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Mathematical knowledge for teaching: dialogue about practice of mathematics teachers in basic education

Conocimiento matemático para la enseñanza: diálogo sobre la labor docente de profesores de matemáticas que actúan en la educación básica

Connaissances mathématiques pour l'enseignement : dialogue sur le travail d'enseignement des enseignants de mathématiques dans l'éducation de base

Conhecimento matemático para o ensino: diálogo sobre o trabalho docente de professores de matemática que atuam na educação básica

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Abstract

This article is part of the first author's doctoral research and its guiding question is the following: What contributions can the professional teaching experience of teachers who have graduated from UEPB's Mathematics degree program offer to a reflection on the relationship between the knowledge needed to teach mathematics in primary and secondary education the training offered in the course? To this end, we used as a theoretical reference some studies that point to the contributions of Mathematical Knowledge for Teaching (MKT) and its implications in the process of initial training for mathematics teachers. The aim of this text, in turn, was to analyze elements identified in the speech of teachers working in primary and secondary education that characterized Mathematical Knowledge for Teaching (MKT) and its relationship with the initial training offered by the UEPB degree program. The research carried out was a qualitative investigation whose participants were seven recently graduated teachers, graduates of the Mathematics Degree program at UEPB. The analysis considered information obtained from the interviews with the participants, following the steps and guidelines described by Content Analysis. The results showed that the subdomains that make

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up MKT were identified in the teachers' classroom experiences and episodes, with special emphasis on Knowledge of Content and Students. The teachers investigated recognize the importance of mastery of the content in the work of mathematics teachers, but ensure that the work of teaching requires a multiplicity of knowledge that goes far beyond the specific content.

Keywords: Mathematical knowledge for teaching, Teaching knowledge, Mathematics degree.

Resumen

Este artículo forma parte de la investigación doctoral de la primera autora y su pregunta guía es la siguiente: ¿Qué aportaciones puede ofrecer la experiencia profesional docente de los profesores egresados de la carrera de Matemáticas de la UEPB a una reflexión sobre la relación entre los conocimientos necesarios para enseñar matemáticas en educación primaria y la formación ofrecida en la carrera? Para ello, utilizamos como referencia teórica algunos estudios que señalan las contribuciones del Conocimiento Matemático para la Enseñanza (CMD) y sus implicaciones en el proceso de formación inicial de profesores de matemáticas. El objetivo de este texto, a su vez, fue analizar elementos identificados en los discursos de profesores que trabajan en la educación básica que caracterizan el Conocimiento Matemático para la Enseñanza (MKT) y su relación con la formación inicial ofrecida por la carrera de UEPB. L'analyse des données a pris en compte les informations obtenues lors des entretiens avec les participants, en suivant les étapes et les lignes directrices décrites par l'analyse de contenu. Les résultats ont montré que les sous-domaines qui composent la MKT ont été identifiés dans les expériences et les épisodes de classe des enseignants, avec un accent particulier sur la connaissance du contenu et des élèves. Les enseignants interrogés reconnaissent l'importance de la maîtrise du contenu dans le travail des enseignants de mathématiques, mais assurent que le travail d'enseignement requiert une multiplicité de connaissances qui vont bien au-delà du contenu spécifique.

Palabras clave: Conocimientos matemáticos para la enseñanza, Conocimientos didácticos, Licenciatura en matemáticas.

Résumé

Cet article s'inscrit dans le cadre de la recherche doctorale du premier auteur et sa question directrice est la suivante : Quelles contributions l'expérience professionnelle d'enseignants diplômés de la licence de mathématiques de l'UEPB peut-elle apporter à une réflexion sur la relation entre les savoirs nécessaires à l'enseignement des mathématiques dans l'enseignement

primaire et la formation proposée dans le cadre de la licence ? Pour ce faire, nous avons utilisé comme référence théorique quelques études qui soulignent les apports de la Connaissance Mathématique pour l'Enseignement (CME) et ses implications dans le processus de formation initiale des enseignants de mathématiques. L'objectif de ce texte, à son tour, était d'analyser les éléments identifiés dans le discours des enseignants travaillant dans l'éducation de base qui caractérisent la Connaissance Mathématique pour l'Enseignement (CME) et sa relation avec la formation initiale offerte par le cours de licence de l'UEPB. La recherche menée est une enquête qualitative dont les participants sont sept enseignants récemment diplômés, issus du programme de licence en mathématiques de l'UEPB. L'analyse des données a été faite à partir des informations obtenues lors des entretiens avec les enseignants participant à l'étude. Les étapes et les lignes directrices décrites par l'Analyse de contenu ont été suivies. Les résultats obtenus ont montré que les sous-domaines qui composent le MKT ont été identifiés dans les expériences et épisodes vécus en classe par les enseignants, notamment la Connaissance du contenu et des élèves. Ce sous-domaine a fourni des indices importants sur la façon dont la connaissance de la relation entre les élèves et les mathématiques peut influencer les choix méthodologiques de l'enseignant. Les enseignants interrogés reconnaissent l'importance de la maîtrise de contenu pour la performance de l'enseignant de mathématiques. Cependant, ils affirment que le travail d'enseignement exige une multiplicité de connaissances qui va au-delà du contenu spécifique.

Mots-clés : Connaissances mathématiques pour l'enseignement, Savoirs des enseignants, Diplôme en mathématiques.

Resumo

Este artigo é um recorte de uma pesquisa de doutorado da primeira autora e tem como questão orientadora da pesquisa a seguinte: Quais contribuições a experiência profissional no ensino, vivenciada por professores egressos da Licenciatura em Matemática da UEPB, pode oferecer para uma reflexão sobre as relações existentes entre o conhecimento necessário para ensinar matemática na Educação Básica e a formação oferecida no curso? Para tanto, tomou-se como referencial teórico alguns estudos que apontam para as contribuições do *Mathematical Knowledge for Teaching - MKT* e suas implicações no processo de formação inicial de professores de matemática. Objetivou-se analisar elementos identificados na fala de professores que atuam na Educação Básica que caracterizavam o Conhecimento Matemático para o Ensino (MKT) e sua relação com a formação inicial oferecida pela Licenciatura da UEPB. A pesquisa realizada configurou-se como uma investigação qualitativa cujos

participantes foram sete professores recém-formados, egressos da Licenciatura em Matemática da UEPB. A Análise dos dados considerou informações obtidas nas entrevistas realizadas com os participantes, seguindo as etapas e orientações descritas pela Análise de Conteúdo. Os resultados obtidos demonstraram que os subdomínios que compõem o MKT foram identificados nas experiências e episódios de sala de aula vivenciados pelos professores, com destaque especial para o Conhecimento do Conteúdo e dos Estudantes. Os docentes investigados reconhecem a importância do professor de matemática conhecer em profundidade o conteúdo a ser ensinado, todavia asseguram que o trabalho de ensinar demanda uma multiplicidade de saberes que vão muito além do conteúdo específico.

