EMP Educação Matemática Pesquisa
ISSN 1983-3156
Qualis A1

http://dx.doi.org/10.23925/1983-3156.2025v27i2p226-251

Knowledge for the education of teachers who teach statistics in basic education

Conocimientos para la formación de profesores que enseñan estadística en la enseñanza básica

Connaissances pour former les enseignants qui enseignent les statistiques dans l'éducation de base

Conhecimentos para a formação do professor que ensina estatística na educação básica

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Abstract

With teaching knowledge as the object of study, research was carried out in mathematics education journals focusing on works on the knowledge of teachers who teach statistics in basic education. To this end, the research focused on articles addressing knowledge and knowings for statistics teaching from 2019 to 2023. The categories established were specific content knowledge, pedagogical content knowledge, curriculum knowledge, knowings of the subjects, and knowings from experience. In the formative processes reported in the articles analyzed, content knowledge was more prevalent than pedagogical knowledge. The study concluded that the initial and continuing formative processes of teachers who will teach statistics in basic education must assume that the teacher must not only have content knowledge but also explore learning situations in social, occupational, and scientific contexts.

Keywords: Knowledge, Knowing, Teacher education, Teaching statistics.

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Resumen

Con el conocimiento docente como objeto de estudio, se realizó una búsqueda en revistas de educación matemática centrada en el conocimiento de los profesores que imparten estadística en la enseñanza básica. Para ello, se han buscado artículos que aborden los conocimientos docentes para la enseñanza de la estadística, desde 2019 hasta 2023. Se establecieron las siguientes categorías: conocimiento del contenido específico; conocimiento pedagógico del contenido; conocimiento curricular; saberes del área; saberes de la experiencia. En los procesos de formación relatados en los artículos analizados, el conocimiento del contenido era más prevalente que el conocimiento pedagógico. Se concluye que es necesario, en los procesos de formación inicial y continua de los profesores que van a enseñar estadística en la enseñanza básica, asumir que los profesores, además de conocer los contenidos, necesito explorar situaciones de aprendizaje en contextos sociales, laborales y científicos.

Palabras clave: Conocimiento, Formación de profesores, Enseñanza de estadística.

Résumé

Avec les connaissances pédagogiques comme objet d'étude, des recherches ont été menées dans des revues d'enseignement des mathématiques en se concentrant sur les travaux sur les connaissances des enseignants qui enseignent la statistique dans l'éducation de base. À cette fin, des articles ont été recherchés sur les connaissances pédagogiques et les connaissances pour l'enseignement de la statistique, de 2019 à 2023. Les catégories ont été établies : connaissances de contenu spécifiques ; connaissance du contenu pédagogique ; connaissances curriculaires ; connaissances disciplinaires ; connaissances issues de l'expérience. Dans les processus de formation rapportés dans les articles analysés, il y avait une plus grande prévalence des connaissances de contenu par rapport aux connaissances pédagogiques. On conclut qu'il est nécessaire, dans les processus de formation – initiale et continue – des enseignants qui enseigneront la statistique dans l'éducation de base, de partir du postulat que l'enseignant, en plus de la connaissance du contenu, besoin d'explorer des situations d'apprentissage dans des contextes sociaux, professionnels et scientifiques.

Mots-clés : Connaissances, Savoirs, Formation des enseignants, Enseignement de la statistique.

Resumo

Tendo o conhecimento docente como objeto de estudo, foi realizada uma pesquisa em periódicos da educação matemática com foco em trabalhos sobre o conhecimento do professor

que ensina estatística na educação básica. Para tanto, buscou-se artigos que abordam conhecimentos e saberes docentes para o ensino de estatística, no período de 2019 a 2023. Estabeleceu-se as categorias: conhecimento do conteúdo específico; conhecimento pedagógico do conteúdo; conhecimento curricular; saberes disciplinares; saberes da experiência. Nos processos formativos relatados nos artigos analisados, houve maior prevalência dos conhecimentos do conteúdo comparativamente aos conhecimentos pedagógicos. Conclui-se que nos processos de formação inicial e continuada de professores que vão ensinar estatística na educação básica é necessário ter como pressuposto que o professor, além do conhecimento do conteúdo, precisa explorar situações de aprendizagem em contextos sociais, ocupacionais e científicos.

Palavras-chave: Conhecimentos, Saberes, Formação de professores, Ensino de estatística.

Knowledge for the education of teachers who teach statistics in basic education

This article presents an excerpt from the research *Specific mathematics for teachers who teach mathematics in middle and high school* [Matemática específica do professor que ensina matemática nos anos finais do ensino fundamental e no ensino médio[, linked to the Working Group for the Education of Teachers who Teach Mathematics [Grupo de Trabalho de Formação de Professores que Ensinam Matemática - GT07] of the Brazilian Society for Mathematical Education [Sociedade Brasileira de Educação Matemática – Sbem][4]. This research aims to build an overview of the most used theoretical frameworks in recent national research focusing on mathematics mobilized and produced by teachers who teach this subject in middle and high school.

Thus, this text aims to inventory, describe, and analyze studies published in A1 or A2 Qualis journals from 2019 to 2023 on the education of mathematics teachers who teach statistical concepts in basic education. The perspective of this analysis will be the discussion of knowings and knowledge for teaching according to the points of view of experience, content, pedagogy, and curriculum. The analysis of these articles will have two concepts as its starting point: teacher knowledge for teaching, according to Shulman (1986), and knowledge/knowing for teaching, according to Tardif (2002).

