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Technological transformations in the teaching of mathematics: a study on the several faces of the development of pedagogical technological knowledge of content

Transformaciones tecnológicas en la enseñanza de las matemáticas: un estudio sobre las multifacéticas del desarrollo del conocimiento tecnológico pedagógico y de contenido

Transformations technologiques dans l'enseignement des mathématiques : une étude sur les multifacettes du développement des connaissances du contenu pédagogique technologique

Transformações tecnológicas no ensino de matemática: um estudo sobre as multifaces do desenvolvimento do conhecimento tecnológico pedagógico do conteúdo

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Abstract

Different studies have shown how the meeting of formal mathematical language and digital technologies influence and transform the production of teaching and learning processes in the subject. Understanding the relevance of Digital Technology (DT) in education, this research aims to analyze how Technological Pedagogical Content Knowledge (TPACK) is developed by teachers who work in Basic Education, as well as its influences on teaching processes. and learning mathematics. It was decided to analyze the corpus made up of ten scientific articles published between 2019 and 2023 belonging to broader research coordinated by researchers linked to GT 07 of the Brazilian Society of Mathematics Education (Sbem) that explained aspects related to DT. This is, therefore, qualitative research of a descriptive nature. The data point to the recognition of TPACK in the articles as a conceptual structure that allows

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understanding how teachers integrate technology into mathematics teaching and learning scenarios and boost teaching action to adapt to the demands of society and promote student learning.

Keywords: Technological Pedagogical Content Knowledge, Basic education, Mathematics teaching.

Resumen

Diferentes estudios han demostrado cómo el encuentro del lenguaje matemático formal y las tecnologías digitales influyen y transforman la producción de procesos de enseñanza y aprendizaje en la asignatura. Entendiendo la relevancia de la Tecnología Digital (DT) en la educación, esta investigación tiene como objetivo analizar cómo el Conocimiento Tecnológico Pedagógico de los Contenidos (TPACK) es desarrollado por los docentes que actúan en la Educación Básica, así como sus influencias en los procesos de enseñanza y aprendizaje de las matemáticas. Se decidió analizar el corpus compuesto por diez artículos científicos publicados entre 2019 y 2023 pertenecientes a una investigación más amplia coordinada por investigadores vinculados al GT 07 de la Sociedad Brasileña de Educación Matemática (Sbem) que explicaron aspectos relacionados con la DT. Se trata, por tanto, de una investigación cualitativa de carácter descriptivo. Los datos apuntan al reconocimiento del TPACK en los artículos como una estructura conceptual que permite comprender cómo los profesores integran la tecnología en los escenarios de enseñanza y aprendizaje de las matemáticas e impulsan la acción docente para adaptarse a las demandas de la sociedad y promover el aprendizaje de los estudiantes.

Palabras clave: Conocimientos tecnológicos, pedagógicos y sobre los contenidos, Educación básica, Enseñanza de matemáticas.

Résumé

Différentes études ont montré comment la rencontre du langage mathématique formel et des technologies numériques influence et transforme la production des processus d’enseignement et d’apprentissage dans la matière. Comprenant la pertinence de la technologie numérique (DT) dans l'éducation, cette recherche vise à analyser comment les connaissances du contenu pédagogique technologique (TPACK) sont développées par les enseignants qui travaillent dans l'éducation de base, ainsi que ses influences sur les processus d'enseignement. et apprendre les mathématiques. Il a été décidé d'analyser le corpus composé de dix articles scientifiques publiés entre 2019 et 2023 appartenant à une recherche plus large coordonnée par des chercheurs liés au GT 07 de la Société brésilienne d'enseignement des mathématiques (Sbem) qui expliquait les aspects liés à l'ED. Il s'agit donc d'une recherche qualitative à caractère descriptif. Les données soulignent la reconnaissance de TPACK dans les articles comme une structure conceptuelle qui permet de comprendre comment les enseignants intègrent la technologie dans les scénarios d'enseignement et d'apprentissage des mathématiques et de stimuler l'action pédagogique pour s'adapter aux demandes de la société et promouvoir l'apprentissage des élèves.

Mots-clés : Connaissances technologiques, pédagogiques et disciplinaires, Éducation de base, Enseignement des mathématiques.

Resumo

Diferentes estudos têm evidenciado como o encontro da linguagem matemática formal e as tecnologias digitais influenciam e transformam a produção de formas de ensinar a disciplina. Entendendo a relevância da Tecnologia Digital (TD) na educação, esta pesquisa tem como objetivo analisar como o Conhecimento Tecnológico Pedagógico do Conteúdo (*Technological Pedagogical Content Knowledge* – TPACK) é desenvolvido por professores que atuam na Educação Básica, bem como suas influências nos processos de ensino e aprendizagem de matemática. Decidiu-se realizar a análise do *corpus* composto por dez artigos científicos publicados entre 2019 e 2023 pertencentes a uma pesquisa mais ampla coordenada por pesquisadores vinculados ao GT 07 da Sociedade Brasileira de Educação Matemática (Sbem) que explanavam aspectos relacionados a TD. Trata-se, portanto de uma pesquisa qualitativa de caráter descritivo. Os dados apontam para o reconhecimento do TPACK nos artigos como uma estrutura conceitual que permite compreender como os professores integram a tecnologia nos cenários de ensino da matemática e impulsionam a ação docente para se adaptar às demandas da sociedade e promover a aprendizagem dos estudantes.

