiment lasted approximately two months.

We emphasize that six students fully participated in the experiment. In the dialogues, the students are identified by the pseudonyms: Amanda, Nathália, Beatriz, Gabriel, Miguel, and César.

To develop the didactic-formative experiment, Study Tasks were created based on the logical-historical development of the content *Systems of Linear Equations*, as well as a literature review and document analysis conducted as part of the study. Contributions from high school teachers at the participating school were also incorporated.

The experiment was structured into three Study Tasks, each aiming at developing the essential actions and operations necessary for forming the concept of System of Linear Equations. The three Tasks proposed in the didactic-formative experiment were: Task 1: Introduces comparison as a necessary human action, which leads to the concept of equality and inequality; Task 2: Addresses the concept of the equation and its judgments; Task 3: Aims to contribute to the formation of the concept of System of Linear Equations.

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[3] The project was submitted to the CEP on December 13, 2017, and approved on the 21st of the same month.

Each Task is treated as a unit of analysis. In this article, we discuss the results of Task 3, focusing on the following aspects: 1) The transformation of Task data in guiding students to identify the general principle of the concept of System of Linear Equations; 2) From modeling to transforming a model into the concept of linear equations; 3) The appropriation of the System of Linear Equations concept as a tool of theoretical thinking—control and evaluation.

Before moving on to the analysis of Task 3, we present a concise overview of the logical-historical development of the content to be addressed—System of Linear Equations—as this is fundamental when working with concept formation from this perspective.

Systems of Linear Equations and Their Logical-Historical Development: An Overview

When analyzing the historical development of algebra, we find that equations are a fundamental component of this area of mathematics. Regarding our object of study—Systems of Linear Equations—some textbooks offer the following definition by Iezzi & Hazzan (1985, p.116): "It is a set of m (m1) linear equations, in the unknowns $x\geq_1$, x_2, x_3, ..., x_n". Thus, the system is linear, if it is of the type:

$$S\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n = b_3 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n = b_m \end{cases}$$

We draw attention to this modern definition of a system of linear equations, which is the result of a historical and logical process through which mathematical knowledge developed. However, this process is often not made explicit to students, who may encounter the definition without grasping its meaning or significance.

The development of knowledge about systems of linear equations, along with linear algebra, can be traced back to Babylonian mathematics, with records of problems involving two equations with two unknowns as far back as 4,000 years ago. The societal advances in the Babylonian region—particularly in agriculture, trade, and administration—also contributed to refining their methods for solving mathematical problems. According to Eves (2011), the Babylonians used calculation tablets to solve quadratic, cubic, and biquadratic equations.

According to Santos and Zuin (2017), Babylonian solutions displayed a form of rhetorical algebra, using ordinary language to express mathematical relationships. Many of the problems reflected everyday life and were solved using what we now call systems of two equations with two unknowns, employing methods we recognize today as substitution and change of variables. This rhetorical algebra relied on language tied to numbers, but as Panossian (2008) points out, there was a clear difficulty in using words—without symbols—to represent unknown quantities.

The Chinese also made significant contributions to the development of systems of linear equations. Around 200 BCE, they were solving systems of two and three equations, and their method of solving these systems later served as a basis for the development of Gaussian elimination. This method—also known as row reduction—is both elementary and time-tested, and remains widely used today. It is a highly efficient approach for solving linear systems, based on the fact that every system can be reduced to a row-echelon form. As Lima et al. (2006, p.135) describe:

> [...] An equivalent system in row-echelon form is obtained through a sequence of elementary operations, which are as follows: (1) Swapping the order of the equations in the system; (2) Replacing an equation in the system with the sum of itself and a multiple of another equation in the same system (Lima et al, 2006, p.135).

By applying these elementary operations, we obtain an equivalent system. This technique is similar to what we now call elimination by addition (Boyer, 2003).

The Indians used a rhetorical method for solving systems of equations—multiple equations with multiple unknowns. Their method of resolution could be classified as *Elimination* (Eves, 2004). In Greece, geometric linear problems were identified whose resolution process required the use of systems of linear equations (Eves, 2004).

As previously discussed, Arabic mathematics made significant contributions to algebra. Eves (2004) points out that during this period, problem-solving methods involving systems of equations with various unknowns were developed, and even undetermined solutions were considered.

The modern study of systems of linear equations may have originated with the German mathematician Leibniz around 1693, through the introduction of the concept of the determinant as a contribution to solving such systems. This leads us to reflect on the organization of knowledge for the teaching-learning process, since the most efficient way to teach a concept does not necessarily align with its historical development.

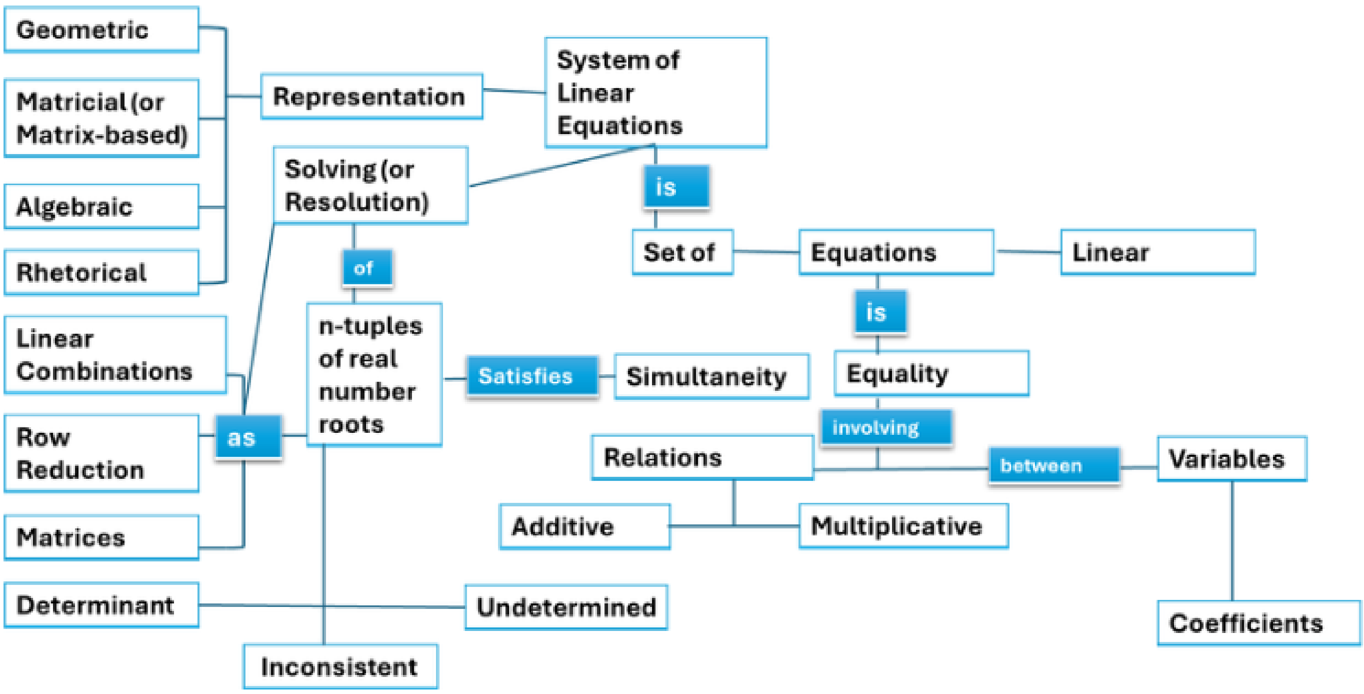
Regarding the teaching of systems of linear equations in basic education, few studies have evaluated the current reality of how this content is taught. Classroom observations have shown that the emphasis is often placed solely on the resolution method.

The content of Systems of Linear Equations is characterized as a topic of practical interest. This characteristic supports what is proposed in the curricular documents, which advocate the need for school content to present relevance to students' daily lives.

By analyzing the historical development and the logical definition of systems of linear equations, we can establish connections between the elements illustrated in the following Figure.

Figure 1

Elements and/or words that characterize the System of Linear Equations (Research data).*



The concept of System of Linear Equations is closely related to the concept of equation, which is its essence. In the teaching-learning processes, as well as in the historical development of this knowledge, the systematization and/or the method of solving a linear system is highlighted, appearing to be its main characteristic.

We understand that the process of solving a system and, notably, the use of algorithms to find the value of a given unknown does not reflect the essence of the concept of System of Linear Equations. To understand the essence of this knowledge, we would need to identify and clarify what directions and/or properties are considered in the process of solving a System of Linear Equations.

In the presented Figure, we highlight that a set of equations and the interrelationship between them constitute a system. And the process of solving a system must consider that the value of each unknown to be found must simultaneously satisfy the "conditions" of all the equations that make up the system.

