

The question-answer map as an alternative model for writing a thesis

El mapa de preguntas y respuestas como modelo alternativo para la redacción de una tesis doctoral

La carte des questions et réponses comme modèle alternatif pour la rédaction d'une thèse de doctorat

O mapa de questões e respostas como um modelo alternativo de redação de uma tese de doutorado

Kleber Ramos Gonçalves,¹
Universidade Federal de Mato Grosso do Sul
Doutor em Educação Matemática
<https://orcid.org/0000-0002-6242-180X>

Marilena Bittar,²
Universidade Federal de Mato Grosso do Sul
Doutora em Didática da Matemática
<https://orcid.org/0000-0001-9989-7871>

Abstract

In this paper we present reflections on challenges faced in the use of an alternative writing model in our doctoral thesis, showing its relevance as a coherent and insubordinate model. The model mobilized was the questions-answers map, developed within the framework of the anthropological theory of the didactic. Due to the limits of the exhibition, we present some clippings of our investigation, whose objective was to answer the following generative question: What teaching proposal for Z is possible to build through the interaction with a study group of teachers in the perspective of the paradigm of questioning of the world (PQW)? The heart answer was divided in two, one referring to the study group constituted during the research and the other referring to the other studies of the thesis. We built a reference epistemological model (REM) that allowed us to analyze the dominant model of the relative integers, which, in turn, was complemented with analyses of previous research data. We mobilized aspects of the PQW, more specifically, conditions and constraints, identified from the conclusions related to continuing education with the group of teachers and from the studies on the study and research path, analyzed through the dialectics question/answers, media/*milieu*, and collective/individual.

¹ kleberemic@gmail.com

² marilenabittar@gmail.com

These analyses also helped us to finalize the reference epistemological model and the teaching proposal initiated together with the members of the study group.

Keywords: Questions-answer maps, Alternative writing template, Paradigm of questioning the world, Reference epistemological model, Relative integers.

Resumen

En este artículo reflexionamos sobre los retos a los que nos enfrentamos al utilizar un modelo alternativo de escritura en nuestra tesis doctoral, mostrando su relevancia como modelo coherente e insubordinado. El modelo movilizado fue el mapa de preguntas y respuestas, desarrollado en el marco de la teoría antropológica de la didáctica. Dados los límites de este artículo, presentaremos algunos extractos de nuestra investigación, cuyo objetivo era responder a la siguiente pregunta orientadora: ¿Qué propuesta didáctica para Z puede construirse a través de la interacción con un grupo de estudio de profesores desde la perspectiva del paradigma del cuestionamiento del mundo (PCM)? La respuesta se dividió en dos: una referida al grupo de estudio conformado durante la investigación y otra referida a los demás estudios de la tesis.

Construimos un modelo epistemológico de referencia (MER) que nos permitió analizar el modelo dominante de enteros relativos que, a su vez, se complementó con análisis de datos de investigaciones anteriores. Movilizamos aspectos del PCM, más específicamente, condiciones y restricciones, identificadas a partir de conclusiones relativas a la formación continuada con el grupo de profesores y estudios sobre el recorrido de estudio e investigación, analizados utilizando las dialécticas pregunta/respuesta, medios/*milieu* y colectivo/individuo. Estos análisis también nos ayudaron a finalizar el modelo epistemológico de referencia y la propuesta de enseñanza que se inició con los miembros del grupo de estudio.

Palabras clave: Mapa de preguntas y respuestas, Modelo de escritura alternativa, Paradigma del cuestionamiento del mundo, Modelo epistemológico de referencia, Enteros relativos.

Résumé

Cet article présente les défis auxquels nous avons été confrontés lors de l'utilisation d'un modèle alternatif de rédaction de notre thèse de doctorat, en montrant sa pertinence en tant que modèle cohérent et insubordonné. Le modèle mobilisé est la carte des questions et réponses, développée dans le cadre de la Théorie Anthropologique du Didactique. Compte tenu des limites de cet article, nous présenterons quelques extraits de notre enquête, dont l'objectif était de répondre à la question directrice suivante : Quelle proposition d'enseignement pour Z peut être construite

à travers l'interaction avec un groupe d'étude d'enseignants dans la perspective du Paradigme du Questionnement du Monde (PQM) ? La réponse a été divisée en deux : l'une se référant au groupe d'étude mis en place lors de la recherche, l'autre se référant aux autres études de la thèse. Nous avons construit un modèle épistémologique de référence (MER) qui nous a permis d'analyser le modèle dominant des nombres entiers relatifs qui, à son tour, a été complété par des analyses de données provenant de recherches antérieures. Nous avons mobilisé des aspects du PQM, plus précisément des conditions et des contraintes, identifiées sur la base des conclusions relatives à la formation continue avec le groupe d'enseignants et des études sur le Parcours d'étude et de recherche, analysées à l'aide des dialectiques Question/Réponse, Médias/Milieus et Collectif/Individu. Ces analyses ont également permis de compléter le Modèle de référence épistémologique et la proposition pédagogique initiée avec les participants du groupe d'étude.

Mots-clés : Carte des questions et réponses, Modèle d'écriture alternatif, Paradigme du questionnement du monde, Modèle épistémologique de référence, Nombres entiers relatifs.

