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Plea for the introduction of distorting transformations in high school programs in Senegal: the case of inversion

Llamado a favor de la introducción de transformaciones distorsionantes en los programas de secundaria en Senegal: el caso de la inversión

Apelo à introdução de transformações distorcidas nos programas do ensino médio no Senegal: o caso da inversão

Plaidoyer pour l'introduction de transformations déformantes dans les programmes de lycée au Sénégal : le cas de l'inversion

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Abstract

The Senegalese Mathematics program of 2006 (current program) invites the teacher to give examples of transformations not preserve the barycentre in Terminal S1 and S3. This is why we consider it necessary to provide teacher with a lesson on geometric inversion. In this article, we will on the one hand experiment and analyze activities aimed at introducing geometric inversion in Terminale S1 class. On the other hand, we will conduct a survey of teachers to get an idea of the place occupied by distorting transformations in the teaching of mathematics in Senegal.

Keywords: Geometric inversion, mathematic program.

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Resumen

Los programas de Matemáticas de Senegal de 2006 (programa vigente) invitan al docente a dar ejemplos de transformaciones que no conservan el baricentro en Terminales S1-S3. Es por esto que consideramos necesario brindar a los docentes un curso sobre inversión geométrica. En este artículo, por un lado, experimentaremos y analizaremos actividades destinadas a introducir la inversión geométrica en la clase Terminale S1. Por otro lado, realizaremos una encuesta entre profesores para tener una idea del lugar que ocupan las transformaciones distorsionantes en la enseñanza de las matemáticas en Senegal.

Palabras-clave: Inversión geométrica, programa de matemáticas.

Resumo

Os programas senegaleses de matemática de 2006 (programa em vigor) convidam o professor a dar exemplos de transformações que não preservam o baricentro nos Terminais S1-S3. É por isso que consideramos necessário oferecer aos professores um curso sobre inversão geométrica. Neste artigo iremos, por um lado, experimentar e analisar atividades que visam introduzir a inversão geométrica na classe Terminale S1. Por outro lado, realizaremos uma pesquisa entre professores para ter uma ideia do lugar ocupado pelas transformações distorcidas no ensino da matemática no Senegal.

Palavras-chave: Inversão geométrica, programa de matemática.

Résumé

Les programmes de Mathématiques Sénégalais de 2006 (programme en vigueur) invitent l'enseignant à donner des exemples de transformations ne conservant pas le barycentre en Terminales S1-S3. C'est ainsi que nous jugeons nécessaire de doter les enseignants d'un cours sur l'inversion géométrique. Dans cet article, nous allons d'une part expérimenter et analyser des activités visant à introduire l'inversion géométrique en classe de Terminale S1. D'autres parts, nous mènerons une enquête auprès des enseignants pour avoir une idée sur la place qu'occupent les transformations déformantes dans l'enseignement des mathématiques au Sénégal.

***Mots clés* :** Inversion géométrique, programme de mathématiques.

Plea for the introduction of distorting transformations in high school programs in Senegal: the case of inversion

In our article Diack and al. (2022), we found that geometric transformations have always been a focal point in mathematics education in Senegal. We also showed that in the mathematics programs of Senegal, the habitat of transformations that was algebraic in the reform of modern mathematics has gradually migrated to a geometric habitat. This work also allowed us to see that the niches of transformations began with the solution of geometric problems (conic theory), then they widened in the hierarchy of Klein geometry and finally in the graphical representation of simple functions in analysis.

More specifically, in the 2006 mathematics programs (current program) of the Terminale S1-S3), it is asked to give examples of applications of transformations that do not preserve the barycenter. Despite everything, the observation is that there are no examples to illustrate this request. And yet inversion is a perfect example of a transformation that does not preserve the barycenter. Thus, would it not be appropriate to study the place of distorting transformations in the Senegalese mathematics curriculum?

Subsequently, after having identified the problem and made a brief historical and epistemological overview of the inversions, we will briefly analyze the Senegalese programs from 1960 up to now. In addition, we will conduct a survey of teachers who once held a class of Terminale S1 in order to study their conceptions on this subject to know what geometric inversion is then, we will analyze a sequence of courses on geometric inversion in Terminal S1 class.