Palavras-chave: Conhecimento tecnológico pedagógico do conteúdo, Educação básica, Ensino de matemática.

Technological Transformations in Mathematics Teaching: a Multifaceted Study on the Development of Pedagogical Technological Knowledge of Content

The technological advancement of the last decades has implied the insertion of DT in various environments, including schools. Research has highlighted the importance of integrating technology in the teaching and learning process (Sousa; Moita & Carvalho, 2011; Cunha & Javaroni, 2020). Thus, it is necessary for teachers to seek to adapt to new knowledge, new technological tools, and their approaches, to meet the demands of a generation of students immersed in digital culture (Sousa; Moita & Carvalho, 2011; Colling & Richit, 2019).

This need is not only about the acquisition of technical skills, but also about understanding how to use them pedagogically, being aware of the process that involves combining technical knowledge, as well as knowledge about how to teach and the skills to adapt technology to the context of the specific content. Koehler and Mishra, for example, point out that "teachers' knowledge of technology is important, but not separate and detached from teaching contexts" (Koehler & Mishra, 2005a, p. 132).

For the development of such skills, it is essential that there is a reflection on the knowledge necessary for teaching. Authors such as Mishra and Koehler (2006), supported by the ideas of Shulman (1986), highlight the importance of the theoretical construct Technological Pedagogical Content Knowledge (TPACK[4]) for the integration of DT in teaching practices. DT is directed by these authors as an essential knowledge that extends Shulman's idea of Pedagogical Content Knowledge to the domain of technology.

The theory emphasizes that the use of technologies must be supported by a dense understanding of how they can be articulated in the teaching and learning process, in the case of Mathematics, this articulation can offer possibilities to explore concepts, develop reasoning and build problem-solving alternatives. In addition, this study sought to analyze how the interactions of pedagogical, technological, and content knowledge can influence the construction of theoretically sustained practices, thus promoting an approach that goes beyond the instrumental use of technology to integrate it as an epistemological and didactic element.

In this sense, this research analyzed how this theory has been applied in the training and practice of Basic Education teachers, as reported in the articles that make up the corpus of the study. With this, it is intended to understand in a deeper way how TPACK is developed and how it influences the teaching and learning processes of the discipline. Such analysis not only

[4] Originally named TPCK, the acronym popularized by Koehler and Mishra (2005b) was modified to TPACK because of the ease of pronunciation emphasized by some members of the research community (Thompson, 2008).

contributes to the broadening of the theoretical debate, but also provides elements to support the initial and continuing training of teachers, in view of the potentialities and challenges faced in the educational context.

We emphasize that this study was motivated by discussions and surveys of a broader research[5] developed by 14 members of the Working Group GT 07 - Training of Teachers who Teach Mathematics, of SBEM, regarding the research carried out and published in journals in the area of Mathematics Education, especially with works focused on the Teaching of Mathematics with directions of activities related to the final years of elementary school and high school. Therefore, we emphasize that the corpus that we will analyze is an excerpt of this investigation.

In the next section, we introduce TPACK.

About TPACK

The National Council of Education (CNE) instituted the National Curriculum Guidelines for the Training of Basic Education Teachers at higher level (Brazil, 2002), which guided that DT should be included in the projects of initial teacher training courses. As an active professional in Basic Education, it is important that the teacher is trained to know how to use DT in his work routine.

The knowledge base for teaching elaborated by Shulman (1986, 1987) was also built considering the specificity of the discipline. For him, content knowledge is built by the learner through pedagogical strategies and teaching and learning methods used by the teacher: it is the pedagogical knowledge of content. Thus, the teacher's knowledge was organized into categories that should include, at least: Content Knowledge, General Pedagogical Knowledge, Curriculum Knowledge, Pedagogical Content Knowledge (PCK), Knowledge of Students and their Characteristics, Knowledge of Educational Contexts and Knowledge of the Purposes, Purposes and Values of Education and its Historical and Philosophical Basis. But the author points out that the PCK is the most important category.

According to Mizukami (2004), PCK is a knowledge developed by the teacher, of his authorship, learned in professional practice, but without dispensing with other types of knowledge. This knowledge of specialized content of the teacher involves "[...] different types of knowledge, including specific knowledge, pedagogical knowledge of the content and curricular knowledge" (Mizukami, 2004, p. 37).