Resumo

Neste artigo apresentamos reflexões sobre desafios enfrentados na utilização de um modelo de redação alternativo em nossa tese de doutorado, mostrando sua pertinência como modelo coerente e insubordinado. O modelo mobilizado foi o mapa de questões e respostas, desenvolvido no âmbito da teoria antropológica do didático. Pelos limites da exposição, apresentamos alguns recortes de nossa investigação, cujo objetivo foi responder a seguinte questão geratriz: Que proposta de ensino para Z é possível construir por meio da interação com um grupo de estudos de professores na perspectiva do Paradigma Questionamento do Mundo (PQM)? A resposta coração foi dividida em duas: uma se refere ao grupo de estudos constituído durante a pesquisa, e outra aos demais estudos da tese. Construímos um modelo epistemológico de referência (MER) que nos permitiu analisar o modelo dominante dos inteiros relativos que, por sua vez, foi complementado com análises dos dados de pesquisa anterior. Mobilizamos aspectos do PQM, mais especificamente, condições e restrições, identificadas a partir de conclusões relativas à formação continuada com o grupo de professores e dos estudos sobre o percurso de estudo e pesquisa, analisados por meio das dialéticas questão/resposta, mídia/meio e coletivo/individual. Essas análises também nos auxiliaram na finalização do modelo epistemológico de referência e na proposta de ensino iniciada juntamente com os integrantes do grupo de estudos.

Palavras-chave: Mapa de questões e respostas, Modelo de redação alternativo, Paradigma questionamento do mundo, Modelo epistemológico de referência, Inteiros relativos.

The question-answer map as an alternative model for writing a doctoral thesis

Why mobilize an alternative writing model? The entry, *alternative*, is defined as “representing an option outside conventional institutions, customs, values, and ideas.” (Alternativo, 2009). Thus, alternative writing of a doctoral thesis report may represent a way of questioning the established parameters, an act of courage, as it goes against many aspects of the traditional writing model. Generally, alternative essays aim to present political positions: they are written to position social struggle acts for recognition, acceptance, and legality of rights usurped throughout history. In our research, we initially chose an alternative model because we wanted to adopt writing aligned with the main theoretical framework mobilized in the investigation: the *anthropological theory of didactics* (Chevallard, 1999). Such writing should reflect the entire research process, portraying the *study and research path* (SRP), in light of the *paradigm of questioning the world* (PQW) (Chevallard, 2009b).

Study and research path³: It is carried out to answer a generative question, aiming to construct a heart answer, R^{\heartsuit} . This construction is done through new studies, searching in existing literature for ready, known answers, R^{\diamond} . Starting from them, other answers, and partial answers, are constructed because they do not fully answer the initial question and must be validated by revealing how important they are to the study. Therefore, the combination of the investigation of ready and partial responses, R^{\diamond} , with the various works W , tools used to study and construct the answer to the generative question, produces the movement called by Chevallard (2009a) of study and research path (SRP) (Gonçalves, 2022).

The paradigm of questioning the world and the paradigm of visiting works: The *paradigm of questioning the world* (PQW) proposes a critical view of the world, questioning existing crystallized models. Thus, the primary difference between these didactic systems is their objectives: while one is concerned with answering and creating new questions, the other focuses on teaching praxeology, on content studies, which is a conclusive difference between the paradigms.

Based on the reflections brought by Bosch (2018a)⁴, we understand that the development of a doctoral thesis could be considered an example of a *study and research path* (SRP). Given the above, we ask: How can we write a thesis report consistent with our choices, reflecting the process experienced and, in addition to showing our research question, highlighting the questions and answers derived from it? The solution we found was writing in the form of a *question-answer map* (QA map) (Bosch & Winsløw, 2016).

A question-answer map is a diagram that presents and highlights the relationships between questions and their answers arising in the *study and research path*.

In addition to building the QA map, we created a *reference epistemological model* (REM) to support the analysis of the *dominant model* (DM) of relative integers. The results of the analyses showed a teaching based on concrete models, whose *technological-theoretical*

³ To make the text easier to read, we chose not to present a “theoretical foundation;” however, when necessary, the concepts will be highlighted in smaller, bold font.

⁴ Conference given to PPGEducMat/UFMS academics in 2018.

block consists of didactic creations produced by textbook authors and teachers. Some teachers in the study group, participants in this research, defended this scenario.

Reference epistemological model (REM): In formulating any teaching problem, the teacher always uses, even if only implicitly, a description and an interpretation, i.e., an epistemological model of the mathematical domain at stake. From the beginning, the *anthropological theory of didactics (ATD)* has emphasized the need to make this model explicit and use it as a *reference* for the analysis of didactic-mathematical facts (Gascón, 1993, 1994, 1994, 1998, 1999a, 2001a). [...] there is always a *provisional character* (Gascón, 2011, p. 208, our translation, emphasis added).

The **dominant epistemological model** is a description and interpretation of the predominant didactic and mathematical facts of a mathematical object in a given educational institution (Chevallard, 1999; Gascón, 2011).

Concrete Model: In this model, for relative integers, we find situations and activities taken from everyday life based on games or concrete materials, such as decks of cards, goal differences, temperatures, and monetary systems. They are also based on students' prior knowledge, which allows them to organize introductory activities and favors understanding the various dimensions of this content.