Palavras-chave: Conhecimento matemático para o ensino, Saberes docentes, Licenciatura em matemática.

Mathematical knowledge for teaching: Dialogue on the professional practice of mathematics teachers working in basic education

Teacher education in Brazil has always been fertile ground for research and discussions that involve questions about how it is carried out, the teaching degree role in preparing prospective teachers, and the rapprochement between universities and schools to provide immersion for future professionals in their field of activity.

These concerns suggest reflections on how undergraduate curricula in Brazil are organized, leading to the conclusion that "there is a great discrepancy between the pedagogical projects of these courses and the curricular structure actually offered" (Gatti, 2014, p. 39). The formation of pedagogical knowledge, professional teaching practices, studies of issues linked to the school environment, and didactics have historically occupied minimal space in the curriculum. Associated with this, there is a robust attachment to specific content subjects (and a resistance to dissolving the dichotomy between theory and practice and reducing the separation of specific and pedagogical content (Fiorentini & Oliveira, 2013; Gatti, 2014; Junqueira & Manrique, 2015; Moreira & David, 2016).

According to Segatto (2019), several countries are implementing changes in initial teacher education that minimize issues that involve the lack of connection between specific and pedagogical content, the lack of articulation between theory and practice, and the fragmentation of content. The author says this discussion has influenced debates on policy changes and initial teacher education programs (Segatto, 2019). In mathematics education, we have noticed a consensus among researchers and scholars focused on initial education that a teaching degree that mobilizes knowledge acquired in practice is necessary (Ball & Forzani, 2009; Fiorentini & Oliveira, 2013; Moreira & David, 2016; Santos & Lins, 2016).

Upon completing their degree and finally beginning their professional career, teachers often realize that teaching requires special knowledge and skills not adequately experienced in the preparatory course (Ball & Forzani, 2009; Moreira & David, 2016). For Rocha and Fiorentini (2005), it is usual in this initial phase of a teacher's career to be overcome by a feeling of professional unpreparedness, often related to the distance between the theory studied in the course and the daily routine of the school. Fear and insecurity are prevalent at this stage, and questions about whether or not to stay in the profession are not far away.

The aspects that make up the teacher education process must be experienced for the work to acquire meaning and significance; otherwise, there is a risk that the education process experienced by the prospective teacher will only occur at a superficial level. The core of this

discussion focuses on how important it is for prospective teachers to experience their future profession more meaningfully, which justifies and legitimizes their education. That said, it seems appropriate to reflect on the knowledge necessary for the mathematics teacher's work as central in the discussion and reflection on the curricula of teaching degrees and the overcoming of a tradition of valuing formal content based on dichotomies that should have been overcome long ago.

In this direction, this article is the result of a doctoral work[3] aimed at research focused on the problems involving teaching knowledge and the initial education of mathematics teachers, more specifically focused on the mathematics teaching degree offered at the Center for Human and Exact Sciences of the State University of Paraíba, Campus VI. The question that guided the investigation was: What contributions can the teaching professional experience by teachers who graduated from the mathematics teaching degree course at UEPB offer for a reflection on the relationships between the knowledge necessary to teach mathematics in basic school and the education offered in the course?

This text will relate the analysis of the elements identified in the basic school teachers' speeches that characterized the *mathematical knowledge for teaching* (MKT) and its relationship with the initial education offered by the UEPB teaching degree. We present a theoretical explanation of the literature that deals with the theme of teaching knowledge, which is associated with the initial education of mathematics teachers and its relationship with the teaching demands experienced in their work environment. We emphasize that the literature that addresses this topic is quite extensive. However, we direct our attention to the work of Ball, Thames, and Phelps (2008), who introduced and developed the concept of mathematical knowledge for teaching (MKT).

The research described in this article identified evidence of the presence of MKT from the perspective of newly graduated teachers from the UEPB teaching degree course working in basic education when the study was carried out. This study brought to light reflections on the need for a curriculum for initial education centered on the work that the mathematics teacher actually develops in basic school.

The text is divided into four sections following this introduction. Next, we discuss the central aspects of the theoretical framework that guided data production and analysis. The "Methodological aspects that guided the investigation" section presents the research and the

[3] This research was analyzed and approved by the Ethics and Research Committee of the Federal University of Rio Grande do Norte (Opinion n. 3.683.960) and carried out within the scope of the Research Group Contar, from the Postgraduate Program in Education at this university. It was supported by the State University of Paraíba.

methodological and analysis procedures adopted. The last two sections refer to the data analysis obtained from the interviews with the teachers and to the final considerations, respectively.

Mathematical knowledge for teaching: from Shulman's pioneering work to the studies of Ball and collaborators

We begin this section by recalling a concept introduced in the mid-1980s, widely disseminated by many national and international studies and in various areas of scientific knowledge: the *pedagogical content knowledge (PCK*), proposed by Lee Shulman in his famous articles, *Those who understand: Knowledge growth in teaching* (Shulman, 1986) and *Knowledge and teaching: Foundations of the new reform* (Shulman, 1987).

Shulman (1986) made a critical analysis of the tests used in the past, about a hundred years earlier, which aimed to measure the teacher's competence in relation to issues involving content and pedagogical skills. When surveying these mechanisms for measuring teacher effectiveness, the author found that the emphasis was on content; that is, a teacher's good performance was restricted to mastering the content he or she would teach.

On the other hand, teacher evaluation policies in the 1980s abruptly reversed this bias, as they were centered on purely pedagogical issues, mainly related to the teacher's success in classroom management and the organization of activities. Although recognizing the relevance of these aspects, Shulman (1986) identified that essential questions regarding the content to be taught had disappeared from these discussions. Shulman (1986) called this absence of content in research on teacher cognition in the 1980s the "missing paradigm."

When questioning this obvious separation between pedagogy and content, Shulman (1986) highlighted a visible imbalance between pedagogy and content issues. For him (Shulman, 1986, p. 8), "Mere content knowledge is likely to be as useless pedagogically-ally as content-free skill." In this sense, the author ensures that it is necessary to adequately combine the two aspects, paying attention to both the content and the teaching process.

