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Problematizing approaches for mobilizing mathematics teachers' knowledge: contributions, limitations and challenges

Enfoques problematizadores para movilizar el conocimiento de los profesores de matemáticas: contribuciones, limitaciones y desafíos

Approches problématisantes pour mobiliser les connaissances des enseignants de mathématiques : contributions, limites et défis

Abordagens problematizadoras para a mobilização de conhecimentos de professores de matemática: contribuições, limitações e desafios

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Resumo

Este artigo apresenta desdobramentos da pesquisa intitulada "Matemática específica do professor que ensina matemática nos Anos Finais do Ensino Fundamental e no Ensino Médio", realizada por pesquisadoras(es) da Sociedade Brasileira de Educação Matemática (Sbem). De 2021 a 2023, esse grupo selecionou artigos acadêmicos de sete periódicos considerados representativos do campo da Educação Matemática nacional. Partimos de um *corpus* geral constituído por 89 artigos, obtidos em periódicos representativos do campo da Educação Matemática, com o recorte temporal de 2019-2023. Os pesquisadores, organizados em subgrupos, enveredaram para produções temáticas específicas, com base nesse *corpus*. Para a escrita deste artigo, três pesquisadores buscaram *compreender as contribuições, os limites e os*

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desafios da implementação de abordagens problematizadoras para a mobilização e produção de saberes e conhecimentos do professor de Matemática. Para isso, realizaram uma metassíntese de 24 trabalhos que abordavam a temática da problematização no ensino e na formação de professores. A partir dos quatro agrupamentos construídos, dentre os resultados encontrados, destacamos: (i) o número expressivo de trabalhos baseados em tarefas; (ii) o silenciamento de referenciais teóricos clássicos da área de resolução de problemas; (iii) uma multiplicidade significativa de referenciais teóricos nos estudos integrantes do agrupamento Modelagem Matemática; e (iv) a preocupação com o ensino na educação básica nos trabalhos que abordam a utilização de problemas, mesmo sem um referencial sobre a abordagem.

Palavras-chave: Matemática, Ensino, Conhecimentos, Problemas, Tarefas.

Abstract

This article presents developments of the study titled "Mathematics Specific to Teachers who Teach Mathematics in Later Primary and Secondary Education," carried out by researchers at the Brazilian Society of Mathematical Education (SBEM). From 2021 to 2023, this group selected academic articles from seven journals considered representative of the field of national mathematics education. We base our study on a general corpus consisting of 89 articles, drawn from journals representative of the field of Mathematics Education, covering the period 2019–2023. Organized into subgroups, the researchers proceeded to examine productions on specific topics, based on this corpus. To write this article, three researchers undertook *to understand the contributions, limits, and challenges of implementing problematizing approaches for mobilizing and producing knowledge for mathematics teacher*s. To that end, they conducted a meta-synthesis of 24 works that addressed the issue of problematization in education and teacher training. Among the findings of the four groups formed, it is worth highlighting: (I) the significant number of studies based on tasks; (ii) the silence of classical theoretical references in problem-solving; (iii) a significant multiplicity of theoretical references in the studies forming the Mathematical Modeling group; and (iv) a concern with teaching in basic education in the studies addressing the use of problems, even without a framework on the approach.

Keywords: Mathematics, Teaching, Knowledge, Problems, Tasks.

Resumen

Este artículo presenta desarrollos de la investigación titulada "Matemáticas específicas del profesor que enseña matemáticas en los últimos años de la enseñanza primaria y secundaria", realizada por investigadores de la Sociedad Brasileña de Educación Matemática (Sbem). De

2021 a 2023, este grupo seleccionó artículos académicos de siete revistas consideradas representativas del campo de la Educación Matemática nacional. Después de constituir el corpus de 89 artículos, los investigadores, organizados en subgrupos, se embarcaron en producciones temáticas específicas. Para escribir este artículo, tres investigadores buscaron comprender las contribuciones, límites y desafíos de implementar enfoques problematizadores para la movilización y producción de conocimientos de los profesores de Matemáticas. Para ello, realizaron una metasíntesis de los trabajos del corpus que abordaron la cuestión de la problematización en la enseñanza y la formación docente. De los cuatro agrupamientos construidos, entre los resultados encontrados destacamos: (i) el importante número de trabajos por tareas; (ii) el silenciamiento de referentes teóricos clásicos en el área de resolución de problemas y (iii) una importante multiplicidad de referentes teóricos en los estudios que integran el grupo de Modelación Matemática (iv) la preocupación por la enseñanza en educación básica en trabajos que abordar el uso de problemas, incluso sin un marco para su abordaje.

Palabras clave: Matemáticas, Enseñanza, Conocimientos, Problemas, Tareas.

Résumé

Cet article présente les développements de la recherche intitulée « Mathématiques spécifiques de l'enseignant qui enseigne les mathématiques dans les années terminales de l'école primaire et secondaire », réalisée par des chercheurs de la Société brésilienne d'enseignement des mathématiques (Sbem). De 2021 à 2023, ce groupe a sélectionné des articles académiques issus de sept revues considérées comme représentatives du domaine de l'enseignement mathématique national. Après avoir constitué le corpus de 89 articles, les chercheurs, organisés en sous-groupes, se sont lancés dans des productions thématiques spécifiques. Pour rédiger cet article, trois chercheurs ont cherché à comprendre les apports, les limites et les défis de la mise en œuvre d'approches problématisantes pour la mobilisation et la production des savoirs des enseignants de mathématiques. Pour ce faire, ils ont réalisé une méta-synthèse des travaux du corpus abordant la problématique de la problématisation dans l'enseignement et la formation des enseignants. A partir des quatre regroupements construits, parmi les résultats trouvés, nous soulignons : (i) le nombre important de travaux par tâches ; (ii) le silence des références théoriques classiques dans le domaine de la résolution de problèmes et (iii) une multiplicité significative de références théoriques dans les études qui composent le groupe de modélisation mathématique (iv) le souci de l'enseignement de l'éducation de base dans des ouvrages qui aborder l'utilisation des problèmes, même sans cadre d'approche.

Mots-clés : Mathématiques, enseignement, connaissances, problèmes, tâches

Problematizing approaches for mobilizing mathematics teachers' knowledge: contributions, limitations and challenges

This article is the result of a broader research titled *Mathematics Specific to Teachers who Teach Mathematics in Latter Primary and Secondary Education*, carried out by researchers from the Working Group Training Teachers who Teach Mathematics (GT-07), at the Brazilian Society of Mathematical Education (SBEM). In its final configuration, 15 researchers[4] from several Brazilian institutions and regions undertook to build a panorama of the main theoretical framework of recent research, focusing on Mathematics (knowledge) that is mobilized and produced by teachers who teach Mathematics (TTM) in the later grades of primary education and secondary education.