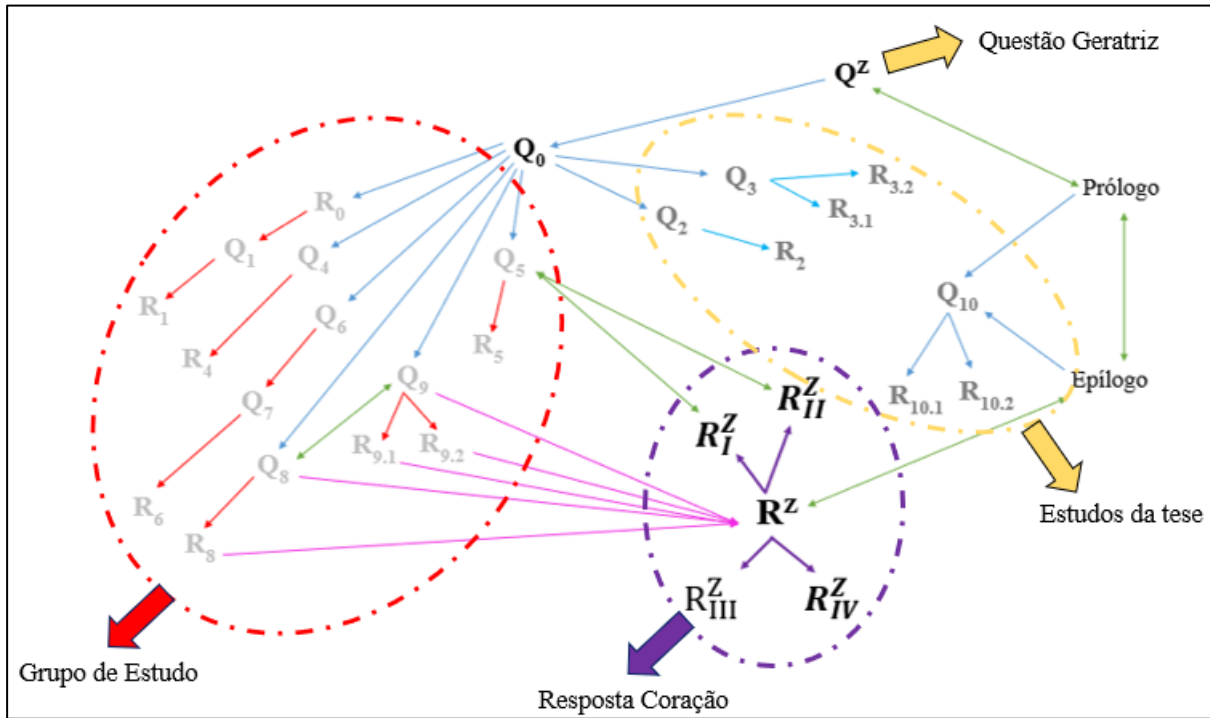
We carried out a continuing education activity inspired by the *paradigm of questioning the world* and organized as a study group. The members were teachers teaching mathematics laboratories⁵ in Campo Grande/MS municipal schools. Thus, the *question-answer map* created consisted of two branches, one with questions and answers relating to the process experienced by the study group and the other composed of questions and answers relating to the process experienced by the researchers throughout the research development. The answers created in the group's branch were decisive for the construction of the activities of the alternative proposal⁶.

⁵ The teachers' role was to conduct all activities in these laboratories, always with joint actions with the teachers at their school. They developed projects, games, teaching materials, and more playful activities, the aim of which was for students to be active during the learning processes.

⁶ Essentially supported by teaching given by the input of relative integers via school algebra studies, combined with concrete models in the properties that justify the techniques for these numbers, using different contexts beyond the idea of measurement.

Figure 1.

Survey Questions and Answers Map (Adapted from Gonçalves, 2022, p.149).



In the following sections, we present elements that justify and allow discussions and reflections on the choices regarding the writing of the research report. These elements are linked to theoretical-methodological choices.

Motivations for studying writing in alternative/insubordinate formats

The research writing scenario presented many questions. The main one was related to the alternative/insubordinate choice that highlighted the different aspects of the work carried out and was coherent with the work developed⁷. We searched in D'Ambrósio (2015) for something that would help us, and the answer found was the concept of creative insubordination. According to this author, courage and confidence are important qualities when considering the risks an innovation can generate. Thus, the results give rise to new and interesting possibilities. Another path that helped us a lot was partnerships: advisee and advisor, the Didactics of Mathematics Study Group (Grupo de Estudos em Didática da Matemática -

⁷ The paradigm of questioning the world, as proposed by Chevallard, can be seen as insubordination to the paradigm of visiting the works.

DDMat⁸), and other researchers⁹. We understand that our choice to write the research report was political since we took a position that defines our education proposal, which DDMat also defends.

D'Ambrósio's studies (2015) warn about teacher education. They state that teachers are usually subjected to some rules that they must uphold, defending laws and policies often without due critical analysis. In this sense, the effects and chains caused by these *conditions* and *restrictions* are disregarded and, sometimes, not even noticed, especially those that affect students' learning. Teachers and students feel this situation.

Our teachers have no voice or agency and follow the rules uncritically without considering whether they are good or conducive to children's good. We prepare educational managers who advocate for laws and institutional rules instead of advocating for children and teachers participating in their education systems. Teachers are expected to blindly follow the rules without worrying about the results of their actions in the education of children, much less with the concern of understanding how they are contributing to the formation of a generation of young people prepared and inspired to seek a new social order [...]. (D'Ambrósio, 2015, p. 3).

***Restrictions*, according to Chevallard (2002), are *conditions* that cannot be changed. The same *restrictions* can and should be modified over time or in other *institutions*, as some restrictions may become just a *condition* (which can be modified), and a *condition* can become a *restriction*.**

Thus, changes always serve the interests of specific groups, reinforcing what D'Ambrósio (2015) states about teachers and managers being trapped and, consequently, students reproducing and following rules without due interpretations. By meeting the interests of some, the possibility and right of students to reach their full human potential are curtailed, as several dimensions of society, science, and their knowledge will not be included in these documents. An example of this is the construction of curricula: we realize that even though they are constantly changed, we reproduce teaching from a few centuries ago. This construction is also a political act, reproducing the ideals of those who hold power at that moment.

On the foundations of the *paradigm of questioning the world*, we find an investigative spirit of questioning the surrounding reality, which leads to an approach and articulation with the ideas of creative insubordination. In such an approach and articulation, the key point is questioning, as this spirit is part of the essence of the human being, which schools have often hindered.