[5] Under the coordination of Dr.ª Flávia Cristina de Macêdo Santana, Dr.ª Roberta D'Angela Menduni-Bortoloti, and Dr. Victor Giraldo.

Koehler and Mishra (2005a) integrated technological knowledge into Shulman's knowledge base for teaching, generating TPACK. Thus, TPACK emphasizes the connections that exist between technologies, pedagogical approaches, and curricular content. This connection is valuable. According to Mazon (2012, p.20), before the constitution of TPACK, "In relation to these three types of knowledge (content, pedagogical and technological), what occurred was an analysis of them in isolation." Let us look at the concept of each of the three types of knowledge (Figure 1) first in isolation, then the concept of two-by-two interactions (Figure 2, 3 and 4) and the connection of the three (Figure 5)[6].

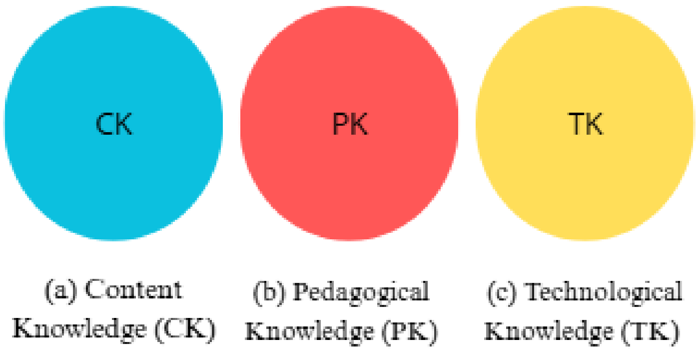

Figure 1.

Representation of the three types of knowledge that make up the TPACK in isolation.

About the CK, represented in Figure 1(a), Mishra, and Koehler (2006, p. 1026) state that it "[...] it is the knowledge about the subject to be taught or learned." But it goes further. It is necessary to consider the knowledge of concepts used in the discipline, the methods, and procedures within a given field, as well as the main facts, ideas and theories, organizational structures, evidence, evidence, established practices, and approaches to the development of that subject (Mishra & Koehler, 2006).

Figure 1(b) represents PK which refers to the processes, practices and methods of teaching and learning and how they are involved, in general, with educational purposes, values and objectives. It covers all issues of student learning and assessment, classroom management, and lesson plan development. That is, it requires the teacher to have an understanding of cognitive, social, and developmental learning capacities and how they apply to students in the classroom (Mishra & Koehler, 2006).

TK, depicted in Figure 1(c), is knowledge about earlier technologies, such as books and whiteboards, and more advanced technologies, such as the Internet and digital video, which

[6] All the figures in this work were made on Canvas.

also involves the skills needed to operate these technologies. As technology continually changes, the nature of TK also needs to change over time. The ability to learn and adapt to modern technologies, no matter what the specific modern technologies are, will still be important (Mishra & Koehler, 2006).

Analyzing the peer interactions between knowledge, let us start with the PCK, which is compatible with Shulman's concept for Mishra and Koehler (2006): PCK is concerned with the representation and formulation of concepts, pedagogical techniques, the knowledge of what makes concepts difficult or easy to learn, the understanding of students' prior knowledge, and the theories of epistemology. It also includes the knowledge that students bring to the learning situation, knowledge that can be facilitating or dysfunctional for manual task learning (Mishra & Koehler, 2006, p. 1027). It is represented in Figure 2 where the interaction between the representative colors of CK and PK generate a new color, PCK.

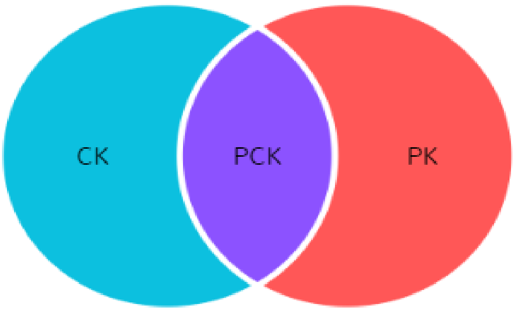

Figure 2.

Representation of the interaction of the CK and PK pair composing the PCK

Technological Pedagogical Knowledge (TPK) is defined as the understanding of how best teachers to use certain technologies to develop teaching and learning procedures. It refers to the knowledge of the existence of various technological components and resources and how they can be used in the teaching and learning scenario, being aware of how teaching can change because of the use of specific technologies (Mishra & Koehler, 2006).

Harris, Mishra, and Koehler (2009) highlight the fact that performing learning activities based on technologies such as PowerPoint and projectors just to present the content are not considered TPK. After all, according to Mazon (2012), TPK refers to the teacher's ability to critically use technological resources in a pedagogical context. TPK is represented in Figure 3, where the interaction between the representative colors of TK and PK generates a new color, TPK.