For approximately three years (2021-2023), this group of researchers used three selection criteria for the articles that would be included in the corpus, namely: (I) being focused on teaching (and not exclusively on basic education students' learning); (ii) their research topic or subjects should be teachers who teach in the later grades of primary or secondary education; and (iii) they should somehow address knowledge of mathematical content for teaching (with its different denominations, depending on the theoretical framework). The research corpus was selected from seven journals considered representative of the field of national mathematics education: *Acta Scientiae* (ISSN:2178-7727), *Boletim de Educação Matemática* (ISSN:1980-4415), *Educação Matemática Pesquisa* (ISSN:1983-3156), *Educação Matemática em Revista* (ISSN:2317-904X), *Perspectivas da Educação Matemática* (ISSN:2359-2842), *Revista Internacional de Pesquisa em Educação Matemática* (ISSN:2238-0345), and *Zetetiké* (ISSN:2176-1744). Due to the volume of journals to be analyzed, the team also adopted a time frame, i.e., the period from 2019 to 2023, which includes the last 5 years of publications, considering the date of completion of the project (2024). Thus, the team built *a* corpus of 89

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articles. From then on, the participants searched for productions on specific topics, and this is the context in which we are situated.

To write this article, three members of this team focused on those works in the study's general corpus that addressed the issue of problematization in teaching and teacher training. This was aimed at *understanding the contributions, limits, and challenges of the implementation of problematizing approaches for mobilizing and producing mathematics teachers' knowledge.*

In Brazil, undergraduate Mathematics Licensure programs (ML) begin in the 1930s, under a transmissive paradigm, centered on the teacher, leaving the student to accept that learning was to receive that transmission without too much noise (Moreira & Ferreira, 2021). The curriculum of that teacher training has been described as a "3+1 Model," as it simply adds another year of pedagogical training to the baccalaureate program. Over the years, changes in society have led to changes in education and, consequently, in teacher training (Novoa, 2022). These changes in school, based on the so-called reproduction theories by Bourdieu and other scholars of the sociological dimension of education, contribute to reflections on the social role of the school and, consequently, of teachers and teacher education (Moreira & Ferreira, 2021)

Different ways of thinking about the social role of school imply different perspectives on training. According to Moreira and Ferreira (2021), a significant contribution to the debate on undergraduate courses, especially to discuss the role of mathematics disciplines, was provided by the notions of re-contextualization and didactic transposition, and the concept of pedagogical content knowledge, developed by Lee Shulman, especially in two articles published in 1986 and 1987. Shulman presents a knowledge base for teaching, and the main contribution of that base is the idea that there is a *pedagogical content knowledge* (PCK)[5]. For Shulman (1986, p. 9) "the PCK refers to the most appropriate forms of representation of ideas, the most powerful analogies, illustrations, explanations, and demonstrations [...] PCK also includes understanding what makes learning a given topic easier or more difficult."

In his studies on knowledge and teaching knowledge, Fernandez (2015) identifies two dominant lines: One approaches Shulman's studies; another follows the line of Elbaz (1983 apud Fernandez, 2015) and Connelly and Clandinin (1985 apud Fernandez, 2015), concerning the theoretical strand of the "teacher's thought," heavily influenced by Schön and his epistemology of practice.

[5] Commonly translated into Portuguese as "conhecimento pedagógico do conteúdo."

The broader study in which this article is situated is based on the notion of mathematics teachers' knowledge. That study is thus permeated by these theories and the models derived from them. Thus, the references adopted for this article do not exclude any of these theories, models, or lines, but rather examine how the studies into mathematics teachers' professional knowledge relate this topic with problematizing teaching approaches.

By problematizing approaches, we mean the training actions that use methodological strategies to strengthen reflection and questioning about mathematical knowledge, be that knowledge training-related, practice-related, or both. These approaches have as characteristic elements the use of problems, tasks, teaching cases, real-life situations, or the problematization of concepts in their epistemological origin.

Given the proposed goal, in addition to this introduction in which we present the context, we organize the article in four sections, namely: (I) *Methodological directions*, where we outline the study and present the process of identification of the articles that form our corpus of analysis; (ii) *Interpretative synthesis of the groups identified in the corpus*, where we highlight the studies' approaches and the connections between the knowledge perspectives and their problematizing approaches; (iii) *corpus synthesis*, where we highlight contributions, limits, and challenges of the implementation of each problematizing approach for mobilizing and producing mathematics teachers' knowledge; and (iv) our *final stitches*, where we indicate elements that emerged from the study and highlight limitations and challenges in the field of research on mathematics teachers' teaching knowledge, particularly those limitations related with approach that problematize mathematics.

Methodological directions

The research process was qualitative (Garnica, 2004) and exploratory, and can be characterized as a meta-synthesis about the topics that consisted "first, in obtaining qualitative evidence from first-order studies that compose the corpus [...] producing an interpretative synthesis [...] to then conduct a second-order study [...] producing other interpretations and other results, which allows arriving at another level of possible synthesis" (Cristovão & Fiorentini, 2021, p.38). A meta-synthesis aims to achieve "a synthesis that goes beyond what was obtained by primary studies, producing new insights and perspectives" (GEPPM, 2018, p. 247). In order to achieve this goal, we define the following specific objectives: (i) identifying, in each study, the associations between the problematizing approaches investigated and the knowledge mobilized and/or produced; (ii) analyzing the contributions, limits, and challenges pointed out by the researchers for implementing these approaches; (iii) building an integrating

synthesis of these associations in connection with the contributions, limits, and challenges identified.

It should be noted that the interpretative syntheses of the studies (Fiorentini & Crecci, 2017; GEPPM, 2018) were carried out according to the first two specific goals. Having being related to each other, these small syntheses were regrouped, producing an integrating synthesis, based on the aspects identified in the interpretive syntheses, showing fundamental elements that stem from the connections between the delimited associations and the contributions, limits, and challenges pointed out.

Thus, from the general corpus (89 articles), to write this article, we identified the works that relate this knowledge to problematizing approaches to mathematics, including the following: Problematized Approach to Mathematics; Scenarios for Research; Exploratory Teaching; Design-Based Research; Mathematical Research; Mathematical Modeling; Problem Solving, and others therefrom.

Therefore, at first, from an evidential perspective (Ginzburg, 1989), we conducted a search (using the search tool from the general spreadsheet[6]) with the terms "*design,*" "*explor,*" "*investig," "*problem, " *"model*," and " task," which relate to the approaches mentioned earlier, and we identified 32 potential studies. Based on the annotated summaries previously made in the broader research, we obtained more general data and then read the full papers to assess their relevance to our study goal and to deepen the analyzes that constituted the interpretive synthesis. Thus, 24 works remained that relate knowledge with some problematizing approach to Mathematics, or at least with the proposition of problems as triggers of formative practices.