Shulman (1987) then suggested the following categories of teachers' knowledge base: content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of students and their characteristics, knowledge of educational contexts, and knowledge of the ends, purposes, and values of education and its historical and philosophical basis. In this work, we focus mainly on how the author defined pedagogical content knowledge (PCK).

For Shulman (1986), the PCK goes far beyond understanding the content itself; it is a type of knowledge that incorporates the aspects of the content most relevant to the teaching activity. In the author's words, the PCK comprises:

> the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations-in a word, the ways of representing and formulating the subject that make it comprehensible to others. Since there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation, some of which derive from research whereas others originate in the wisdom of practice. (Shulman, 1986, p. 9)

The author refers to how teachers relate their pedagogical knowledge to disciplinary knowledge to teach students, not in a juxtaposed way, but amalgamated. The implications of the discussion presented by Shulman (1986; 1987) denote the understanding that teaching and teacher education must understand the teacher as someone capable of acting self-consciously and not just someone who applies techniques or procedures developed by specialists.

The ideas brought by Shulman (1986; 1987) mark the beginning of a new look at the teachers' persona and their role in the teaching process. However, like every ideological construction originating from its time, the PCK belongs to a specific historical moment and is the result of the circumstances that characterize it. Over the years, this construct has been widely discussed, reflected upon, and restructured. Shulman himself, in a more recent publication, agrees that the form the concept took was a response to an ongoing discourse at the time whose prevailing view treated teaching as a "process without content, and teachers as skilled actors without minds, emotions, and careers." (Shulman, 2015, p. 9).

The limitations Shulman (2015) listed when rereading the PCK are flaws many academics and researchers are trying to repair in more recent studies, which can provide important results that were not sufficiently achieved back then, mainly because they consider the relevance of how teachers think and develop their practices. The investigations carried out by Ball and collaborators on the teaching knowledge involved in teaching mathematics are part of this range of research and constitute the theoretical foundation that underpinned our study. Here, our attention is directed to the work of Ball, Thames, and Phelps (2008), dealing with the MKT.

Ball, Thames, and Phelps (2008) based their work on studies developed by Shulman (1986; 1987) on PCK, whose conceptual understanding suggests that there is exclusive content knowledge for teaching. For Ball, Thames, and Phelps (2008), there was widespread use of PCK in research in various areas. However, in many cases, this use was incorrect

because there were differences in the understanding of what the concept encompassed regarding knowledge and teaching, and in how the construct related content knowledge to teaching practice. PCK has often been misunderstood and inappropriately used, so even twenty years after its publication, there has been little progress in the initial proposal to develop a coherent theory for the content knowledge needed for teaching (Ball, Thames, & Phelps, 2008).

In the more specific context of mathematics, Ball and his collaborators developed several projects that investigated both the teaching of mathematics and the mathematics used in teaching. In these investigations, instead of starting from a previously determined school curriculum that covered the topics the teacher needed to know, Ball and collaborators worked from a perspective that they called "bottom-up," that is, starting with practice based on the analysis of what occurs in teaching situations and what their demands are, to subsequently evaluate the formative process. These studies are based on empirical experiments, observing and recording teachers in practical work situations, alternating shorter periods with longer periods, and investigating what teaching is required of these professionals.

The work by Ball, Thames, and Phelps (2008) aimed to advance the development of a functional theory about the content knowledge needed for teaching, in particular, the teaching of mathematics. Questions about what teachers need to know and do to teach effectively require a solid theoretical understanding of the mathematical knowledge necessary for teaching.

The consequences of this interpretation are aimed at focusing on the use of knowledge in and for teaching, not on the teacher's central figure. With this, concerns and questions turn to the mathematics teacher's use of this content, i.e., the mathematical knowledge necessary to carry out teaching tasks successfully. With this, the authors define MKT as:

> [...] The mathematical knowledge needed to carry out the work of teaching mathematics. Important to note here is that our definition begins with teaching, not teachers. It is concerned with the tasks involved in teaching and the mathematical demands of these tasks. (Ball, Thames, & Phelps, 2008, p. 395)

The analysis of these studies allowed the authors to observe evidence that teaching requires a unique form of "pure" and "specialized" knowledge that emerges from the practical situation itself. Teachers' practice reveals that mathematical demands are numerous and substantial. Thus, mathematical knowledge for teaching requires teachers to have a good knowledge of mathematics in different forms. For example, teaching demands that the teacher not only identify that the student made a mistake but also explain the origin of the error. This

investigation allows the teacher to recognize students' paths and the procedures that led him to make a mistake and, therefore, guide him in the search for a solution to the problem.

The work of a teacher in the classroom is a complex task, requiring this professional to have a varied framework of techniques, procedures, language, teaching methods, and much more. All these aspects are connected and related to the ultimate goal of teaching, which is to make learning happen. Obviously, teachers must know the content —and know it in depth and uncompressed way— so they may be able to foresee the most diverse or unexpected situations and be prepared to evaluate whether the student's thinking was mathematically correct.

As research progressed, Ball and his collaborators realized that the mathematical demands required for teaching were rarely addressed in university formative mathematics courses. These authors observed that there were aspects of content knowledge revealed in teaching situations that needed to be mapped, organized, and included in teacher education courses (Ball, Thames, & Phelps, 2008) and presented a model for understanding MKT based on a refinement of two of the categories proposed by Shulman (1986; 1987): content knowledge and pedagogical content knowledge, so that each of these categories was subdivided into three:

• Content knowledge — subdivided into *common content knowledge* (CCK), *specialized content knowledge* (SCK), and *horizon content knowledge* (HCK); and

• Pedagogical content knowledge — subdivided into k*nowledge of content and teaching* (KCT), *knowledge of content and students* (KCS), and *knowledge of content and curriculum* (KCC).

These categories are illustrated in Figure 1:

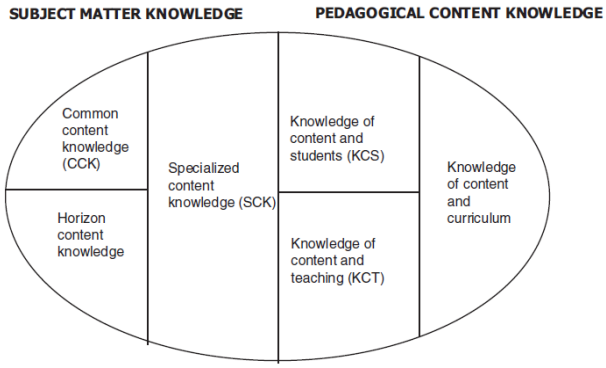

Figure 1.