Theoretical framework

For decades, educators in many countries have paid little or no attention to research on teacher education at different levels of education. The complex education of teachers was largely neglected in the educational research scenario until the mid-1980s, as the interests seemed to be elsewhere: studies on assessment, academic performance, learning, and studies centered on students or teaching resources (Curi & Pires, 2008). However, from that decade onwards, theories and concepts were disseminated based on works such as Shulman's (1986, 1987), which ask, for example: "What do teachers know?"; "What knowledge is essential for

4 Currently, researchers from different universities in Brazil are part of this initiative. The research is coordinated by researchers Flávia Cristina de Macêdo Santana (State University of Feira de Santana, UEFS) and Roberta D'Angela Menduni Bortoloti (State University of Bahia, UNEB) and by researcher Victor Augusto Giraldo (Federal University of Rio de Janeiro, UFRJ). The team includes: Eliana Matesco Cristovao (Federal University of Itajubá, UNIFEI), Enio Fernandes de Paula (Federal Institute of São Paulo, IFSP), Henrique Rizek Elias (Federal University of Technology of Paraná, UTFPR), Lúcia Cristina Silveira Monteiro (Federal University of Alagoas, UFAL), Lya Raquel Oliveira dos Santos (Federal University of Piauí, UFPI), Marta Élid Amorim (Federal University of Sergipe, UFS), Mayara de Miranda Santos (Federal Institute of Piauí, IFPI), Renata Camacho Bezerra (State University of Western Paraná, Unioeste), Sabrina Salazar (Federal University of Pelotas, UFPel), Silvânia da Silva Costa (UFS) and Vânia Cristina da Silva Rodrigues (Federal University of Triângulo Mineiro, UFTM).

teaching?"; "How are teachers educated?"; "Who produces knowledge of teaching?" A possible justification for the significant increase in the number of investigations to answer these questions may be related to the fact that, in recent decades, many researchers have recognized that these professionals not only transmit information and reproduce teaching techniques but also reflect on their own practices.

One of the most relevant contributions of research on teachers' knowledge is the recognition of the importance of practical experience and reflection as essential elements for professional development. António Nóvoa is one of the educators who have contributed significantly to this field, discussing valuable perspectives on the complexity and depth of teaching knowledge. Thus, investigating the tacit knowledge and non-formal strategies teachers employ in their daily classroom routine is also important for promoting excellent education.

Therefore, it became relevant to identify what teachers know and, notably, how they act. This fact highlights a relevant conclusion: teacher education must be considered a fundamental research object in the teaching and learning field.

Regarding the knowledge necessary for teaching, we embrace the research carried out by Shulman (1986). In his investigations on how teachers construct their practices, this researcher identifies different types of knowledge relevant to teaching: content knowledge, pedagogical content knowledge, and curriculum knowledge.

About content knowledge, which encompasses the understanding of the principles of organization of the teaching object and the fundamental ideas related to that object, Shulman (1987) emphasizes that "the teacher has a special responsibility toward content knowledge, serving as the first source of students' understanding of the content" (p. 9). Given students' diversity, the teacher must have a flexible and multifaceted understanding and be capable of providing alternative explanations of the same concepts or principles.

Pedagogical content knowledge refers to the ability to teach that content, including selecting, organizing, and managing components that will make the approach more understandable to the student, such as representations, explanations, analogies, arguments, and evidence. Therefore, the teacher must have a varied repertoire of examples and approaches from research and experience. Identifying students' preconceptions and knowing strategies to overcome and transform them is also part of pedagogical content knowledge, as is predicting errors and their influence on learning new content.

Curriculum content knowledge, in turn, is related to the teacher's familiarity with curriculum guidelines and recommendations for the introduction and development of content, as well as the exploration of teaching materials, resources, and alternative strategies for

necessary interventions. The teacher must also be aware of the distribution of this content in the prescribed curricula, foresee possible connections between the content and other simultaneous subjects, and articulate issues related to the content in previous or subsequent years.

Regarding teaching knowledge, we rely on Tardif (2002), who highlights the complexity of the knowledge that teachers have and mobilize in their professional practice. Therefore, among such knowings, we highlight that of academic education and that of experience: the subject and pedagogical knowledge acquired during the teacher's initial education at university or in teacher education programs and the knowledge acquired by the teacher throughout their career through interaction with students and colleagues and reflection on their own practices.

Subject knowledge refers to the teacher's understanding of the area of knowledge they teach, which certainly includes mastering concepts and procedures specific to their subject and understanding theories and methods associated with the area. In turn, pedagogical knowledge concerns the teaching practices and strategies used for student learning. This type of knowledge encompasses understanding learning theories, teaching methods, lesson planning, assessment of learning, and adapting teaching to students' needs.

Tardif (2002) also includes the knowledge of the programs and textbooks used for work. It is important to highlight the importance Tardif (2002) attributed to the knowledge teachers acquire during their journey as basic education students. Thus, an important part of teachers' professional competence would have roots in pre-professional schooling, and this legacy of school socialization would remain strong and stable for a reasonable time beyond the beginning of the career. It is also worth highlighting that, for this author, teachers' knowledge from experience (which we will call here knowing from experience or experiential knowing) is the result of an individual construction, but, at the same time, it is shared and legitimized through professional socialization.

Thus, when considering Shulman's (1986) and Tardif's (2002) concepts, we would briefly state that *knowledge* refers to what the teacher understands about the subject and the strategies to teach and *knowing* covers a broader range of experiences and knowledge that a teacher accumulates throughout their teaching practice. Thus, *knowledge* is what teachers must know, while *knowing* emphasizes experiences and contexts that shape pedagogical practice. This distinction is relevant for understanding how teachers develop professionally and how they can be supported in their continuing education.

Based on both authors' conceptions of knowledge and knowing, this analysis will focus on the categories *knowledge of specific content, pedagogical content knowledge, curriculum knowledge, and knowing from experience* because we understand they encompass fundamental aspects of the teaching exercise.

Methodology

This study included the following phases: a) Obtaining the research *corpus*; b) Analysis of the articles regarding the theoretical references adopted to identify the teacher's knowledge and knowing. To obtain the *corpus* of the work, we looked for articles that address teaching knowledge and knowing for teaching statistics, published between 2019 and 2023. To this end, we defined some recognized journals in the mathematics education area, i.e., *Boletim de Educação Matemática* (Bolema), *Revista Educação Matemática Pesquisa* (EMP), *Acta Scientiae*, *Revista Internacional de Pesquisa em Educação Matemática* (Ripem), *Zetetiké*, and *Educação Matemática em Revista* (EMR).