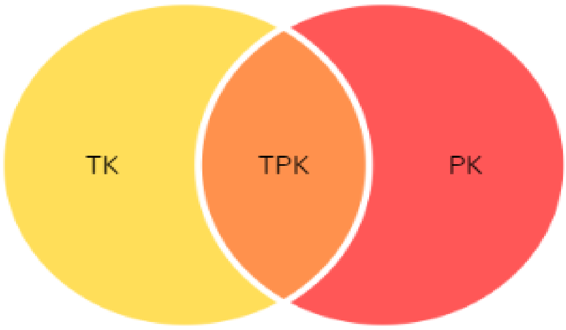

Figure 3.

Representation of the interaction of the TK and PK pair composing the TPK

Technological Content Knowledge (TCK) is the relationship between content and technology; it includes the understanding of the way in which technology and content are related to each other. For Mishra and Koehler (2006), even if technology restricts the possible types of representations, modern technologies can provide new, more varied representations and greater flexibility in the navigation between these representations. In addition, according to the authors, with the use of technology, there is the possibility of changing the nature of learning a concept, which, without the use of this technology, would be more difficult for the learner to understand.

As Mazon (2012) concluded, it is part of the TCK for the teacher to know how to select the most appropriate technologies for the content to be taught, as it is the relationship between content and technology. So, the teacher needs to know how students can learn through different tools appropriate to that content. The TCK is represented in Figure 4, where the interaction between the representative colors of the CK and the TK generate a new color, the TCK

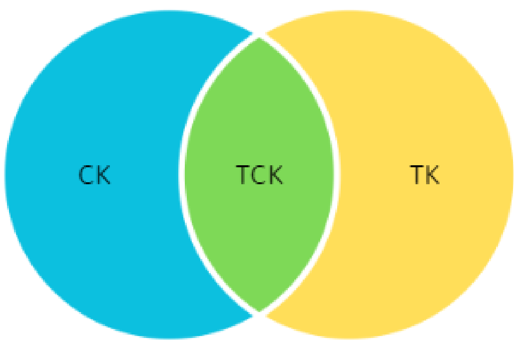

Figure 4.

Representation of the interaction of the pair CK and TK composing the TCK

Finally, the interaction of the trio of knowledge generates TPACK, which is an emergent form of knowledge that goes beyond the interactions of the three components (content, pedagogy, and technology) explained earlier. But there is also a context in question (light gray part of Figure 5).

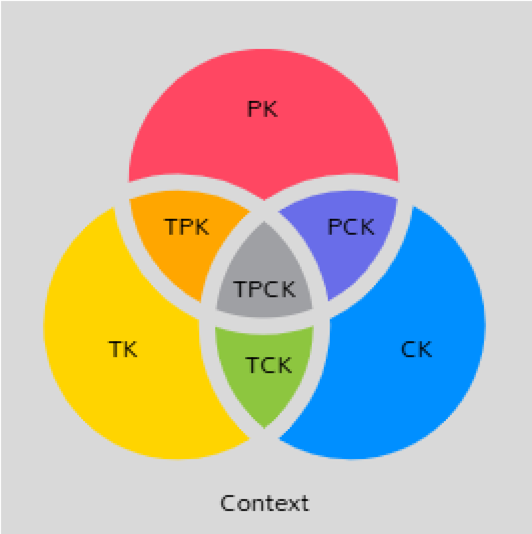

Figure 5.

Representation of the interaction of the threesome TK, PK and CK in a context, composing the TPACK

Mishra and Koehler (2006) define TPACK as the knowledge necessary for the teacher to decide how to use technology for the teaching of quality content, using its bases in an integrated way and observing its complex relationships. According to the authors, it is the basis of good teaching with technology, and for that, it requires an understanding of the representation of concepts through the use of technologies, pedagogical techniques that use technologies in a constructive way to teach the content, knowledge about the students' previous knowledge and the theories of epistemology, knowledge about what to do with difficult or easy-to-learn concepts and how technology can help fix some of the problems students face, and finally, knowledge of how technologies can be used to build on existing knowledge and develop new epistemologies or strengthen old ones.

To be able to teach with quality, according to the authors, the teacher needs to develop a differentiated understanding of the complex relationships between technology, content, and pedagogy, and be able to use this understanding to develop specific and appropriate strategies for each context and representation. The three issues cannot be considered in isolation, but within the relationships of the system defined by the three key elements, in order to achieve the integration of productive technology in teaching. The main objective of TPACK is to articulate the three knowledges that form the basis for its structuring within the existing context.

We opted for the model developed by Mishra and Koehler (2006) to analyze the materials, as the concepts that make up the TPACK can contribute to facing the challenge of bringing education and technology closer together, in addition to guiding research regarding the use of DT by teachers.

In the next section, we present how the corpus of the research was constructed.