Domains of mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008, p. 403)

C*ommon content knowledge* (CCK) refers to the knowledge acquired by the subject who has had access to some education in mathematics as a student or a professional from

other areas. This knowledge is not exclusive to mathematics teachers. Many people from the most diverse areas of knowledge can correctly solve a mathematical problem or identify a wrong answer without necessarily having higher education in mathematics.

When we consider the pedagogical context, it becomes clear that the teacher needs to know the content they are teaching to recognize, for example, when a student proposes a wrong solution or when a textbook offers an imprecise or incoherent definition. The teacher should also be able to use mathematical language and symbols correctly otherwise their teaching is likely to suffer.

However, all of this requires mathematical skills that other professionals also have, and therefore, recognizing that a mathematical operation is incorrect or that a symbol has been used incorrectly in a sentence is not a skill that only the mathematics teacher holds. The use of the term "usual" in the definition of the concept does not mean that it is knowledge that everyone has. However, it indicates that it is knowledge used in a wide variety of scenarios, i.e., it is not something exclusive to teachers or the work of mathematics teaching (Ball, Thames, & Phelps, 2008).

Unlike the previous one, the *specialized content knowledge* (SCK) is a specific to the teaching situation and is not normally used for purposes other than teaching. When, for example, the teacher investigates patterns in students' errors, identifying their origins and the paths that led them to the error and guiding them toward the correct solution, they are using a deep and very particular knowledge of mathematics. It is a special type of work that others generally do not do and which requires a unique mathematical understanding and reasoning (Ball, Thames, & Phelps, 2008). This subdomain of knowledge is of special interest to the cited authors since teaching requires mathematical knowledge that is unnecessary in other contexts.

Horizon content knowledge (HCK) is the understanding of how mathematical topics in the curriculum are related. This subdomain is linked to the teacher's perception of which mathematical content will be subsequent and what relationships can be established between the content worked on and what is to come (Ball, Thames, & Phelps, 2008).

The horizon content allows the teacher to provide the mathematical bases necessary for students to envision the sequence of content. For example, a preschool teacher must know how mathematics at this level of education is related to elementary school mathematics, both in terms of establishing relationships between the content being taught and organizing teaching with the following topics to be covered in mind. This knowledge helps the teacher

decide which terms or symbols are appropriate to use or what conceptual associations they can or cannot make at a given moment.

Knowledge of the content and students refers to knowledge that relates students to mathematics. When proposing a mathematical task in the classroom, the teacher must predict how students will deal with that situation, the possible errors, at what moments they will encounter difficulties or where understanding will be clear, and which situations may motivate or discourage them. This way, the teacher can anticipate how their students will think and outline strategies for teaching. For Almeida (2015, p. 44), "Knowledge of content and students translates into knowing how students learn certain content, what difficulties they face, what the most common mistakes are and how to avoid them."

Knowledge of content and teaching (KCT) is the subdomain that combines the teacher's knowledge of mathematics and teaching, that is, the instruction designer (Ball, Thames, & Phelps, 2008). When teachers choose which examples will be used to introduce a given content, which ones will be used to deepen it, or when they analyze which representations are appropriate in teaching a specific concept, they use a pedagogical understanding of learning that allows them to evaluate the advantages and disadvantages of certain methodological choices during instruction and, with that, opt for those that they believe are most appropriate for each situation.

Knowledge of content and curriculum is the same category proposed by Shulman (1986; 1987) and includes knowledge of the curriculum programs of school institutions. It refers to understanding how the contents are organized in the curriculum matrix, the most recent guidelines that regulate teaching, and what instructional materials are suggested.

Ball et al.'s empirical results propose that the MKT is a multidimensional knowledge domain. However, one is not sure whether the categories previously proposed are entirely correct; the line that separates one category from another is fragile. In observing practical teaching situations, one should consider the variability and unpredictability of the natural environment in which they occur. Therefore, the limits between each proposed category may not be sufficiently clear (Ball, Thames, & Phelps, 2008). Ball, Thames, and Phelps (2008) recognize that this model has problems of definition and precision and that refining these categories is still in progress.

We must emphasize here that the relevance of these authors' work does not necessarily lie in establishing categories of teachers' knowledge or in creating more structuralist models. According to Giraldo (2018), their contribution is directed toward recognizing the specificities of the knowledge that teachers mobilize and construct in teaching

situations. In this way, teaching is central to research into teaching knowledge and improving the work carried out by teachers.

Ball and Forzani (2009) believe a consensus that improvement in teacher education is essential if we want to achieve success in student learning. However, some initiatives do not achieve the expected results because they are insufficient in promoting essential reforms in the curriculum of these courses. The authors start from the argument that teaching is not a natural activity and, therefore, learning must be carefully designed, i.e., intentional (Ball & Forzani, 2009).

The notion that teaching is not natural is difficult for society to assimilate because there is a belief that most people teach something. For example, parents teach their children many different things, coworkers teach their peers how to do something, a cook teaches a step-by-step recipe, and so on. However, Ball and Forzani (2009) draw attention to the fact that there is a difference between teaching in the sense of helping someone do something in informal contexts and professional teaching.

For the authors, teaching in the sense of helping others is an activity in which most people engage on a daily basis. However, "Professional classroom teaching, on the other hand, is specialized work that is distinct from informal, commonplace showing, telling, or helping" (Ball & Forzani, 2009, p. 498). Therefore, the complexity of the work that teachers carry out in the classroom leads us to reflect on how initial education can contribute more effectively to preparing these prospective teachers.

Methodological aspects that guided the investigation

To elucidate the research question now stated, we sought *to give voice* to newly graduated mathematics teachers to learn how they were developing the work of teaching mathematics in their classrooms. The reports from the teachers participating in this study provided important information about real experiences with mathematics taught in school, the challenges and difficulties faced at the beginning of the career, the education received in the teaching degree course, and the demands perceived in the complex work of teaching.

Therefore, this research is qualitative, with concerns surrounding the subjectivity of the object investigated. The organization and implementation of the research stages and the methods used to produce and analyze the data with the participating subjects and the institutions involved allowed us to understand more generally the individuals and their interaction with others in the search for the knowledge necessary to develop their work and considering the environment of which they are a part.