The initial selection of articles was based on reading the titles, keywords, and an abstract, assessing whether the articles were related to mathematical knowledge for teaching. Below, we define the following criteria for permanence: 1) focus on teaching; 2) have as a research topic or as research participants teachers (or prospective teachers) who teach mathematics in middle school or high school; 3) address, in some way, mathematical knowledge/knowings for teaching (with its different names, depending on the theoretical framework adopted by the authors); and 4) include Brazilian researchers in its authorship. Based on these choices, we initially selected articles published within the established period. Sometimes, we had to read the full text to decide whether to integrate it into the research *corpus*.

We decided to look into the teacher's knowledge for statistics teaching from the works that followed the listed criteria. Thus, we obtained nine works, as shown in Table 1.

Table 1.

Works on teacher knowledge for teaching statistics (own authorship, 2024)

Articles	Titles	Journal
Fernandes & Santos Junior (2020)	"Combinação pedagógica entre letramento estatístico e compreensão gráfica" [Pedagogical combination between statistical literacy and graphic understanding]	EMP
Rodrigues & Ponte	"Investigação Baseada em Design: uma experiência	EMP

(2020a)	de formação de professores em Estatística" [Design-based research: an experience in teacher education in statistics]	
Araujo & Carvalho (2021)	"Conhecimentos didático-matemáticos de licenciandos e professores de matemática para abordagem da curva normal" [Didactic-mathematical knowledge of teaching degree students and mathematics teachers for approaching the normal curve]	EMP
Silva, Prado, Pietropaolo, & Alves (2021)	"Letramento Estatístico: análise de um processo formativo do professor que ensina Matemática" [Statistical literacy: analysis of a formative process for teachers who teach mathematics]	EMP
Assemany & Figueiredo (2022)	"Autonomia e Insubordinação Criativa no Ensino de Tendências de Medida Central" [Autonomy and creative insubordination in the teaching of central measure tendencies]	EMP
Brandelero & Estevam (2023)	"Reflexões Compartilhadas em uma investigação sobre a própria prática: trajetória de aprendizagem de uma professora envolvendo ensino exploratório de estatística" [Shared reflections in an investigation of one's own practice: A teacher's learning trajectory involving exploratory statistics Tteaching]	EMP
Santos, Barbosa, Tinti, & Lopes (2023)	"Narrativas (auto)biográficas e o desenvolvimento profissional de professores de matemática que ensinam probabilidade e estatística" [(Auto)biographical narratives and the professional development of mathematics teachers who teach probability and statistics]	Acta Scientiae
Rodrigues & Ponte (2020b)	"Desenvolvimento do conhecimento didático de professores em Estatística: uma experiência formativa" [Development of didactic knowledge of teachers in statistics: a formative experience]	Zetetiké
Mello & Basso (2023)	"Formação de professores e criatividade: uma experiência com licenciandos de Matemática" [Teacher education and creativity: an experience with mathematics teaching degree students]	Zetetiké
Total of articles		09

After such a decision, we analyzed the articles, looking for the theoretical references adopted regarding the teacher's knowledge. Finally, we analyzed the works based on Shulman (1986) and Tardif (2002). Such analyses will be presented in the following sections.

References used in research on the knowledge and knowings of teachers who teach statistics

This section will briefly present the works that make up the *corpus*. We highlight the theoretical frameworks adopted by the authors for data analysis of teachers' knowledge and knowings. As the studies they mention are not necessarily our references, we cite them without the year of publication.

We begin with Fernandes and Santos Júnior (2020) and their analysis of the contributions of a pedagogical strategy articulating statistical literacy and graphic understanding for middle school. In this sense, we promoted a continuing education course with 12 mathematics teachers from the public school system in Curitiba, Paraná. The theoretical concepts adopted in the research were Gal's statistical literacy and Curcio's graphic understanding regarding statistical education and teaching knowledge, who used the knowledge base conceived by Shulman (1986, 1987).

In the article "Investigação Baseada em Design: uma experiência de formação de professores em Estatística" [Design-based research: an experience in teacher education in statistics], Rodrigues and Ponte (2020a) present the results of a research project that seeks to understand, based on an intervention cycle of a DBR, how the articulation of a set of design principles in the context of a formative experience can promote the didactic knowledge of participants in a specialization course for mathematics teachers. Thus, some aspects of the design were used to identify dimensions in the knowledge expressed by teachers, such as the use of authentic classroom situations (Smith), the use of exploratory tasks (Ponte), and the inclusion of technological resources (Ben-Zvi). As for a model for the didactic knowledge of the mathematics teacher, we use Ponte, who directs four dimensions, as shown in Figure 1: knowledge of mathematics for teaching, knowledge of the student and learning, knowledge of educational practice, and curriculum knowledge.

Figure 1.

Dimensions of the teacher's didactic knowledge in statistics (Rodrigues & Ponte, 2020a, p. 147)

Araújo and Carvalho (2021) discuss the results of a study that aimed to investigate the didactic-mathematical knowledge of teaching degree mathematics students and high school mathematics teachers for the articulated approach between statistics and probability based on the normal curve. The authors used the teacher's didactic-mathematical knowledge and competencies (DMKC model), anchored in the ideas of Pino Fan and Godino and Godino, Batanero, Font, and Giacomone (2016). In this model, didactic-mathematical knowledge is seen as teaching knowledge related to mathematics and didactic aspects, and competence refers to the teacher's effective action in a specific context and with a determined purpose. The components of this model encompass mathematical knowledge (common and advanced) and epistemic, cognitive, affective, mediational, interactional, and ecological facets, all related to each other, as shown in Figure 2 below.