Thus, the appropriation of the concept of System of Linear Equations presupposes the assimilation of the concept of equations and has as a characteristic a method of resolution that allows one to find simultaneous solutions for a set of equations. Therefore, a linear system may have a single solution – determined; an infinite number of solutions – undetermined; or no solution – inconsistent. A system is classified as inconsistent when the information provided to find the unknowns is incompatible.

Certain situations involving Systems of Linear Equations can also be interpreted graphically/geometrically. For example, a system of two equations has as a solution for each of them the coordinates (x, y) of the points on a line. The system is undetermined, inconsistent, or determined, if the lines represented by the equations coincide, are parallel, or are intersecting, respectively.

Thus, in the necessary movement for the appropriation of the concept of System of Linear Equations, the study of equations and their logical-historical development is also essential.

System of Linear Equations: Conducting the Experiment

Task 3, chosen for presentation, had as its main goal the development of judgments and concepts related to the topic of Systems of Linear Equations. Considering the previously completed Tasks—focused on comparisons and equations—and based on the Study Activity Theory, a more consistent performance from students was expected from the outset.

The organization of this task, as described in Table 1, aimed to address the need for a deeper study of Systems of Linear Equations by establishing connections with students' everyday experiences. The proposal included conceptual discussions on the definition of a System of Linear Equations, involving the formation of fundamental judgments and concepts; the development of algebraic and geometric modeling supported by videos and digital technologies; the study of equivalent systems and their properties; and the classification, analysis, and resolution of problem situations involving Systems of Linear equations. In this way, the proposed Task required students to engage in different types of actions that contributed to the formation of the concept under study, grounded in theoretical thinking. Such Tasks, therefore, differ from traditional classroom approaches, which tend to emphasize solution methods at the expense of discussions about the underlying concepts.

Table 1

Organization of the Tasks on Judgments and the Concept of Systems of Linear Equations (Research Data).

The study of Systems of Linear Equations in High school. Task 3: Developing the concept of a system of equations		
Task	**Actions**	**Objective**
3.1, 3.2 and 3.3: What future profession do you intend to pursue? Introduction to the idea of Systems of Linear Equations.	Finding an unknown “value(s)” that meets certain conditions, aiming at problem-solving	Introduce the "concept" of a System of Linear Equations, as knowledge that enables students to solve a problem whose solution essentially involves responding simultaneously to given conditions. Verify the Transformation of the study Task data to discover the universal relationship of the object, which should be reflected in the corresponding theoretical concept.
3.4: Giving meaning to the concept/definition of System of Linear Equations	Establish the general relationships – and the essence, of the judgments and concepts related to a System of Linear Equations.	Translate the meaning of System of Linear Equations and the meaning of the elements that constitute its definition.
3.5: Algebraic Resolution of a System of Linear Equations	Algebraically model problem situations using Systems of Linear Equations.	Verify the ability to use algebraic language to represent everyday situations. Verify how students solve a System of Linear Equations, making sense of the operations performed and identifying potential difficulties.
3.6, 3.7, 3.8, 3.9 and 3.10: Equivalent Systems	Understand the properties involved in solving Systems of Linear Equations.	Develop students' theoretical thinking in the process of solving Systems of Linear Equations.
3.11 and 3.12 Geometric Interpretation of Systems of Linear Equations	Interpret Systems of Linear Equations represented geometrically.	Check the student's ability to algebraically interpret a geometric representation.
3.13, 3.14 and 3.15 - Discussion on Systems of Linear Equations	Understand the classifications and properties of Systems of Linear Equations considering their	Check whether students can apply the studied concept – its general relationship, in specific situations/problems

	different forms of representation.	
3.16, 3.17 and 3.18 Problem situation involving Systems of Linear Equations	Mobilize judgments and the concept of Systems of Linear Equations to solve a given problem situation.	Check whether students can apply the studied concept – its general relationship, in specific situations/problems

Source: Prepared by the author

We began the first task on Systems of Linear Equations by proposing that students read a news article[4] about how to choose a future profession. High school students, who are in the process of forming their identities, often have doubts about which "career path" to follow after completing basic education. The article sparked discussions about the criteria and conditions they would need to consider in order to "choose" a future profession in a way that would bring them satisfaction.

This task demonstrates the possibility of fostering discussions in Math classes that go beyond using Mathematics for its own sake, beyond memorizing and applying formulas. The social role of the teacher — who teaches various subjects, including Mathematics — involves promoting human development through the teaching of school content.

Thus, the discussions with students about choosing a profession were very enriching, as they shared their interests, family expectations, peer influence, and more. Some students were unaware of all the courses offered by universities and colleges in their regions. Others didn't know the responsibilities of certain professions, or how to access higher education, among other aspects

Figure 2

Task 3.1, with responses from student Beatriz (Research Data).

Tarefa 3 – Sistemas de Equações Lineares
Task 3 – Linear Equation Systems

1. What is the future profession you think of pursuing? ARCHITECTURE
2. Read the attached report and discuss with your classmates the proposed questions.

Link to the Report:
https://guiadoestudante.abril.com.br/orientacao-profissional/como-escolher-o-curso-que-melhor-se-encaixa-em-seu-perfil/

[4] News article available at: https://guiadoestudante.abril.com.br/orientacao-profissional/como-escolher-o-curso-que-melhor-se-encaixa-em-seu-perfil/. Access: Feb. 13, 2020.

Choosing a future profession is not simple.
To do well, it is necessary to weigh a series of factors, such as affinities, skills, well-being, financial return, among others.
Based on the report, we developed the following diagram to help you think about your future profession.
To find the "ideal" profession, it is important that "the answer" meets all the conditions presented.

ABOUT THE PROFESSION YOU THINK OF PURSUING:
→ What areas of knowledge do I most like studying?
→ In which professions could I use the skills I already have?
→ Which universities do I intend to study at? What courses do they offer?
→ In what places, companies, and positions could I apply the knowledge acquired in college?
Profession
ABOUT THE PROFESSION YOU THINK OF PURSUING:
– Condition 1
– Condition 2
– Condition 3
– Condition 4
Thus, answer:
a) What profession do you intend to pursue? What conditions made it possible for you to decide on it?
ARCHITECTURE

b) Considering your answers to the initial problem conditions, among the possible solutions presented below, which profession would be the most suitable? Justify.

Professions
Lawyer
Doctor
Engineer

(*handwritten note:* **Maybe**)

c) Nathalia intends to choose her profession by following the guidance from the *Guia do Estudante* (Student Guide) report.

Courses
Medicine
Nursing
Physiotherapy
Occupational Therapy
Dentistry

(*handwritten note:* **All**)

d) Gabriel likes the exact sciences and is skilled in computer language. He wants to study at the same university as Nathalia. Under these conditions, which of these courses meets his interests? Justify.

Courses
Medicine
Nursing
Physiotherapy
Occupational Therapy
Dentistry

Regarding the students' discussions about the task, we observed that some of them were already certain about the path they wanted to follow in higher education. Beatriz wanted to study Architecture; Amanda and Nathalia, Medicine; Miguel wanted to pursue some kind of Engineering, though he wasn't sure yet which one he preferred; César and Gabriel were more undecided. During the discussion, there was a lot of interaction among the students, where they explained why they would like to pursue a certain profession in the future. They also sought to clarify doubts about universities and courses with the teacher.

They analyzed and considered certain conditions for choosing a profession, such as whether or not to move to another city, whether to attend a public or private institution. For example, they noted that students who liked Mathematics tended to pursue some type of engineering, while those who enjoyed Biology would opt for a health-related field.

In Beatriz's response to Task item 3.1, among the course options presented, the only one she might consider taking was Engineering, since she enjoys the exact sciences.

In item *(c),* in the problem situation presented, the students indicated that all the courses could be chosen, considering the given conditions. In this context, the teacher asked whether it was possible to relate that situation to a System of Linear Equations. The students were unsure, so the teacher guided them to think about the possible solutions of a linear system. Amanda then observed that the given problem situation could have more than one solution, just like Systems of Linear Equations that can have infinitely many solutions. In item (d), the students had no difficulty identifying that the problem had no solution. Gabriel quickly made the connection with Systems of Linear Equations — which can also have no solution, and are thus classified as "inconsistent."

Figure 3

Task 3.2 (Research data).

a) What is the general characteristic of this problem? b) What is the relationship between the previous problem and the topic of linear equation systems?

Task 3.2 continued the discussions around the problem situation presented and Systems of Linear Equations. With the teacher's guidance, the class was able to relate the discussion about which profession they intended to pursue to a problem situation to be solved — for example, as a System of Linear Equations. The students reasoned that, to solve the problem, they would need to analyze certain conditions that could influence the outcome. To do this, they would have to find a solution that simultaneously satisfies all the conditions presented.