For the objective of this article, we focus on the reports provided by seven teachers who graduated with a teaching degree at UEPB working with basic education classes. These reports were obtained through interviews based on the subjects' descriptions of their experiences in the work environment and the challenges arising from the activity of mathematics teaching.

Thus, during the research, we investigated the details that characterize teachers' work in mathematics, how they mobilize their knowledge in everyday school life, and their real needs. Therefore, more than the final result or the search for an exact answer to the question presented, we are interested in the consequences of this movement carried out by teachers and what reflections this can generate for initial education.

We agree with Flick (2009, p. 24) when he argues that, in qualitative research, "objects are not reduced to simple variables, but rather represented in their entirety, within their everyday contexts." For this reason, we conducted the interviews in an environment familiar to the participating teachers, i.e., at the university where they completed their degree. We provided them with a viable location in terms of physical space and room availability and an environment where they felt comfortable expressing their opinions.

Generally, the interview sought to collect information about the teaching work developed by these newly graduated teachers, from their descriptions of experiences lived in the classroom and their conception of the mathematical content necessary in basic school, and its approach. This information can help with the investigation and reflection on the existing relationships between the mathematics experienced in the undergraduate course and the mathematics necessary for teaching.

Among the seven participants, six teachers were interviewed in December 2019 and one in March 2020. In the latter's case, the interview was conducted remotely via Skype due to the social isolation measures adopted in Brazil during the COVID-19 pandemic. The face-to-face interviews were held at Campus VI of the State University of Paraíba.

Bardin (2016) believes that since we use content analysis for the data obtained, there is a richness in the interview as a source of data collection, given its singularity, since we deal with relatively spontaneous speech the interviewees orchestrate more or less at will. Thus, subjectivity is very present. The interviews were recorded in audio, typed, and saved in the researcher's personal files. We transcribed all statements meticulously, including the interviewees' hesitations, laughter, and pauses. The care taken with these records allowed the researcher to analyze the data in all its richness, faithfully describing the issues discussed and preventing important information from being lost.

The research data analysis by Alcantara (2021) was organized in two stages. The first stage consisted of a comparative study of the curriculum matrices in force in the mathematics teaching degree at UEPB (Campus VI) at the time of the research; the second moment comprised the analysis of the interviews with the teachers to examine signs of the presence of MKT in the respondents' speech when recalling episodes or teaching situations experienced at work, and their relationship with the education received. This article focuses on the second moment of the analysis carried out in Alcantara (2021), detailed in the following section.

Mathematical knowledge for teaching in the voice of teachers working in basic education

The analysis of interviews with teachers participating in Alcantara's study (2021) sought to identify signs that characterized mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008) based on the experiences those teachers had in the work environment, i.e., when they recalled episodes or teaching situations experienced in their classrooms.

Following the guidelines proposed by Bardin (2016), the analysis of the interviews selected context units related to the following topics of interest: the importance of the mathematics teacher knowing the content they will teach, evidence of the presence of mathematical knowledge for teaching in teaching work, and the identification of the knowledge necessary, in the participants' view, for the mathematics teacher to develop their teaching activities effectively.

Regarding the first topic of interest, i.e., *the importance of the math teacher knowing the content they will teach*, we sought to identify passages in the reports in which these teachers discussed the importance of knowing the mathematical content well. All the teachers interviewed highlighted that knowledge about the content to teach is essential. The teachers highlighted this aspect at various points during the interview:

> This is something extremely important, <u>because the teacher cannot teach what they do not know, they have to know [the content]</u>, even if they say: "Ah, I don't know that much!" as some teachers use about geometry... the excuse for not teaching saying they don't know... Even if you don't know much, if you're going to teach, you have to seek knowledge, you have to have the autonomy to go and seek it. (Interview with teacher Gisele cf. Alcantara, 2021, p. 152, our emphasis)
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> We have to know what we are doing. If we don't know... How can I do something I don't know? I have to get to know it. This is crucial. It's a priority. Am I going to talk about something I know "in passing"? If I didn't learn it at university, I have to figure out how to learn it on my own, but I have to know what I'm passing on to my student, because within a class there's always someone who leaves with that question... And

> are you going to say: “I don’t know”? Sure, the teacher is not obliged to know everything, we know that! Because we are human beings and not machines, but we at least have to know the essentials, it is extremely important! (Interview with teacher Rebeca cf. Alcantara, 2021, p. 152)

The statements described demonstrate that teachers agree that one cannot teach what one does not know, as discussed by Shulman (1986; 1987) and Ball, Thames, and Phelps (2008). Although the reports show that, in some situations, maybe the teacher does not know all the content that they will teach, the interviewees understand that they must try to fill these gaps by researching other sources or talking to colleagues, but that the student cannot be deprived of studying the content due to the teacher’s lack of preparation.

Content knowledge gives the teacher a framework of knowledge capable of giving them the confidence to answer unexpected questions that may arise in the teaching situation, preventing them from being afraid to discuss a specific subject in class. Content knowledge also allows the teacher to have more options to work with, i.e., from the moment the teacher has in-depth knowledge of what they intend to teach, they will have a broader range of possibilities when handling the work in the classroom (Ball; Thames, & Phelps, 2008; Ball & Forzani, 2009; Santos & Lins, 2016).

Regarding the analysis of the *evidence of the presence of mathematical knowledge for teaching in teaching work*, we highlight teachers’ reports in which it was possible to identify traces of the presence of MKT and its subdomains when describing situations experienced by them in the classroom. We begin with teacher Carlos’ report on the work with the fractions content:

> I really like giving this content because I don’t just go straight to telling you who the numerator and denominator are, no way! I arrive with some material, show it, and ask: How many parts did I divide it into? So I collected, how many did I collect? So, so ... From there I will build with them [...] I’ll bring a pizza. Or maybe I myself... When there is no time, I draw and erase, I first divide, then I erase, I ask: How much did I divide? And I keep asking them... I even brought a pizza to work on fractions and they made a huge fuss in the room. Real pizza! (Interview with teacher Carlos cf. Alcantara, 2021, p. 159)

In the report above, teacher Carlos stated that he likes to teach classes on fractions and that this content is covered in practically all of his classes. Pizzas are one of the main resources he uses to introduce working with fractions. Based on the description presented, we observe that after choosing this material, the teacher continues the instruction by asking questions to the class based on the handling of the chosen material, in this case, the use of pizzas, whenever possible and when not, using illustrations made on the board. This

methodological choice is very specific to the teaching situation and which the teacher believes to be more appropriate for initiating the fractions content. We observed other reports:

> The book has several expressions with square roots, with cubic roots, and it's something they have to memorize because the method for finding square roots and cubic roots involves a lot of that... (demonstration) So, I make a poster, I take it to the classroom with the perfect squares, but there it only has up to, let's say... up to the square root of 400, but what about after that? (Interview with teacher Carlos cf. Alcantara, 2021, p. 159)
> Last semester, I worked on a project on geometric design, where I used a lot of the knowledge I acquired here in the Geometric Design course with Professor Alex, and it was really cool, I could do some really cool work. (Interview with teacher Fabricio cf. Alcantara, 2021, p. 159)
> Each class is a new experience. So, for example, with some classes, if you only have the lecture, some will find it works, others won't. (Interview with teacher Isaac cf. Alcantara, 2021, p. 160)

The teachers mentioned the use of posters to work on perfect square numbers when teaching classes on radication, the preparation and development of projects on geometric drawing to work on geometry content, and evaluating the viability or not of expository classes; that is, teacher Isaque's opinion for some classes, the expository class is viable and for some other classes, this form of presenting the content is not a good option. All these reports refer to methodological choices and teaching processes carried out by teachers to work on specific content, suggesting evidence of mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008).

We now observe the following report:

> There in Pernambuco, at the school where I work, the curriculum is predetermined by a system, there it is the SIEP, and in the third grade, there is much content that reviews the second grade, but it brings, for example, probability content before combinatorial analysis, and it's horrible that you teach probability without having worked on combinatorial analysis [...] The curriculum there doesn't help either. To give you an idea, I barely use the textbook there because it has nothing to do with what is asked for in the required curriculum. (Interview with teacher Marcelo cf. Alcantara, 2021, p. 152, our emphasis)

When this interview was conducted, teacher Marcelo was working as a teacher at a technical school in the state of Pernambuco and used the Sistema de Informações da Educação de Pernambuco (SIEP) [Pernambuco Education Information System], which provided members of the school community with institutional information, access to decrees and ordinances, normative instructions, information on democratic management, internal regulations, and school calendar, among other materials and documents. SIEP also provides

the curriculum matrices to be followed by schools in the state education network in Pernambuco for full-time and semi-full-time schools, as well as high schools integrated with vocational education.

In his account, the teacher Marcelo pointed out difficulties in working with this curriculum matrix, citing as an example the fact of having to teach the content of probability before having worked on combinatorial analysis, as established in the matrix. For this teacher, this arrangement of content in the curriculum makes teaching work difficult.

It is possible to assume that the teacher's opinion is justified by the fact that the resolution of problems involving probability calculations extensively uses the fundamental counting principle studied within the scope of combinatorial analysis. Marcelo also refers to the incompatibility of the textbook adopted at the school with what is proposed in the curriculum. Although the report does not provide detailed elements about the relationship between the curriculum matrix and the teaching material adopted at the school, it is possible to see that the teacher knows the school's curriculum program and the arrangement of the contents in the curriculum.

Concerning the contents of combinatorial analysis and probability, Marcelo continues:

> Probability appears in the first unit, and combinatorial analysis appears only in the third. That is very bad. So, there must be communication between topics that need others. This influences our work. And the teacher, because he/she knows this content, can get around the situation and teach the content that really must come first. (Interview with teacher Marcelo cf. Alcantara, 2021, p. 162, our emphasis)

Here, we see that Marcelo's discomfort with the position these two topics occupy in the curriculum matrix is due to his perception that they are related and, in some way, there must be what he calls "communication" between them. This report reinforces our earlier inference that some results and concepts studied within the scope of combinatorial analysis are widely used in calculating probabilities.

Other reports point in the same direction, as teachers deal with aspects of knowledge of the curriculum program of the schools in which they work, understanding how the contents are placed in the curricula, and knowledge of external assessments that guide the school's actions and planning. In this sense, speeches of this nature suggest signs of horizon knowledge (Ball, Thames, & Phelps, 2008) since teachers understand how the contents are related.

Let us now see what Carlos says about the relationship between sixth-grade students and the content of fractions and some concepts involving flat and spatial figures:

> It is content that students have a lot of difficulty with. They can't relate the part to the whole, and so... they come from the fourth grade with that idea that a fraction is just a numerator and denominator, they can't... From the materials, like a pizza, for example, and I take a part, they can't relate the whole to the part that I take out first, they want me to say who is the numerator and who is the denominator, so, like, it's very mechanized [...] It has to be divided there, the whole has to be divided into whatever the denominator is, otherwise they can't make connections. I say: take the whole and divide it by the quantity of the denominator because they cannot make connections between part and whole. (Interview with teacher Carlos cf. Alcantara, 2021, p. 165, our emphasis)

The report above shows that teacher Carlos highlights a situation involving the teaching of fractions, stating that this is content that students find very difficult to learn. The teacher argues that students are more concerned with identifying the numerator and denominator of fractions, demonstrating difficulties in establishing relationships between part and whole. In Carlos's understanding, students come from the fifth grade with a very mechanized idea about fractions, attached to the structural form of a fractional number and not its meaning. Although Carlos uses manipulative materials to explain problems involving the notion of part and whole, he states that, even so, students still cannot understand clearly.

At another point in the interview, Carlos says that students have a mistaken notion of concepts associated with flat and spatial figures, such as circumference, circle, and sphere. "Students think that a circumference is a circle, that a circle is a sphere" (Interview with teacher Carlos cf. Alcantara, 2021, p. 165).

Students' confusion between the concepts of circumference, circle, and sphere was also mentioned by other teachers. By highlighting situations in the classroom where students' difficulties with the mathematical concepts studied are perceived, the reports suggest that teachers know how their students relate to mathematics, how they learn or have learned some content, their main difficulties, and the mistakes made in understanding the content. Therefore, it is possible to perceive in the reports signs of the presence of knowledge of content and student, as pointed out by Ball, Thames, and Phelps (2008).

