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Contributions of a linguistic approach to the initial education of mathematics teachers

Aportes de un enfoque lingüístico en la formación inicial de profesores de matemáticas

Apports d'une approche linguistique dans la formation initiale des professeurs de mathématiques

Contribuições de uma abordagem linguística na formação inicial de professores de matemática

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Abstract

This work is an excerpt from a master's degree research in mathematics education. It aims to analyze the contributions of a formative activity focused on a linguistic approach to mathematics and its teaching to mathematics teachers' initial education. This objective seeks to answer the following research problem: How can a formative activity focused on a linguistic approach to mathematics and its teaching contribute to the initial education of mathematics teachers? To this end, a qualitative field study was conducted with mathematics teaching degree students at the Federal University of Pará (UFPA). Data were produced through a questionnaire and audio recordings during the development of the formative activity, which was a mini-course on a linguistic approach to teaching mathematics, using discursive textual analysis (DTA) as the methodology for analyzing the data produced. The results indicate that the activity contributed to those students' education regarding i) the identification of linguistic challenges in teaching mathematics, ii) reflections on the nature of mathematical knowledge, iii) encouragement of research, and iv) theoretical studies in different areas of mathematics. In conclusion, discussions about language in mathematics teaching can present several contributions to teacher education, serving as a starting point to prepare teachers who consider

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these discussions in their professional practices, which can mitigate the challenges present in the classroom.

Keywords: Mathematical language, Teacher training, Pedagogical content knowledge, Training activity.

Resumen

Este trabajo es un extracto de una investigación de maestría en educación matemática. Tiene como objetivo analizar las contribuciones de una actividad formativa centrada en un enfoque lingüístico de las matemáticas y su enseñanza a la formación inicial de profesores de matemáticas. Este objetivo busca responder al siguiente problema de investigación: ¿Cómo una actividad formativa centrada en un enfoque lingüístico de las matemáticas y su enseñanza puede contribuir a la formación inicial de profesores de matemáticas? Para ello, se realizó un estudio de campo cualitativo con estudiantes de la carrera de profesorado de matemáticas de la Universidad Federal de Pará (UFPA). Los datos se produjeron a través de un cuestionario y grabaciones de audio durante el desarrollo de la actividad formativa, que fue un minicurso sobre un enfoque lingüístico de la enseñanza de las matemáticas, utilizando el análisis textual discursivo (ADD) como metodología para analizar los datos producidos. Los resultados indican que la actividad contribuyó a la formación de esos estudiantes en lo que respecta a i) la identificación de desafíos lingüísticos en la enseñanza de las matemáticas, ii) reflexiones sobre la naturaleza del conocimiento matemático, iii) estímulo a la investigación y iv) estudios teóricos en diferentes áreas de las matemáticas. En conclusión, las discusiones sobre el lenguaje en la enseñanza de las matemáticas pueden presentar diversas contribuciones a la formación docente, sirviendo como punto de partida para preparar profesores que consideren estas discusiones en sus prácticas profesionales, lo que puede mitigar los desafíos presentes en el aula.

Palabras clave: Lenguaje matemático, Formación de enseñanzas, Conocimiento pedagógico del contenido, Actividad de formación.

Résumé

Le présent travail est extrait d'une recherche de master en Enseignement des Mathématiques et vise à analyser les apports d'une activité de formation axée sur une approche linguistique des Mathématiques et son enseignement pour la formation initiale des professeurs de Mathématiques. Cet objectif cherche à répondre à la problématique de recherche suivante : comment une activité de formation axée sur une approche linguistique des Mathématiques et

de son enseignement peut-elle contribuer à la formation initiale des professeurs de Mathématiques ? À cette fin, une recherche de terrain avec une approche qualitative a été réalisée avec des étudiants du cursus de mathématiques de l'Université fédérale du Pará (UFPA). La production de données s'est faite au moyen d'un questionnaire et d'enregistrements audio lors du développement de l'activité de formation, qui était un mini-cours sur une approche linguistique de l'enseignement des mathématiques, utilisant l'analyse textuelle discursive (ATD) comme méthodologie d'analyse des données produites. Les résultats indiquent que l'activité développée a contribué à la formation des étudiants du premier cycle en ce qui concerne : i) l'identification des défis linguistiques dans l'enseignement des mathématiques ; ii) des réflexions sur la nature des connaissances mathématiques ; iii) encourager la recherche ; et iv) des études théoriques de différents domaines des mathématiques. En conclusion, les discussions sur la langue dans l'enseignement des mathématiques peuvent apporter plusieurs contributions à la formation des enseignants, en servant de point de départ pour préparer les enseignants à considérer ces discussions dans leurs pratiques professionnelles, ce qui peut atténuer les défis présents en classe.

Mots-clés: Langage mathématique, Formation des enseignants, Connaissance des contenus pédagogiques, Activité de formation.

Resumo

O presente trabalho é um recorte de uma pesquisa de mestrado em educação matemática e tem como objetivo analisar as contribuições de uma atividade formativa com foco em uma abordagem linguística da matemática e do seu ensino para a formação inicial de professores de matemática. Esse objetivo busca responder o seguinte problema de pesquisa: de que maneira uma atividade formativa com foco em uma abordagem linguística da matemática e do seu ensino pode contribuir para a formação inicial de professores de matemática? Para tanto, realizou-se uma pesquisa de campo de abordagem qualitativa com alunos do curso de licenciatura em matemática na Universidade Federal do Pará (UFPA). A produção dos dados ocorreu por meio de um questionário e de gravações de áudio durante o desenvolvimento da atividade formativa, que foi um minicurso sobre uma abordagem linguística do ensino de matemática, tendo a análise textual discursiva (ATD) como metodologia de análise dos dados produzidos. Os resultados apontam que a atividade desenvolvida contribuiu para a formação dos licenciandos no que diz respeito: i) à identificação de desafios linguísticos no ensino de Matemática; ii) às reflexões sobre a natureza do conhecimento matemático; iii) ao incentivo à

pesquisa; e iv) aos estudos teóricos de diferentes áreas da matemática. Em conclusão, têm-se que as discussões sobre a linguagem no ensino de matemática podem apresentar diversas contribuições para a formação docente, servindo como ponto de partida preparar professores que considerem essas discussões em suas práticas profissionais, o que pode mitigar os desafios presentes em sala de aula.

Palavras-chave: Linguagem matemática, Formação de professores, Conhecimento pedagógico do conteúdo, Atividade formativa.