The movement to build the PQW, linked to the research movement, can also be

⁸ Group in which we participate, led by Marilena Bittar and José Luiz Magalhães de Freitas. <http://grupoddm.pro.br/index.php/home/>

⁹ As an example, we cite Professor Marianna Bosch (2018a) and her conference. The debate was fundamental for the choice and development of the research.

interpreted as a political act, as it was initially more focused on the precepts of teaching processes. Throughout the development of the research, it took on the character of an insubordinate process, firstly, regarding the forms of continuing education pre-determined by the Municipal Department of Education [Secretaria Municipal de Educação - Semed] of Campo Grande/MS, secondly, regarding the construction of an alternative proposal to the one in the *dominant model*. However, we could not represent this fact in our *question-answer map*; instead, we contemplate it in the answer at the heart of the thesis (**R♥**).

Thus, we dealt with two forms of insubordination: one focused on the teaching and learning processes during the preparation of the teaching proposal, and the other referred to the choices of the writing format when we decided to mobilize the PQW as an alternative/insubordinate writing.

Thinking about teaching and learning processes, insubordination acts occur mainly when teachers prioritize their studies to the detriment of established standards. For example, “create alternative arguments to explain differences in student achievement [...]; question how mathematics is presented at school; emphasize the humanity and uncertainty of the mathematics subject; position students as mathematics authors [...]” (D’Ambrósio & Lopes, 2015, pp. 3-4).

Thinking about writing formats, Barbosa (2015, p. 350) warns that an insubordinate format for writing research reports is one that breaks with the “traditional representation of educational research in these types of academic work. I assume that authors who adhere to these formats have fundamental reasons [...]”. In our case, this founding reason was the coherent and natural way of expressing the research as *question-answer map*, keeping in mind our main theoretical framework.

Given this scenario, to write this article, we constructed some justifications based on the ideas of insubordination, in addition to those already presented in the doctoral thesis (Gonçalves, 2022).

What question-answer map could we construct?

Researching is an action that requires much questioning about the context of the investigation. Hence, surveying other research works whose results are close to the theme investigated is a recurring action. Such surveys, followed by studies and analyses, also allow the elaboration of new questions and, consequently, the construction of new answers. These, in turn, are provisional, helping to construct the final answer, the heart answer. The elements of the PQW, whose focus is questioning reality, are indicated by the *Herbartian* system and analyzed through *dialectics*.

O *Herbartian scheme* was proposed as a tool for “analysis of study and research processes. By describing mathematical themes or domains as a tree of questions and answers [or *question-answer map*], it offers an alternative to the common view of mathematical content as a hierarchical structure of concepts and results” (Bosch & Winsløw, 2016, p. 33, our translation).

Dialectics: The dynamics of a Herbartian scheme can be “captured in terms of some dialectics that describe the production, validation, and dissemination of the heart response” (Bosch, 2018b, p. 4040).

Milieu: similar definition to the one given to the didactic *milieu* in the theory of didactical situations. “A milieu is any system that can be considered to have no intention in the answer it can give, explicitly or implicitly, to a given question. The system then considered behaves in this respect as a fragment of ‘nature’” (Chevallard, 2007, p. 344).

The metaphor of the *study and research path* allowed us to organize and structure an alternative writing in which the thesis chapters were sometimes given by questions, sometimes by answers. It is worth highlighting that the generative question of the thesis, “*Q^Z: What model is it possible to build considering conditions and restrictions of the Brazilian education system and the reflections of teachers for teaching Z?*”, articulated with the other studies, indicated as an initial response the need to establish a study group with teachers from Campo Grande/MS municipal schools. In this group, we debated the teaching of relative integers and began to develop an alternative proposal to those contained in the *dominant model*. The knowledge produced was also auxiliary and complemented both the *reference epistemological model* as the *dominant model* of relative integers (description started with the results of previous research (Gonçalves, 2016) and with the texts by Cid (2015) and Borba (2009)).

Establishing such a group also helped to question the *conditions* and *restrictions* of the Campo Grande/MS education system, present in the teaching and learning processes of relative integers and the possibilities of an emancipatory process of some of the elements of the *dominant model*. Our studies were based on the precepts of the PQW, which states that everyone should play the role of a mediator, proposing new questions and exposing their experiences. We planned to ‘plant the seed’ of the PQW, allowing them to experience the characteristics of a *Herbartian citizen with procognitive attitudes*. This paradigm shift caused some challenges for us researchers, mainly the changes in roles that occurred during the meetings, especially when we compared our work proposal with the training offered by the Municipal Department of Education¹⁰. As we did not follow this model, a problematic situation was created regarding the protagonism of the participating teachers. This protagonism is fundamental for the development of the *paradigm of questioning the world*.

Herbartian citizens with procognitive attitudes: In the scenario portrayed by the *paradigm of questioning the world*, a fundamental change is that to learn something, one must study it. Generally, Herbartian citizens abandon questions without immediate answers. From *Herbartian citizens* (Chevallard, 2013), a different attitude, called *procognitive*, is required. Thus, they are led to act as if knowledge were about to be discovered

¹⁰ These formative courses usually have a theoretical part, followed by practical activities.

again, always seeking to answer questions never asked before.

The mediation of debates with teachers also revealed challenges and questions: How do we encourage teachers to take a stand and express their ideas in silent moments? Would our opinions influence them? How can we mediate discussions without imposing elements of our studies on the group? How can we get teachers to break their silence? What should our attitude be if the group went in a direction contrary to some results of our studies and research that we considered necessary?