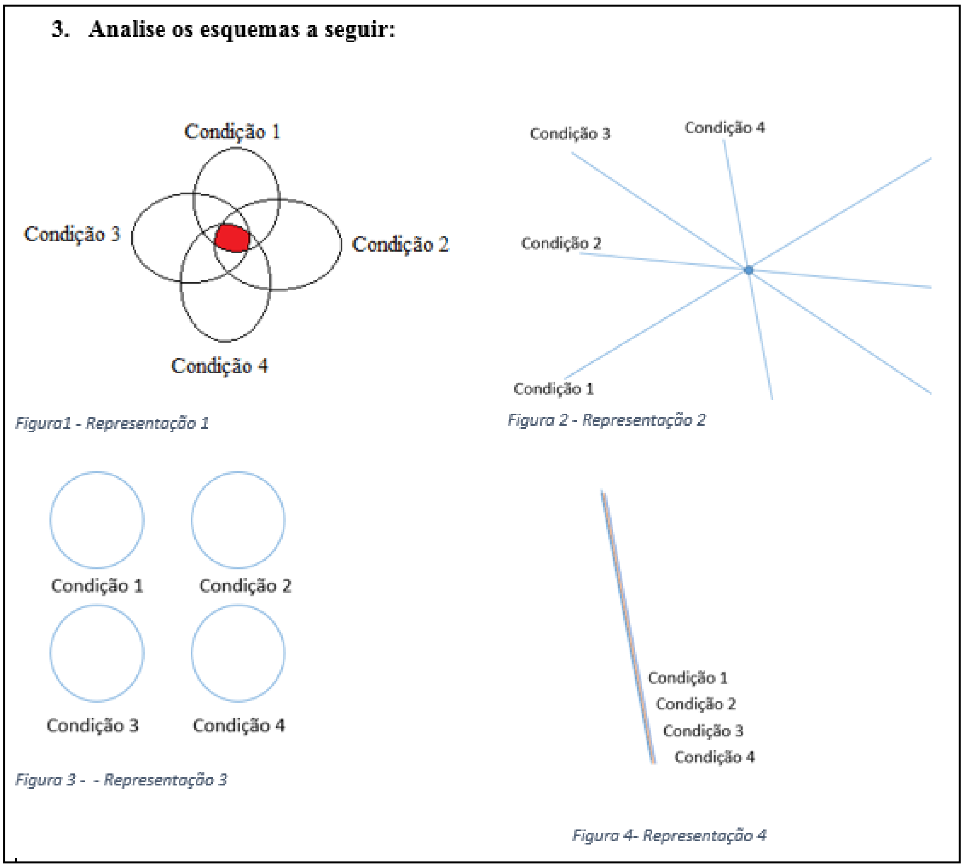
For instance, the convergence of answers to questions such as: "Which field of knowledge do I want to study?", "In which professions will I use the skills I already have?", "At which universities do I want to study?", and "In which locations or companies could I work?" — would point to a suitable career path. Miguel commented: "I want to study something in the exact sciences field, I like Math, I'm willing to move to another city, I want to work for a big company; those are my conditions […] so the solution to my problem is to study engineering, but I still don't know which kind." Amanda noted that the general idea was to find a single "answer" that matched all the questions (conditions). Gabriel remarked: "Why haven't we learned this way before?"

The teacher then asked: "What is the relationship between Systems of Linear Equations and this discussion?" The students responded that the situation is similar to a System of Linear Equations because each condition in the situation resembles an equation, and the set of conditions would represent a system. In order to solve both the problem situation and the system of equations, it would be necessary to find a common solution that satisfies all conditions/equations. Nathalia reflected: "Just like in the task we did, systems can have more than one solution, a single solution, or no solution at all — that's how the system relates to the problem."

At the teacher's request, the students began Task 3.3. In Figure 4, this task is presented, illustrating how the conditions of a problem can be represented using diagrams. Following that, the students were asked to make connections in order to standardize the perspective from which the article is written.

Figure 4

Task 3.3 (Research data).



Translation:

Figure 1 – Representation 1: Condition 1, Condition 2, Condition 3, Condition 4 (four overlapping circles with a red area in the center).

Figure 2 – Representation 2: Condition 1, Condition 2, Condition 3, Condition 4 (four lines intersecting at one point).

Figure 3 – Representation 3: Condition 1, Condition 2, Condition 3, Condition 4 (four separate circles with no intersections).

Figure 4 – Representation 4: Condition 1, Condition 2, Condition 3, Condition 4 (four nearly parallel lines).

Reply:

a) What relationships can be established among these figures — that is, what is general and what is specific?

b) What is the relationship between them and the problems from question 1? Justify.

c) Establish relationships between these figures and systems of linear equations. What are their similarities? What characterizes these relationships? Justify.

Continuing with the task, the students had no difficulty interpreting the diagrams and relating them to Systems of Linear Equations. In Figures 1 and 2 (from the task), they observed that the representations had only one point in common and associated this with a single solution. However, they did not recall the correct classification —

consistent and determined. In Figure 3, they noted that the “conditions” had no relationship, which could indicate a system with no solution — the students remembered that such systems are classified as inconsistent. In Figure 4, the students observed that the lines overlapped, indicating that the diagram could represent a system with many solutions — although they did not recall that this type of system is classified as *consistent and undetermined*.

It was observed that these tasks, with their variety of questions, analyses, and reflections, helped students understand that both representation and language — in this case, algebraic language — play a fundamental role in the transformation of images (Davydov, 1988).

In Task 3.4, the students were presented with a formal definition of a System of Linear Equations, and a discussion was proposed about the elements that compose such systems.

Figure 4

Task 3.4 (Research Data).

4. Giving meaning to the concept/definition of a system of linear equations.

Analyze this representation of a system of linear equations in its general form:

$$S\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots + a_{3n}x_n = b_3 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n = b_m \end{cases}$$

Discuss with your colleagues and record:

a) What does this definition have to do with the previously studied activities?

b) What are the meanings of *n* and *m* in this representation?

c) What are the meanings of a_{11}, a_{12}, a_{13}, and a_{1n}?

d) How many equations make up this system?

e) What are the meanings of x_1, x_2, x_3, and x_n? How do these letters behave/repeat in this representation?

f) What are the similarities and differences between this system and its elements $\begin{cases} ax + by = r \\ cx + dy = s \end{cases}$ compared with the one presented earlier?

g) What does each equation express in the system?

h) What is the relationship between the equations and the solution of the system? What does the relationship among the equations express in the system?

i) What mathematical properties are essential in the process of solving a system of equations? Justify.

The tasks on comparisons and equations (not addressed in this article), especially those involving the discussion of the definition of equations with the students, contributed significantly to their ability to respond to the questions presented. Gabriel commented: "Wow, teacher, this “Task” is similar to the one on equations." The teacher confirmed that the two were indeed similar, as both involved formal definitions — generalizations — but while one was about equations, this one referred specifically to Systems of Linear Equations.

Regarding the relationship between this definition and the previous Tasks, César responded: "Since it's the definition of a System of Linear Equations, it's completely related to the previous Task, because we had already made that connection between the problem and the system." The teacher then asked: "But in what specific aspects can we make that relationship explicit?" Beatriz replied: "The equations define the conditions for the result of the problem." The following dialogue took place:

> **Teacher**: What does this activity have to do with the ones we've worked on previously?
> **César**: Each condition is an equation, and within that condition there can be several variables — all the conditions together form the solution.
> **Teacher**: Imagine that career-choice problem as a system — does that make sense now?
> **Miguel**: Each question would be an equation/condition of the system.
> **Teacher:** With just the definition, would you be able to establish a relationship with the problem?
> **Miguel**: No, after the career-choice activity, it became easier.
> *(Research Data)*.

The teacher then explained to the students that the definition represented the general, formalized form of a System of Linear Equations. This general definition could be used to construct a System with two equations and two unknowns — or even systems with infinite equations and unknown variables.

Discussions continued regarding the definition of a System of Linear Equations:

> **Teacher:** What do *n* and *m* mean?
> **Gabriel**: *n* is the number of variables and *m* is the number of coefficients.
> **Teacher**: How many equations are in this system?
> **Gabriel**: There are *n* equations because there are many equations.
> **Teacher**: Why is x_1 repeated in all rows?
> **Miguel:** Because the unknown has to be the same.
> **Teacher**: When there's no unknown, what is the value of the coefficient?
> **Miguel:** Zero
> *(Research Data).*

Furthermore, César understood that "$a_{11}, a_{12}, a_{13} e\ a_{1n}$" represented the values of the coefficients multiplying the unknowns and that they could be different.