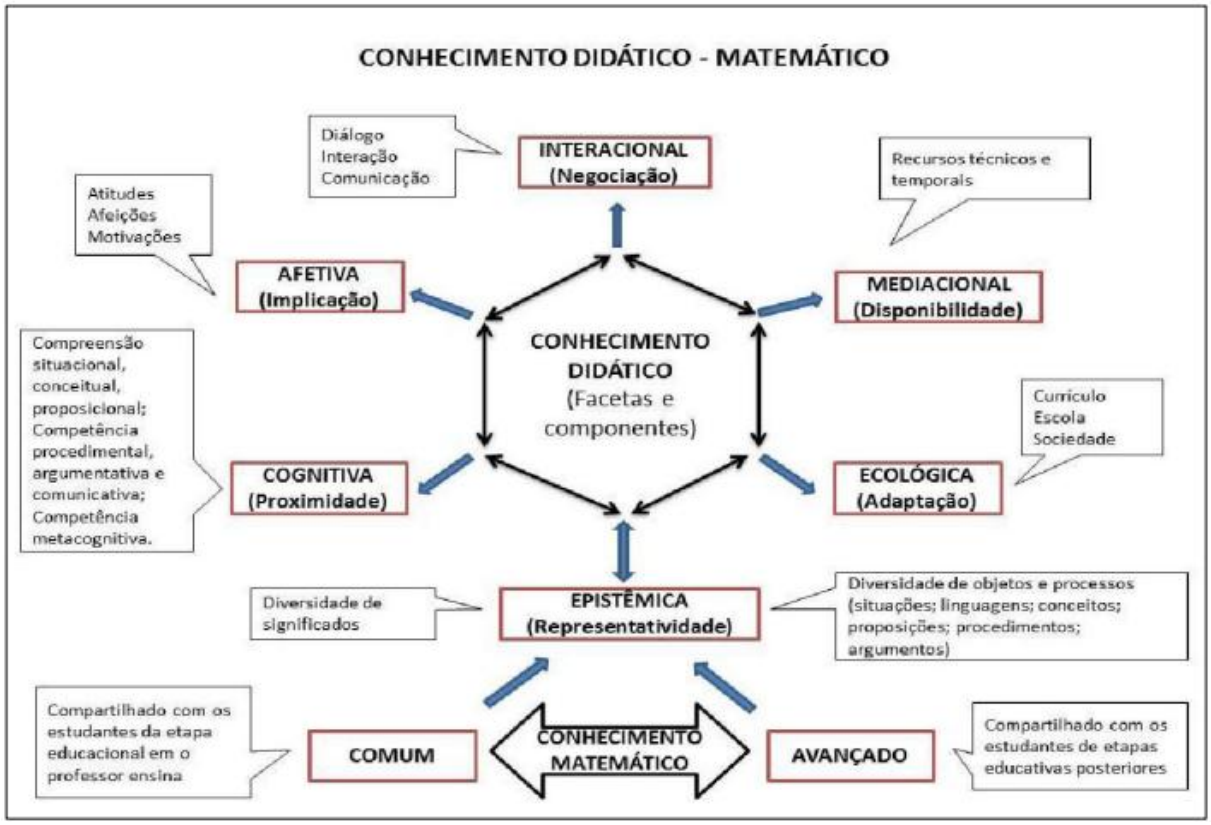

Figure 2.

Didactic-mathematical knowledge (Araújo & Carvalho, 2021, p. 284)

The study of Silva et al. (2021) analyzes the knowledge of a group of teachers during a formative process on teaching measures of central tendency (MCT) within the scope of the Education Observatory Program. The qualitative research was developed in the action research modality with 12 mathematics teachers from the São Paulo state public school system. For this purpose, statistical literacy (Gal), teaching knowledge (Ball, Thames, & Phelps, 2008), and teacher reflection (Zeichner) were used.

Shulman (1986, 1987) addressed teacher knowledge by bringing a knowledge base that interrelates pedagogical and content aspects with a view to teaching. Ball, Thames, and Phelps (2008) began to work with the concept of *mathematical knowledge for teaching* (MKT) –and refined the initial categories proposed by Shulman (1987), as shown in Figure 3.

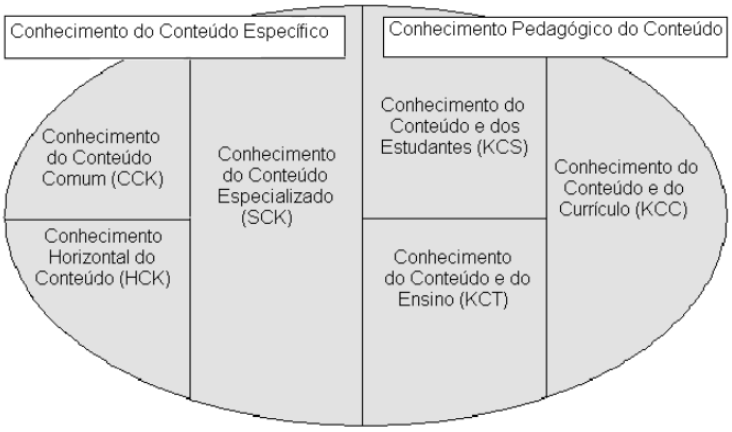

Figure 3.

Mastery of mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008, p. 403, free translation)

Assemany and Figueiredo (2022) highlighted the creative insubordination actions of a mathematics teacher from the municipal school system of Niterói when teaching measures of central tendency in a didactic intervention. This case study analyzed interviews and documents, observing the teacher's behaviors when preparing and developing a didactic sequence for her students. The work was based on Freire's conception of teaching autonomy and Sá-Chaves regarding transgressions. D'Ambrósio, Lopes, and Gutiérrez have their analyses rooted in creative insubordination, evidenced in work from the relevance given to the uncertainty of statistics and how the teacher interpreted, disagreed with, reflected on, and reorganized the learning situations.

The focus of this work is on the autonomy (Freire) and reflection (Schön) of the teacher who interprets, disagrees with and organizes instruction in a subversive, responsible, and ethical way (Sá-Chaves). The dimensions of knowledge go beyond mathematical and pedagogical knowledge, encompassing knowledge with students/communities and political knowledge (Gutiérrez), as per the model illustrated in Figure 4.

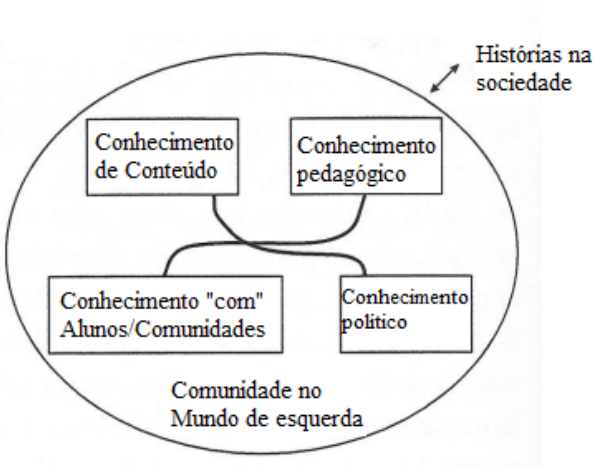

Figure 4.