These questions revealed the internal conflicts faced with the challenges posed by working with teachers. Teachers' construction of new questions was also a challenge because several ready-made answers in textbooks and on the Internet are institutionalized and used without the necessary reorganization of the spaces in each classroom. This feature of the *paradigm of visiting works* (PVW) is necessary for the PQW, but in a different way: it must seek answers already produced to the questions derived from the generatrix; after all, it is not about "reinventing the wheel." However, they cannot be configured as acceptance of finalized results but rather mobilized as assistance in the construction of provisional answers to the heart response.

Another important theoretical tool for our research was the scale of *levels of codetermination*, a device that helps identify and understand *conditions* and *restrictions* that weigh on teaching practice.

The *levels of codetermination* can be separated into two: higher levels (Civilization, Society, School, and Pedagogy) and lower levels (Subject, Domain, Sector, Theme, and Object). The upper levels can be considered the *conditions and restrictions* coming from spheres other than the classroom; however, they directly influence teaching actions. The lower levels, related to praxeology studies, seek ways to detach themselves from the *paradigm of visiting works*, i.e., through the action of "construction-deconstruction of works, silently replaces a centuries-old tradition, in which the student waits attentively, according to the immemorial problem of copying the works and cultural mimicry, in which the teacher teaches -i.e., shows what it is, how to do it, and why to do it that way" (Chevallard, 2002, p. 1, our translation).

At the *school* level, the continuing education offered by REME in Campo Grande/MS took place once every two months. Our meetings were fortnightly during the teachers' working hours to provide more study time, which means that the actions related to our project would only be carried out during face-to-face meetings because teachers should be preparing their lesson planning otherwise.

At that point, we understood that using the planning time was not a good choice. Overall, the results were "fruitful," but teachers should, by right, have a specific workload for those studies. We realized that such a paradigm shift would be beneficial if there were such a schedule. In addition to teachers' involvement, structural changes in the school and society are necessary

for the change recommended in the PQW to occur in its entirety. Furthermore, the COVID-19 pandemic exposed other problems reported during the research writing (Gonçalves, 2022), which, in a way, were problems beyond the contexts experienced in the study group that took place before the pandemic period.

This entire journey also allowed us to sketch an action diagram, which guided the possible paths for preparing the REM. The actions described in the diagram are directly related to the questions and answers in our QA map, as the diagram described all the investigative paths we saw. However, some were more in-depth, represented in the QA map, and for the others, we indicated possible investigative movements. The diagram was divided into three major contexts: the mathematical (M), arising from doctoral studies (branch of thesis studies), given by the search for alternative models to concrete models; the everyday/concrete (C), coming from the results of the master's degree, given by the studies of concrete models, and the mathematical and routine (MEvd) context, coming from the consensus of the study group with the teachers (branch of the study group).

Below, we present the action diagram, divided into three major contexts: the mathematical (M) context, arising from doctoral studies, given by the search for alternative models to concrete models; the everyday/concrete (C) context, coming from the results of the master's degree, given by the studies of concrete models and; the mathematical and routine (MC) context, arising from the consensus of the study group, and the questions and answers that made up the question-and-answer maps of our doctoral research