> **César**: *a* is the number that multiplies *x*, which is the unknown, and *b* is the result of the equation.
> **Miguel**: One question, why is it written as a_{11}, a_{12}, a_{13}, and not just a_1, a_2, a_3?

Teacher: Because it's a representation that can be arranged in the form of a matrix, for example – first row; first column [...].
(Research Data).

The teacher also asked if the values $(a_{11}, a_{12}, a_{13} e\ a_{1n})$ could be equal. Miguel said: "I think so, like 2x +2y".

In this process, we observed the development of substantive abstraction and generalization, which are fundamental for the development of theoretical thinking, "[...] the Task is a unit between the goal (objective) and the conditions for achieving it" (Davydov, 2019, p. 293).

Regarding the similarities and differences between the general definition of a system and the one presented in item *(f)* - $\begin{cases} ax + by = r \\ cx + dy = s \end{cases}$, Beatriz responded: " one (the genral definition) represents *n* conditions and the other (example in item f) only point to two". Thus, the dialogue and discussions about the Tasks and the definition of a System of Linear Equations continued.

Teacher: Do you think the class is getting repetitive?
Miguel: A little. Because it seems like it's the same questions.
Teacher: Yes. But earlier, what topic were we dealing with?
César: Comparisons and Equations
Teacher: Are these the same questions as for equations? The same content? If not, why does it feel repetitive?
Students: Because the system is a set of equations.
Teacher: So, if you understand equations well, the chance of understanding systems well is high. That's why equations come before systems.
(Students: Agreed)
Teacher: What does each equation express in a system??
Beatriz: A condition.
Teacher: What is the relationship between the equations and the solution to the system?
Amanda: They both have a simultaneous result.
Teacher: Why is it important to understand the most general formula?
Amanda: Because it's from that one that the more specific ones come.
(Research Data).

We observed that the Tasks carried out up to this point in the discussions began to contribute to the development of theoretical thinking. According to Aquino and Rodrigues (2019, p. 256), based on Engels[5] (1974), "[...] theoretical thinking is a

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[5] ENGEL, F. Viejo prólogo para el [Anti-]Dühring. Sobre la dialéctica. In: **Carlos Marx y Federico Engels. Obras Escogidas (en tres tomos)**. T. III, Moscú: Progreso, 1974, p. 57-65).

capacity that is cultivated through theoretical abstractions that express the synthesis of knowledge of the reality of life”.

In this sense, we consider that this discussion on judgments and the concept of a System of Linear Equations — which started with the analysis of concrete reality through comparisons and is now advancing toward synthesized representations — suggests that the development of scientific concepts is essential for the formation of thinking.

Davydov (1988, p. 164) highlights the basis related to the assimilation of scientific knowledge,

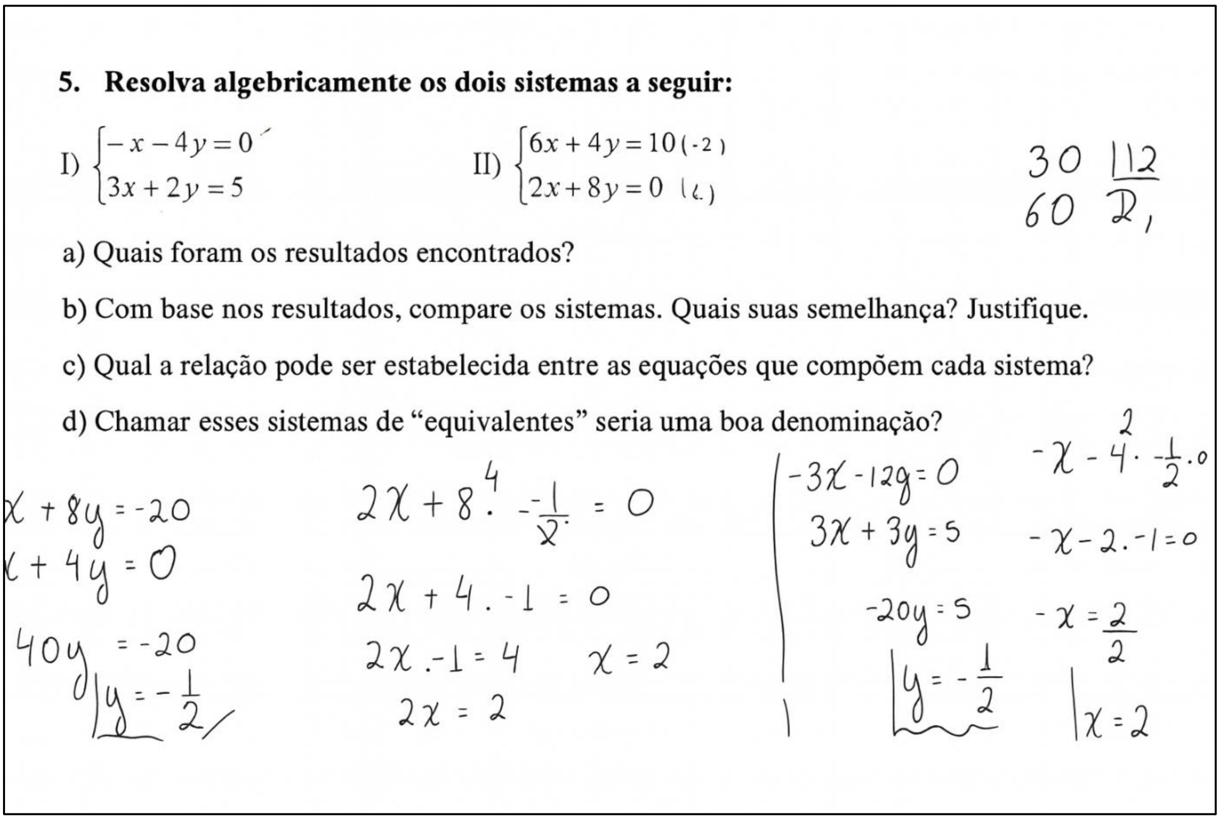
> [..] upon assimilating this knowledge, the person no longer deals with the reality that immediately surrounds them, because the 'object of cognition' is mediated by science as a social formation, by its history and experience. In this object of distinct aspects, presented to the individual in the form of generalized, abstract content, lies their thinking

Thus, we agree with Aquino and Rodrigues (2019), when they analyze that scientific knowledge is the universal form of expression of theoretical thinking and is developed through object-sensory activity, giving meaning to judgments and concepts.

In Task 3.5, we proposed that the students algebraically solve two Systems of Linear Equations with two unknowns. The students had no difficulty solving the Systems — as shown in the example in the figure below, all students used the addition method to solve it.

Figure 5

Task solved by Miguel (Research data).


5. Resolva algebricamente os dois sistemas a seguir:

I) $\begin{cases} -x-4y=0 \\ 3x+2y=5 \end{cases}$ II) $\begin{cases} 6x+4y=10 \\ 2x+8y=0 \end{cases}$

a) Quais foram os resultados encontrados?

b) Com base nos resultados, compare os sistemas. Quais suas semelhança? Justifique.

c) Qual a relação pode ser estabelecida entre as equações que compõem cada sistema?

d) Chamar esses sistemas de "equivalentes" seria uma boa denominação?

Translation:

> a) What results were found?
>
> b) Based on the results, compare the systems. What are their similarities? Justify.
>
> c) What relationship can be established between the equations that make up each system?
>
> d) Would calling these systems "equivalent" be a good term?

After the students found the solutions to both Systems, they discussed the Task with the teacher. Beatriz was the first to find the results and immediately commented that the solutions were the same. As indicated in item *(b)* of the Task, the teacher guided the students to analyze the equations and the Systems, highlighting whether they had any relationship. Gabriel identified that, in the first equation of *System II (6x + 4y = 10),* the coefficients were twice the values of the coefficients in the second equation of *System I (3x + 2y = 5)*. Similarly, the equation *2x + 8y = 0* was the same as *–x – 4y = 0 multiplied by (–2)*. Gabriel then remarked: "the equations from System I are similar to those from System II; we just need to multiply each one by a value, and they become the same."

At that point, the teacher asked whether they could "classify" the relationship between these Systems as equivalent and whether they were familiar with that term. The students agreed with the teacher that the Systems could be called equivalent but did not recall having heard the term before. Based on this context, the teacher introduced the Tasks about Equivalent Systems.

The Task presented in Figure 6 began with the definition of Equivalent Systems, which the teacher asked the students to read. Then, a video[6] was shown explaining how to identify when two Systems are equivalent.

Figure 6

Task guidelines on Equivalent Systems (Research data).