Contributions of a linguistic approach to the initial education of mathematics teachers

It is not news that many basic education students consider mathematics one of the most difficult subjects. This feeling can lead them to lose interest in studying the subject and/or believe they cannot learn it. This fact, together with issues related to teaching practice, curriculum, and the like, has an impact on the results of educational assessments, such as the Programme for International Student Assessment (PISA) and the Basic Education Assessment System (Sistema de Avaliação da Educação Básica - SAEB). These results highlight the need for us to (re)discuss the weaknesses present in the educational scenario and propose ways to alleviate them.

In the field of mathematics education, over the last few decades, many authors have discussed mathematics teaching and its challenges, such as Garcia (2009) and Machado et al. (2014). In addition to highlighting several challenges present in school environments, these discussions make clear the importance of looking at the teaching and learning of mathematical concepts from different perspectives, considering that there are no single solutions to alleviate the difficulties present in the classroom.

Thus, several research trends emerge to explore mathematics teaching from different theoretical, philosophical, methodological, epistemological, and related perspectives, such as mathematical modeling, history of mathematics, ethnomathematics, and others. Among these trends, we highlight studies on language in mathematics teaching[3], given that "language plays an important role in the constitution of mathematical knowledge" (Zuchi, 2004, p. 49).

The National Common Curriculum Base (Base Nacional Comum Curricular - BNCC) (Brasil, 2018) highlights the importance of using different types of language, including mathematical language, in its second general competence, which states that it is necessary to:

> Use different languages – verbal (oral or visual-motor, such as Libras and writing), corporal, visual, auditory, and digital language, as well as knowledge of artistic, **mathematical**, and scientific languages to express and share information, experiences, ideas, and feelings in different contexts and produce meanings that lead to mutual understanding. (Brasil, 2018, p. 9, our emphasis)

This highlights the need for teachers to be aware of discussions about mathematical language to develop students' skills and abilities related to its use.

[3] Language in mathematics teaching constitutes a trend that presents different approaches and theoretical perspectives (Radford; Barwell, 2016). However, in this work, we refer to discussions arising from the understanding of mathematics as a language.

According to Feio (2009), mathematics teaching and learning face linguistic[4] challenges at all levels of education; i.e., challenges arise from language use, which, if not taken into account, can contribute to the continued lack of knowledge and incorrect utilization of mathematical language. Silva (2019) adds the following note on mathematics:

> Understanding how such language works and how it relates to the language shared between the master and his apprentice is of fundamental importance for the success of teaching and learning in this complex field of knowledge. Therefore, it is necessary to know this language better and understand its appropriation process. (Silva, 2019, p. 33)

In this context, there is an urgent need for initiatives to bring these discussions closer to in-service and prospective mathematics teachers so that they can critically appropriate the knowledge[5] in their teacher (self)education process, identify obstacles, and (re)think their educational practices. In addition to the possibility of contributing to teacher education, these initiatives can favor the development and enrichment of research in mathematics education.

Based on the above, we highlight the potential of initial education courses as possible starting points for promoting discussions about mathematics teaching from a linguistic approach, that is, considering linguistic knowledge such as the characteristics of mathematical language and issues involving reading and writing. This is relevant because, during initial education, the set of attitudes, values, and functions that prospective teachers confer to the profession will be subjected to a series of changes and transformations in line with the socializing process that occurs in this education process (Imbernón, 2000).

It is important to emphasize that we are not disregarding the possibilities offered by continuing education courses but rather directing our attention to teaching degree courses, considering that these are responsible for certification and initial preparation for practicing the profession.

About implementing formative activities, which we understand as being the set of activities that seek to contribute to professional and/or academic education in some aspect, such as mini-courses, subject matters, workshops, and the like, there is a need for investigations on the contributions of these activities, so that they can support similar approaches, as well as identify positive and negative aspects to improve future initiatives. This may be an important step for in-service and prospective teachers to understand the importance of these discussions and relate them to their future experiences and professional practices, which may contribute to alleviating linguistic difficulties in educational environments.

[4] In this work, the use of the term “linguistics” concerns only discussions about language in mathematics teaching.
[5] In this work, we will consider “knowing” and “knowledge” as synonyms.

Given the importance of discussions about language and mathematics teaching in teaching degree courses, we searched the following databases: Portal of the Coordination for the Improvement of Higher Education Personnel (Portal de Periódicos da Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - PC/Capes), Capes Theses and Dissertations Catalog (Catálogo de Teses e Dissertações da Capes - CTD) and the Digital Library of Theses and Dissertations (Biblioteca Digital de Teses e Dissertações - BDTD) This search aimed to understand what is usually discussed and investigated in research on language in the education of mathematics teachers.

Thus, to analyze recent research, we delimited the period of the last ten years (2013-2023) and used the following combinations of keywords: "*linguagem + atividade formativa + matemática*," "*linguagem matemática + formação de professores de matemática*," "*linguagem matemática + desenvolvimento profissional*," "*linguagem matemática + identidade profissional docente*" ["language + formative activity + mathematics," "mathematical language + mathematics teacher's education," "mathematical language + professional development," "mathematical language + teacher professional identity"]. To analyze and select the works presented by the repositories, we considered reading the title, abstract, introduction, final considerations, and, when necessary, the body of the works.

Based on the initial delimitations, we defined the following exclusion criteria: i) works that do not address the researched topic; ii) works that are not available in their respective repositories; iii) repeated works, that is, that have already been selected previously; iv) literature reviews. Those criteria were defined to filter, as precisely as possible, the works that deal with language and the education of mathematics teachers. In the search, we did not find any works on the topic investigated in the BDTD and CTD, which represents a lack, in the last ten years, of Brazilian theses and dissertations addressing linguistic issues in the initial education of mathematics teachers.

Considering the possibility of articles dealing with the topic investigated, we conducted an online search for combinations of keywords using articles published in peer-reviewed journals as a filter. Thus, the only combination that revealed works on the topic investigated was "*linguagem matemática + formação de professores de matemática*" ["mathematical language + mathematics teacher's education"] which resulted in 180 articles, of which two address the investigated topic: the works by Silva and Silveira (2013) and Oliveira (2021).

Furthermore, considering international works, we conducted the online search using English keywords. Thus, the combination "mathematical language + mathematics teacher's education" resulted in 141 articles, of which three are related to the topic studied, namely

Jaarsveld (2016), Caro and Planas (2021), and Hernández-Suárez et al. (2017). Table 1 below lists the works found, organized in chronological order:

Table 1.