Method

In this article, we present an excerpt from research carried out by members of the area of Mathematics Education linked to WG 07 - Training of Teachers who Teach Mathematics, of Sbem. These researchers are involved in a larger production, which aims to build an overview of the theoretical references most used in recent national research works focused on Mathematics (knowledge, knowledge), mobilized and produced by teachers who teach mathematics in the final years of Elementary School and High School.

The research materials, the reference of our study, consisted of 63 scientific articles registered in journals Mathematics Education Research (EMP), Journal of Science and Mathematics Teaching (Acta Scientiae), Perspectives of Mathematics Education (PEM), Zetetiké, Mathematics Education Bulletin (Bolema), International Journal of Research in Mathematics Education (RIPEM), Mathematics Education in Review (EMR) published between 2019 and 2023. This database used as a reference production of international excellence (Qualis A1/Capes) that, by analysis of title, abstract and keywords, met the following criteria for inclusion and exclusion:

Table 1.

Criteria for corpus selection

Criterion 1	Articles should focus on teaching (and not exclusively on learning by primary school students).
Criterion 2	The articles must have as a research theme or as (future) subjects-teachers who teach in the final years of Elementary School or High School.
Criterion 3	The articles must address, in some way, knowledge/knowledge of mathematical content for teaching (with their different denominations, depending on the theoretical framework).

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From this corpus of the broader research, meeting the objective of this work, we searched for scientific articles that discussed DT, to constitute the corpus to be analyzed: we observed if the title, abstract, keywords and objectives had words related to DT such as technology, digital technologies, remote classes, software, GeoGebra.

In Table 2, we present the ten works that were aligned with our object of study. It is, therefore, qualitative research, considering that studies of this nature enable us to interpret and understand phenomena and attribution of meanings (Prodanov & Freitas, 2013).

Table 2.

Works selected to compose the corpus of this study

Magazine	**Authors**	**Year**	**Article Title**
Bolema	Aparecida Santana de Souza Chiari Marcelo de Carvalho Borba Daise Lago Pereira Souto	2019	The Theory of Activity in the Production of Interactive Digital Mathematics Didactic Material
	Juan Luis Prieto G. José Ortiz Buitrago	2019	Knowledge necessary for the management of mathematical work in the development of simulators with GeoGebra
	Bruna Maria Vieira Gonçalves Francisco José de Lima	2020	Teacher Learning and Development of Methodological Strategies in the Context of PIBID: reflections on GeoGebra as a resource for teaching functions
EMP	Adriana Richit Juliane Colling	2019	Pedagogical, Technological and Content Knowledge in the Initial Training of Mathematics Teachers
	Fernando Oliveira Garcia Caroline Subirá Pereira Antônio Carlos Frasson Virginia Ostroski Salles	2020	Mobile technologies in the initial training of mathematics teachers
	Carolina Cordeiro Batista Rosa Monteiro Paulo	2022	"I didn't see that in class!": Becoming aware of yourself in the class study discussion
Acta	Fábio Henrique Patriarca Nielce Meneguelo Lobo da Costa Samira Fayez Kfouri da Silva	2019	The Continuing Distance Education Program M@tmídias: Contributions to the teaching of trigonometry
PEM	Sandro Ricardo Pinto da Silva Sueli Liberatti Javaroni	2020	The Didactic Video: some perceptions of teaching practice inserted between academic mathematics and school mathematic
Zetetiké	Jéssica Adriane de Mello Marcus Vinicius de Azevedo Basso	2023	Teacher training and creativity: experience with mathematics undergraduates
EMR	Salete Maria Chalub Bandeira Eliete Lima	2022	Virtual learning during the pandemic: learning how to teach geometry with GeoGebra

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Considering the journals listed in the second paragraph of this section, we emphasize that no RIPEM article met the inclusion criteria in this research.

From the reading of these articles, we analyze the approaches related to the development of the TPACK developed by teachers who work in Basic Education. We sought to identify theoretical approximations and divergences, as well as the transformation that occurred in the production of the teaching process of Mathematics teachers using DT present in the corpus of this work. And it is this analysis that we will describe and discuss below, right after presenting, succinctly, the corpus of the research.

Description of the articles selected in the research.

After identifying the articles related to DT, we will make a brief presentation and subsequent analysis of each one below.

Chiari, Borba and Souto (2019) conducted a collective case study among distance learning Mathematics Degree students enrolled in Linear Algebra to discuss how formal mathematical language, DT, the Internet, virtual learning environments and the interaction between participants of distance learning courses can influence and transform the production of knowledge in this discipline. The theoretical framework of teacher knowledge used was the Activity Theory proposed by Engeström (1987, 2001).