Interpretative synthesis of the groups identified in the corpus

In the process of making the initial interpretative synthesis, we formed four paper groups with the problematizing approaches identified. Based on this perspective, we decided to signal the topics focused on by each article, the theoretical framework of knowledge assumed in these studies, and the theoretical framework directly connected to the topics of our search terms ("*design*," "*explor*," *"investig,"* "problem," *"model,*" and "*task*"). Based on this, we built four groups, namely: (1*) Approaches based on exploratory or investigative tasks*; (2*) Tasks directed to teacher learning*; (3) *Mathematical modeling*; and (4*) Other approaches using mathematical problems.* At the end of the section, we provide an overview of the connections and/or

[6] Data from the general corpus (89 articles) were organized in a spreadsheet in which information about adherence to the criteria for inclusion in the *corpus* was recorded, including titles, authors, objectives, theoretical frameworks regarding mathematical knowledge, aspects of methodological directions, and the research context.

convergence between the theoretical frameworks on knowledge and on the problematizing approaches found in each group of the corpus.

The first group comprises nine studies that connect exploratory or investigative tasks in contexts where intentionality permeates the construction, planning, and analysis of their uses.

Table 1

Group 1: Approaches based on exploratory or investigative tasks (the authors. The initials 'KN' indicate knowledge references presented by the papers' authors. Likewise, the initials 'PA' indicate references of problematizing approaches. For space optimization, we used acronyms for: Didactic Knowledge (DK); Mathematical Knowledge for Teaching (MKT), Pedagogical Content Knowledge (PCK), Mathematical Reasoning (MR), Hypothetical Trajectory of Learning (HTA).)

Theory	**Article**	**Focus**	**Theoretical framework**
MR	Martins; Henriques and Caetano (2023)	Discussing the knowledge of teachers to promote students' **RM**. The work highlights the importance of designing and adapting **tasks that promote RM**, as well as conducting these tasks in the classroom. It also emphasizes the need for planning and reflection on pedagogical practice to promote RM.	KN: Herbert Davidson, Bragg (2019) PA: Ponte (2005) and Ponte, Quaresma, and Mata-Pereira (2020)
MKT	Marins, Teixeira, and Savioli (2021)	Investigating professional knowledge that is mobilized by PIBID participants in a formative process based on an **exploratory teaching** perspective. The authors point out several practices related to the proposition of exploratory tasks and which contributed to the mobilization or development of Mathematical Knowledge for Teaching (**MKT**), particularly the pedagogical knowledge as per Ball content and collaborators.	KN: Ball, Thames, and Phelps (2008); PA: Oliveira; Carvalho (2014); Oliveira; Menezes; Canavarro (2013) and Ponte and Quaresma (2015)
CD	Rodrigues and Ponte (2020a)	To problematizing the results of a training experience occurred in a discipline of Statistics from a specialization course taught to 13 teachers in Rio de Janeiro. The purpose was to understand how the participants develop **DK** regarding representations and **statistical investigation**.	KN: Batanero (2002) PA: Ponte (2012) and Smith (2001)
DK	Rodrigues and Ponte (2020b)	Understanding how the connection of a set of ***Design Experiment*** principles in the context of a training experience can contribute to promoting participant's **KD**.	CO: Shulman (1986), and Ponte (2012) PA: Cobb et al. (2003) and Ponte et al. (2016)

DK	Molina and Samper (2019)	Studying the relationship between types of open **conjecture problems** in geometry, within Dynamic Geometry Environments, and classes of argument produced by future mathematics teachers. Problems are presented in a plane geometry course for future mathematics teachers in Colombia. Focusing on **DK** on argumentation and proof in geometry.	KN: Ball; Lubienski, and Mewborn (2001) PA: Ponte (2003, 2004) and Ponte, Fonseca, and Brunheira (1999)
HTL	Vieira, Trevisan, and Baldini (2020)	Highlighting **elements valued** by seven Mathematics teachers (five working in the Paraná state education system, one graduate student, and one teacher trainer) who are members of a study group focused on implementing **algebraic thinking tasks** for the 6th-grade in their classrooms.	KN: Simon and Tzur (2004) and Thompson (2009); PA: Stein and Smith (2009) and Ponte et al. (2013)
PCK	Amorim, Pietrocolo, Galvão, and Silva (2020)	To analyze **the knowledge base for the teaching** of Diophantine equations of a group of future mathematics teachers. In developing this study, some assumptions of the **Design Experiments** methodology were adopted**.**	KN: Ball, Thames, and Phelps (2008) and Shulman (1986) PA: Cobb et al. (2003)
PCK	Teixeira (2020)	Identifying, knowing, and reporting the teachers' pedagogical practices about their experiences during the **resolution of a variety of counting problems** that were proposed to be solved also by students of the later grades of primary school in regular classes. The teachers were advised to put themselves in the students' position to **reflect on their pedagogical practice.**	KN: Shulman (1986) PA: Cobb et al. (2013) and Fischbein (1994)
PCK	Silva and Costa (2019)	To investigate **teacher knowledge** in a training action guided by the **investigative approach** in Mathematics classes. The analysis of the results obtained using teaching cases as a pedagogical tool in teacher training points to the potential of these cases for future teachers' understanding of the investigative approach.	KN: Shulman (1986) and Misukami (2004) PA: Ponte et al. (1998); Ponte et al. (2003) and Alro and Skovsmose (2006)

In the paper titled *Knowledge of Teachers to Promote Mathematical Reasoning: An Experience of Continuing Education*, the authors Martins, Henriques and Caetano (2023) investigate the context of continuing teacher education within a curricular unit of a master's program in teaching, aiming to develop the knowledge of six practicing teachers to promote their students' mathematical reasoning (MR). The activities held resulted in the design and implementation of mathematical tasks in basic education classrooms, as well as a final reflection on what they experienced in the process. The authors consider that teacher knowledge for promoting MR can be associated with four main dimensions: "(i) theoretical and methodological aspects; (ii) curricular aspects; (iii) material aspects; and (iv) context aspects.

According to the authors, exploratory teaching is characterized as an alternative to pedagogical work in the classroom, with a view to MR capacity development by the students (Ponte, 2005). For the authors, the mathematical tasks are activities proposed to students to promote mathematical reasoning, generalization, and justification through construction and observation, considering previous knowledge.

In the article "Exploratory Mathematics Teaching Practices and the Mobilization/Development of Mathematical Knowledge for Teaching by PIBID Participants", Marins, Teixeira, and Savioli (2021) seek to investigate professional knowledge that is mobilized/developed by PIBID participants in a training process based on exploratory teaching; they also seek to answer the following guiding question: What practices carried out in a training process and based on an exploratory approach to mathematics teaching can contribute to mobilize/develop professional knowledge? The authors analyze the results of a training action composed of six face-to-face meetings, as well as remote actions.

Based mainly on Ball, Thames, and Phelps[7] (2008), they conclude that certain practices contributed to the following purposes: Mobilization/development of mathematical knowledge for Teaching (Specialized Knowledge of Content (SCK); Knowledge of Content and Students (KCS); and Knowledge of Content and Teaching (KCT)). And they conclude that the practices that contributed to these purposes were: Choosing a challenging and interesting task to students; anticipating their possible resolutions; seeking to know the mathematical object in detail; explaining the class dynamics; using a manipulable material; monitoring task accomplishment; selecting the resolutions to be discussed in class; sequencing them in order to provide a logical thread of ideas; maintaining a harmonious climate for discussing mathematical ideas; and and connecting students' answers.