Papers found in the search

Year	Title of the paper	Authors	Type
2013	A compreensão de regras matemáticas na formação docente: Uma pesquisa sob o ponto de vista da linguagem. [Understanding mathematical rules in teacher education: A research from the point of view of language]	Silva and Silveira (2013)	Article published in a journal
2016	Making a case for exact language as an aspect of rigour in initial teacher education mathematics programmes.	Jaarsveld (2016)	Article published in a journal
2017	Conocimiento y uso del lenguaje matemático en la formación inicial de docentes en matemáticas. [Knowledge and use of mathematical language in the initial education of mathematics teachers]	Hernandez-Suárez et al. (2017).	Article published in a journal
2021	Sobre a linguagem matemática na formação inicial de professores. [About mathematical language in initial teacher education]	Oliveira (2021)	Article published in a journal
2021	Estudio exploratorio con futuras maestras sobre lenguas matemáticas para enseñar la relación entre área y volumen. [Exploratory study with prospective teachers on mathematical languages to teach the relationship between area and volume]	Caro e Planas (2021)	Article published in a journal

We initially acknowledged the few works addressing language in mathematics teachers' education, highlighting the need for more research to enable reflections and different ways of understanding it and its implications in the current educational scenario.

By reading the works, we found that only Jaarsveld (2016) presented an intervention to alleviate the linguistic challenges faced by prospective teachers regarding the content of quadratic functions; meanwhile, the other studies are limited to pointing out weaknesses in how undergraduate students use mathematical language. In other words, in most works, there is no concern with discussions that can lead prospective teachers to reflect on how they will teach mathematical concepts, nor on the different linguistic manifestations that can influence mathematics teaching.

Therefore, although the literature highlights the importance of discussions on language and mathematics education, mathematics in-service and prospective teachers are not expected to acquire this knowledge if they are not familiar with the aspects of those discussions. The different uses and perspectives of language in teaching mathematical concepts are not usually addressed in teaching degrees, and the relationship between discussions about language and teacher education lacks further contributions.

Based on the above, aiming to contribute to the theme, the objective of this work, which is an excerpt from a master's research in mathematics education, is to analyze the contributions of a formative activity focusing on a linguistic approach to mathematics and its teaching for the initial education of mathematics teachers. The objective seeks to answer the following research problem: How can a formative activity focusing on a linguistic approach to mathematics and its teaching contribute to the initial education of mathematics teachers?

Thus, we will discuss some aspects of mathematics teaching from a linguistic approach, i.e., starting from an approach that considers mathematics a language. Next, we will discuss the knowledge teachers mobilize, seeking a relationship with mathematics teachers' linguistic knowledge. Finally, we will present the research methodology, results, and discussions.

Teaching mathematics from a linguistic approach

Pimm (2002) identifies three possible interpretations for discussions on language in mathematics teaching: a) mathematics and language; b) the mathematics of language or the language of mathematics; c) mathematics as language. In this work, we will adopt the last interpretation, mathematics as a language, as this perspective allows for in-depth analysis and discussion of the linguistic aspects that permeate the teaching of mathematical concepts (Pimm, 2002).

In international literature, many authors point out that learning mathematics is similar to learning a foreign language, as both have similarities, such as acquiring new vocabulary (Pimm, 2002; Rojano, 1994). Thus, there is a broad discussion about what it means to consider mathematics a language and the particularities of this language. Although there is no closed and accepted definition for "mathematical language," there are several notes about what constitutes or should constitute this conceptualization.

Firstly, we highlight that understanding mathematics as language means seeing it as a human creation, i.e., as something not predetermined in a Platonic reality or empiricism (Gottschalk, 2004). Furthermore, this understanding also considers the communicative skills that are part of mathematical language usage, such as speaking, listening, reading, and writing

(Pimm, 2002). In other words, throughout mathematics teaching and learning, we constantly mobilize and develop linguistic skills associated with the appropriate use of mathematical concepts and procedures.

Furthermore, understanding mathematics as a language means considering that it has a **grammar**, which is the set of rules that regulates the appropriate use of notations, words, processes, and other mathematical knowledge elements. This grammar determines particular characteristics of mathematical texts, such as monosemy, objectivity, formality, impersonality, timelessness, and specialized vocabulary (Barata, 2017; Silveira, 2020a).

> To communicate well in mathematical language, it is also important to know its grammar, which requires knowing its *letters* (the alphabet) so that we can form *words*, then *phrases*, and then study its *grammar rules*. The grammar rules of a language have the same function as the rules of a game; in both cases, we must distinguish the right moment for using the pieces and the rules. (Cunha & Velasco, 2019, p. 9)

In addition to the characteristics mentioned above, mathematical language presents a set of symbols responsible for expressing ideas in an "economical" way, i.e., few symbols can express many ideas. Pimm (2002) divides the symbols of mathematical language into four types: logogram, pictogram, punctuation symbols, and alphabetic symbols. Logograms are symbols created to express mathematical ideas, such as$\forall, \sqrt{}, \cup, \div, \infty, \equiv, \therefore$, while pictograms are pictorial figures created for the same purpose, such as $\odot$ and $\triangle$. Punctuation and alphabetic symbols are borrowed from the native language[6] for the expression of mathematical ideas, such as "!" expressing the factorial, and the "X" representing unknowns and variables.

Thus, thinking about mathematics teaching from a linguistic approach means thinking about an educational process that considers these linguistic issues that permeate mathematics and its teaching. Thus, we will highlight some issues that emerge from the consideration above.

At the outset, it is important to point out some thoughts on the process of translating mathematical texts in the classroom. This process is significant for the understanding and use of mathematical language (Silveira, 2014; Costa et al., 2016) since it is exclusively written and does not have an orality of its own, making it necessary to be translated into a mother tongue to enable its communication in teaching and learning situations.

Silveira (2014) defines translation as the process of passing from one language to another, prioritizing maintaining the meaning present in the original language. The author highlights that interpreting mathematical texts in teaching and learning situations involves

[6] Also known as natural language, it is the term used to refer to a person's first language, such as Portuguese for those born in Brazil and English for those born in the United States.

"translating the symbols into natural language and, subsequently, giving meaning to the words inserted in grammar and mathematical rules" (Silveira, 2014, p. 58). Following this premise, this translation process allows mathematical language to be verbal and meanings to be attributed to the mathematical text, which is fundamental in acquiring this language.

For example, let us suppose that the teacher will teach the inclusion operation, which is studied in set theory. To do this, he begins the class by presenting the expression "A ⊃ B." At first, this expression will not make mathematical sense to students; they will observe it without knowing what it means or what grammar rules govern its use. However, throughout the teacher's explanation, students will understand that the symbol "⊃" represents an inclusion relationship and the rules regulating its use. In other words, the teacher will seek to construct a meaning for the expression presented, enabling it to be interpreted and learned by the students.

The translation of mathematical language is not just about reading/decoding or transliteration term by term but understanding the rules governing mathematical texts/concepts (Silveira, 2014; Teixeira Júnior et al., 2023). Therefore, simply reading mathematical texts is not a guarantee of learning, nor can it be considered as translation since, for this, the rules must also be understood.