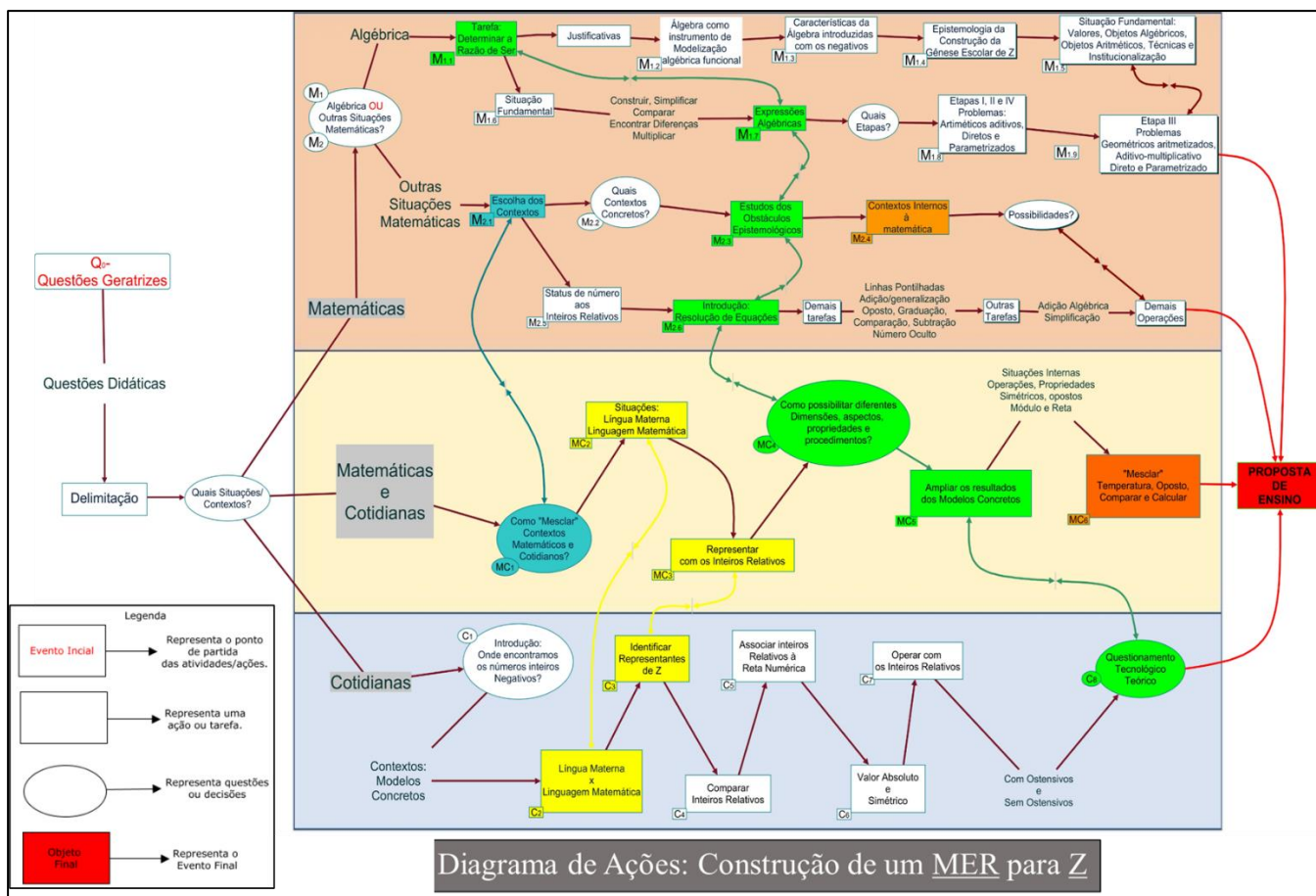


Figure 2.

Action Diagram (Gonçalves, 2022, p. 157)

Questions and answers that composed the doctoral thesis

The doctoral research began shortly after I finished my master’s degree in mathematics education, in the same program and line of research (Gonçalves, 2016), as a possibility to expand the investigation both about the questions raised by the research already completed and new questions on the teaching of relative integers. The didactic phenomenon observed – reminiscent of the master’s degree– that worried us was: “How to teach content –relative integers– whose mathematical justifications exist at a teaching level far from students attending the final years of elementary school?¹¹” Besides this initial question, we also highlight: Can we observe this fact in other mathematical content? Are there ways other than the constant in the *dominant model* that mathematically justify some rules?

¹¹ It is possible to justify, for example, the rules for adding and subtracting integers, but the same does not happen with multiplication: how to justify, for example, that $(-2) \times (-5)$ is equal to $+10$?

In this scenario, we constructed the following generating question Q^Z : *What model is it possible to build considering the conditions and restrictions of the Brazilian education system and teachers' reflections for teaching Z?* The model designed to answer this question was the REM for relative integers. This decision was based on Gascón's (2011) remarks that to prepare a research problem (also called a didactic problem), one must mobilize a REM, which, in turn, is an integral part of the *epistemological* dimension of research on didactics of mathematics. This author considers this dimension basic, as some elements of the REM are used to determine the other two dimensions: *economic-institutional* and *ecological*.

Economic-institutional and ecological dimensions: To characterize the object of study of didactics of mathematics, a heuristic pattern [was created] to develop didactic problems [...]. The standard is structured around the three basic dimensions [...] when constructed within the scope of the anthropological theory of the didactic: the epistemological dimension, which places “what is mathematical” at the heart of the problem; the economic-institutional dimension, which depersonalizes the didactic problem and delimits the minimum unit of analysis of the study processes, and the ecological dimension, which emphasizes the necessary conditions for the institutionalized study of mathematics to be possible and reveals restrictions of all kinds that affect this study (Gascón, 2011, p. 204, our translation, emphasis added).

According to Gascón (2011), in an investigation, one must check how the elements and characteristics of the investigated object are: in his words, how “are things”? To obtain answers, one must take actions that promote changes in its elements and characteristics. This movement, linked to studies of the *paradigm of questioning the world*, particularly to *conditions* and *restrictions* of the Campo Grande/MS education system, made us realize how essential it would be to interact with teachers from this education network.

This movement established new questions, such as Q_0 : *What teaching proposal for Z can be built through interaction with a teachers' study group from the PQW perspective?* and Q_1 : *How do we set up a study group in light of the PQW? Would it be plausible to set up this study group? Which teachers should we invite?*

To overcome some *restrictions*, we invited teachers from the mathematics laboratories from Reme from Campo Grande/MS. Due to their positions in schools, we believed we could overcome some restrictions, helping design responses Q_0 and Q_1 . And due to the desire to build an alternative proposal to the existing ones with the teachers, some other questions arose: Q_2 : *What aspects of PQW should one mobilize with the group of teachers?* Q_3 : *What are the first discussions that should be raised with the group of teachers?* Q_4 : *Considering what the research says, how can we teach relative integers?* Q_5 : *How important is it to build a REM?* Q_6 : *Why teach relative integers?*

The studies necessary to plan the meetings and the data from discussions with the group helped us design the answers to our questions. For example, answer R_2^{12} was built from some of our studies on ATD: concepts of *institution*, *didactic system*, *Herbartian scheme*, *didactic milieu*; particularities and differences between PQW and PVW; the position of Y in *teaching systems*; *study and research activities* (SRA); *study and research paths* (SRP); and *question/answer*, *media/milieu*, and *collective/individual dialectics*.

Before the pedagogy of the *study and research path* (SRP), the pedagogy of *study and research activities* (SRA) was mobilized. The SRA is grounded on the concept of fundamental situations given in the *theory of didactical situations* (TDS) (Brousseau, 1999). The mobilization of a fundamental situation “[...] is a well-founded epistemological requirement, which defines a project for developing a mathematical infrastructure that is didactically suitable for an SRA pedagogy. However, an SRA pedagogy requires less radical conditions. (Chevallard, 2009c, p. 7, our translation).

For answers $R_{3.1}$ and $R_{3.2}^{13}$, we deal with the historical and epistemological development of the set of relative integers, as well as proposals and school genesis of relative integers. Such answers were essential to allow us to think about the paradigmatic change we would experience with the teachers throughout our meetings.