Problematic and Etymology

1. Problematic

In the etymological sense, if we refer to the LAROUSSE dictionary, a transformation is defined either by a passage from one form to another or by a modification, or even a change. Better yet, according to Wikipedia, transformation can be defined as a general rule as follows: "the action or process through which a thing is modified, transformed or changed in form while retaining its identity"

However, in the Senegalese mathematics programs, from the sixth to the first, all the transformations studied retain the shapes, that is to say, they retain the geometric angles, alignment, parallelism, orthogonality, barycenter, contact, etc. Thus, throughout his or her schooling, the learner might think that there are no distorting transformations. In other words, that transform a figure into a non-similar figure.

To give students a better understanding of the transformations studied and the meaning of their properties, it is desirable that students encounter and study various deforming or non-deforming transformations. This is why it is important to propose from the terminal a punctual transformation that does not retain the barycenter as the geometric inversion In geometry, a plane inversion of pole O (O is a point of the plane) and power (is a non-zero real number) is a transformation such as: $f: (P)\backslash\{O\}? (P)\backslash\{O\}$, which has any point associates the point such that :

$$\begin{cases} O, M \text{ et } M' \text{alignés} \\ \overline{OM} \times \overline{OM'} = k \end{cases}$$

It should be noted that in the mathematics programs in force in Senegal, a rather summary passage is dedicated to deforming transformations. Specifically, in the S1-S3 Terminale program, in the study of affine applications, it is suggested in comments to "give examples of applications that do not retain the barycenter"

However, the discussions conducted in the different pedagogical cells lead us to think that in practice, this comment is misunderstood by most teachers. However, these non-affine applications may have properties that the student is not used to encountering; in particular, they may distort figures. This is the case of inversion.

In addition, one can wisely choose the inversion to simplify a geometry problem. For example, any two circles can be transformed into two concentric circles.

Some authors have been interested in geometric inversion. Among them Bkouche (1991) who emphasizes its importance in conformal geometry considered as the study of all transformations preserving angles.:

> Among the deforming transformations, mention should also be made of the inversion which makes the straight line appear as a particular circle, and which plays an important role in conformal geometry. (ibid. p. 146)

There is also KUNTZ (1998) who considers inversion as a forgotten transformation of French programs explains that it can be introduced from the second with the computer tool to continue in the first with a more theoretical study and become an excellent subject of work directed in Terminale with complex numbers.

According to Walter (2001), from a didactic point of view, inversions have their limits, so far as their use as a problem-solving and demonstration tool seems difficult to master, or even rejected by students. Indeed, the mental process that makes it possible, from the reading of a statement or a figure, to reinvest the adequate knowledge with a view to solving a problem or developing a demonstration constitutes a real difficulty for students. These difficulties for which the translation of a perceptual understanding of geometric properties into theoretical knowledge is a very strong cognitive challenge.

> It is not enough for a property to be demonstrated to become mobilizable, it is also and above all necessary for it to be constructed a mental image, a figurative representation of this property, which may or may not be part of an explicit personal lexicon, in the same way as an enunciated and justified theorem. (Daniel 1995, p.76)

Taking into account what is said above, would it not be appropriate to integrate the distorting transformations into the mathematics curricula in Senegal and if so, how could teachers support them in a sequence of courses? What could be the contribution of inversion in the learning of geometry?

We believe on the one hand that it is indeed possible to insert a theme on geometric inversion in the mathematics program of Terminals S1-S3, more precisely in the flat GEOMETRIE PLANE just after the theme on barycenter and affine application. On the other hand, inversion is an important tool to facilitate evidence in planar geometry since transforming straight lines into circles and vice versa and preserving angles. For example, by choosing a relevant inversion circle, it is possible to transform one geometric configuration into a simpler one in which a proof is easier.

We believe that little importance has been given to transformations that do not retain the barycenter in Senegal's mathematics curriculum. And this leads teachers to neglect these transformations in their teaching-learning sessions. This neglect causes an ignorance of distorting transformations by learners.

2. Methodology

To answer the questions raised in the issue, we will firstly conduct a questionnaire survey among teachers who held a terminal S1 class. This survey will allow us to know what prevented the deforming transformations in particular the geometric inversion from living in the Senegalese mathematics programs, but also to study teachers' conceptions of these transformations and how they are supported in teaching-learning sessions.