Furthermore, translation also occurs through figures, graphs, and other mathematical language elements. For example, to apply Pythagoras' theorem, we must know how to identify the right triangle, the legs, and the hypotenuse, among other elements, just as we need to analyze the function graph to define the formation laws or vice versa.

Silveira (2018) explains that there is a residue in the process of translating mathematical texts, that is, "that which was extinguished by the process of formalizing natural language" (Silveira, 2018, p. 164). For example, look at the following text:

$$x \in \mathbb{N} \ / \ x \geq 3$$

Through it, we know that x could equal 3, 4, 5, 6, and the other subsequent natural values. However, this information is implicit and is characterized as a residue, i.e., something that must be interpreted so that an adequate meaning can be attributed to the text. In this sense, one of the main difficulties in learning mathematics is the recovery of the residues in mathematical texts, the recovery of the implied information that makes up these texts, which is important for understanding them.

Although mathematical texts seek to present a single meaning, natural language is polysemic and ambiguous, which can baffle the communication and understanding of mathematical propositions. As an example, we highlight situations in which teachers say "four x squared," which does not make it clear whether they are referring to the expression $4x^2$ or

$(4x)^2$. Therefore, "concern with the words used to teach mathematical concepts should be constant because the success of the teacher's explanation depends on how he/she gives meaning to what he/she intends to teach" (Silveira, 2020a, p. 5).

Another challenge present in the teaching and learning of mathematical concepts is the confusion in the use of terms shared between mathematical language and the mother tongue since mathematical vocabulary presents a category of words that mathematics and the mother tongue share, but with different meanings (Pimm, 2002; Scheleppegrell, 2007).

As an example, Pimm (2002) highlights a situation in which the teacher asks for the difference between 24 and 9, in order to make students answer 15, i.e., the result of expression $24 - 9$. However, students did not give the answer the teacher expected, which occurred due to confusion in the understanding of the term "difference," which has different meanings in mathematics and in the mother tongue.

We emphasize that there is no way to communicate without misunderstandings, but this does not mean that teachers should or cannot do anything. In this regard, Silva et al. (2019, p. 153) highlight that:

> (...) If, on the one hand, we do not have direct access to the student's thoughts, it is through language –when the student speaks, writes, and describes what he/she thinks or understands– that we can understand what the student does not assimilate. Thus, depending on the occasion, we can formulate new explanations, point to objects, use gestures, use technology, give new examples, etc., aiming to solve difficulties.

Following this premise, teachers should provide situations that allow students to express themselves mathematically. This will help them analyze whether students understood the rules taught, i.e., whether they attributed the appropriate meaning to the presented mathematical texts.

> Through natural language, the teacher explains mathematical concepts and describes them through representations and examples. However, what he/she expresses through explanations receives a meaning from the student that may or may not agree with what the teacher intends to teach. In short, for teachers to be successful in their teaching practice, they must be attentive to the words used, as our language is polysemic, and we cannot predict how students might interpret what we teach. (Silveira, 2020b, p. 10)

Another challenge in teaching mathematical concepts is the procrastination of specific language terms. As an example, Silva (2019) highlights a situation in which the teacher in the early years teaches the term "digit" as a synonym for "number," which, later, during the teaching of the numbering system, caused students to face difficulties in understanding that

numbers are formed by digits. As another example, Silveira (2020b) presents the case of the terms “simplify” and “cut,” mentioning that:

> Saying “cut” instead of “simplify” can baffle learning since cutting is not the same as simplifying, just as saying that in an equation, a term passes from one side of the equation to the other by changing the sign can also cause confusion since the sign is not changed; instead, the operation is changed to its inverse. (Silveira, 2020b, p. 2)

Therefore, it is important to highlight the correct meaning of mathematical terms so that the student can learn them appropriately. This issue has raised many discussions worldwide since, in many situations, teachers do not value the formal learning of mathematical concepts (Schleppegrell, 2007; Silveira, 2020b; Jaarsveld, 2016).

Therefore, we can notice that discussing mathematics teaching from a linguistic perspective contributes to observing linguistic issues that permeate the classroom, which helps teachers to (re)think their practices. This highlights the importance of analyzing how these discussions can contribute to teacher education, which will be our focus in the next section.

Language and education of mathematics teachers

Many discussions in the teacher education field address the knowledge teachers must have for their profession that is mobilized during their formative process. Those studies intensified in the 1980s, motivated by the belief in a knowledge base for teaching, i.e., a knowledge base teachers should have for the profession (Almeida & Biajone, 2007). This foundation could be of utmost importance for (re)thinking teacher education, both in general and specific areas, which encouraged researchers to study teachers’ knowledge related to pedagogical practices, curricular knowledge, knowledge about how schools work, etc.

Thus, over the last few decades, different conceptions and theoretical models have emerged to explain the knowledge teachers mobilized in general and specific areas, such as mathematics and biology, relevant for discussions and investigations on teacher education. Following this premise, we will use the dimensions present in the pedagogical content knowledge model proposed by Shulman (1986) to establish relationships with the knowledge teachers mobilize.

Lee S. Shulman is an emeritus professor at Stanford. He was also a professor at the University of Michigan. Shulman is known for being one of the pioneers of teaching work research and for his theoretical model entitled pedagogical content knowledge (PCK). Over the last few decades, this model has inspired the creation of other models that seek to list the

knowledge teachers mobilize in general or specific areas. This motivated us to use it to discuss the language and knowledge mathematics teachers mobilize.

In his conception of how knowledge develops in teachers' minds, Shulman (1986) proposes three categories (or domains) of knowledge mobilized in teaching practice: content knowledge, pedagogical content knowledge, and curriculum content knowledge. We will explain each category individually to relate them to the linguistic approach highlighted previously.

Content knowledge refers to "the amount and organization of knowledge perse in the mind of the teacher." (Shulman, 1986, p. 9). In this sense, content knowledge concerns knowledge of the subject matter that will be taught, including concepts, methods, procedures, theories, evidence, tests, etc. If we are going to teach mathematics, for example, we must master the contents that will be worked on, such as algebra, geometry, and trigonometry, which is related to knowing the terms, properties, and application of rules, among other elements related to the topic to be taught.

It is important to emphasize that content knowledge is also related to the interrelationships between different subjects, which may be from the same area or different areas. For example, we have algebraic knowledge, which can be worked on in some geometry and trigonometry subjects.

We emphasize that linguistic knowledge can be included in content knowledge since it can allow the teacher, for example, to reflect on the appropriate use of notations, specific terms, terms shared with the mother tongue, etc. This can be important when the teacher teaches terms with different meanings but are often treated as synonyms, such as "addition" and "sum."