Equivalent systems - Definition
Two systems are equivalent when they have the same solution set.
For example, given the systems:

$$S_1 = \begin{cases} x + y = 3 \\ 2x + 3y = 8 \end{cases} \quad \text{e} \quad S_2 = \begin{cases} x + y = 3 \\ x + 2y = 5 \end{cases}$$

We verify that the ordered pair (x, y) = (1, 2) satisfies both and is unique.
Therefore, S_1 and S_2 are equivalent: $S_1 \sim S_2$.
Example of how to identify Equivalent Systems: https://www,youtube.com/watch?v=cpwBxz-vw5A

After reading the definition of Equivalent Systems and analyzing the video, the teachers and students had a discussion:

> **Teacher:** The video presented dealt with equivalent systems. What does that have to do with solving Systems of Linear Equations?
> **César and Miguel**: When systems are equivalent, the same solution you find for one system will also be valid for the other system.
> **Teacher:** If, for example, the equations of a system are multiplied—one by −2 and the other by 6—would that result in an equivalent system?
> **Miguel:** Is row reduction the same thing?
> **Teacher:** Row reduction and the addition method, for example, are based on this property (Equivalent Systems). We look for an equivalent system that is "easier" to solve (*Research data*).

Tasks 3.6, 3.7, 3.8, and 3.9, therefore, aimed to promote the students' assimilation of the actions and properties necessary for developing Equivalent Systems. This involved a series of concrete, particular practical tasks that are solved based on the general mode of action.

[6] Vídeo available at: https://www.youtube.com/watch?v=cpwBxz-vw5A. Accessed on Feb. 14, 2020

We highlight Tasks 3.8 and 3.9, in which we continued working with Equivalent Systems, but added another unknown—meaning we began to study Systems of Linear Equations with three equations and three unknowns. Additionally, theorems were introduced that translate the general procedure for obtaining Equivalent Systems, as presented below.

Figure 7

Tasks 3.8 and 3. 9 (Research data).

8. Analyze the equivalent systems:

$$\text{I)}\begin{cases}2x+4y+2z=18\\2x+y-z=2\\-3x+y+2z=4\end{cases}\qquad \text{II)}\begin{cases}4x+2y-2z=4\\6x-2y-4z=-8\\x+2y+z=9\end{cases}$$

a) What are the characteristics of these equivalent systems?

b) Based on the system presented and the activities carried out, explain this theorem:

Theorem I: If we multiply all terms of any linear system S by a number K ≠ 0, the new system S' obtained will be equivalent to S.

9. Analyze the equivalent systems

$$\text{I)}\begin{cases}x+3y+2z=2\\3x+5y+4z=4\\5x+3y+4z=-10\end{cases}\qquad \text{II)}\begin{cases}x+3y+2z=2\\-4y-2z=-2\\-12y-6z=-20\end{cases}$$

a) Why are these systems equivalent? Justify.

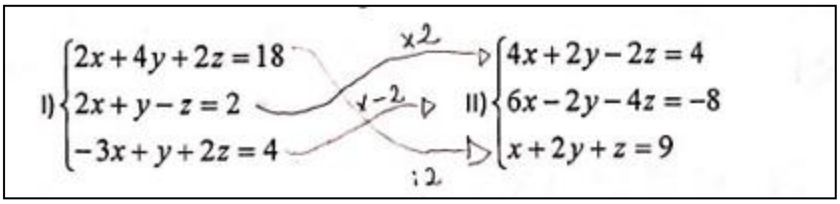
b) Based on the system presented and the activities carried out, explain this theorem:

Theorem II: If we replace one equation of a linear system S with the sum (term by term) of it and another equation, the new system S' obtained will be equivalent to S.

Even in tasks involving systems with three equations and three unknowns — 3x3 systems — students had no difficulty identifying and justifying why the systems were equivalent, as shown in the figure below. We observed in their written work that they established the relationships between the equations that were configured as equivalent — indicating the operation that needed to be performed on the equation.

Figure 8

Beatriz's response to Task 3.8 (Research data).



Regarding the theorems presented, students understood that these theorems formalized the actions they had already been performing. On Theorem I, Gabriel commented: "Teacher, when we multiply the whole equation by a number and create a system, that's what we're doing." Beatriz analyzed: "When we solve a system using row reduction, we use these theorems." César noted that he didn't remember ever studying these theorems and said he used to solve systems without thinking about it, but now he understands what he's doing and it makes sense. This shows that there is a movement between the general and the particular, and that theoretical thinking develops when this movement occurs — through contradictions and new syntheses.

Thus, in Task 3.10, students were asked to use Equivalent Systems to solve the given Systems of Linear Equations.

Figure 9

Task 3.10 (Research data).

10. Using Equivalent Systems, Solve:

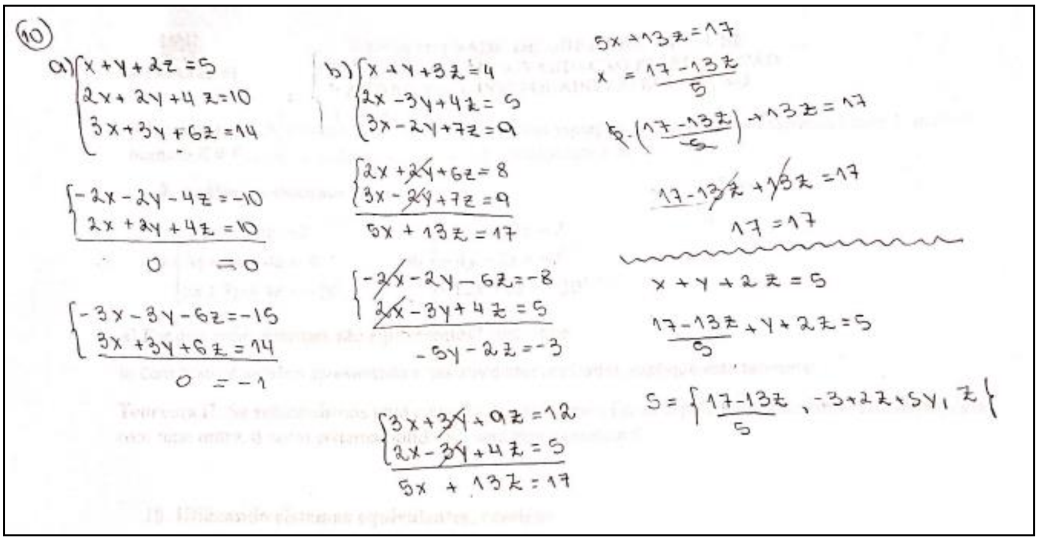
a) $\begin{cases} x+y+2z=5 \\ 2x+2y+4z=10 \\ 3x+3y+6z=14 \end{cases}$

b) $\begin{cases} x+y+3z=4 \\ 2x-3y+4z=5 \\ 3x-2y+7z=9 \end{cases}$

While solving these systems, César said that "it wasn't working." The teacher then asked why he was having difficulty and, in that context, reminded the students that systems can have one solution, infinitely many solutions, or no solution at all. The other students had no difficulty solving the problems. It is worth noting that they had studied this content in March and April and completed the Task in October and November of the same year.

Figure 10

Amanda's response to Task 3.10 (Research data).



In Amanda's work, we observed that she used the row reduction method — applying the properties of Equivalent Systems. She concluded that the result of the system in item *(a)* was inconsistent, and in item (b), consistent and undetermined; however, she did not explicitly record these classifications. In addition, she made a mistake in item *(b),* at the end of the solution, by expressing *y* in terms of *z*, but incorrectly performed a division operation.

Although the error affected the final result of the system, it did not indicate that the student was not developing judgments and the concept of a System of Linear Equations; rather, it represented a lack of attention and focus during the solution process.

In Tasks 3.11 and 3.12, students were required to represent and interpret Systems of Linear Equations geometrically. To support this, we proposed using a technological tool — GeoGebra[7].

Figure 11

Tasks 3.11 and 3.12 (Research data).

Using Geogebra as a resource

11. In GeoGebra, construct the graphs of these systems:

I) $\begin{cases} -x-4y=0 \\ 3x+2y=5 \end{cases}$ II) $\begin{cases} 6x+4y=10 \\ 2x+8y=0 \end{cases}$

a) What do these graphs have in common?

b) How are equivalent systems represented graphically?

| 12. Using GeoGebra, solve these systems: |

[7] GeoGebra is a dynamic mathematics application that combines geometry and algebra concepts in a single interface – it is freely distributed.

a) $\begin{cases} x+y=7 \\ x-y=1 \end{cases}$ b) $\begin{cases} 2x + 3y + z = 11 \\ x + y + z = 6 \\ 5x + 2y + 3z = 18 \end{cases}$ c) $\begin{cases} 5x+y-z=0 \\ -x-y+z=1 \\ 3x-y+z=2 \end{cases}$

Discuss with your classmates:
a) Is it possible to solve this problem using this method? What difficulties did you encounter?
b) What do this solving method and row reduction (Gaussian elimination) have in common?