With regard to the contribution of inversion, we conducted a sequence of courses in a terminal S1 class. During these different sessions, we offered activities to students during which we introduced them to deforming transformations, in particular geometric inversion.

Brief historical overview

In the geometry of transformations, the study of inversion dates back to the first half of the 19th century according to Chasles (1870).

According to some authors, Jacob Steiner (1796-1863) was the first to use the notion of inversion in 1826. But at that time, the notion of transformation had not appeared yet. That is why he talked more about homographic figures. For Chasles, the discovery of inversion is attributed to the Italian geometer Giusto Bellavitis (1803-1880) who, in a thesis in 1845, illustrated inverse figures. Chasles also approved of the use of transformations by the English physicist William Thomson: *«If the point A belongs to a sphere, the point A' is on a second sphere, which Mr. Thomson calls the image of the first."*

Nevertheless, the first complete theory of inversion is dedicated to Joseph Liouville when he established that the group of transformations conserving the angles of a space of dimension greater than or equal to is generated by similarities and inversions. This theory evolved over time until it led to the invention of Peaucellier's mechanical device.

Geometric inversion and its reason of existence.

Geometric inversion is an advanced mathematical concept that is of great importance in STEM[4]. In a geometric inversion, the distance between a point and its image is inversely proportional to the square of the distance between the point and the center of the inversion. It retains many important geometric properties.

For example, by choosing a relevant inversion circle, it is possible to transform one geometric configuration into a simpler one in which a proof is easier. This makes it a powerful tool in flat geometry. Moreover, it is the inversion that is at the base of several mechanical devices including that of Peaucellier which makes it possible to convert a circular movement. This is why deforming transformations should occupy an important place in the teaching of Mathematics.

Place of inversion in Senegalese programs

Until 1971, Senegal did not yet have a specific mathematics curriculum. Thus, because Senegal was a former French colony, then the French curriculum was the only foundation for mathematics education.

[4] Science, Technology, Engineering and Mathematic

In the 1972 reform, no place was reserved for geometric transformations for the sixth and fifth grades. But from the fourth, a large part was reserved for transformations and all its forms. But all studies of transformations resulted in transformation groups (rotation groups, isometry groups, similarity groups, which gave more importance to the algebraic point of view.

On the other hand, in the 1986 reform homographic functions were used and implicitly inversions in descriptive geometry (in the fifth form C and in the sixth C). Indeed, an inversion is the composition of the application "orthogonal symmetry" $(z ? \bar{z})$ and the homography.

It is finally in the period of the counter-reform of modern mathematics with the dominant paradigm of the geometric framework, and only in the S1-S3 Terminals classes that inversion appears implicitly as a non-affine application, i.e. not retaining the barycenter. This implicit aspect could explain the lack of interest. This implicit aspect could explain the little interest given to this object of knowledge by teachers.

Given the importance of distorting transformations in the teaching of geometry, we believe it is appropriate to integrate them into mathematics curricula in Senegal. It is with this in mind that we carried out a course on geometric inversion with students from Terminal S1 in Lamine Gueye High School in Dakar.

Analysis of a sequence of courses on inversion

In Senegal, according to the baccalaureate office, only 494 students are enrolled in the S1 BACCALAUREATE for the 2021-2022 school year, or 0.33% of the total number of students enrolled in the BACCALAUREATE. That is why our experiment was carried out with few students, because the number of students in each class of terminal S1 rarely exceeds, on average, 5 students.

The sequence of courses on inversion took place in the Terminal S1 class of eight students from Lamine GUEYE High School in Dakar. It consists of three sessions of two hours each. The first session consists of discovering the notion of inversion and giving the first properties. For the second session, we announce some immediate properties that will flow directly from the definition.

In practice, the sessions took place on Thursday 22nd and Monday 26th June 2023 in Lamine GUEYE High School in Dakar with the students of the Terminal S1 class. The first session was done on Thursday, June 22nd from 10am to 12 (2h of time) and the other two sessions on Monday, June 26th, 2023. There are eight students in the class: six boys and two girls

1. Session One :

The first session involves getting students to determine the characteristics of an inversion (pole and power) just based on figures transformed by inversion. Note that the student of Terminale S1 or S3 has already seen in the previous classes certain geometric transformations (isometries and homothety). So we proposed the following activity

Activity

The graph opposite (see figure 01) consists of a circle (c) with center O and radius 2 and two figures and in the figure, the points A', B', C' and D' are respectively the transforms of the points A, B, C and D of a certain transformation.