Political knowledge for mathematics teachers (Gutiérrez, 2018, p. 20)

Brandelero and Estevam (2023) investigate learning evidenced in the reflections of a basic education teacher about her first teaching experience researching her own practice involving exploratory teaching of statistics. In this sense, the teacher-researcher presented reflections she shared with a research group in which she participates. The authors rely theoretically on Cyrino and Oliveira for exploratory teaching, on Gal for statistical literacy, and on Rodgers for the professional development of reflective teachers, addressing four stages: spontaneous interpretation of experience, description of experience, analysis of experience, and intelligent action/experimentation.

Santos et al. (2023) analyzes (auto)biographical narratives of mathematics teachers who teach probability and statistics in basic education and their professional teaching development processes. To this end, textual and lexical analyses of the content are used through the IRaMuTeQ software from the narratives of ten teachers from the states of São Paulo and Espírito Santo, participants in a continuing education course offered in a virtual learning environment. Focused on learning how to teach, the teachers' narratives allowed them to create four categories of analysis: professional challenges, initial education, school practice, and continuing education.

Rodrigues and Ponte (2020b) study the development of didactic knowledge on teachers' representations and statistical investigations in continuing education. The participants were 13 elementary school mathematics teachers who took a statistics subject in a specialization course. The data sources were the field diary, audio recordings, documentary collection, and interviews.

For content analysis (Bardin), they defined categories of analysis based on the dimensions of the stated teacher's didactic knowledge (Ponte) and units of analysis for each

category based on the highlighted specifications of knowledge related to statistics (Batanero), as shown in Table 2 below.

Table 2.

Categories and units of analysis (Rodrigues & Ponte, 2020b, p. 8)

Categories	Units of analysis
Knowledge of the statistics curriculum	- What aspects of the curriculum they recommend - How they relate the materials to the curriculum proposal
Knowledge of statistics	- The meanings they attribute to statistical representations - How the different stages of the investigative cycle work
Knowledge of educational practices in statistics	- Perceptions about tasks related to representations - Knowledge of how to conduct investigations in class
Student knowledge and learning	- How they predict student answers - How they understand students' answers and errors

Mello and Basso (2023) investigate the creative process of four mathematics students with teaching degrees who developed teaching practices with sixth- and seventh-grade students at an application school during the pandemic. Aiming to understand how creativity can favor the construction of teaching knowledge, they analyze plans and portfolios of participants who, in 2020, created the character Ellie, who interacted with basic education students. Additionally, the files made available to students featured a variety of colors and elements. There was also an approach to everyday situations. In 2021, the undergraduates continued working with Ellie through video production. The authors are theoretically anchored in Piaget's reflective abstraction, which explains how the construction of knowledge develops specifically with human beings, and in Borges and Fagundes' "cyclone of creation," which sees the creative process in the following stages: asking, knowing, understanding, and creating, in a cycle that drives innovation and progress to new levels of complexity.

Below, the adopted references related to the teacher's knowledge are summarized.

Table 3.

References related to the teacher's knowledge (own authorship, 2024)

Articles	**Theoretical framework(s) adopted**
1. Fernandes & Santos Junior (2020)	Conhecimentos docentes [Teaching knowledge] - Shulman (1986, 1987).
2. Rodrigues & Ponte (2020a)	Conhecimento didático do professor [Teacher's didactic knowledge] - Ponte (2012); Dimensões nos conhecimentos expressos pelos professores [Dimensions in the knowledge expressed by teachers] - Smith (2001), Ponte (2005) and Ben-Zvi (2000).
3. Araujo & Carvalho (2021)	Modelo de Conhecimento e Competências didático-matemático (CCDM) [Didactic-mathematical knowledge and competencies model (DMKC)] - Pino Fan and Godino (2015) and Godino, Batanero, Font and Giacomone (2016).
4. Silva *et al.* (2021)	Domínio do conhecimento matemático para o ensino [Mastery of mathematical knowledge for teaching] – Ball, Thames, and Phelps (2008).
5. Assemany & Figueiredo (2022)	Conhecimento político para professores de Matemática [Political knowledge for mathematics teachers] (Gutiérrez, 2012).
6. Brandelero & Estevam (2023)	--------
7. Santos et al. (2023)	--------
8. Rodrigues & Ponte (2020b)	Dimensões do conhecimento didático do professor em Estatística [Dimensions of the teacher's didactic knowledge in statistics] - based on Ponte (2012) and Batanero (2002).
9. Mello & Basso (2023)	--------

We noticed in the analysis of the articles that a plurality of theoretical references were adopted to analyze the teacher's knowledge, taking a look at the content and pedagogical issues, as already proposed by Shulman (1986, 1987). We also observed a focus on students and the curriculum, which can be seen from the categories proposed by the authors. We also highlight

the perspective given by Gutiérrez to the so-called political knowledge for mathematics teachers, which is a difference when noting the other models for the analysis of teaching knowledge and the non-observance of aspects related to a model of teacher knowledge in three of the works analyzed, that is, Brandelero and Santos (2003), Santos et al. (2023), and Mello and Basso (2023).

A look at the knowledge and knowings of the teacher who teaches statistics

In this section, we will present an analysis of the *corpus* of our study in light of the teacher's knowledge for teaching according to Shulman (1986) and knowledge/knowings for teaching according to Tardif (2002). This way, we sought to identify the teaching knowledge/knowings evidenced in each article selected for the present investigation.

Fernandes and Santos Júnior (2020) developed a formative process in a combined proposal that involved statistical literacy and graphic understanding. The first activity consisted of a question that proposed the analysis of a graph with curves, presenting growth patterns stipulated by the World Health Organization (WHO) according to the child's age, and asked about the percentage increase. A discussion was also encouraged about other problem situations in real contexts. According to the authors, the participants could determine the solution to the proposed problem, highlighting *content knowledge*. A participant teacher also presented an alternative representation of the proposed problem.