In "Development of didactic knowledge of teachers in statistics: A formative experience," Rodrigues and Ponte (2020a) list Ponte's (2012) four domains of didactic knowledge, namely: (I) knowledge of Mathematics for teaching it; (ii) knowledge of the

[7] The acronyms of the MKT (Mathematical Knowledge for Teaching) model refer to expressions in English: Subject Matter Knowledge (SMK) domain: Common Content Knowledge (CCK), Horizon Content Knowledge (HCK), and Specialized Content Knowledge (SCK). PCK domain: Knowledge of Content and Teaching (KCT), Knowledge of Content and Students (KCS), and Knowledge of Content and Curriculum (KCC).

curriculum; (iii) the knowledge of the students and their learning; and (iv) knowledge of educational practice with knowledge of Statistics for teaching it, by Batanero (2002). The activities developed in the training context (including analyzing didactic materials, exploratory tasks, student productions material, and statistical investigation) focused on promoting statistical literacy[8] in the school environment.

In an article titled "Design-Based Research[9]: a teacher training experiment in Statistics," Rodrigues and Ponte (2020b) address the model proposed by Ponte (2012) for Mathematics teachers' didactic knowledge and affirm that it is formed of four dimensions: knowledge of Mathematics for its teaching; knowledge of the students and learning; knowledge of the educational practice; and knowledge of the curriculum. The authors propose a training experiment. They seek to connect these dimensions with the following principles of Design Experiments: use of tasks of an exploratory nature; use of authentic classroom situations; and use of technology. The results suggest that these principles contributed to developing teachers' knowledge of the exploratory approach to Statistics.

In the article "Types of Problems that Prompt the Generation of Inductive, Abductive, and Deductive Arguments," Molina and Samper (2019) analyze students' resolution strategies, identifying the types of arguments associated with each type of problem, and they point out how the typology of problems can contribute for the didactic-thematic knowledge of mathematics teachers. The authors argue that knowing the different types of arguments and recognizing the difference between empirical ones (for example, inductive and abductive) and formal ones (deductive) should be part of the knowledge of a mathematics teacher, and they argue that knowing different types of tasks related to argumentation and how to approach them, recognizing the relationship between these and the mathematical activity implied in it, is also essential to the elements of teaching knowledge.

The innovation described by the authors is based on problems such as those proposed in class by Ponte and collaborators, when dealing with the so-called *mathematical*

[8] Based Martins and Ponte (2011) and Steen (2001), the authors say that "in general, literacy is understood to refer to a set of interpretation and communication principles, ideas, and skills that are necessary to efficiently treat information that involves quantitative data emerging daily and in professional contexts" (Rodrigue; Ponte, 2020a).
[9] From the English: Design Experiments .

investigations (MI). At the end of the analyses, the authors present the relationship between three types of problems. Problems of searching for the consequent are related to inductive arguments; problems of searching for the antecedent are related to abductive arguments; and problems of determining dependence are related to both inductive and abductive arguments. They conclude that the future teachers were involved in the process of solving open problems, and it was based on their assumptions and associated mathematical conversations (to accept or reject mathematical ideas exposed) that the mathematical content was organized in geometry classes. They formulated arguments of different kinds during the resolution process, presented their conjectures, verified their results, and tried to support the veracity of their conjectures.

The article by Vieira, Trevisan, and Baldini (2020), which is titled " Elements valued by teachers when implementing tasks in math classes," is the result of interweaving activities that connect university and school through actions/projects interested in investigating the professional practices of practicing teachers. The central focus is the problematization of elements related to the creation of non-routine[10] tasks (compared to those present in textbooks) directed to the teaching of Algebra. However, in this context, the references related to the theme (Stein & Smith, 2009; Ponte et al. 2013) are articulated to the discussion about THA (Simon & Tzur, 2004) and the appreciation of students' mathematical knowledge in socialization processes and the discussion of resolution strategies, as a teaching action (Thompson, 2009).

The work titled "A Sequence of Activities for the Teaching of Diophantine Equations: A possibility to expand the knowledge base of future Mathematics teachers," by Amorim, Pietropaolo, Galvao, and Silva (2020), analyzes, in the light of the assumptions established by Ball, Thames, and Phelps (2008), the knowledge base for teaching Diophantine equations of a group of future Mathematics teachers attending a training program. To that end, the principles of the Design Experiments methodology were used. The study involved a group of 10 students from the ML course at a UFS campus. The authors analyzed the group's reflections in written records and audio recordings that were generated by their experience with a teaching sequence on Diophantine equations. The activity explored the relationship between a content of Number

[10] The task under discussion in the article is based on a manipulation material, inspired by Vieira (2018), on the representation of geometric figures and algebraic language.

Theory and topics that would be taught in basic education. According to the authors, the sequence allowed the group to reflect on the difficulties that basic education students can have in situations involving equations, as well as how they could help their students overcome them.

Teixeira (2020), in the paper titled *Practices of teachers of basic education during the resolution of counting problems*, undertakes to answer the following research question: What experiences should a teacher of Mathematics have in a continuing education activity in order to select and direct learning situations intended to develop their students' combinatorial reasoning by using counting problems, so as to understand those students' difficulties and help them overcome them? The research also investigated whether a sequence of activities that explores the resolution of counting problems, without the use of formulas, could help teachers (re)signify their knowledge on the subject. The *Design Experiment* methodology by Cobb et al. (2003) was chosen to guide the development of activities, because it allows for flexibility in adapting the initial design proposed in a cyclical movement. The research involved the continuing education of twenty teachers who, at the time, were teaching mathematics in primary and/or secondary education classes of the state education system. The program was developed over eight meetings with an average duration of five hours. The research found that teachers have difficulty exploring the four meanings of the concept of multiplication and addressing combinatorial reasoning.

Silva and Costa (2019), in the article titled *Investigative approach in Mathematics classes: An investigation with teaching cases in teacher training*, present reflections and analyses from a meeting that was part of a teacher training program guided by and for an investigative approach to Mathematics classes, using teaching cases. The teaching cases are addressed from the perspective of Misukami (2000), who, influenced by Lee Shulman's studies, highlights its potential as an instrument for reflecting on pedagogical practice. According to the authors, investigative activities involve open-ended, unstructured situations or problems that allow for resolution through different paths. The analyses were carried out in the light of the insights achieved by the teachers, and highlighted aspects that can be systematized and categorized according to an investigative approach in Mathematics classes.