And to analyze how formal mathematical language, DT, the internet, virtual learning environments and their interactions can influence and transform the production of knowledge in distance courses, Chiari, Borba and Souto (2019) used the theoretical construction of human beings-with-media, which, according to Borba and Villarreal (2005), emphasizes not only the collective production of knowledge, but also the role of the media in this production. This was the only article in this research that culminated in the production of Interactive Digital Didactic Material.

Prieto G. and Buitrago (2019) analyzed the knowledge needed to manage the modeling processes that students of Club GeoGebra (a project aimed at teacher training developed in secondary education institutions in Venezuela) go through when developing simulators. The authors identified three types of knowledge: knowing how to analyze the inconsistency of a construction, knowing how to communicate a construction technique to others, and knowing how to anticipate a technique before it is used. The theoretical framework of knowledge/teaching knowledge used was that of Tardif (2002).

The article by Colling and Richit (2019) has in its title the theoretical framework that we used to analyze the articles: TPACK by Mishra and Koehler. It addresses the perspectives of use of DT in initial training emphasized in the teaching practice of Basic Education. In addition to analyzing curricular documents of a Mathematics Degree Course and the teaching plans of the different disciplines, the authors applied questionnaires and conducted interviews with the participants and identified four perspectives of articulation of technologies with content and pedagogical knowledge in the course: Perspective associated with the development of teaching strategies and methodologies for Basic Education; Perspective associated with the development of the curricular content of Basic Education; Perspective associated with the development of pedagogical knowledge for teaching in Basic Education; and perspective associated with the motivating role assumed by technologies.

Garcia, Pereira, Frasson and Salles (2020) emphasized the importance of investigating the use of applications for mobile devices in the pedagogical practices of Mathematics Education professionals. They analyzed the perception of students of the mathematics degree course regarding the use of mobile technologies for teaching functions. It was identified that the association of these technology resources promotes improvement in professional training, as the presence of essential knowledge related to the teaching profession was verified. The theoretical references of knowledge/teaching knowledge used were Tardif (2002), Shulman (1987) and Pimenta (1996).

The authors Batista and Paulo (2022) chose a title that expresses the teacher's moment of strangeness when perceiving his own teaching action: "I didn't see that in class!". They explained the way in which the teacher understands that turning to the practice of teaching with technology favors becoming aware of his way of being a teacher. The theoretical framework of knowledge/teaching knowledge used was Bicudo (2003).

The article written by Patriarca, Costa and Silva (2019) aimed to identify the contributions of the use of technology for teaching Trigonometry in the training of 600 High School Mathematics teachers, working in state public schools in São Paulo. The authors also drew on Mishra and Koehler's TPACK and Imbernón's (2000; 2010) ideas regarding continuing education. Its results showed the construction of knowledge of the mathematical content, the pedagogical knowledge of the content and the technological knowledge of the content, signaling possibilities for the construction of the TPACK of the participants.

Silva and Javaroni (2020) investigated how Mathematics is presented in one of the videos prepared by undergraduates, students of the Supervised Internship disciplines, of a distance learning course of Undergraduate Mathematics. In the video in question, the licentiate student teaches how to calculate square root by subtraction. Through their analyses, the authors inferred that there is a relationship between Academic Mathematics and School Mathematics and that students use videos as didactic and pedagogical resources. The theoretical framework of knowledge/teaching knowledge used was Pimenta and Lima (2012).

Mello and Basso (2023) sought to understand how creativity can favor the construction of teaching knowledge. To this end, they analyzed the teaching plans and portfolios of four undergraduates who developed teaching practice with sixth and seventh grade students in the context of the COVID-19 pandemic. The "cyclone of creation" proposed by Borges and Fagundes (2016) contributed to the analysis of records, as did Piaget (1995). The case study indicated that the practice experienced by the group points to a creative process related to Piaget's (1995) process of reflective abstraction. In addition, the authors concluded that the creation process evidenced is continuous, can be expanded, and improved.

Lima and Bandeira (2022), based on training offered to Sbem's partners, reported the search for professional improvement to teach geometry remotely to students. They used the framework of Mishira and Koehler (2006) to analyze the data and concluded that it was possible to learn how to manipulate the GeoGebra application (both on the computer and on the smartphone) as well as to plan activities for teaching scientific concepts with the group of teachers.

Finally, Gonçalves and Lima (2020) discussed the possibility of (re)elaborating the teaching of Mathematics, through reflection on the effects of initial training on the development of the teaching practice of the licentiate, through the Institutional Program of Scholarships for Initiation to Teaching (Pibid). The authors concluded that the context of PIBID can contribute to the mobilization and appropriation of theoretical and practical knowledge, as well as indicate that the GeoGebra software can cooperate for the development of learning of students at Basic School and favor the professional development of the future teacher. We highlight that in this article the pedagogical knowledge of the content on page 1072 is cited, however, Shulman is not cited, he only cites the work developed by Errobidart and Rosa (2019).