Among the transformations that you know, is there one that transforms the figure into? justify

What is the relative position of the lines; and?

Calculate, what can you conjecture?

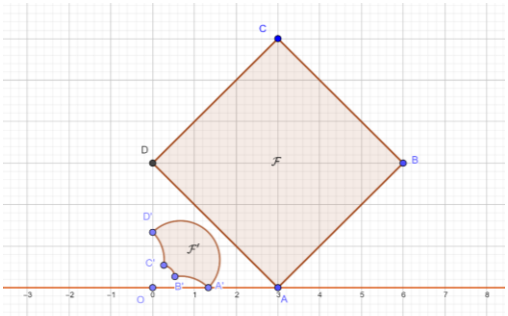

Figure 1.

First session activities

A priori analysis

This activity is about discovering inversion through its characteristics: pole and` power. The workspace is a sheet of paper and the material used is the pencil and possibly an eraser.

First, for the first question, the expected answer is that such a transformation is not known because students only know the similarities. We can also expect some of them to think of homothetic because of the reduction of the image figure. However, the difference in shape of the two figures would make this response infrequent.

Then, with regard to the second question of the activity, it could be that some students answer that the straight lines (AA') and (DD') are perpendicular without alluding to the expected answer that is to say that the four straight lines are concurrent in O. Some students could confuse the segments with the straight lines on the drawing; which would not allow them to see the straight lines are concurrent in O. This point O is a characteristic of the studied transformation: the pole.

Finally, for the third question, we think that students will show approximately equal values, which leads us to the discovery of another characteristic of inversion which is power. To answer the third question of the statement, students must rely on Figure 01 where we have drawn and in a direct orthonormal reference, thus facilitating the reading of the coordinates of the points in question. Students should be expected to use the ruler, to measure distances or, since the figure is gridded, apply the Pythagorean theorem to calculate distances.

2. A posteriori analysis

After implementing the first activity, we collected responses from students who presented themselves as follows:

For the first question, all students answered no, Six students did not justify their answers and as an illustration here are the justifications of the other two remaining.

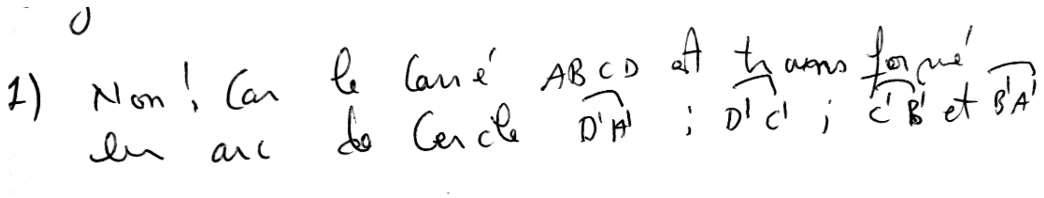
1) Non ! Car le Carré ABCD est transformé en arc de Cercle D'A' ; D'C' ; C'B' et B'A'

Figure 2.

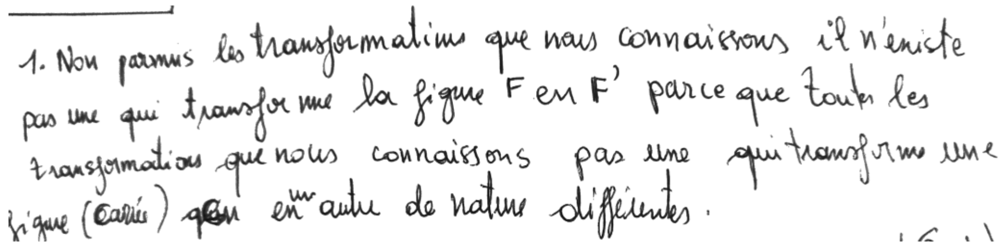
1. Non parmis les transformations que nous connaissons il n'existe pas une qui transforme la figure F en F' parce que toutes les transformations que nous connaissons pas une qui transforme une figure (Carrée) en un autre de nature différentes.