The second group includes six works, of which three are part of a special issue of the journal *Perspectivas de Educação Matemática*, titled *Mathematical Tasks and Professional Teacher Learning in Initial and Continuing Education.*

Table 2

Group 2: Tasks directed to teacher learning (the authors. The initials 'KN' indicate knowledge references presented by the papers' authors. Likewise, the initials 'PA' indicate references of problematizing approaches. For space optimization, we used acronyms for: Didactic Knowledge (DK), Mathematics Teacher Specialized Knowledge (MTSK), School-Related Content Knowledge (SRCK), Professional Learning Task (PLT), and Training Task (TT)

Theory	Article	Focus	Theoretical framework
PLT	Silva, Albrecht, and Neves (2023)	Involving an ML trainer in processes of theoretical study, mathematical task creation, and **formative tasks**, as well as in planning, developing and reflecting on actions that develop didactic plans. In addition to references about this topic, collaborative work and the exploratory perspective are problematized.	KN: Ribeiro and Ponte (2020) PA: Fiorentini (2004) and Canavarro (2011)
PLT/ PLOT	Gross et al. (2023)	Analyzing a **PLT** plan used in the training context of a discipline in a master's course, based on **the PLOT model** proposed by Ribeiro and Ponte (2020).	KN: Ball, Thames, Phelps (2008) and Silver (2007); PA: Ribeiro and Ponte (2020).
PLT/ SRCK	Jardim, Aguiar, and Ribeiro (2023)	Based on the **SRCK** by Dreher and collaborators, to understand how the **professional knowledge** of students at an ML program is modified using two **PLTs** related with the discussion of algebraic structures of groups and with the connections between that knowledge and the teaching of school contents.	KN: McCrory et al. (2012), Dreher et al. (2018) and Ribeiro and Ponte (2020) PA: Canavarro et al. (2012)
PLT/ DK	Aguiar, Ponte, and Ribeiro (2021)	To investigate, based on a few **PLTs**, the **mathematical and didactic knowledge** (Ponte, 1999) mobilized and deepened by mathematics teachers when preparing, developing, and collectively analyzing a class about patterns and regularities in a secondary education class.	KN: Ponte (1999), PA: Ball; Cohen (1999)
TT/ MTSK	Ribeiro, Almeida, and Mellone (2021)	To present the concept of TT , developed in the context of the Research and Training Group **Interpretative and Specialized Knowledge of teachers** who teach mathematics.	KN: Ribeiro (2021) PA: Ribeiro, Almeida, and Mellone (2021).
TT/ MTSK	Ribeiro, Gibim, and Alves (2021)	Discussing, through **TT** about fractions, the importance of changing the focus in initial and continuing teacher education, based on the **MTSK** model.	KN: Carrillo et al. (2018) and Di Martino, Mellone, and Ribeiro (2020)

PA: Ribeiro, Almeida, and Mellone (2021).

Gross et al. (2023), in the paper titled *A Plan for Developing and Analyzing Professional Learning Tasks*, when discussing Professional Learning Opportunities (PLO), built on the theoretical and methodological model called Professional Learning Opportunities for Teachers (PLOT), proposed by Ribeiro and Ponte (2020) . The authors outline information that guides readers about the construction and analysis of a Professional Learning Task (PLT), in addition to discussing examples for that purpose. The article data are analyzed from the design experiments perspective (Cobb et al., 2016).

Jardim, Aguiar and Ribeiro (2023), in the paper titled *Professional learning tasks and mathematical knowledge involving the algebraic structure of groups: An experiment in a Mathematics licensure program*, used research cycles based on design experiments, from an exploratory teaching perspective (Canavarro, *et al*, 2012), for a PLT (Ribeiro & Ponte, 2020) in an Algebra discipline of an undergraduate program. Regarding the theoretical framework about knowledge, the authors start from an interesting problematization of the concepts of school mathematics and academic mathematics, to highlight studies that seek to demarcate the connections (and their benefits) between these two terms. By treating the research participants as future secondary education practicing teachers, the authors used the School-Related Content Knowledge (SRCK) by Dreher and his collaborators (2018). Translated to Portuguese by the authors as *Conhecimento do Conteúdo Relacionado à Escola*, this approach is said to consider non-trivial relationships between school mathematics (SM) and academic mathematic (MA), comprehending the three aspects: to (I) *Knowledge of the curricular structure and its legitimation* as regards *metamathematical reasons*; (ii) *Knowledge of the relationship between SM and AM* ; and (iii*) Knowledge of the relationship between SM and* AM. In this context, the study by McCrocy et al. (2012) is also listed as an important theoretical reference for analyzing the knowledge of future teachers regarding the teaching of Algebra.[11]

[11] The authors point out that, according to McCrory et al. (2012), "[...] there are three practices that can assist in understanding and evaluating teachers' knowledge for teaching algebra: connecting algebra topics,

Algebra as a subject also appears in the article by Silva, Albrecht and Neves (2023), titled *The Construction of a Mathematical Task on Linear Systems: Collaborative Work in the Context of the Training of a Teacher Trainer*, in which another PLT is the subject of analysis. Aligned with references of critical mathematical education by Ole Skovsmosis, of collaborative work (Fiorentini, 2004), and of authors who discuss the perspective of Exploratory Teaching (Canavarro, 2011; Cyrino & Teixeira, 2016), the study involved, in addition to its authors, a trainer responsible for the discipline of linear algebra in an ML course and the students enrolled in it.

In the paper *Mathematical and Didactic Knowledge of Basic School Teachers about Patterns and Regularities in a Training Process Anchored in Practice*, Aguiar, Ponte, and Ribeiro (2021) use a PLT by Ball and Cohen (1999) that is specially designed for training to investigate the mathematical and didactic knowledge (Ponte, 1999) mobilized and deepened by mathematics teachers when preparing, developing, and collectively analyzing a class on patterns and regularities in a secondary education class. The training process was carried out over 15 weekly meetings of four hours with Mathematics teachers from basic education schools in the metropolitan area of Sao Paulo. The practice records, more precisely the use of videos that formed the design of PLTs and were intended to promote collective discussions among teachers, proved to be powerful tools for teachers to leave their isolation to think, build, and reflect collectively. The participant teachers mobilized and expanded their knowledge about the interpretation of different ways of generalizing a mathematical pattern. They also expanded their own mathematical knowledge about an important topic to be discussed in basic education, Algebra.

In the paper titled *The necessary change in focus in the training of teachers of, and who teach mathematics: A discussion of tasks for creating and developing interpretative knowledge*, Ribeiro, Gibim, and Alves (2021) uses the *Mathematics Teachers' Specialized* Knowledge (MTSK) (Carrillo et al., 2018) and the Interpretative Knowledge[12] by Di Martino, Mellone, and

representations, and domains; trimming the complexity of an advanced mathematics topic to obtain an understanding of school mathematics; and breaking down hidden meanings in formulas and procedures, as well as directing and clarifying possible constraints" (Jardim, Aguiar, and Ribeiro, 2023, p. 5) .

[12] In Brazil, researchers have translated it as: Specialized Knowledge of Mathematics Teachers.