Furthermore, Shulman (1986) highlights the importance of teachers knowing more about what they are going to teach, enabling not only the presentation of "truths" but a critical discussion of why the "truths" are the way they are, as well as the relevance of their study.

> Teachers must not only be capable of defining for students the accepted truths in a domain. They must also be able to explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions, both within the discipline and without, both in theory and in practice. (Shulman, 1986, p. 9)

Curriculum content knowledge is knowledge related to the organization of content according to the level of education. For example, if we want to teach a specific content to a specific class, we must know how it is (or can be) structured to be taught at that level, which includes, for example, the curriculum resources available in that particular context.

> The curriculum is represented by the full range of programs designed for the teaching of particular subjects and topics at, a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances. (Shulman, 1986, p. 9)

Furthermore, this knowledge can be related to mathematics teaching from a linguistic approach, considering that the teacher cannot/should not, for example, use terms and notations that are not in accordance with the level of the class, as well as procrastinate terms and knowledge that students need to learn at a certain level.

Finally, pedagogical content knowledge is related to all the possibilities of teaching the content: examples, materials, and explanations, among others. Shulman (1986) emphasizes that teachers must seek alternatives for teaching specific content, constituting a set of possibilities for carrying out the "conversion" of academic knowledge into school knowledge.

> Within the category of pedagogical content knowledge I include, for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations in a word, the ways of representing and formulating the subject that make it comprehensible to others. Since there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation, some of which derive from research whereas others originate in the wisdom of practice. (Shulman, 1986, p. 9)

The author also highlights the need for teachers to critically examine their practices, seeking strategies to promote their students' understanding and build their "arsenal" of possibilities.

> How many individuals whom we prepare for teaching biology, for example, understand well the materials for that instruction, the alternative texts, software, programs, visual materials, singleconcept films, laboratory demonstrations, or "invitations to enquiry?" Would we trust a physician who did not really understand the alternative ways of dealing with categories of infectious disease, but who knew only one way? (Shulman, 1986, p. 10)

This knowledge can be related to discussions of language in mathematics teaching, given that these enable reflections on strategies that can favor the translation of mathematical language, such as the strategies presented by Jourdain and Sharma (2016) and Thompson and Rubenstein (2000), which aimed to favor the acquisition and use of mathematical vocabulary. In this sense, these discussions can make up the "arsenal" of possibilities that mathematics teachers can and should have.

We can conclude that knowledge about language in mathematics teaching, following the interpretation of mathematics as a language, can be glimpsed in the domains of pedagogical content knowledge. Therefore, language cannot/should not be understood as a domain and/or part of just one domain but as knowledge that can support all domains in PCK. This points to the possibility that these discussions contribute to reflections both in relation to mathematical knowledge and its teaching, which can be fruitful for teacher education.

Research methodology

We reiterate that our research aims to analyze the contributions of a formative activity focused on a linguistic approach to mathematics and its teaching for mathematics teachers' initial education. Furthermore, we highlight that this research project was submitted and approved by the Research Ethics Committee (CEP) of UFPA, under opinion number 6640066 and Certificate of Presentation of Ethical Appreciation (CAAE) number 74250123.0.0000.0018.

To achieve our objective, we carried out field research, which consists of "observing facts and phenomena as they occur spontaneously, collecting data related to them, and recording variables that are presumed to be relevant for their analysis" (Marconi & Lakatos, 2003, p. 186). This type of research seeks information and knowledge to find answers to problems, validate hypotheses, and/or discover phenomena, in addition to collecting or producing data in the place where the event occurs, carrying out comparison and analysis through relevant literature (Marconi & Lakatos, 2003; Moraes & Fonseca, 2017).

The research approach is qualitative, which consists of "a process of reflection and analysis of reality through methods and techniques for a detailed understanding of the object of study in its historical context and/or according to its structure (...)" (Oliveira, 2013, p. 37). This approach is concerned with producing and analyzing data that cannot be analyzed solely through quantification, allowing a detailed visualization of the object of study and the contexts that permeate what is being investigated.

The locus of the research was the mathematics teaching degree at the Federal University of Pará (UFPA), Belém campus. This choice stems from the relevance of the UFPA as a higher

education institution in the North region and its expressiveness in the education of professionals and scientific production in Pará.

The research subjects were 12 students attending a mathematics teaching degree at UFPA, enrolled from the fifth period onwards. This delimitation owes to the importance of participants already being more familiar with the course and the possibility of having had contact with discussions and experiences related to the linguistic approach presented in the formative activity, which can favor more discussions and critical reflections on the proposed theme.

We chose the subject matter "Supervised Practicum II" for the formative activity because the responsible professor made one day's room for the research without compromising his planning. Furthermore, students in this subject attend the second half of the course, meeting the requirement for participation in the proposed investigation.

Notably, "Supervised Practicum II" and other subjects in the mathematics teaching degree at UFPA were composed of students admitted in 2019 through 2021 because of the pandemic that delayed subject completion. Thus, students from different years also motivated the choice of this subject for the research since the participants' different educational paths could favor sharing experiences, opinions, and other elements relevant to achieving our objective.

After choosing the subject, we informed students about the day of the mini-course and informed them that participation in the research would be optional. Thus, of the 20 students enrolled, 12 agreed to engage in the research, constituting the participants in the investigation.

To compile the research data, we chose a questionnaire that is "a data collection instrument, consisting of an ordered series of questions, which must be answered in writing and without the presence of the interviewer" (Marconi & Lakatos, 2003, p. 201). Questionnaires can present advantages, such as greater freedom of answers due to anonymity, lower risk of distortion due to researcher influence, and faster and more accurate answers (Marconi & Lakatos, 2003). Thus, we analyzed a questionnaire about participants' perceptions of the mini-course discussions.

Furthermore, considering the possibility of participants presenting doubts, questions, reflections, and other elements that can contribute to constituting the data, we also chose audio recording as a means of constructing empirical data, taking into account the importance and need to ensure adequate obtaining of participants' perspectives, which is essential in qualitative research (Bogdan & Biklen, 1994). Thus, we transcribed students' oral considerations and separated and analyzed the excerpts relevant to the research.

To analyze the data obtained, we used discursive textual analysis (DTA), which corresponds to "a methodology for analyzing data and information of a qualitative nature to produce new understandings about phenomena and discourses" (Moraes & Galiazzi, 2011, p. 7). The steps of this methodology correspond to the division of the empirical data into empirical units, to the categorization of the empirical units, resulting in the initial categories, to the categorization of the initial categories, resulting in intermediate and/or final categories, and to the analysis of the categories created. In this way, we organized the written contributions and transcribed participants' speeches, enabling the application of the DTA stages, which included the creation of empirical units, the creation of categories, and their analyses based on the literature in the area.