Another activity also required knowledge of graphic representations and analysis of the proposed problem situation when they were asked to analyze a graph to prescribe a diet for a person. With this, the levels of two substances were verified according to the graph presented, and they took into account that, for an effective weekly diet, a parameter whose value is given by the number of times the levels of these two substances are equal and greater than the minimum of one of these substances during the diet should be established. Once again, the focus is on *content knowledge*.

The authors state that "the participant teachers solved the problems in a dialogical and interactive way, seeking to visualize the teaching and learning process, as well as its developments, in a general context" (Fernandes & Santos Júnior, 2020, p. 528). Furthermore, the participants indicated teaching actions and approaches, reflecting on the "most appropriate strategy for teaching specific content, in order to make it understandable for students" (Fernandes & Santos Júnior, 2020, p. 530). Thus, in the discussions held during the education process, reflections on *pedagogical knowledge* were promoted. About that type of knowledge, we highlight a change in teachers' understanding of teachers, with a new focus on the

perspective of statistical education as no longer “merely restricted to the application of operational procedures for representing data and/or information” (Fernandes & Santos Júnior, 2020, p. 532), which can be reinforced by one of the participants' speech: “One possibility for teaching statistics is to use the ideas of the author we studied in the course on the investigative cycle stages (P4N2’s excerpt)” (Fernandes & Santos Júnior, 2020, p. 534).

About *experiential knowings*, Fernandes and Santos Júnior (2020) highlight a perspective of future use of the combination between elements of statistical education and graphic understanding by participant teachers, emphasizing that “it favors students’ cognitive expansion on the topic” (p. 536). P8N5’s excerpt discussing research in statistics activities also points to a look into students: “Therefore, research leads students to think, analyze, inquire, and reflect on different aspects of the chosen topic (P4N2’s excerpt)” (Fernandes & Santos Júnior, 2020, p. 534)

Finally, regarding *curriculum knowledge*, teachers had curriculum knowledge of the curriculum organization of the State Education Guidelines (*Diretrizes Estaduais de Educação*) related to mathematics; however, the authors understand that it was necessary to revisit the Curriculum Parameters and Guidelines (*Parâmetros Curriculares e Diretrizes*) related to teacher education. However, they do not highlight the National Common Curriculum Base (*Base Nacional Comum Curricular* - BNCC), the current normative document in Brazilian education.

We also highlight the authors’ emphasis during the analysis on the articulation between statistical literacy and graphic understanding, pointing out aspects of both so that teaching knowledge was not too stressed in light of Shulman. The term *knowings* also appears constantly in the analysis of the education process offered, not being part of Shulman’s (1986, 1987) concept of knowledge base.

The research by Rodrigues and Ponte (2020a) includes results from an intervention in a teacher education course consisting of ten sessions. In the first session, a diagnostic questionnaire was applied. The second focused on analyzing curriculum guidelines and school materials linked to the *curriculum knowledge* (Shulman, 1986) and knowings from programs and textbooks used at work (Tardif, 2002). The third involved analyzing a video-recorded class, observing the teacher's actions and the student's reasoning, which can establish relationships with *pedagogical knowledge* and *experiential knowings*. Sessions four and five aimed to analyze and carry out exploratory tasks using technological resources, which can increase the students' repertoire by relating *content and pedagogical knowledge*. The sixth session proposed the analysis of students' answers, favoring the discussion of aspects of

experiential knowings. In the seventh session, inferential reasoning and probabilistic distribution models were discussed. A statistical investigation was carried out in the eighth and ninth sessions, providing a possible expansion of the *content knowledge*. Finally, in the tenth session, the participating teachers answered a final questionnaire.

For data analysis, Rodrigues and Ponte (2020a) focus on the second and fourth sessions, considering them representative. In particular, in the second, they notice the part related to the selection of teaching materials. One of the participants initially chose a book focusing on the number of exercises without reflecting more deeply on the aspects related to teaching statistics: "*I picked up a 9th-grade book I've already used much* (...). *I like it because it gives you a lot of exercises*" (Rodrigues & Ponte, 2020a, pp. 154-155). However, following the educator's intervention, there was a redefinition of the choice of materials, focusing on their potential in approaching statistics. This highlights an expansion in elements aimed at *didactic and curriculum knowledge.* In this sense, we highlight an excerpt from another participant's speech:

> Statistics at school must be used to evaluate data that is useful to society (...) I think that inside the classroom we have to take magazines, newspapers… Teach students how to collect [data] in a more real context (Rodrigues & Ponte, 2020a, p. 154).

During the discussion about the choice of materials, one of the participants listed the guidelines contained in the National Curriculum Parameters (*Parâmetros Curriculares Nacionais* - PCN) about varied and exploratory tasks, which underscores the *curriculum knowledge.* Finally, one discussion reflects that the teacher must go beyond the textbook and must be attentive while preparing tasks so that they make sense to students.

In the fourth session, there was a proposal for analysis and exploration tasks. Rodrigues and Ponte (2020a) highlight one involving the concept of weighted average, observing the difference between this and the arithmetic average and involving percentage, relative frequency, and the influence of the concept of proportionality when using the weighted average, therefore involving the *content knowledge*. It should be noted that, at first, the participant teachers did not understand the complementarity between the proposed items, not seeing an interrelationship between them, despite solving the items without difficulty (except for the last one, which requested them to construct a graph), which the authors suggest is related to the frequent contact with more direct questions.

Regarding graph construction, all participants disregarded variability. However, when redoing the graph using Excel, they could observe and reconsider the presence of variability and the meaning of average. Teachers blamed this error on the fact that they "*deal more with*

graph analysis than with its construction. Furthermore, the task in question was not part of the type of activity they usually did or took to their students" (Rodrigues & Ponte, 2020a, p. 161), i.e., the average weight of ten people in an elevator, with the average weight of the four women present being 60 kg and of the six men being 80 kg. Finally, we realized that this task favors the expansion of the *content knowledge* since subsequent discussions mediated by the educator made the students realize the existing proportional relationship, which had not been achieved previously.