Ribeiro (2020) to create Training Tasks (or Tasks for professional training). This last concept is characterized by the authors in another paper that is also part of our corpus, titled *Conceptualizing Training Tasks for Developing the Specificities of Teacher's Interpretative and Specialized Knowledge*, by Ribeiro, Almeida, and Mellone (2021). One can see in the discussions of both articles that the tasks for teacher training should not be restricted to implementing a specific knowledge for students, but should intentionally consider "[...] the specificities of the professional context and aim to develop the specificities of the specialized knowledge of the teacher of, and who teaches, or will teach mathematics" (Ribeiro, Gibim, and Alves, 2021, p. 9). By providing an example, Ribeiro, Almeida, and Mellone (2021) present a structure characterized by two parts: (i) a proposal for a task that may be used by teachers with their students, when questions are also included to teachers, intentionally concerned to promote the specialized knowledge of the topic in question; and (ii) a set of student outputs (real or hypothetical), which may originate from various sources (such as videos, classroom episodes, or written records) regarding the task previously presented. The purpose is to establish links between interpretive knowledge, with a view to building constructive feedback.

The third group comprises studies that were based on the use of mathematical modeling activities, which have very diverse theoretical references.

Table 3

Group 3: Mathematical Modeling (the authors. The initials 'KN' indicate knowledge references presented by the papers' authors. Likewise, the initials 'PA' indicate references of problematizing approaches. For space optimization, we used acronyms for: Language Games (LG), Mathematics Teacher Specialized Knowledge (MTSK), Hypothetical Learning Trajectory (HLT) and Objectivation Theory (OT))

Theory	Article	Focus	Theoretical framework
MTSK	Escorcia, Acevedo-Rincón, and Montes (2023)	It discusses the specialized knowledge of mathematics teachers, which incorporates information and communication technologies (ICT) in mathematics teaching, using **modeling**. The **MTSK** model is presented as an analytical tool to understand this knowledge.	KN: Ball, Thames, Phelps, (2008); Carrillo et al. (2018) and Padilla-Escorcia and Acevedo-Rincón (2020, 2021, 2022) PA: Villa-Ochoa (2007, 2015)
HLT	Ferreira and Silva (2019)	It analyzes the outputs presented by participants in **a modeling activity**, specifically with regard to each of the **steps of the Modeling: Integration, Mathematization, Resolution, Interpretation of Results**, and **Validation** are related to a previously planned THA, observing how each of these phases was hypothetically anticipated by the researchers.	KN: Simon and Tzur (2004) PA: Almeida, Silva, and Vertuan (2012) and Borromeo Ferri, 2006
OT	Prieto and Buitrago (2019)	Based on Radford's **OT**, the authors aim to identify and describe the practical knowledge of simulator developers for managing mathematical work, as recognized by a group of GeoGebra Club promoters.	KN: Radford (2013), PA: Villarreal and Borba (2010) and Gutiérrez, Prieto, and Ortiz (2017)
LG	Sousa and Almeida (2019)	It investigates the various **LGs** associated with **Mathematical Modeling activities** and the meanings constructed by mathematics students within these language games in relation to first-order ordinary differential equations.	KN: Wittgenstein (1996, 2013) PA: Pollak (2012)

The work titled *Mathematics Teachers' Specialized Knowledge for Teaching through Modeling Using ICT*, by Escorcia, Acevedo-Rincon, and Montes (2023), discusses the specialized knowledge of mathematics teachers who adopt ICT in mathematics teaching, using modeling. The *Mathematics Teachers' Specialized* Knowledge (MTSK) model is presented as an analytical tool to understand the knowledge a mathematics teacher uses when developing

tasks related to their profession. The characteristics of the specialized knowledge of teachers who adopt ICT in the teaching of mathematics, using modeling, include the need to know in depth the discipline of mathematics, relate it with didactic-pedagogical knowledge, and develop mathematical modeling processes using ICT.

The study by Ferreira, and Silva (2019), titled *Mathematical Modeling and a Hypothetical Learning Trajectory Plan*, investigates the use of an HLT designed by the authors, in the process of solving a mathematical modeling problem, from the perspective of mathematical education, in teacher training. The context of the study was a mini course involving basic education teachers, future mathematics teachers, and teacher trainers, and the problem to be solved by the teachers focused on the topic "Reaction speed of antacid tablets." For the authors, the plan to use a HLT as a resource in planning mathematical modeling activities can be presented as a strategy in initial or continuing teacher education, considering that a HLT is an instrument for guiding teaching practice. However, they analyze teachers' resolutions by comparing them with what was planned in their HLT. In a footnote, they point out that "the HLT theoretical assumptions provided guidance for the work carried out by the researchers. However, the participants were given an overview of a HLT, which was theoretically presented in the mini course, as well as the information that it had been used as a basis for planning the modeling task".

The study by Prieto and Buitrago (2019), titled *Necessary knowledge for managing mathematical work in designing simulators with GeoGebra*, was not directly related to teaching. However, in the context of a simulator project for *GeoGebra*, it investigates the knowledge for managing mathematical work as recognized by a group of promoters from the *GeoGebra Club*. The participants were seven Mathematics and Physics teachers, four women and three men, who worked as promoters for the GeoGebra Club during the school year 2016-2017. The results of the study indicate that this knowledge was mobilized in response to an ethical need by teachers, as expressed by the promoters: to ensure that students learn geometry while solving construction tasks, communicate to others the techniques used to solve tasks, or anticipate a technique to ensure better performance in the future. For the researchers, this result

confirms that knowledge of mathematical management for work is strongly rooted in the promoters' concrete experiences.

The study by Sousa and Almeida (2019), titled *Linguistic Appropriation and Meaning in Mathematical Modeling Activities*, builds on Pollak's (2012) Mathematical Modeling (2012), and its philosophical basis is the studies on the language games by Wittgenstein (1996, 2013) and some of his interpreters. Considering these theoretical-philosophical elements, mathematical modeling activities were developed in the discipline of ordinary differential equations of a Mathematics undergraduate licensure program. The authors consider the students' discourse practices for discourse analysis, thereby enabling the construction of idea trees. The results indicate that the construction of meaning within modeling activities is associated with the students' linguistic appropriation regarding the rules and techniques that are configured in specific language games identified in the mathematical modeling activities.

The fourth group comprises five studies generally based on the use of the term 'problem' which mostly do not have problematizing approaches in their theoretical framework.