Our formative activity consisted of a mini-course on language in mathematics teaching, starting from the interpretation of mathematics as a language. Considering that participants may not be familiar with linguistic discussions, the mini-course promotes debates on linguistic approaches in teaching degrees, contributing to investigating how these approaches can impact initial teacher education. Furthermore, creating mini-courses, workshops, and subjects can be a starting point for bringing together initial teacher education and discussions that can contribute to teaching practice, including discussions on language.

Before the mini-course onset, students interested in participating in the research signed the Informed Consent Form (ICF) and the authorization form for voice recording to ensure the ethical guarantees of the research.

The mini-course lasted three hours and was conducted in an expository and dialogic manner, enabling dialogue between participants and instructors, and was divided into three formative moments: a) mathematical language and its characteristics; b) uses of language in

mathematics teaching; c) research on language and mathematics education. During the first two moments, participants performed oral and written activities to present examples of situations that could happen in the classroom.

In the first formative moment, we discussed mathematical language, highlighting what it means to interpret mathematics as a language and its main characteristics, such as timelessness, formality, and specialized vocabulary. Next, we carried out the first oral activity with the students, which discussed the situation presented by Pimm (2002), relating to the confusion caused by the term "difference" in a class, as mentioned in the previous section. In this way, the students discussed the reasons for this confusion.

The written activity developed at the end of the first formative moment consisted of answering a statement: "Try to remember and write one to four terms from mathematical language that are borrowed from natural language." This statement aims to promote reflection on the words shared between natural language and mathematics, constituting an important step in the search for strategies to alleviate the confusion caused by the inadequate understanding of these terms.

The second formative moment consisted of a discussion about language in mathematics teaching, highlighting issues regarding translation, literacy, and some linguistic challenges present in the classroom, such as the procrastination of specific terms in mathematical language and the polysemy of the mother tongue. Next, we developed the second oral activity, which consisted of presenting the following sentence: "There are twice as many boys (G) as girls in the room (F)." From this sentence, students should discuss whether the expression that represented what was in the sentence was $2G = F$ or $G = 2F$, which is directly related to how the sentence was interpreted and will be translated.

Two written activities were developed at the end of the second formative moment. The first one said: "Translate the text (in natural language) into mathematical language." An example is provided to guide students: translating the text "8 added to the sextuple of a number" into "$8 + 6x$". For this activity, we selected only three items for students to answer: i) three times the square of the value of a real variable; ii) the sum of 4 and the product of 5 by 6; iii) the product of the sum of 4 and 5, by 6. This task aimed to make students observe situations in

which challenges related to the translation of natural language into mathematical language may occur, as well as to lead them to identify the presence of translation residues, such as the parentheses that must be placed in the expression that represents the third item, which is: (4+5).6.

The second written activity developed at the end of the second formative moment consists of responding to the following statement: "Translate mathematical language into natural language." This statement proposes the opposite process to what was done in the previous statement. The example given to guide the students was: "X .(X+1)", which translates into "the product of a number and its successor." For this activity, we separated, again, three items: a) $(3x)^2$; b) $(x+y)^2$; c) x^2+y^2. This activity and the previous one were designed to encourage reflections on the translation of mathematical language in teaching and learning situations and on obstacles that may occur during this process.

Finally, in the last formative moment, we discussed research involving language and mathematics education, highlighting some of the main authors generally used in these investigations, such as Wittgenstein and Vygotsky, and recommending some works for further study on the topic, such as Pimm (2002).

The participants' perceptions constitute one of the most important points of our investigation since this will allow the analysis and description of the contributions of the formative activity developed and its relevance for teacher education. To this end, we created a questionnaire with five open questions, which were answered by participants anonymously after the mini-course ended.

For this article, we will analyze only two questions from the questionnaire and the participants' statements since the other questions have purposes that are not related to the objective we intend to achieve. To maintain participants' anonymity, as stated in the documents they signed and which were accepted by the research ethics committee, we will refer to the students as A, B, C, and so on. Any institution mentioned by students will also be identified by the letters of the alphabet.

Results and discussions

The first question to be analyzed is: “Do you think this mini-course contributed to your education? Why?” Based on the answers, we organized five initial categories and one final category, as shown in Table 2 below.

Table 2.

Categories of the first question.

INITIAL CATEGORIES	FINAL CATEGORY
Care regarding the use of mathematical language in the classroom.	General contributions of the mini-course to teachers’ practice and education.
Improvement of teaching practice.	
Encourage research.	
Contributions to mathematics theoretical studies	
Reflections on the nature of mathematical knowledge	

Students A, B, D, E, J, H, and G explain their contributions regarding the care that must be taken when using mathematical language in the classroom. Although mathematics has monosemy as one of its main characteristics, it is taught through the mother tongue, which is ambiguous and polysemic and can lead to confusion in understanding what is being communicated (Silveira, 2018; Pimm, 2002; Schleppegrell, 2007). Furthermore, mathematics has several particularities, such as specific mathematical vocabulary composed of different categories of words, which can cause students to confuse terms and teachers to procrastinate on words specifically used in mathematical contexts (Silva, 2019; Pimm, 2002; Schleppegrell, 2007).

> Mathematics as a language has some striking peculiarities: it is objective, monosemic, and loaded with its own symbolism. Full of precise terms, grammar structures, formality, impersonality, and timelessness, in addition to a mutual impregnation with natural language, as it does not have orality. It consists of a written symbolic system, while the orality used to read mathematical texts is borrowed from natural language, thus justifying the relationship of mutual dependence. (Cockroach, 2017, p. 21)

In this way, the mini-course helped to draw students' attention to the care needed in teaching mathematical concepts because mathematics teachers do not usually perceive the different linguistic challenges that permeate the acquisition of mathematical language (Pimm, 2002; Jaarsveld, 2016). This enables a critical and reflective look at how teachers use the multiple linguistic manifestations that permeate the classroom, such as the mother tongue, sign language, and mathematical language itself, which can make a difference in teaching practice (Silveira, 2020a; 2020b).

Students K and I demonstrated interest in deepening their research on the topic for professional development and postgraduate studies. This motivation reflects a significant contribution of the mini-course, encouraging research and teacher self-education. This critical and reflective approach is essential for teachers to develop autonomous thinking and constantly seek to construct knowledge, as Nóvoa (1992) highlighted. This initiative can be a helpful starting point for incorporating those discussions into professional practices and expanding investigations into language in mathematics teaching in Brazil.