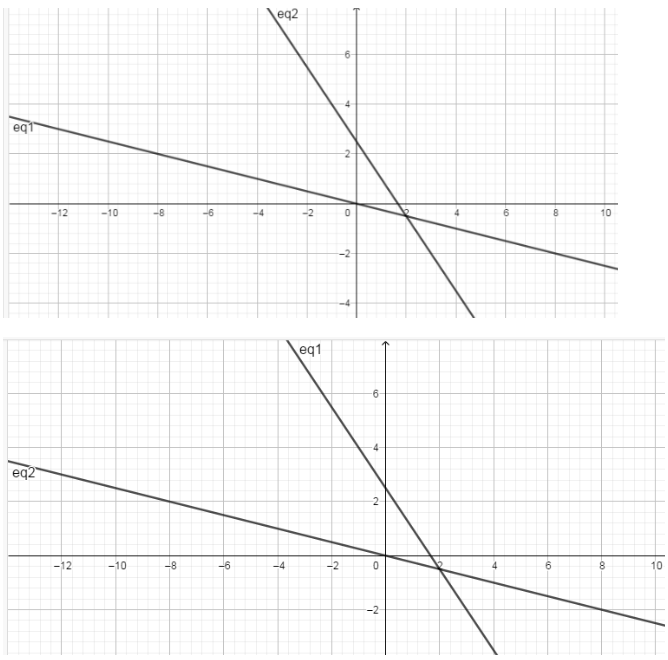
In this context, we reflect on Davydov's view (1991/2019), who noted that one of the unresolved issues in the Study Activity Theory relates to the use of computers. The author criticizes the use of digital technologies in a traditional way; however, he acknowledges the importance of seeking to integrate such technologies into the teaching-learning processes in a purposeful and meaningful way.

Given the difficulty of geometrically representing systems with three equations and three unknowns, we recognized the necessity of using GeoGebra as an alternative for students to explore and understand the geometric representations of Systems of Linear Equations.

In Figure 12, we present César's solution using GeoGebra. Students were not familiar with this tool, so the teacher had to explain how it worked — how to construct graphs, among other functionalities. The students quickly understood how to use it and first built the representations of the Systems of Linear Equations (Items I and II of Task 11).

Figure 12

César's GeoGebra solutions for Systems I) $\begin{cases} -x-4y=0 \\ 3x+2y=5 \end{cases}$ *and II)* $\begin{cases} 6x+4y=10 \\ 2x+8y=0 \end{cases}$*, respectively (Research data).*



Upon viewing the graphs, students first noticed that the point of intersection of the lines was the same in both representations. They also observed that the arrangement of the lines was identical — in other words, the graphs were the same. Thus, they concluded that Equivalent Systems have the same geometric representation.

It is also worth noting that César and Miguel solved the Systems algebraically to verify whether the point of intersection truly matched the solution. They enjoyed using GeoGebra and commented that it made it much easier to find and understand the solution of systems.

In Task 12, GeoGebra was also required. However, in items *(b)* and *(c),* students had to represent 3x3 systems. When constructing the graphs — as Miguel did (see figure below) — they reacted with surprise.

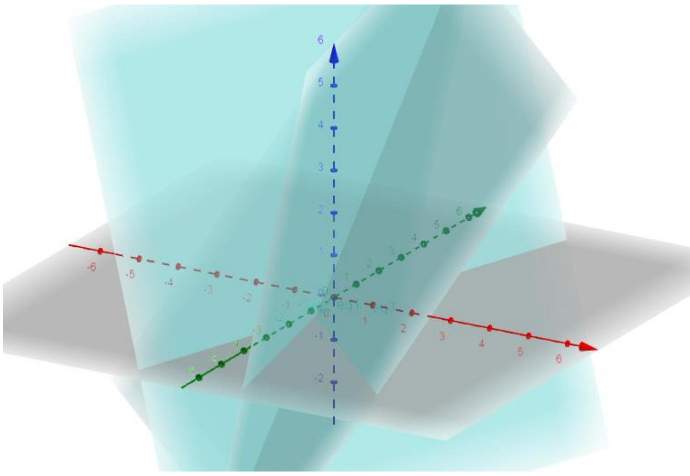
The teacher then explained that although 3x3 systems are widely studied in the classroom, their geometric representation is rarely discussed. A discussion followed:

> **Teacher**: How is each equation represented?
> **César:** As a plane?!
> **Teacher**: And is there any relationship among these planes?
> **César:** It seems so! (Research data).

Using GeoGebra, students explored the representations by rotating and zooming in and verified that the intersection points of the planes corresponded to the solutions of the system. They also noticed, as they completed the rest of the tasks, that when the planes are parallel, there is no solution; and when the planes overlap, there are infinitely many solutions. They commented that the general principle underlying the analysis of 2x2 systems also applied to 3x3 systems, the only difference being in the geometric representation — using lines in the former, and planes in the latter.

Figure 13

Miguel’s work - item c - Task 3.12 (Research data).



In Tasks 3.13, 3.14, and 3.15, the goal was to promote discussion of systems of linear equations — consistent and determined, consistent and undetermined, and inconsistent — and understanding of a homogeneous system, considering the different forms of representation.

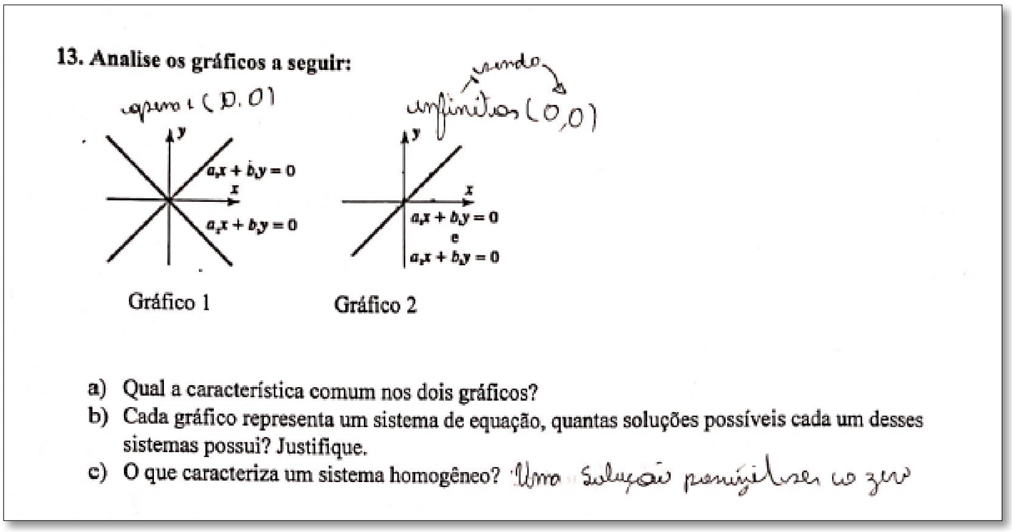
Na Tarefa 3.13, apresentamos dois gráficos juntamente com suas representações algébricas – na forma geral. Os alunos observaram que as retas de ambos os gráficos passavam pela coordenada (0,0). Indicaram, sem dificuldades, que, no Gráfico 1, tinha apenas uma solução (0,0), e, no Gráfico 2, teria infinitas soluções.

The teacher then asked the students: “Among the infinite solutions of the system represented by Graph 2, can you tell me at least one?”

The students were unable to respond, so the teacher guided them to analyze the algebraic representation, suggesting they substitute 0 for both x and y. After the students identified that (0,0) could be one of the infinite solutions of the system represented by Graph 2 — as shown in Figure 14 — the teacher defined and explained the characteristics of a homogeneous system.

Figure 14

Student Gabriel’s response (Research data).



Translation

Graph 1

Graph 2

a) What is the common feature in both graphs?

b) Each graph represents a system of equations. How many possible solutions does each system have? Justify your answer.

c) What characterizes a homogeneous system?