Figure 3.

Answers from both students for the first question

With regard to the second question, all the students affirm that the lines (AA'), (BB'), (CC') *and* (DD') are concurrent in O. But we noticed that they have a problem illustrating this. Indeed, one of the students explains it in the following way

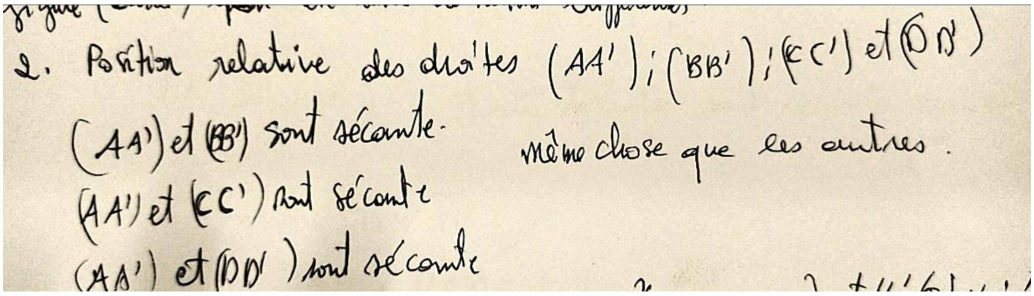

Figure 4.

Answers from both students for the first question

We also noticed that seven of the students say that the lines (AA') and (DD') are perpendicular. Here's how one of them justifies it.

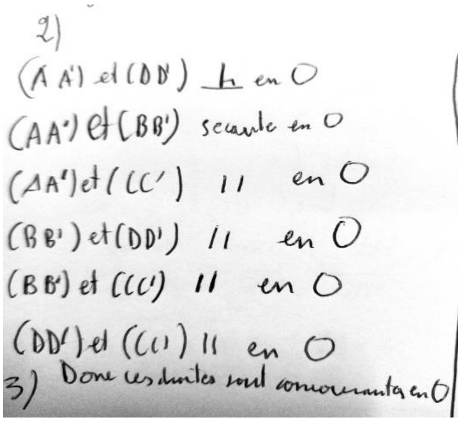

Figure 5.

Student response to second question

We believe that this is due to the use of an orthonormal coordinate system. Indeed, since the axes of the coordinate system are perpendicular, then the (AA') and (DD') are perpendicular.

Regarding the third question, six students noticed that the value of the real $\overline{OM} \times \overline{OM'}$ where is the counterpart of is approximately equal to 4.On the other hand, the other two found different values. This is because instead of using the marker graduations, they used a graduated ruler to measure the actual distances in accordance with our predictions in the analysis.

In relation to the conjecture, the following student's deduction caught our attention.

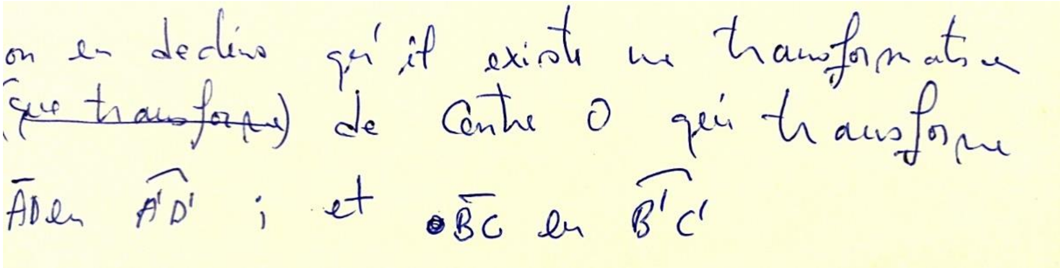

Figure 6.

Student response to the third question

Although he has put algebraic measures in place of segments, we can still see that the student has noticed that the segments are transformed into arcs of a circle and this highlights the notion of deforming transformation.