The knowledge of content and students influences the teacher's perception of how he or she can address specific content based on students' life experiences and accumulated experience. Let us look at the following reports:

> I work with fractions in the sixth, I work with fractions in the fifth and I work in different ways. They are students of different levels and it is good to know the specificities of each class, of each student to see what you can bring to class. For example, I have a student whose father sells things, so from that point on, you can make connections with the content, for example, in operations with decimal numbers,

> you come to the shop and buy a piece of clothing for 49.90. Even he (the student), at that moment, will develop more, because this way, as it is in his day to day life, he can see that, which influences a lot. (Interview with teacher Carlos cf. Alcantara, 2021, p. 167)
>
> (It is important) to know how to relate the content to the student's real experience, to his practice, especially to regional peculiarities such as, for example, my situation: I teach at a school that is located in a municipality with a very large income flow, Santa Cruz do Capibaribe. For you to have an idea, I believe it was this weekend, the city received more than 150 thousand people at the fashion hub. And there, the student needs [...] I always try to relate it to the content, with financial mathematics, exemplifying problem situations in clothing production, all that, which is something very usual there. (Interview with teacher Marcelo cf. Alcantara, 2021, p. 167)
>
> [...] When you know (the class), you can prepare the lesson in such a way that you, for example, create groups and put a student who is more familiar with the content to guide that group, to help those who have more difficulties. So, in this sense, it is very important to know [...] Because when they manage to solve an easy question, they feel more motivated to continue, and then you gradually raise the level according to this "thermometer" that you perceive in the class. (Interview with teacher Marcelo cf. Alcantara 2021, p. 168)

As we have observed, Carlos's and Marcelo's reports demonstrate the importance of familiarity with their students' social and economic realities. Evidently, the fact that the teacher knows some particularities about his students, such as knowing that one of them is the son of a merchant, or that they live in the rural area of the municipality, or that the city they live in has an important textile hub for the region, can influence teachers' teaching work whether in the choice of examples, the language used, or even in other aspects of a methodological nature.

All teachers interviewed in the research criticized more traditional teaching models in which the teacher usually begins approaching the content with a ready definition of the concepts that will be studied. In these teachers' view, students will only truly understand what they intend to study through more practical examples, preferably through problem situations.

According to Marcelo's report, knowledge of the class, particularly of the students' relationship with mathematical content, represents a "thermometer" that can guide the teacher in the work in class for a more precise assessment of when to advance and when to go slower. The composition of small study groups to organize work in the classroom is an example of a methodological choice influenced by the teacher's knowledge of students' level of learning.

The teachers interviewed in the research highlighted that the relationship students have with mathematics is also influenced by aspects related to contexts of socioeconomic vulnerability and difficulty in accessing digital information vehicles. Although currently there is a global expansion of digital information media and the internet, some students still have

limited access to these media. Some of the teachers interviewed demonstrated knowledge of this aspect and argued that it impacts their work with content in class, especially when in tasks that require these digital means of information from students. These teachers' concern is that working without excluding these students is necessary.

Let us now look at a report on the work with teaching fractions:

> I divided (the pizza), so I say: it is not in equal parts, but consider that they are equal parts. And from there, we'll work... Then I ask: How many slices did I divide it into? In eight. I took that one, so how many are left there? Seven. What part of the whole does the one I took correspond to? So, we work like this, I really like teaching fractions, I know there comes a time when it seems mechanical because I can't find other ways to work, like operations [...] It's for them to know, for them to understand the concept of fraction, for them to know what a fraction is, for them to know what is represented there. For example, that 1/3 that is represented there so that they know what that 1/3 means. (Interview with teacher Carlos cf. Alcantara, 2021, p. 173)

Carlos explains here how he teaches fractions to students by manipulating a pizza, more particularly, by describing the meaning of a fraction as a relationship between part and whole. As the teacher manipulates the material, he asks the class questions: "How many slices did I divide (the pizza) into?" The teacher then removes one of the slices and asks the class again: "How many are left there?" And he continues to ask questions as he manipulates the material. We note that these questions are intentionally asked to the class with the specific objective of leading them to understand the procedures being carried out and the fractional representations obtained at each stage of the process.

Teacher Carlos' report demonstrates his concern in making students understand the concept of fractions and the associated meanings. More than knowing how to represent a number in fractional form by identifying the numerator and denominator, the teacher is concerned with making the student understand the meaning of the fraction obtained at each stage. This intentional chaining of procedures and questions carried out with the class is exclusive to the teacher and the task of teaching and, therefore, suggests the presence of specialized content knowledge (Ball, Thames, & Phelps, 2008).

In other reports, the interviewed teachers describe situations that have occurred in the classroom in which they critically evaluate the textbook, making changes or adaptations in the presentation of the content in order to develop representations that are more favorable to students' understanding; they suggest choosing simpler or more complex examples when working on specific content and the order in which these examples are proposed to the students; and they indicate the use of appropriate language when interacting with the students,

expressing themselves through terms that facilitate understanding, both in speech and in writing. Reports of this nature also suggest the presence of specialized content knowledge (Ball, Thames, & Phelps, 2008) on the part of the teachers participating in the research.

Regarding the *knowledge necessary for the mathematics teacher to carry out his/her teaching activities well,* all the teachers interviewed highlighted in-depth knowledge of the content as something fundamental for the teacher to effectively develop teaching work (Ball, Thames, & Phelps, 2008; Shulman, 1986; 1987). However, teachers are also unanimous in stating that knowing the content is not enough for a mathematics teacher to be successful in this important task. Teachers believe that other aspects must be considered, such as establishing connections between what is being studied and students' daily lives, cultivating close and friendly relationships with students, using teaching methods appropriate to the demands of their work context, not becoming complacent, enjoying the profession, and considering the knowledge that students bring.

Based on the interviews, teachers recognize the complexity of teaching work and that teaching requires a multiplicity of knowledge that goes far beyond disciplinary knowledge, as stated by Ball, Thames, and Phelps (2008). They must acquire a whole set of knowledge and skills that allow the teacher to propose situations in the classroom that facilitate the understanding of the content.

The professional work of teaching involves knowledge and skills that are not acquired naturally but need to be learned through vocational education (Ball & Forzani, 2009). This education should provide the prospective teacher with an approach to real practice and should recognize this practice as the source of an entire repertoire that needs to be analyzed and discussed in the teaching degree course (Ball & Forzani, 2009; Moreira & David, 2016; Santos & Lins, 2016).

We understand that the situations that teachers experience at work shape their relationship with the content and improve the task of teaching. However, for Ball and Forzani (2009), professional education cannot be based solely on intuition or experience. Formal preparation in the teaching degree should provide opportunities for the mathematics teacher to begin their career with a minimum base that gives them the conditions to work on the content, with the necessary security and preparation to face teaching demands (Ball & Forzani, 2009).