Writing answer R_4^{14} was based on the analysis, together with the teachers, of the text by Cid (2015) about a teaching proposal based on the concomitant study between algebra and relative integers.

Therefore, in constructing this answer, the study group members addressed the difficulties that teachers usually encounter in teaching relative integers, besides those arising from student learning.

We discussed the bibliographic review carried out by Cid, which was determined and categorized considering the students’ errors and difficulties. This proposal presented by Cid (2015) was considered a counterpoint to those described in the *dominant model*.

Thus, we move toward the objective set for the first meetings to discuss experiences, difficulties, and expectations regarding the teaching and learning processes of relative integers.

After that, we addressed elementary issues of teaching paradigms, creating a scenario that would take us to a brief presentation of the concepts of *Q-A map*, *dialectics*, and *heart response*, and finally, a preview of the elements of the *PQW*.

In answer R_5^{15} we describe some episodes from the seven meetings with study group

¹² R_2 : Study Chevallard's texts on PQW, REM, and SRP.

¹³ $R_{3.1}$: Study the historical and epistemological development of relative integers and $R_{3.2}$: Study teaching proposals and school genesis for relative integers.

¹⁴ R_4 : Compare the teaching of relative integers through concrete models with teaching through the algebraic model.

¹⁵ R_5 : Identify and describe the dominant model, paradigms of teaching, and learning beliefs.

participants.

Table 1.

Summary of the meetings(Research author)

Meetings	Summary
I	I work with the denaturalization process. Elements of construction of Q_5 : <i>How important is it to build a REM?</i> and of R_5 : <i>Identify and describe the dominant model, paradigms of teaching, and learning beliefs.</i>
II	Analysis of two teaching proposals and construction elements of Q_6 : <i>Why teach relative integers?</i>
III	Studies of the reason for being of relative integers and elements of the construction of Q_7 : <i>What is the reason for being of the relative integers?</i> and R_6 : <i>Reasons for being: everyday life, games, and arithmetic surroundings versus mathematical contexts, generalized arithmetic, and algebraic surroundings.</i>
IV	Studies of difficulties, errors, methodologies, games, articles on relative integers, and the BNCC. Building elements of Q_8 : <i>What are the difficulties, the most common errors, and the most used interventions in the teaching and learning processes of Z?</i> And, R_8 : <i>Difficulties and errors: signs, number line, operations, and comparisons. Interventions: games and everyday procedures.</i>
V	Lecture – theoretical issues and teaching processes: I - historical and epistemological studies, with an emphasis on relative integers; II - definitions, concepts, properties, a more formal treatment.
VI	Analysis of the third poll and studies of games and activities, for example, “Geometric idea” and “More positive and more negative.”
VII	Presentation of proposals by teachers – introduction. Construction elements of Q_9 : <i>What activities, problems, and strategies should make up an introduction to a proposal... relative?</i> and from answers $R_{9,1}$: <i>activities, problems, and exercises... and algebraic.</i> $R_{9,2}$: <i>Heart response of the study group.</i> End of the study group.

In the general context of our research, the study group would construct questions and develop their solutions. For example, how can we “know” a concept? What must be mobilized? Formulas, algorithms, procedures? What problems must be resolved? Therefore, the meetings with teachers served as education based on the principles of an SRP.

Changing teaching practice becomes a great difficulty because, behind this process, we see a change in paradigms. This process is not simple; it takes a few hours of study. This change/revolution would be something to be achieved over time in its “totality,” so in these meetings, we sought to “plant the seed” of the *PQW*. In this sense, one of the most impactful aspects for them and the researcher would be the absence of an educator, as everyone in the group should assume this role. We see that at Reme of Campo Grande/MS, pre-service teachers usually assume the role of students and, therefore, in this scenario, proposing new questions and building a new proposal does not seem easy.

Constructing the *dominant model*, supported by studies of the two teaching paradigms

and portraying the teachers' beliefs, helped us write answer R_6^{16} , which served to answer questions Q_6 and Q_7 . In this response, we also present conflicts arising from the paradigmatic change, identified by the higher levels of *codetermination*, for example,

I understand if the justification was the lack of time; although teachers participated in project meetings during planning hours, this time was not enough to meet all the formative demands. As they were in person at the meeting, they should plan for another period. In this sense, despite having managed to arrange for teachers to participate during their activity hours, I did not think about and could not organize these other moments arising from both the obligations as leader of the mathematics laboratory and the possible tasks of our meetings. These conditions, not materialized by our rulers, come from the upper levels of the codetermination scale [...]. (Gonçalves, 2022, p. 98, emphasis added).

Another example of the analyses carried out from higher levels of *codetermination* was the closure of mathematics laboratories for political and economic reasons in Campo Grande/MS. Not only were the mathematics laboratories closed, but also the science and technology laboratories, besides the closure of some programs focused on special education. The justifications were the financial difficulties faced by municipal management. Because of this *restriction* imposed at the *Society* level, our studies were concluded before the deadline agreed upon with the teachers, which prevented the group from functioning and demonstrated the relevance and importance of the analysis in light of the *levels of codetermination*. So as not to lose the constructed data, we describe the group's *heart response*, a fact driven by theoretical and methodological choices. Based on this response, we finalized the alternative proposal to those contained in the *dominant model* and the *REM* for these numbers.

Questions Q_8 and Q_9^{17} and answers R_8^{18} , $R_{9,1}$ and $R_{9,2}$, were the last ones discussed with the teachers' group. We were already finalizing our studies to begin the construction of the alternative proposal, and, in the last meeting, before the news of the closing of the laboratories,

¹⁶ R_6 : Reasons for being: everyday life, games, and arithmetic surroundings x mathematical contexts, generalized arithmetic, and algebraic surroundings. Q_6 : Why teach relative integers? Q_7 : What is the reason for being for relative integers?