Regarding the participants of the research under analysis, we identified in the articles, in addition to teachers of Basic Education, teachers of higher education, from other areas, in training, as well as teachers who had already graduated (Table 3).

Table 3.

Professional characteristics of the participants of the articles analyzed.

Participant	Items No.
Higher education teacher	3
Professor of other areas	1
Teacher in training	6
Teacher already graduated	4
Associate Professor at SBEM	1

The sum of the number of articles exceeds the total of ten because some of them had participants in more than one category described in Table 3. The largest representation is of teachers in training (six), perhaps because of the ease of contacting participants for research in Higher Education Institutions. One of the articles also had the participation of a professor from another area (Physics), three of them involved a professor from Higher Education. In four articles we identified that the participants were already trained teachers and one of them involved teachers associated with SBEM who enrolled in a training course, and in this case, they can be mathematics teachers from Basic Education to higher education, this information was not detailed in the article, therefore, we reproduce it as it was described in the text.

Let us now see what was identified with TPACK in the articles researched.

Article Analysis: Exploring the Multifaceted TPACK

When investigating the various approaches related to the development of TPACK by teachers who work or will work in Basic Education, we found different developments that intersect in this field. These are studies that highlight how the encounter of formal mathematical language and DT influence and transform the production of the processes of teaching the discipline, that is, the development of teaching practice. In this set of articles, different theoretical bases are shown that converge to the development of knowledge aimed at taking advantage of the potential of technologies in the promotion of mathematics teaching.

TPACK has been recognized as a conceptual framework that contributes to understanding how teachers effectively integrate DT into the teaching of subjects such as mathematics. In this sense, the analysis of data from different scientific studies showed multiple faces, as well as different factors, such as formal mathematical language, DT, use of software, production of digital educational resources, virtual learning environments, interaction between participants of distance courses, and knowledge necessary for the management of mathematical work are intertwined in the search for different ways of teaching.

The effectiveness, i.e., the associated achievement, because of the ability to achieve the objectives, of the integration of technology in mathematics teaching is evidenced using GeoGebra software for the construction of simulators that help students understand abstract concepts and perform virtual experiments. The studies by Pietro G. and Buitrago (2019), and Gonçalves and Lima (2020) highlight the importance of technology as a tool that enables and boosts the understanding of mathematical concepts, making them more accessible and evident to students, thus being a tool that facilitates learning.

To exemplify, we cite Gonçalves and Lima (2020, p. 1068-1069) who stated that they chose the software for "its dynamicity, whose main purpose was to assist in the construction of graphs and graphical analysis of functions, in order to understand its behavior, one of the difficulties presented by the students in the aforementioned study". The authors also highlighted that the significance in the teaching and learning process lies in the teacher's awareness of its use and in a plan that defines the objectives, directing them to the actions to be developed.

In this example, we observe the interaction between the content (functions), the technology of software and a lesson plan defined as a function of overcoming students' difficulties in the discipline. This interaction highlights the connections between technologies, pedagogical approaches, and specific curriculum content, exemplifying how these three elements can influence each other to promote teaching based on educational technologies (Harris; Mishra & Koehler, 2009).

The integration of mobile technologies in education, such as apps, also plays a significant role in teaching practice. The participants' perception of the improvement of learning through technological resources highlights the importance of adapting pedagogical actions to the needs of students. This was shown by research by Garcia, Pereira, Frasson and Salles (2020) that analyzed the perception of academics in the Mathematics Degree course, regarding the use of mobile technologies for teaching functions.

By boosting learning through practice and in a more accessible way, these tools facilitate the incorporation of more dynamic and engaging teaching methods.

By recognizing the importance of a student-centered approach and allowing students to explore and experiment with technology during lessons, teachers are taking a more flexible and inclusive stance towards the learning process. This demonstrates an important aspect of TPACK, which is the recognition of the existence of various technological components and resources, and how they can be used in teaching and learning, knowing how teaching can change because of the use of specific technologies (Harris; Mishra & Koehler, 2009).

In this sense, it was observed that the online courses offered to train teachers in the use of educational technologies demonstrate a continuous commitment to the search for specific knowledge for the teaching of mathematics. By exploring different multimedia learning objects and providing opportunities for reflection and practical application, these courses help teachers strengthen their strategies and integrate technological resources into the subject curriculum.

Another example of the student-centered approach is the study proposed by Batista and Paulo (2022) brought situations experienced by a group of Mathematics teachers who watched videos with clippings of their classes, with excerpts in which students were using software, talking to colleagues to define strategies for solving the task, asking questions and accessing the internet. The authors point out that when analyzing the recordings, the teachers understood that being a teacher with technology can bring a new meaning to the acts of teaching and learning.

This example points to an important characteristic of TPACK, the availability on the part of the teacher, in terms of the flexibility and fluency of the curricular content, pedagogy, technology and the context involved, in which each one directly influences the other (Harris; Mishra & Koehler, 2009; Graham et. al, 2009; Graham, 2011; Harris & Hofer, 2011).