Some aspects caught the attention of the teachers' speech and left reflections not only on the course curriculum but also on teaching and learning during the course, as is the case of Isaac's speech (cf. Alcantara, 2021, p. 139) who, when comparing the expository method commonly used in teaching in teaching degrees with the school needs, exposes its

insufficiency in the face of students' necessities at school levels. All interviewees agree that there is a certain discrepancy between what the course prioritizes as content and what teachers need in practice, as teacher Rebeca's statement makes clear when commenting on the subjects Differential and Integral Calculus:

> There, I learned to integrate [referring to the course], but here [in the school classroom] it is not to integrate, here I will have to teach Pythagoras' theorem, Thales' theorem, etc... So you think, "What the hell did I do at university for four years? I'm lost!". [...] They aren't related at all. The situation is sad. It's very sad, like... it makes us think: "What did I do at university for four years if I don't need this?" Few things that you see at university you need there [in the school classroom]. (Teacher Rebecca cf. Alcantara, 2021, p. 145)

Likewise, the following reports demonstrate some concern with the education received in the degree and the preparation for professional activity:

> We see the contents of basic education in the basic subjects... but we don't see the relationships between content and anything. We only see formulas, it's to learn formulas, formulas, and formulas... to drill, drill, and drill... We don't even see applications! We only see mechanized exercises. (Teacher Carlos cf. Alcantara, 2021, p. 131)
>
> Perhaps one of the biggest challenges is the issue of contextualization at the university... Because this is more abstract mathematics, we see much technique, so we are not prepared to deal with contextualized situations such as, for example, problem solving [...]. So in some cases where textbooks bring up problems like this... that require a more refined interpretation from you, so it demands more, and the degree doesn't prepare you for that [...]. (Teacher Gisele cf. Alcantara, 2021, p. 143)

In the teachers' statements, the subjects most closely linked to pure mathematics were mentioned, mostly referring to those whose contents and methodologies are required, in some way, in their work environment. In this regard, Ball, Thames, and Phelps (2008) point out that aspects of mathematical knowledge are revealed in teaching situations and must be mapped, organized, and included in formative courses.

Preparing a teacher for the range of situations they will face when starting their career is not easy. However, the fact that teacher education must be geared toward the demands of teaching is already a consensus among many teachers and researchers on the subject (Ball, Thames, & Phelps, 2008; Ball & Forzani, 2009; Fiorentini & Oliveira, 2013; Moreira, 2012; Moreira & David, 2016; Santos & Lins, 2016). In our understanding, it becomes increasingly necessary to establish safer bridges between what is discussed at university and what is experienced on the "school floor."

Final Considerations

This article aimed to analyze elements identified in the speech of teachers working in basic education that characterized the mathematical knowledge for teaching (MKT) and its relationship with the initial education offered by the teaching degree in mathematics at UEPB. It sought to support reflections on mathematics teachers' initial education, especially related to the context that is the object of the research.

We took as our main theoretical reference Ball, Thames, and Phelps' (2008) contribution regarding the MKT, a construct launched in 2008 that refers to the mathematical knowledge necessary for the teacher to carry out the task of teaching effectively. The MKT is made up of the various types of knowledge that the mathematics teacher mobilizes in the teaching situation.

Data analysis was constructed based on reports of teaching situations experienced by seven newly graduated teachers working as teachers in basic education when the research was carried out. The information was obtained through a semi-structured interview and analyzed based on the assumptions of content analysis (Bardin, 2016).

The analysis of the reports of the teachers who participated in this study allowed the identification of elements that characterized the MKT and its subdomains based on real situations experienced in their classrooms. The reports indicated evidence of the presence of MKT. Based on these indications, we could characterize this knowledge better in its multiple dimensions, which involve, among other aspects, knowledge of the curriculum and teaching materials, including the evaluation of those most appropriate to each context, knowledge of didactic representations and teaching methodologies, including the evaluation of the advantages and disadvantages of their use, understanding how mathematical topics are related to each other, among other areas of knowledge and considering the different levels of education, investigating patterns in students' errors, identifying their origins, the possible paths that led them to the errors and what are the guidelines in the search for solutions, and knowledge about how students relate to the various mathematical contents.

This last aspect is directly related to the knowledge of content and students, which was given special emphasis in relation to the other subdomains. This is because the teachers' reports unveiled the strong presence of the elements that characterize this subdomain, namely, evidence of situations involving the understanding of how students learn specific mathematical content, episodes that demonstrated what the students' main difficulties were with specific content, which situations were motivating and challenging, moments in which students' most usual errors were identified when discussing specific mathematical content

worked on in class, evidence of knowledge of students' context both in the family and social aspects, and the need to cultivate relationships of familiarity and affection with these students.

This entire set of elements influenced and continues to influence the choices made by the teachers participating in this study regarding the development of their teaching work in their respective classrooms. However, these aspects were not the only ones. The participating teachers were unanimous in recognizing that in-depth knowledge of the content to teach is fundamental for teachers to build a theoretical framework capable of providing them with the security to deal with unexpected situations that arise in the classroom. In other words, interviewees agreed that teachers cannot teach what they do not know or are unfamiliar with (Ball, Thames, & Phelps, 2008; Shulman, 1986; 1987). However, the interviewees stated that knowing the content is not enough for a mathematics teacher to teach successfully, meaning that other aspects must be considered.

Based on the work carried out and described in this text, we conclude that the study on mathematical knowledge for teaching based on the practical work developed by recently graduated teachers provided relevant evidence for rethinking the teaching degree course at the State University of Paraíba (UEPB) and guided a deeper reflection on the relationship between the education offered and the demands of teaching mathematics in basic education.

Based on the studies by Ball, Thames, and Phelps (2008), we understand that the MKT is configured as the knowledge necessary for the teacher to develop the work of teaching effectively and is composed of a multiplicity of knowledge that goes far beyond disciplinary knowledge. It is unique and specialized knowledge, specific to the teacher, situated in a context, and validated in the work.

We hope that our research can contribute to expanding debates around this topic, not only in theoretical terms but mainly in encouraging teaching practices to create spaces for discussion and reflection on their work environment. Likewise, promoting reflection on prospective teacher educators' role and the implications of their decisions, encouraging initiatives that can contribute to strengthening teacher education courses in Brazil, and developing curriculum proposals more aligned with teaching mathematics in basic education seem to be urgent issues.

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