Table 4

Group 4: Other approaches using mathematical problems (the authors. The initials 'KN' indicate knowledge references presented by the papers' authors. Likewise, the initials 'PA' indicate references of problematizing approaches. For space optimization, we used acronyms for: Creativity (CRE), Onto-Semiotic Approach to Knowledge (OSA), Knowledge Quartet (KQ), Pedagogical Content Knowledge (PCK), Anthropological Theory of Didactic (ATD))

Theory	Article	Focus	Theoretical framework
CRE	Marcatto (2023)	Discussing the implementation of educational problem-solving tasks from the perspective of the Exploratory Problem-Solving Model (EPSM) (Koichu, 2019) in the teacher education of teachers who teach Mathematics, so as to establish a knowledge base with a focus on promoting particularly participants' **creativity** in the resolution processes.	KN: Gontijo and Fonseca (2020); Koichu (2019), and Sriraman (2009); PA: Cobb et al. (2003)
KQ	Perez and Piquet (2022)	In this study, we will see how teachers establish connections between representations to help students build algebraic language. To this end, the authors analyzed three episodes extracted from two classes by a particular teacher. The teacher's interventions occurred within a lesson that develops in a **problem-solving environment.** The analyzes were carried out based on the theoretical framework provided by the **Knowledge**	KN: Shulman (1986) and De La Fuente; Rowland; Deulofe (2016)

		Quartet (KQ), a tool that, according to the authors, allows observing how teacher knowledge emerges when a teacher helps their students learn mathematics.	
PCK	Etcheverria; Almeida, and Amorim (2021)	The goal of this study was to discuss an initial teacher education experience with students from an undergraduate licensure program at UFS, focusing on the teaching of addition and subtraction operations. The training process was based on the conception of **pedagogical action and reasoning** presented by **Shulman**, and was developed in the discipline of Mathematics teaching topics	KN: Shulman (2014) PA: Vergnaud (1996) and Magina et al. (2008)
EOC	Uribe and Oliva (2021)	Based on Godino's **onto-semiotic approach to knowledge (OSA)**, the goal of this study was to investigate the types of **subject-matter and didactic** knowledge that future mathematics teachers draw upon when performing **evaluative activities on trigonometric functions** in an introductory analysis course at a university in southern Chile.	KN: Godino (2002, 2009)
TAD	Costa and Del Rio (2019)	The study presents the description and **praxeological analysis (knowledge and know-how)** of a geometry problem that allows to begin studying the notion of function. Based on **ATD**, they perform a prior analysis of the knowledge and know-how that can be drawn upon when studying this problem, and they adapt it by incorporating dynamic geometry into the resolution process, to be presented in numerous workshops and teacher training courses in Argentina.	KN: Brousseau; Chevallard and Otero (2013)

Marcatto (2023) uses the Exploratory Problem Solving Model (EPSM) (Koichu, 2019) approach when presenting the discussion of a classroom implementation, carried out in a context of TTM training on problem-solving, based on the Design Experiments framework, by Cobb et al. (2003). This scenario is understood by the author as capable of enhancing students' learning by valuing their interactions with peers. Another point stressed by the author is, at the same time, the alignment between this perspective and the requirements of the National Common Curriculum Base (Brasil, 2018) and students' and teachers' difficulty understanding this perspective in the development of skills and competences.

In the work titled *The use of connections between representations by the teacher in constructing algebraic language*, Perez and Piquet (2022) present the connections between representations used by teachers to help their students build algebraic language. To this end, the authors analyze three episodes extracted from two classes by a particular teacher. The

teacher's interventions occurred within a class considered a problem-solving environment, which, according to the authors, is an environment where students have a leadership role, the opportunity to connect what they are learning with what they already know, and the ability to connect and have students connect different mathematical representations. The analyzes were carried out based on the theoretical framework provided by Knowledge Quartet (KQ)[13], a tool that, according to the authors, allows observing how teacher knowledge emerges when a teacher helps their students learn mathematics. In addition, the authors propose adding to KQ a new indicator called "connections between representations," referring to the connections the teacher makes to perform a translation between two representations.

In the article *Training Process of the Future Mathematics Teacher: Focusing on Addition and Subtraction Operations*, the authors Etcheverria, Almeida and Amorim (2021) present an initial training experience with students from an undergraduate licensure at UFS. The plan puts into practice the steps of pedagogical reasoning proposed by Shulman (2014), which can be categorized into the following stages: understanding, transformation, instruction, evaluation, reflection, and new understandings. The data were collected through a field journal and portfolios. The study participants were asked to design problems related to addition and subtraction. From discussions about different categories and complexities of problem situations, the problems designed by the licensure students were categorized according to Vergnaud (1996) and Magina (2008). According to the authors, the pedagogical action and reasoning experienced by the students can help in the training process of future mathematics teachers by promoting understanding of the complexities of the pedagogical process.

In the article *Types of knowledge deployed by future mathematics teachers when solving problems about trigonometric functions*, Uribe and Oliva (2021) analyze the knowledge of future mathematics teachers who attended the introductory analysis course at a university in southern Chile, when performing evaluative activities on trigonometric functions; they investigate their answers and establish a specific typology for general and specific mathematical knowledge. The problems were chosen after a previous analysis of the selection of fields and

[13] In Brazil, the authors who use this model do not translate its name, just clarify that Knowledge Quartet (KQ) is a theoretical tool derived from the teacher practice, formed by four dimensions.

sub-fields of problems related to trigonometric functions, based on the bibliographic texts used in the course, as well as the notes produced by the teachers who taught the course in the first semester of 2018 and in previous semesters. This investigation was carried out considering the primary elements and the epistemic facet of the Onto-Semiotic approach, through the decomposition of units and identification of entities that future mathematics teachers draw upon (Godino, 2002). According to the authors, the observations indicate that the students present general knowledge: algorithmic, representative, interpretative, and specific knowledge associated with the field of problems addressed in the activities.

The article *Contributions from Dynamic Geometry to studying the notion of function based on a geometric problem: A praxeological analysis*, by Costa and Del Rio (2019), presents the description and praxeological analysis (knowledge and know-how) of a geometric problem that allows beginning studying the notion of function. The research falls within ATD, which proposes the abandonment of traditional (or monumental) pedagogy in favor of adopting the so-called Pedagogy of Research and of Questioning the World (Chevallard, 2013; Otero, 2013). For the implementation of the proposal, the authors performed a prior analysis of the knowledge and know-how that can be put at stake when studying the proposed problem. After the study, the problem was presented in workshops to teachers from Buenos Aires, Argentina, with the particularity of incorporating dynamic geometry in the resolution process. At the workshops, questions about the problem and its resolution are asked, allowing teachers to reflect on teaching.

Integrative syntheses of the groups formed

The process of forming the groups allowed us to identify connections between knowledge theories and those theories directed to the discussion of problematizing approaches used by the authors of the corpus we formed.

In the first group, i.e., *Approaches based on exploratory and investigative tasks*, we note a significant concern with the development of Didactic Knowledge and Pedagogical Content Knowledge by teachers and future teachers. Most works in the corpus (9 studies) are in this group, which relates problematizing approaches and mobilization/production/deepening of

knowledge. This seems to be an emerging trend for research on teaching knowledge. In the work of Martins, Henriques and Caetano (2023), for example, the teacher knowledge perspective to promoting mathematical reasoning (MR) is connected with contents through participants' pedagogical practice when designing or adapting tasks that promote conjecture, generalization, and justification, by predicting specific actions to support the creation of justifications and the sharing of information in the classroom, and anticipating possible directions that a lesson can take.