Araújo and Carvalho (2021) also integrate the results of an investigation into teaching knowledge based on a formative process, a workshop that included three activities, and a systematization of the topic addressed. The first activity required calculating statistical measures, such as arithmetic mean, mode, median, and standard deviation, in addition to involving probability. The answers presented by the teachers and undergraduates participating in the workshop are classified as adequate, demonstrating their *content knowledge* of the topics listed. After that, they had to explain how they approached these concepts in class and how they considered they should be approached, with answers that demonstrated a mechanized approach emphasizing the application of formulas. The subsequent discussion encouraged reflection on the inclusion of real data that connected with students' social context and could develop skills linked to critical thinking, analysis, and data interpretation.

In the theoretical systematization coordinated by the educators, the conclusion was that the participants had a better command of descriptive statistics than inferential statistics, and, therefore, in their classes, they prioritized elements with which they were more familiar. This fact shows how *content knowledge* is necessary for developing and expanding other knowledge and knowing, in this case, *pedagogical knowledge*, since gaps in the content significantly affect how they organize, prepare, select, and develop activities with their students.

We also listed that the discussions allowed noticing the *pedagogical knowledge* regarding the perspective of the approach to statistics, as can be seen in the following excerpts from P6 and P8, respectively: "*I believe we should give students an opportunity to be researchers*" (Araújo & Carvalho, 2021, p. 291); "*I also think that by carrying out research with the students' context itself, we can work on the concept of samples, in the idea of representing the whole set and also associating it with probability*" (Araújo & Carvalho, 2021, p. 291). In P8's speech, there is also a relationship with the *content knowledge* since the relationship between statistics and probability is emphasized. Furthermore, the authors point out that, when proposing the development of statistical research, there is a consonance with the current curriculum documents, which reflects aspects of *curriculum knowledge*.

At the time of the theoretical systematization of the normal curve, we understand that there was an expansion of the *content knowledge* of the participants since they "(...) *began to understand the concept of the normal curve and to be able to perform statistical readings and assimilate the meanings of each graph presented and the concepts of centrality and dispersion measures covered by each*" (Araújo & Carvalho, 2021, p. 292). Furthermore, the calculation of probabilities using the methods of the standardized normal table was addressed through Geogebra, which, as teaching resources, enables an expansion of teaching and technological repertoire. The meeting ended by addressing the curriculum guidelines that guide the teaching of the normal curve at the national and state levels, contributing to the *curriculum knowledge.* From a student's answer to a problem involving a normal curve, questions such as "*How would you assess and explain that student's answer? Given the answer presented, how would you proceed with the class discussion*?" (Araújo & Carvalho, 2021, p. 296) were encouraged by researchers and enabled discussions from a didactic point of view and reflections on experiential knowings.

Silva et al. (2021) present the results of an investigation carried out in a continuing education course for mathematics teachers, with the theme of measures of central tendency (MCT). In the first phase, the participants' profiles and professional knowledge were surveyed using a questionnaire. In the second moment, the formative process was carried out. In the diagnostic questionnaire, one question requested participants to choose the most favorable situation using the mean, mode, and median of specific values as data. The authors emphasize that "*most did not present arguments that took into account the relationship between the measures of central tendency for decision-making*" (Silva et al., 2021, p. 344). Thus, this situation was taken up again in the formative process, and some conceptual gaps were indeed perceived, indicating weaknesses in *content knowledge.*

Advances in this knowledge were also observed when comparing the answers given in the initial questionnaire and the answers given at a later stage of the formative process, despite some obstacles. Silva et al. (2021) highlight that "*Teacher I improved the interpretation of the measures of central tendency, but the precedence of the values of these measures to the detriment of their meanings was still decisive in his analysis*" (p. 346). Some gaps were identified, such as the view that the mode represents most data. When considering the reflections and discussions promoted by the formative process, the authors state that the participants had doubts about the meanings of the MCT, highlighting that such limitations related to *content knowledge* can compromise the mastery of *pedagogical knowledge.*

Assemany and Figueiredo's (2022) research takes place in a didactic intervention with eighth-grade elementary school students. The teacher first considered that the methodology of the material adopted by the school did not fit her didactic-pedagogical convictions, denoting aspects related to *pedagogical and curriculum knowledge,* since it takes into account the choices and organization presented in the teaching material from the point of view of both content and methodological aspects. About that knowledge, the guidelines during the didactic intervention point to a methodology that emphasizes reflective debate, research guidance, formalization of concepts, and exploratory tasks. The authors highlight that students' knowledge was valued throughout the activities, making them protagonists. We can indicate these aspects as linked to *experiential knowings*.

Brandelero and Estevam (2023) show individual reflections on a teacher's own practice and collective reflections in a research group she participates in. The text presents the teacher's actions during pandemic times, from the preparation of the activity that would support the task with the students, to the guidance, conduct, and questioning attitudes of the teacher with the participating students. Such situations highlight an expansion of *content* and *pedagogical knowledge* based on the teacher's pedagogical practice. The expansion of such knowledge was also encouraged by the new perspective raised by the planned activity, which sought to give meaning to the average, not focusing solely on the calculation procedures and the result found, as shown in the following excerpt:

> For the teacher, the systematization of learning related to the concept of simple arithmetic mean was one of the moments that caused her some strangeness, as she was used to conceptualizing average based on the calculation procedure, and at that moment, she was following the guidelines planned for the class, aiming not only to calculate the mean, but to give meaning to the values found (Brandelero & Estevam, 2023, p. 497).

Brandelero and Estevam (2023) highlight the power of work conducted from practice to implement professional teacher development so that it promotes "advances and expansions based on the problematization of teachers' knowledge, beliefs, understandings, and feelings to the detriment of models and actions inferred from the presentation of (new) knowledge that they do not have, that they lack" (p. 505). This brings us to the appreciation *of knowings from experience* in teacher education.

From this perspective, Santos et al. (2023) analyze (auto)biographical narratives of mathematics teachers who teach probability and statistics, prepared as the first activity of an extension course. Those narratives should consider aspects related to the personal and professional life trajectory, which we believe is linked to what Tardif (2002) calls *experiential*

knowings.