After this first session, we notice that the students did not find it difficult to locate the pole (point of competition of the straight lines (AA') ; (BB') ; $(CC')$and (DD')) and the power of the inversion(Question 04) through a diagram illustrating a figure and its image by a geometric inversion. Nevertheless, they also noticed that among the transformations seen in the previous classes, there is none that allows them to make such a figure. This allowed us to introduce the definition of geometric inversion.

After having institutionalized the notion of geometric inversion, we represented with the Geogebra software, in the form of practical work, some figures (circles, straight lines, squares, etc.) and observed the images with respect to a positive power inversion.

3. Session two:

This session presents two moments:

- The first moment is to discover, through Geogebra, how inversion acts on straight lines and circles, then to state conjectures related to circles and straight lines
- The second moment is to state conjectures related to circles and lines.

This is how we proposed the following activity:

Activity Statement

Activity: 1. Draw a circle with center O and radius 1. 2. Draw a straight line not passing through What is its inverse with respect to the circle? 3. What would happen if the right goes through? What conjecture can you make? 4. Draw a circle that does not pass through. What is its inverse with respect to the circle? 5. What would happen if the circle goes through? What conjecture can you make? 6. What conclusion can be drawn ?

Objectives:

On the one hand, discover the nature of the image of simple figures (straight lines and circles) by a positive power reversal. On the other hand, to determine the position of the inverse of a point in the plane, at the moment when the latter approaches or moves away from the pole of the inversion.

Hardware: Smartphone or computer where Geogebra software is installed.

We expect students to represent the figures without major worries and find that the image of a straight line (not passing through the pole) through a geometric inversion is a circle and that the image of a circle (not passing through the pole) through a geometric inversion is a straight line.

Pour cette activité, les élèves peuvent avoir un problème avec le logiciel Geogebra car peut être pour certain, c'est la première fois qu'ils l'utilisent. Les élèves pourront aussi rencontrer un problème dans l'installation du logiciel de même que son exécution.

A posteriori analysis

Note first that the first question consisted of a geometric figure construction. Then, after completing the activity, we realize that the students did not find any major difficulties in answering questions 2 and 3. Then, for the fourth question, although they observed that the image of a point moves further and further away from the pole of the inversion as its counterpart gets closer, five of them have a problem expressing it. For example, a student tells us: "...when he moves away from the pole, his opposite becomes smaller and smaller...". Another states: "...if the circle goes through, then its inverse.

In this session, students enjoyed observing inverse geometric figures using the Geogebra software. From there, they manipulated inverse figures and were able to conjecture about some geometric properties (the image of a straight line or a circle by an inversion). They were also able to conjecture the extension of the definition of inversion over the entire plane by bringing the image as close as possible to a point on the inversion pole. This is what will make the inversion a transformation of the plan.

Thus, the following properties have been stated:

Property 01

The inverse of a straight line (D) not passing through the pole O is a circle with a center O passing through the pole and vice versa. The straight line (OO) is perpendicular to (D).

Property 02

The inverse of a circle (C) not passing through the pole O is a homothetic circle.

Property 03

The inverse of a circle (C) passing through the pole O is a straight line (D).

Property 04

The inverse of a straight line (D) passing through the pole O is the straight line (D) itself.

At the end of this last activity, we realize that students are beginning to appropriate the notion of geometric inversion. Thanks to the Geogebra software, they were able to observe the image of a straight line or a circle (passing through the pole or not) in relation to a positive power reversal.

In our next work, we intend to rigorously demonstrate these properties and then propose situations using them.

Questions during the course

At the end of the first part of the second session, students saw with the help of the Geogebra software how to determine the image of a point, a straight line or a circle by a positive power reversal. From there, there was a discussion between the students and the teacher.

Teacher: What about the image of a point, if it approaches the pole?
Student 01: The image moves away from the pole.
Student 02: but arrived at the inversion circle the point and its image are confused.
Student 03: The image may come off the screen.
Teacher: What about now, if the point moves away from the pole?
All students: its image is closer to the inversion pole.
Teacher: For illustration, here is an extract
All students: its image is closer to the inversion pole.
Teacher: So how do you think we could define the image of the pole?
All students: total silence for a few moments.
Student 01: It cannot be defined.
Student 02: maybe the infinite limit, laughing...

In this rich discussion, we were able to let the students gradually build an extension in the plane of the definition of an inversion. Thus, the inversion can now be considered as a transformation of the plane.