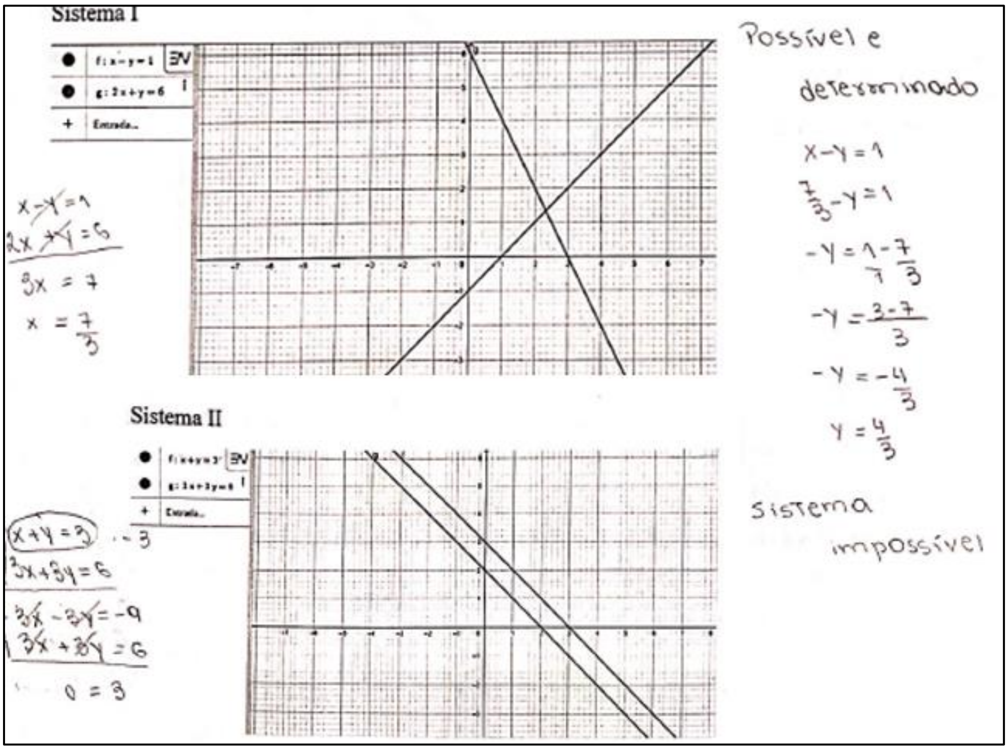
In Tasks 3.14 and 3.15, the students analyzed systems of linear equations, relating their algebraic and geometric representations. In Figure 15, we present Beatriz's notes on Systems I and II from Task 3.14. She solved them algebraically and checked the geometric representation. Afterwards, she classified the systems as consistent and determined, and inconsistent.

In this context, the discussions around Tasks 3.14 and 3.15 reinforced the students’ understanding of the different types of representations of Systems of Linear Equations. The students reported that, when they studied this topic previously, they didn’t recall having received an explanation and/or done any “exercises” involving the geometric representation of a System of Linear Equations.

They commented that previous lessons on systems of linear equations had focused on procedural solving, and that participating in this experiment was contributing to their learning — by clarifying lingering doubts and offering new insights, in connection with other topics like functions.

Figure 15

Beatriz's response (Research data).

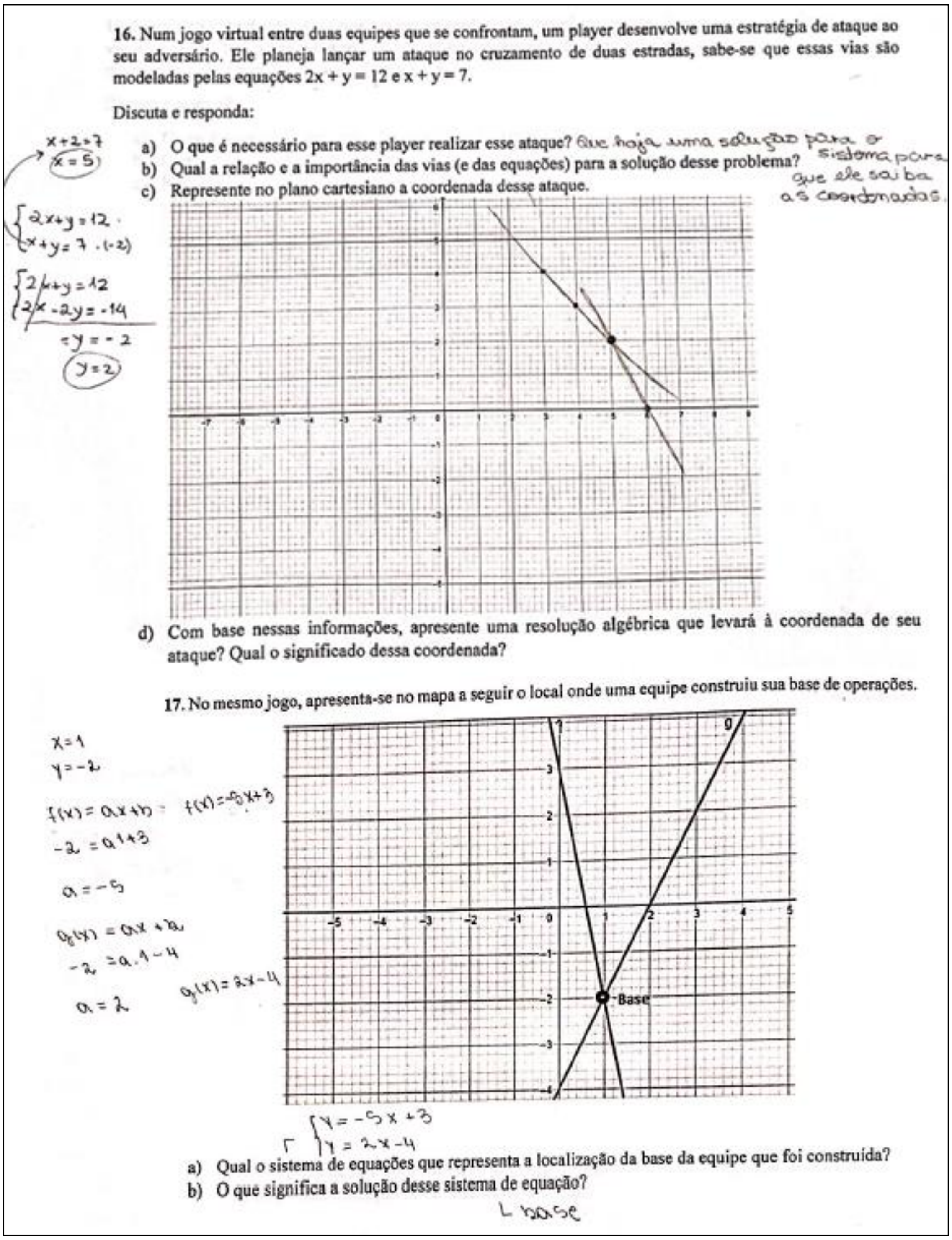


In Tasks 3.16, 3.17, and 3.18, students needed to apply their reasoning and the concept of systems of linear equations to solve a specific problem situation.

We discussed that in Tasks 3.16 and 3.17, the problem situations required knowledge of both algebraic and geometric modeling. We observed, based on Amanda's work (Figure 16), that students did not have difficulty interpreting the problem and solving it. We consider that the development of theoretical thinking promoted by these Tasks was fundamental for students to interpret the situation presented, being aware of the theoretical knowledge they would need to apply to solve it.

Figure 16

Amanda’s responses to Tasks 3.16 and 3.17 (Research data).



Translation:

16.

In a virtual game between two opposing teams, a player develops an attack strategy against the opponent. He plans to launch the attack at the intersection of two roads, which are modeled by the equations 2x + y = 12 and x + y = 7.

Discuss and answer:

a) What is necessary for this player to carry out the attack?

b) What is the relationship and importance of the roads (and the equations) for solving this problem?

c) Represent the coordinates of this attack on the Cartesian plane.

d) Based on this information, present an algebraic solution that leads to the coordinates of the attack. What is the meaning of this coordinate?

17.

In the same game, the map below shows the location where one team built its operations base.

a) What system of equations represents the location of the team's base that was built?

b) What does the solution of this system of equations represent?

With these Tasks, we concluded the development of this proposed approach to teaching systems of linear equations, based on the Study Activity Theory.

The Tasks, which focused on systems of linear equations, connected theoretical knowledge with real-life situations, enabled discussion of reasoning and the concept of linear systems, highlighted problem-solving processes — present in the historical development of algebra, articulated algebraic and geometric representations — including the use of digital technologies, debated equivalent systems, and proposed different problem situations to be solved, in a dialectical movement: from the abstract to the concrete, from the general to the particular, and across different semiotic representations of the same object.

Final Considerations

We assessed that the set of Tasks was effectively developed, showing that students "acclimated" to a teaching organization based on the Study Activity Theory. In this regard, we considered that Tasks 1 and 2 were essential for the students to grasp general relationships present in a network of judgments and concepts related to the System of Linear Equations, as well as to understand a teaching organization different from what they were used to, one that discusses judgments and concepts, aiming to reveal their essence, and not merely reproduce, promote repetition, and memorization of content.