Students I, F, and C highlight the contributions of the mini-course to improving teaching practice, emphasizing the importance of facilitating better learning for students. As mentioned above, linguistic knowledge is intrinsically linked to the domains of pedagogical content knowledge, which suggests that this knowledge can mobilize both content and pedagogical knowledge. This interconnection points to a potential improvement in teaching practice in general, as stated in the second section of this article.

Student L highlights the contribution of the mini-course to the application of linguistic knowledge in theoretical studies of various areas of mathematics. This reiterates that the knowledge can mobilize content knowledge, allowing teachers to pay more attention to the conceptual acquisition of mathematics. For example, they can begin to value mathematical procedures and concepts, explaining the difference between terms such as "number" and "numeral," "unknown" and "variable," "addition," and "sum," among others. The lack of mastery of these distinctions can make it difficult to teach mathematical vocabulary terms and may impair students' ability to give some justifications (Silva, 2019; Jaarsveld, 2016).

Shulman (1986) highlights the importance of teachers knowing more about what they will teach and being able to validate it. Following this premise, linguistic knowledge can be a starting point for teachers to question their mastery of mathematical language, starting to look at content knowledge differently, i.e., seeking greater appropriation of the concepts and procedures present in the different mathematical theories.

During activities and discussions, some students presented notes/questions about mathematical knowledge and its production/creation process. In particular, we highlight the questions asked by student J, which were the same as those asked by other participants in the mini-course:

> If I consider mathematics a science, am I disregarding that it has a language or disregarding its language? If mathematics is a language and theories in mathematics are created, how does this creation process take place? (Student J, 2024).

In this context, we highlight that the discussions held are close to the intuitionist philosophical current and the constructivist pedagogical tendency of mathematics, as it considers that mathematics is not rescued in a Platonic reality nor discovered in the empirical world (Fiorentini, 1995; Gottschalk, 2004). Considering that, in general, the school curriculum and initial teacher education courses are based on the perspective of infallible mathematics, far removed from human practices (D'Ambrosio, 1993), we can observe that one of the possible reasons for doubts and observations regarding mathematics is the "clash" between the student's conceptions, which, in general, are based on the classical formalist tendency, and the conception presented, which shows us another way of looking at mathematics.

Thus, we emphasize that our aim is not to try to convince students that the mini-course presented the "correct" way to understand mathematical knowledge but rather to discuss another perspective, seeking reflections on how this can contribute to teaching practice.

> That's interesting. I had never thought of mathematics this way. Until then, for me, in my studies of the subjects and scientific initiation, the transmitted idea was always that mathematics is something abstract and pre-defined, that we discover on the way. (Student J, 2024).

> I work as a mathematics teacher at the X and Y preparatory course. My stance in the classroom is closer to what is known as the classical formalist tendency. So, I place a

> lot of emphasis on formality in teaching mathematics and realize that this language issue, when used consciously, favors student learning. (Student L, 2024)

Thus, we observe that the mini-course contributed to reflections on the nature of mathematical knowledge. Angelo (2022, p. 80) mentioned that "the philosophical currents that deal with the foundations of mathematics have repercussions in the area of education and, consequently, in the conceptions and methodological postures of the teacher who teaches mathematics." Therefore, enabling reflections on the different perspectives of mathematical knowledge is fundamental for the education of professionals who adopt more critical and reflective attitudes toward their pedagogical practices. This can lead to more conscious teaching, adapted to students' needs, promoting a deeper understanding of mathematical concepts.

The second question in the questionnaire asks: "Throughout your academic and professional career, have you ever noticed any obstacles related to mathematical language in the classroom? If you do, which?" The categorization of empirical units gave us three initial categories: ambiguity and confusion faced by students, uses of language by the mathematics teacher, procrastination of mathematical language terms, and limitation of time to teach mathematical content. Through these initial categories, the final categories were reached, which are i) challenges related to the use of language in teaching mathematics and ii) challenges that are not related to linguistic issues.

Table 3.

Categories of the second question.

INITIAL CATEGORIES	FINAL CATEGORY
Ambiguity and confusion faced by students. Use of language by the mathematics teacher.	General contributions of the mini-course to teachers' practice and education.
Time limitation.	Challenges un related to linguistic issues.

Students A, E, J, and I report that they had noticed linguistic challenges, especially those related to the ambiguity of their native language, which often leads students to confuse concepts in mathematics. This situation highlights the importance of teachers paying attention to communication in mathematics and developing strategies that help identify misunderstandings and other linguistic challenges.

> This does not mean that the teacher can only say meaningful words, as we recognize that our language is flawed and can often be vague. We believe that if the teacher pays attention to the meaning that the student can give to the words he or she speaks, it is already a big step toward an attempt at communication because if both establish a language game, it is likely that they will somehow be able to participate in the same discursive universe. If the teacher identifies something that was vague in his/her explanation, he/she can try to replace the uncertain words with meaningful words, as uncertain words represent a danger to students' learning. (Silveira, 2020b, p. 10)

Therefore, the teacher must promote student oral and written participation in mathematical activities (Schleppegrell, 2007). Through students' linguistic manifestations, the teacher can identify doubts and confusion (Silveira, 2020a, 2020b; Silva et al., 2019). Furthermore, adopting different strategies, such as using games, can favor mathematical learning, making the teaching environment more inclusive and effective.

> Through students' language, we can find the origin of their confusion and errors, and also through language, we can teach them how to correctly translate a mathematical text so that the text provides meaning. The meanings of everyday language do not necessarily converge with those of mathematics. (Silveira, 2014, p. 70)

Students D, G, J, H, F, and C highlight the challenges that arise from the way teachers communicate when they are teaching. One example is the procrastination of mathematical language terms, which educators often use as a strategy to facilitate understanding. However, this approach can harm mathematical learning at later levels (Silva, 2019; Silveira, 2020b). Student G illustrates this issue by reporting that he faced difficulties during his undergraduate studies due to the lack of teaching about certain terms in basic education. Therefore, we reiterate that it is important to highlight the appropriate meaning of mathematical terms so that students have complete learning in the sense of knowing mathematical vocabulary appropriately (Silva, 2019; Jaarsveld, 2016).

Student G reported difficulties during his undergraduate studies due to the lack of teaching about specific mathematical terms in basic education. This situation highlights the importance of teaching that not only simplifies the topics but also prepares students for a deeper and more consistent understanding of mathematics: "Yes, in elementary school, there was a lack of teaching of some mathematical languages and terms. Because of this, I found it difficult to understand mathematical terms used in the undergraduate course" (Student G, 2024).