Survey of teachers

In the problematic of our article, we were interested in the following questions:

Would it not be appropriate to introduce distorting transformations into Senegal's mathematics curricula?

If so,

- What are the teachers' conceptions of the notion of distorting transformations and how could they take it into account in a sequence of lessons?

- What could be the contribution of inversion in the teaching-learning of geometry?

In order to provide answers to these questions, we conducted questionnaire surveys with a few teachers (about twenty) who had held a science final year class at least once. The answers to each question asked in this questionnaire is summarized either by a diagram (see index) or by a sentence.

Thus, the analysis of the questionnaire resulted in the following information:

- In terms of seniority, 23,5% the teachers concerned by the survey have spent less than 5 years teaching mathematics. Similarly, 23,5% they taught between 5 and 10 and finally, 52,9% did more than 10 years. Here we think that the higher the seniority, the more likely it is that the teacher will encounter situations that the teacher encounters situations that deals with the notion of inversion.

- As far as teaching in the S1 or S3, 76,5% series is concerned, a class of the S1 or S3 series has already been 23,5% held against those who have never held one.

- For the use of geometric inversion in mathematics teaching, 60 % teachers interviewed say they have used inversion in their teaching-learning sessions and 40% tell them that they do not make use of geometric inversion. They use these transformations more in geometry (85,7%) and a little in algebra, that is 21,4%.. We think that there can be several situations where the teacher may encounter exercises involving inversions, as in the case of complex numbers.

- The usefulness of geometric inversion in the teaching of mathematics is confirmed by the 59% teachers interviewed. On the other hand, 5,9% only think that it is useless and 35,1% have no ideas about the usefulness of geometric inversion in the teaching of mathematics.

In illustration, here is the answer of some teachers who think that inversion is useful:

a. Geometric inversion is useful in simplifying exercises by transforming circles into straight lines and vice versa;

b. For example, enlarging or decreasing geometric figures ;

c. It allows students to better understand conservative transformations

d. Show that some transformations do not preserve the shape of geometric objects.

On the other hand, for those who think that geometric inversion is not useful in the teaching of mathematics, justify it by the fact that it would make the mathematics curriculum heavier.

- Finally, in the activity that we proposed, 70,6% the teachers interviewed think that it would be adequate in the Terminale S1 class and 5,9% proposed the activity for the

Première S1 in the Transformation part. Regarding the difficulties that students may encounter in answering the questions of the activity, one of the teachers says:

> In the Terminale S1 class, the difficulties can be varied: -Does the transformation in question admit an invariant point? - The transformation does not maintain distances. Does transformation have a relationship? -if the student has once encountered the inversions in exercises in the complex plane, think about the complex writing of the transformation. - Does the transformation keep the angles? Isn't the right one wrapped around the circle ? Whereas we have a transformation.

Conclusion

At the end of the activities, we note that the students took time to understand the notion of inversion. As one of them put it, “this is the first time I've seen a transformation that can turn a straight line into a circle.” But as the course unfolded, they were able to gradually adopt the notion of inversion. When we asked a few questions about geometric inversion to some teachers who held at least once a class of Terminale S1, we were very surprised to find that they did not know the latter as well.

That's why the idea of equipping teachers with a course material on geometric inversion is well justified. In perspective, we think that in perspective, it would be appropriate to give some applications on the use of geometric inversion in the solution of mathematical problems, but this could extend to the point of being the subject of an article on the use of inversion in automobile mechanics, in the operation of engines by transforming a circular movement into a rectilinear movement (or the opposite).

After analyzing the teachers' responses, we realize that the majority of teachers who teach in science series use inversion in the teaching of mathematics, particularly in geometry. Despite everything, the mathematics program of Senegal has reserved only a simple comment in the Terminale S1-S3 part for geometric inversions.

This is in a way what confirms the importance of the reintroduction of geometric inversion in the teaching of mathematics in Senegal. Since60 % around the teachers concerned by the survey confirm the usefulness of geometric inversion in the teaching of mathematics, this supports our idea of inserting a lesson on geometric inversion in the mathematics program of the Terminale S1-S3, more precisely in the PLANE GEOMETRY part just after the barycenter and affine application lesson.

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