Regarding **the transformation of task data**, we analyzed that in the Tasks about the System of Linear Equations, students were able to establish the general relationship of this knowledge. The approach of a Task that addressed a situational problem, such as choosing a profession, with the definition of the System of Linear

Equations – particular Tasks – led the student to develop the substantive abstraction that reveals the essence of the object, which will later be deduced in particular cases.

The teacher’s mediation, acting in the students' Zone of Proximal Development, and the situations presented, with constant moments of reflection, allowed students to demonstrate that *simultaneity* constituted the essence of the concept of System of Linear Equations. The students observed that the equations were interrelated and each of them set a “condition” to be satisfied.

However, it was not enough to independently find the solution of just one equation; the solution of the system needed to simultaneously satisfy the “conditions” of all the equations. *"It is noticeable that the student does not invent the substantial abstraction, but through the mental actions performed in solving Tasks, they develop the content of the essential relationship that should be reflected in the theoretical concept*" (Souza; Ferola & Coelho, 2019, p. 382).

With the teacher's guidance, the student reproduced “[...] the historically performed movement of reducing the concrete to the abstract, which resulted in the scientific elaboration of the essential relationship, the content of the theoretical concept” (Souza; Ferola & Coelho, 2019, p.382-383)

On the **modeling of the universal relationship**, we observed that it is directly related to the transformation of the task data, as it synthesizes the essence of the concept that forms the model. Thus, after students identified the general relationship of the object studied and related it to its more elaborated definition, we engaged in reflections – discussing and making sense of the elements, representations, classifications, and behaviors of the System of Linear Equations, so that the students could assimilate its internal characteristics, leading to the development of theoretical thinking. We were also concerned with modeling in different languages: graphic/geometry, algebraic, and rhetorical.

The theoretical thinking, which occurs through substantive abstractions, requires the integral investigation of the object to highlight its general relationship. In this process, gradually, the experiment acquires a cognitive character, and can be carried out mentally. Thus, “the task of theoretical thinking consists of elaborating the data from contemplation and representation in the form of a concept and with it reproducing the system of internal connections that originate the given concrete, discovering its essence” (Davydov, 1988, p.143).

Therefore, the development of theoretical thinking is anchored in the actions of analysis and synthesis. The Tasks stimulated these aspects of thinking, including studies that focused on comparisons and equations, which were later transposed into Tasks about the System of Linear Equations.

Regarding **control and assessment because of the learning task**, we observed the autonomy that students developed during the Tasks. At first, they were highly dependent on the teacher and, later, when working in pairs, they checked with their peers "what the result was," and if necessary, they observed what they had gotten wrong and/or how the solution was developed, then making the necessary corrections.

Students considered that the Tasks they completed expanded their "vision" of Mathematics, which had been seen from a more "mechanical" and procedural perspective. Their commitment to participating in all the Tasks over two months, voluntarily and enthusiastically, indicates that the study Tasks were providing their human development and the acquisition of algebraic content.

We consider that the set of study Tasks addressing the System of Linear Equations in its specific forms, contextualized and problematized; with its internal and external characteristics, procedural and reflective actions; and classical and traditional approaches; promoted a teaching-learning process that contributed to the human development of the students.

Regarding the formation of the concept of the System of Linear Equations, we verified the importance of students appropriating algebraic representations – judgments and concepts – in their most general forms – unknowns, variables, coefficients, etc. There are indications that the mastery of this scientific knowledge contributed to the formation of theoretical thinking about the System of Linear Equations – which reveals itself through a conceptual network.

The organization of the teaching-learning process, which we developed in the practice of high school education, created conditions for the development of students' theoretical thinking, notably about the System of Linear Equations, based on the results presented and analyzed in this text. The indications of the development of students' theoretical thinking are present in the dialogues between students and teacher, and in the records in the Tasks – solutions, comments, and observations. In them, the interrelations of the judgments and concepts that reveal the essence of the System of Linear Equations were expressed.

Thus, this research presented a possibility of re-signifying the organization of algebra teaching/ System of Linear Equations in High School, showing teachers and the academic community a possible path to be carried out, debated, and refined. It brings to High School the possibility of adapting the principles of the Study Activity Theory to the teaching-learning processes and awaits further studies and contributions aimed at transforming the Brazilian educational reality.

References

Aquino, O. F.& Rodrigues, A. (2019). A formação do pensamento na escola: uma discussão na interseção entre a filosofia, a psicologia e a didática. In R. Valdés Puentes & A. M. Longarezi (Eds.), Ensino Desenvolvimental: sistema Elkonin, Davídov, Repkin (p. xx-xx). Campinas: Mercado de Letras.

Aquino, O. F. (2017). O experimento didático-formativo: contribuições de L. S. Vigotski, L. V. Zankov e V. V. Davidov. In A. M. Longarezi & R. V. Puentes (Orgs.), Fundamentos Psicológicos e Didáticos do Ensino Desenvolvimental (pp. xx-xx). Uberlândia, MG: EDUFU

Boyer, C. B. (2003). História da Matemática. São Paulo: Edgard BencherLtda.

Davidov, V. V. (1982). Tipos de generalización em la enseñanza. Habana: Editorial Pueblo y Educación.

Davidov, V. V. (1986). Problemas do ensino desenvolvimental - A Experiência da Pesquisa Teórica e Experimental na Psicologia. *Revista Soviet Education*, 30(8).

Davidov, V. V. (1987). Análisis de los principios didácticos de la escuela tradicional y posibles principios de enseñanza en el futuro próximo. In La Psicología Evolutiva y Pedagógica en la URSS. Antología. Moscú: Progreso.

Davidov, V. V. (1988). La enseñanza escolar y el desarrollo psíquico – Investigación psicológica teórica y experimental. Moscú: Editorial Progresso.

Davidov, V. V. (1999). O que é a atividade de estudo. Revista “Escola inicial”, 7.

Davídov, V. V. (2019). Desenvolvimento psíquico da criança. In R. Valdés Puentes, C. G. Coelho Cardoso, & P. A. P. Amorim (Orgs.), Teoria da aprendizagem desenvolvimental: contribuições de D. B. Elkonin, V. V. Davidov e V. V. Repkin (Vol. 10, pp. 175-120). Curitiba: CRV.

Davidov, V.& Márkova, A. (1987). La concepción de la actividad de estudio de los escolares. In.: La psicología evolutiva y pedagogica en a URSS – Antología. Moscu: Editorial Progreso.

Elkonin, D. B. (1961). Desarrollo Psíquico de los Escolares. In A. A. Smirnov (Ed.), Psicologia (p. 553-559). Cuba: Imprensa Nacional de Cuba.

Eves, H. (2004). Introdução à História da Matemática. Campinas, SP: Editora da Unicamp.

Eves, H. (2011). Introdução à história da Matemática. Campinas, SP: Editora da Unicamp.

Iezzi, G. & Hazzan (1985). Fundamentos De Matemática Elementar - Volume 4. São Paulo: Atual Editora.

Kopnin, P. V. (1978). A dialética como lógica e teoria do conhecimento. Rio de Janeiro: Civilização Brasileira.

Lima, E. L., et al. (2006). A Matemática do Ensino Médio (Vol. 3). Rio de Janeiro: Sociedade Brasileira de Matemática.

Moura, M. (2000). O educador matemático na coletividade da formação: Uma experiência com a escola pública. Tese de Livre Docência, Faculdade Educação, Universidade de São Paulo, São Paulo.

Panossian, M. L. (2008). Manifestações do pensamento e da linguagem algébrica de estudantes: indicadores para a organização do ensino. Dissertação de mestrado, Faculdade de Educação da Universidade de São Paulo, São Paulo.

Souza, R. C. C. R., Magalhães, S. M. O., & Silveira, M. J. (2014). A tradição do materialismo histórico-dialético na produção acadêmica sobre professores. In C. Cunha, J. V. Sousa, & M. A. Silva (Orgs.), O Método Dialético na Pesquisa em Educação (pp. xx-xx). Campinas, SP: Autores Associados.

Sousa, M. C. De, Panossian, M. L., & Cedro, W. L. (2014). Do movimento lógico e histórico à organização do ensino: o percurso dos conceitos algébricos. Campinas, SP: Mercado de Letras. (Série Educação Matemática).

Santos, C. M., & Zuin, E. S. L. (2017). Sistema de equações lineares em livros didáticos (1930-1970): apontamentos para a formação inicial e continuada de professores de Matemática e áreas afins. Material didático ou instrucional - Material didático.

Vygotski, L. S. (2001). Pensamiento y lenguaje. In Obras Escogidas (Vol. III, 2ª ed., pp. xx-xx). Madrid: Visor.

Vigotski, L. S. (2008). Pensamento e linguagem. São Paulo: Martins Fontes.