Another face found in the articles was the innovation in the creation of educational resources, such as videos with virtual characters and 3D animations, which reflects a creative and adaptive approach to the teaching of mathematics. For Cibotto and Oliveira (2013, p. 8, emphasis added), TPACK "encompasses the teaching of curricular content using pedagogical techniques that appropriately use technologies to teach the content in a differentiated way according to the learning needs of students".

From this perspective, we identified in the work of Mello and Basso (2023) that creativity, combined with technology, can promote new possibilities for teaching and learning. In the research, the authors analyzed a case study in which mathematics undergraduates created a character, a virtual teacher, who was similar in age to the students and who sought to explore visual resources, using tools such as Canva and interactive games. By exploring new ways of presenting mathematical concepts, the text points to other possibilities of teaching mathematics, through the pedagogical use of technology.

During the analysis, it was evident that TPACK has been developed both in the initial and continuing education processes, as well as in the face-to-face and distance modality.

However, some works delve into the exploration of technological knowledge, distancing themselves from the production of specific knowledge for the teaching of mathematics.

Moving towards the influences of TPACK in the teaching and learning processes of mathematics, it was noted that it allows educators to identify, understand and seek possibilities to overcome the challenges that arise when teaching the subject in different contexts. In this direction, we find educators who have developed creative and innovative strategies to successfully integrate technology. This included the selection and justification of software that offers dynamic capabilities to address complex mathematical concepts in a visual and interactive manner.

The school context was also considered in the articles. For example, Gonçalves and Lima (2020) state that it is important to consider the school context, as they identified that one of the difficulties faced by the participants, Pibid scholarship holders, when proposing classes with the use of computer resources, lay in the precariousness of the computer labs in Basic Education Schools with few computers in good condition and almost always without internet access.

One way to avoid this problem emphasized by Gonçalves and Lima (2020, p. 1070) lies in the importance of knowing the school context in advance, as "[...] Situations like this should awaken in the scholarship holder the care to check, in advance, the available resources, since these episodes are already expected. Even so, [...] this fact causes frustration and a certain imbalance in the planning of activities." However, the Pibid scholarship holders found a way to solve the problem: "take the finished material, present it to the students with the help of a data show, so that the students could analyze it and formulate ideas" (Gonçalves & Lima, 2020, p. 1071, emphasis added).

In this perspective, we identified in the work of Mello and Basso (2023) that creativity, combined with technology, can promote new In addition, teachers also explored other digital tools, that is, technological resources, for example, Excel spreadsheets, programs such as Geogebra, big data and artificial intelligence, and mobile applications, taking advantage of their potential to bring teaching closer to students. This involves not only familiarizing yourself with the functionalities of digital tools, but also reflecting on how they can be used in a way that promotes student learning. For example, the production of interactive digital teaching materials involves students in the learning process, increasing their engagement and providing a more meaningful experience.

We believe that TPACK has fostered the development of a culture of continuous reflection among educators, encouraging them to share experiences, identify challenges, and seek collaborative solutions. This circumstance makes it possible to cope with structural limitations and adapt practices to the real needs of students and the conditions of the school environment.

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Final Thoughts

This study sought to analyze how TPACK is developed by teachers who work in Basic Education as well as its influence on the teaching and learning processes of mathematics. By discussing the data from several scientific studies, it was possible to identify a series of approaches and developments that are interconnected in this field that we consider multifaceted.

The data points to the recognition of TPACK as a conceptual framework that allows understanding how teachers integrate technology in mathematics teaching and learning scenarios. In this sense, it revealed multiple interfaces between technological knowledge, pedagogy and specific curricular content, showing how these elements are intertwined to promote more meaningful teaching.

A relevant aspect observed in this study was the importance of initial and continuing teacher training in the development of TPACK. Both face-to-face and distance courses have contributed to training teachers in the targeted use of educational technologies, strengthening their teaching strategies and integrating technological resources in a significant way in the curriculum of the discipline.

We emphasize that some works identified in this research, such as in Lima and Bandeira (2022), seem to distance themselves from the production of specific knowledge for the teaching of mathematics, prioritizing the exploration of technological knowledge in general. Right suggests the need for a greater focus on the integration between pedagogical, technological, and content knowledge, to promote more accessible teaching during the Covid-19 pandemic.

In summary, this study contributes to broadening our understanding of the role of TPACK in the teaching of mathematics in Basic Education, highlighting the importance of integrating technology, in a pedagogically grounded way, in the promotion of new bases directed to specific knowledge for the teaching of the discipline. It is hoped that the results presented here can guide future research from other multifaced such as teacher training curricula, school curricula, evaluation processes, accessibility, didactic materials, and many other possibilities that are presented in the development of TPACK.

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