Vieira, Trevisan, and Baldini (2020), for example, recognize the task elaboration, construction, and implementation steps as fundamental to teaching practice. Molina and Samper (2019) seek to develop the didactic-mathematical knowledge of future teachers in Colombia through the resolution of open problems of conjecture (IM) in geometry, related to argumentation (inductive, deductive, abductive), using GeoGebra. The works by Rodrigues and Ponte (2020a) and Rodrigues and Ponte (2020b), based on the dimensions of the teacher's didactic knowledge described by Ponte (2012), propose a training experience, based on Cobb's Design Experiments, and emphasize "the importance of not conceiving teaching experiences only as tests of pre-conceived theories, but enabling the construction of local theories. These local theories differ from specific and general theories as the former are adapted to a specific topic."

Teixeira (2020) also uses the design experiments approach when proposing a sequence of activities exploring the resolution of problems of counting Elementary School, without the use of formulas, so as to develop the combinatory reasoning of students and help teachers overcome possible difficulties related to this topic. Molina and Samper (2019) advocate that future teachers "know mathematics in a way that is useful for teaching – it is not enough that future teachers know the mathematics they are going to teach" (Molina & Samper, 2019, p. 111-112) – and that future teachers can "conceive the resolution of certain types of problems as a way of involving argumentative activity in teaching and having tools to recognize the characteristics of this type of teaching" (Samper Molina, 2019, p. 112).

In the second group, *Tasks directed to teacher learning*, six works problematize the potential of PLT and TT in teacher training. The studies by Ribeiro, Almeida, and Mellone

(2021) and Ribeiro, Gibim, and Alves (2021), for example, by articulating this problematizing perspective with MTSK, indicate the importance of the collaborative context among teachers working in basic education, mathematics licensure trainers, and licensure students in task discussion processes. The designs for addressing these investigations have significant similarities, considering that all associate the use of tasks directed to teacher learning with different knowledge models, enabling a detailed analysis of that knowledge, as well as reflecting on the need for changes of focus on initial and continuing teacher education.

In the third group, *Approaches using mathematical modeling*, three of the four studies seem to be the ones that least relate to processes of knowledge mobilization/production for teaching by teachers and future teachers who teach mathematics. These studies focused on technical language development (Sousa and Almeida, 2019), the validation of a HTL to predict steps in solving modeling problems followed by teachers and future teachers (Ferreira & Silva, 2019), or the knowledge necessary to manage mathematical work during the creation of simulators in *GeoGebra* (Prieto & Buitrago, 2019). The article by Escorcia, Acevedo-Rincon and Monte (2023) is the most recent and connects the MTSK model and mathematical modeling with *GeoGebra* software to discuss the specialized knowledge of the mathematics teacher who incorporates ICT in the teaching of conic sections. The types of specialized knowledge a mathematics teacher uses when developing tasks include knowledge of topics, knowledge of mathematical practices, and knowledge of mathematics teaching. This change in perspective between the oldest and the most recent works seems to indicate that the theoretical framework of knowledge is closer to knowledge for teacher training.

In the fourth group, titled *Other approaches using mathematical problems*, the five studies that use problems in training mostly do not do so based on a particular approach. However, they bear similarities for being explicitly concerned with teacher training that allows a change in the paradigm (still) in effect in basic education classrooms, which maintains a focus on mathematics and teacher, rather than on social practices, i.e., which does not prioritize dialogue, construction, or problematization. These researchers aim to develop creativity and prepare the teacher to understand the documents that guide teaching (Marcatto, 2023); they aim to establish connections between representations (Perez & Piquet, 2022), a fact that can

facilitate students' understanding. There are also researchers who connect the pedagogical reasoning model and the theory of conceptual fields (Etcheverria, Almeida, and Amorim, 2021) already in initial training, so as to enable future teachers to categorize and design problems with addition and subtraction operations in order to understand fundamental concepts and transform the teaching of these.

By exploring, in the context of a discipline of Mathematical Analysis, the didactic knowledge that future mathematics teachers of a university in southern Chile put at stake when carrying out evaluative activities of trigonometric functions, Uribe and Oliva (2021), seem to question the role of these disciplines in training, advocating for an approximation with the profession. Costa and Del Rio (2019) build on ADT to analyze the praxeologies of a geometric problem that allows to begin studying the notion of function in the way they deem most significant, incorporating dynamic geometry into the resolution process and proposing to abandon the traditional (or monumental) pedagogy for the so-called research and world questioning pedagogy (Chevallard, 2013; Otero, 2013).

For the authors, it is important to work with problems to enhance an understanding of the concept of function, instead of using traditional teaching, which only prioritizes an orderly presentation of aspects of the content, without problematizing it historically. They point to dynamic geometry software as an opportunity to revisit the notion of a function as a model that relates the variation between the measurements of two quantities.

Final stitches: contributions, limitations and challenges

In the process of building our corpus, we sought to establish the terms used as search criteria in an intentionally broad manner ("design," "*explor*," "*investig,"* "*problem,*" "*model,* "and"), in order to identify the topics related to problematizing approaches. In our effort to gather research that looked at mathematics teachers' knowledge and at problematizing approaches to teaching, one of our surprises was the silence of studies that proposed these connections in the field of problem solving (in its various theoretical approaches).

Of the four groups that were built, the number of studies (15 works) that problematize the potential of using tasks is striking: nine studies involve exploratory or investigative tasks,

and six have tasks directed to teacher training. Regarding theoretical perspectives, the diverse framework found in the papers of the *Mathematical modeling* group was remarkable. Still regarding this aspect, in the group *Other approaches that use mathematical problems*, for example, a conspicuous characteristic was the explicit non-demarcation of theoretical framework involving problematizing approaches. While this fact may signal a certain dispersion, we found in every paper a concern with training focused on teaching that is meaningful for students. This group surprised us, because in our search with the term "problem", we expected to find studies focusing on problem-solving methodology or on problematized Mathematics[14], and such works did not appear. Thus, we can infer that researchers using these approaches may not give up specific references on knowledge, which does not mean that the use of these approaches does not aim at a training concerned with teaching quality in basic education.

We conclude this task by observing that our contributions in the effort to identify (possible) connections between what we call problematizing approaches and the mobilization and production of mathematics teachers' knowledge point to possibilities for new investigations. Among these is a more detailed study of the practices (and their contexts) developed within the 24 studies comprising our corpus or, due to the limitations imposed by an academic text, a closer look at a particular group. We understand that responses in this direction would collaborate for a more comprehensive understanding of the challenges of implementing problematizing approaches in teacher training processes.

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[14] To define problematized mathematics, Giraldo (2018) discusses the idea of naturalized mathematics versus problematized mathematics. For the author, naturalized (or non-problematized) exposition considers mathematics as a "body of knowledge that has always been and will always be the way it is today, or that evolves linearly from a state viewed as "backwards" to a "more advanced" state, through the isolated inspiration of "geniuses with innate talent" (Giraldo, 2018, p. 41). Problematized mathematics "corresponds to a conception of mathematics based on its multiple social production processes" (Giraldo, 2018, p. 41). Thus, problematized Mathematics (PM) allows for greater questioning, reflections, and investigations by the student.

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