In addition, this student presented some oral contributions on some linguistic challenges observed throughout his trajectory:

> The teacher does not usually teach certain terms to students, such as commutativity, distributivity, inverse element, and others. He ends up procrastinating or using other terms referring to that property. For example, during this year's monitoring, I noticed that many first-year students still use the term "*chuveirinho*" [handheld shower] to refer to distributivity and have difficulties because they do not know some terms and relations that are being presented in the Sets and Functions subject. (Student G, 2024)

Given what was explained by student G, it is clear that procrastinating terms in mathematical contexts can lead to serious consequences. This practice can prevent students from developing the essential knowledge to learn concepts at more advanced levels, making it difficult to understand the mathematical rules being taught.

Furthermore, this situation creates significant difficulties in higher education since teachers often assume that students already have the basic knowledge necessary to understand

more advanced mathematical concepts, as mentioned by student D. This gap in education can compromise the academic performance and vocational education of future educators.

> I have already witnessed obstacles in the classroom, especially when teachers work with extremely advanced mathematical language made up of several terms that a large part of the class is unfamiliar with. They eventually assume that we already have the theoretical framework to understand how he imparts class, which creates many difficulties. (Student D, 2024)

These situations highlight the importance of developing students' mathematical vocabulary and providing learning that allows them to understand the meaning of mathematical texts. This implies an education in which students learn mathematical knowledge consciously rather than just mechanically applying algorithms. This way, it will be possible to build a learning experience in which students have a true "immersion" in the rules that govern the appropriate use of mathematical elements, allowing a deeper understanding of mathematical concepts. This approach can significantly contribute to educating more prepared and critical educators who can identify and overcome linguistic challenges in teaching mathematics.

Student C explained the challenges related to time limitations and school demands in his answer to the second question, which means that the teacher has to choose what is considered most important. This consideration is also presented orally by student G:

> The teacher ends up finding himself in a scenario where he must make choices and define the most important subjects to be addressed. There is often not enough time to provide solid development of mathematical language. What can be done is to implement strategies so students learn what is important for entrance exams. (Student G, 2024)

Following this premise, the lack of time and the impositions of some schools can be an obstacle to the development of mathematical learning in which students can adequately understand the meaning of mathematical texts. Schools often value the mathematical content required in the entrance exams of the region, which eventually disregards many mathematical concepts, as well as teaching that is in accordance with mathematical vocabulary, given that this is not necessary to take some entrance exams. This situation becomes even more harmful due to the large amount of content that must be taught in a short time; because of that, teachers sometimes choose not to address specific topics considered irrelevant for entrance exams. Thus,

it is possible to identify obstacles that, although not directly linked to teaching practice, can impact it.

From the above, we can see that the second answer was also a contribution from the formative activity: identifying linguistic challenges in the classroom. This can be a differentiator so that in-service and prospective teachers can have a critical and reflective look at linguistic issues that can jeopardize the teaching and learning of mathematical concepts.

From the analysis of the categories highlighted above, we see that linguistic discussions can be highly relevant for (prospective) mathematics teachers to critically analyze the different linguistic manifestations that permeate the teaching of mathematical concepts, considering the discussions carried out in their professional practices. This points to the possibility that these discussions mobilize both content knowledge and pedagogical content knowledge, which can be fruitful for initial teacher education and the search to improve the quality of mathematics teaching at an (inter)national level.

Final considerations

The search for ways to help alleviate obstacles in teaching and learning mathematics is constant. Among the fields of study that propose different approaches to mathematics and its teaching, studies on language in mathematics teaching stand out, considering that many challenges are of a linguistic nature (Feio, 2009; Silveira, 2014; Pimm, 2002; Silva, 2019; Schleppegrell, 2007). Teachers often do not perceive those barriers, which contributes to their persistence in school environments and the lack of critical reflections that could lead to a change in attitude on the part of mathematics educators.

Although the literature presents several discussions about language in mathematics teaching and its contributions to educational environments, we cannot expect in-service and prospective educators to critically appropriate this knowledge without it being part of their continuing teacher education process. Therefore, it is essential to seek strategies that bring in-service and prospective teachers closer to linguistic discussions. Thus, it will be possible to contribute to the continuing education process of those teachers and analyze, in practice, how discussions about language can impact the vocational education of mathematics teachers.

In this context, we direct our attention to mathematics teachers initial education, considering that this is responsible for the formal preparation and certification so that individuals can practice the profession, being composed of several discussions that influence the vision of prospective mathematics teachers about the profession.

We emphasize that, although no education is complete and finished, we argue that initial teacher education should, at least, engage with the demands placed on the profession, enabling teachers to identify these demands and seek ways to contribute to addressing them. This does not mean that all responsibility for the failure in teaching mathematics should be attributed to teaching degree courses. Nevertheless, (re)thinking teacher education can be a way to alleviate obstacles present in the classroom, contributing to educating critical and reflective teachers who can (re)construct themselves from different adversities.

Based on the above, in this work, we analyze the contributions of a formative activity focused on a linguistic approach to mathematics and its teaching for the initial education of mathematics teachers. We discussed teaching from an approach that considers mathematics a language. Furthermore, we drew connections between linguistic knowledge and the pedagogical content knowledge model proposed by Shulman (1986).

The results indicate that the formative activity made significant contributions to: i) the identification of linguistic challenges in teaching mathematics, which was explicit in the answers presented in the second question and the participants' reports; ii) reflections on the nature of mathematical knowledge, when the participants presented questions about mathematical knowledge and its production; iii) the encouragement of research, when the students reported an interest in researching more about language and teaching mathematics; iv) theoretical studies of different areas of mathematics, when one of the individuals reported that questions involving language could be used in his theoretical studies of mathematics.

Thus, we believe we have achieved our objective, which was to analyze the contributions of a formative activity focused on a linguistic approach to mathematics and its teaching to mathematics teachers' initial education, and to have answered our research question: How can a formative activity focused on a linguistic approach to mathematics and its teaching contribute to the initial education of mathematics teachers?

Furthermore, we believe that our research contributes to the mathematics education field, as it brings discussions and investigations on the relationship between language and mathematics teachers' education, which is an aspect that is not consolidated in the literature and can expand studies on this relationship, as well as encourage the search for the promotion of formative activities that aim to expand and disseminate discussions regarding language in mathematics teaching.

It is important to highlight that the formative activity developed lasted only three hours, which, although enabling contributions to teacher education, constitutes a limitation since it did not allow for a broader and more in-depth discussion of the topic. Thus, as a suggestion for future research on this topic, researchers could develop longer-lasting formative activities, such as subject matters or continuing education courses, analyzing the contributions of a linguistic perspective of mathematics and its teaching in the education of mathematics teachers working in basic education.

We also suggest research that seeks to relate linguistic knowledge and the education of (prospective) teachers in the initial years of elementary school, taking into account that these teachers are usually responsible for the process of mathematical literacy, which, as discussed throughout this work, can present linguistic challenges.

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