Some excerpts from the participants' narratives list the search for courses that can contribute to the improvement of *pedagogical knowledge*, such as P1, who studied pedagogy to improve his practice, and P2, who highlighted participation in continuing education:

> [...] In this course, I learned about new teaching methodologies essential for teacher education. Soon after, I entered my second lato sensu specialization in higher education teaching. [...] This course provided me with a vast knowledge of learning theories and expanded my knowledge of constructivist pedagogy [...] (Santos et al., 2023, p. 262).

The context of school practice, the *knowings from experience*, also expands the *pedagogical knowledge*, as identified by P6 when highlighting gaps in initial education: "Contextualizing, problem situations, illustrations, applications in everyday life, social reality, all these terms had to be learned during my professional activity because they never appeared in my formative process" (Santos et al., 2023, p. 264).

Rodrigues and Ponte (2020b) carried out their analysis based on "*three distinct moments of the formative process: analysis of teaching materials; resolution of tasks and analysis of students' answers; and the completion of a mini-statistical investigation by teachers and discussion about carrying out investigations in the classroom*" (p. 8). First, participants were divided into groups to analyze teaching material. Each group was responsible for studying materials from one of the middle school grades (from sixth to ninth grade). Teachers generally criticized a given material for its synthetic form and lack of exploration of aspects of one of the topics. Two participants were concerned about proposing activities that would mobilize the student's critical sense, not focusing only on calculations, as we can see in the excerpt presented by Rodrigues and Pontes (2020b): "*I found the first question interesting and interdisciplinary, but it loses focus on statistics and critical thinking by only doing calculations. The accounts can be made at another time*" (p. 10). Such discussions foster questions about *pedagogical and curriculum knowledge* in view of the critical analysis of teaching material. However, the authors point out some weaknesses since "*the teachers present superficial considerations regarding the tasks*" (Rodrigues & Ponte, 2020b, p. 10). Rodrigues and Ponte (2020b) see the association of tasks with curriculum proposals and their objectives, configuring, from our point of view, *curriculum knowledge*.

In the second stage, referring to the resolution of tasks and the analysis of students' answers, we can highlight the proposals that deal with the *content knowledge*, in which it was possible to identify teachers' difficulties understanding the meanings inherent in various graphic representations. In the part where they analyzed the students' answers, we noticed a collective

reflection on the characteristics of the proposed question. One participant highlights: "*It is always necessary to have moments of discussion between students... I think that way they learn from their errors…* " (Rodrigues & Ponte, 2020b, p. 13), highlighting the appreciation of students' errors, which we associate with *pedagogical knowledge*. In general, regarding this moment, the authors state the following:

> Initially, some teachers showed that they did not know the need to propose varied tasks so that students could develop statistical literacy. Although some teachers still show concerns about the quantity and quality of the proposed tasks, their discomfort is noticeable when they realize that the tasks on data processing are not adequately valued in the materials. With reflection and small interventions from the educator, teachers could identify important characteristics of the tasks, including appreciating aspects of students' context (Rodrigues & Ponte, 2020b, p. 14).

As a result, in the stage involving the teachers carrying out a mini-statistical investigation and discussing the investigations in the classroom, gathered in groups, the teachers were instructed to choose topics that interested them. After this activity, they discussed aspects of conducting statistical investigations in the school context. In this context, the authors observed gaps in content knowledge but also reported the potential of the proposal.

Mello and Basso (2023) look at pedagogical practices carried out by teaching degree students during the pandemic period. Among the actions of the teaching degree students is the creation of the character Ellie, who is "the same age as the students and has a similar language" (Mello & Basso, 2023, p. 6). The activities also included "*a variety of colors and elements, considering that this resource could potentially spark students' interest in learning more about Professor Ellie and, consequently, learning more about mathematics*" (Mello & Basso, 2023, p. 7). This character and the focus given to the students' context and the visual characteristics of the activities were part of the methodological approach chosen to interact with basic education students.

Given the perspective of creating, preparing, and forwarding activities with students, we highlight signs *of content knowledge and pedagogical content knowledge for the* organization and choices for teaching statistics, given the concern and engagement in a proposal to create a character with their characteristics in search of better interaction. The speech of one of the undergraduates portrays this path well: "*The creation of Professor Ellie became quite a challenge, as we must also explain as if we were their age and using their knowledge*" (Mello & Basso, 2023, p. 16).

Conclusions

The perspective of the analysis of the articles was the discussion of knowings and knowledge for teaching according to the points of view of experience, content, pedagogical, and curriculum, having as initial foundations the studies by Shulman (1986) and Tardif (2002). We identified that, in the set of articles analyzed, the following references were used: statistical literacy, by Gal; knowledge of the mathematics teacher, according to Ball, Thames, and Phelps (2008); didactic knowledge of the mthematics teacher, according to Ponte; didactic-mathematical competencies and knowledge of the teacher, according to Pino Fan and Godino (2015); and graphic comprehension, by Curcio (1989).

There seems to be a consensus in the articles analyzed that the mathematics teacher must be able to mobilize the specific contents of their subject to develop students' competencies under the curriculum and that, therefore, a solid knowledge of the subject to be taught, although central, is insufficient. However, in the formative processes proposed by the researchers, authors of the articles, content knowledge prevailed compared to pedagogical knowledge. This observation reiterates some consensus among the chosen theorists that developing didactic knowledge without mastering content knowledge is not possible. However, there is no agreement on how broad and deep the teacher's mathematical knowledge should be. Such emphasis on knowledge of statistical content in continuing education for teachers can be partly justified by the fact that, in the curriculum guidelines for mathematics courses, the subject Probability and Statistics is only included in non-licensing degrees, even though, in some universities, licensing courses in this field present syllabuses involving these topics (Gatti, 2014).

For formative processes –initial and continuing education– of teachers who will teach statistics in basic education, we consider it necessary to start from the assumption that the teacher must have not only knowledge of the content they will teach but also explore learning situations around which they will organize the content to teach based on contexts that are rich for learning, such as fields of social action. These include those of personal life, public life, and the practice of study and research, as well as the worlds of culture, science, technology, and work.

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