¹⁷ Q_8 : What are the difficulties, the most common errors, and the most used interventions in the teaching and learning processes of Z? Q_9 : What activities, problems, and strategies should make up an introduction to a teaching proposal for Z? R_8 : Difficulties and errors: signs, number line, operations and comparisons. Interventions: games and everyday procedures, $R_{9,1}$: activities, problems and exercises of concrete and algebraic models and $R_{9,2}$: heart response from the group of teachers.

¹⁸ R_8 : Difficulties and errors: signs, number line, operations, and comparisons. Interventions: everyday games and procedures; $R_{9,1}$: activities, problems, and exercises using concrete and algebraic models; $R_{9,2}$: the heart response of the group of teachers.

the teachers presented and discussed some drafts of the introductory part of the proposal. Therefore, mainly in $R_{9,2}$, we describe the contents of such a proposal for the relative integers listed in the teachers' group, that is, the agglutination between the genesis of Cid (2015) and the proposals contained in the textbooks, as well as the choice of possible activities and games to be incorporated. All these elements were later used to construct the heart answer of the research. We tried our best to be faithful to the consensus of the study group. However, our interpretations and the influences of the theoretical framework, at times, overcame the teachers' consensus. We do not see this fact as something bad; it is just a natural research process partly interrupted by the decisions of municipal management at that time. The *restrictions* at the *Society* level prevented the work from being carried out as planned. However, as already highlighted, we could design what was proposed, taking into account the study group's discussions.

Finally, we built the *heart response* of the thesis, divided into four parts, R_I^Z , R_{II}^Z , R_{III}^Z and R_{IV}^Z ¹⁹. With the constructed proposal, we sought to overcome some gaps identified from the concrete models, which are constant in the *dominant model*. In this way, the counterpoints constructed with these teachers allowed us to design a proposal focused on the conditions and restrictions of the Campo Grande education system and the study of learning difficulties and problems shared and experienced by group members and the research discussed. Thus, the core of the teaching proposal was the potential of the studies of concrete models combined with the analyses based on the contexts presented by Cid (2015), seeking to emphasize the epistemological *reason for being* of relative integers. (Gonçalves, 2022).

Some final words

The *paradigm of questioning the world* aims to build an investigative spirit. Chevallard (2013) advocates non-passive attitudes for today's citizens and those of the future. One of the objectives is to become a *Herbartian citizen with procognitive attitudes*. Hence, reflecting on

¹⁹ R_I^Z - Reference epistemological model possible to build considering aspects of interaction with the group of teachers and other studies. R_{II}^Z - Design of the reference epistemological model for relative integers. R_{III}^Z - Relevant elements and aspects of the teaching proposal for relative integers and R_{IV}^Z - Design of the teaching proposal for relative integers.

tasks, rules, or their consequences cannot happen through our impulses or the myths to which we are subjected. There is a need for more in-depth, permanent, and cautious analyses of everything around us, especially regarding students' learning process, constantly questioning the foundations and effects that can be produced. Thinking and acting this way encourages research based on experiences, emotions, and intuitions. Nevertheless, these cannot be limiting, and the actions cannot be commonplace, reproducing unjustified rules.

We believe that the model we adopted for developing and writing our research report allowed us to put our investigative spirit into practice and favor our approach to the spirit of a *Herbartian citizen with procognitive attitudes* or reflective thinking.

Reflective Thinking: I- “[...] a state of doubt, vacillation, perplexity, mental difficulty, in which thought originates”; **II-** “act of search, hunting, investigation, to find some material that clarifies the doubt, that dispels the perplexity” (D’Ambrósio & Lopes, 2015, p. 7).

During the research and writing process, the construction of our QA map made us, for example, notice gaps in the teaching of relative integers and connections between the teaching of these numbers and school algebra. Thus, new questions arose, including the need to organize a study group with teachers from the municipal schools to construct a teaching proposal, providing clarity on these elements that needed deepening through our investigation. Furthermore, the purpose of writing this article was to reflect on the use and relevance of this writing model in our research report. The *study and research path* was the tool presented by Chevallard (2013) to assist in developing and implementing the *paradigm of questioning the world*. In this sense, the materialization of this work can occur through the production of a *question-answer map*.

That said, using the metaphor “doctoral research is an excellent example of the *study and research path*,” the writing of the doctoral thesis report as *question-answer map* happened naturally and contributed to describing the research results in more detail, highlighting the uses of other tools. For example, the *levels of codetermination* allowed us to demonstrate the *restrictions* arising from the level of *Society*. Such *restrictions* impaired the functioning of the study group, which, in turn, directly affected the research, causing sudden changes in the teaching models of the municipal schools. They directly impacted the education system, as they caused many changes, teachers' dismissals, and the abrupt closure of laboratories, which

consequently affected student learning. Another example was the analysis of lower levels, which was used in studies with the group's teachers and became very helpful in constructing the teaching proposal for relative integers. Obviously, a more detailed presentation of these results will be the focus of other articles.

Finally, we emphasize that our theoretical and methodological choices allowed us to start from a teaching problem, building our didactic problem, given through its description and interpretation, and configuring our *reference epistemological model*. The elements of this model allowed us to highlight characteristics of the *epistemological* dimension of our *teaching problem*. Furthermore, studies of *levels of codetermination*, allowed us to highlight characteristics of the *ecological* dimension of this didactic problem and answer our generating question for this article: *How can we write a thesis report that is coherent with our choices, that reflects the process experienced and that, in addition to showing our research question, highlights the questions and answers that derived from it?*

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