Editorial

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This issue is the third of the 2024 *Educação Matemática Pesquisa* journal. This year, we are editing volume 26, sharing the results of scientific research in mathematics education with our readers.

Researchers in didactics of mathematics have been concerned with calculus and mathematical analysis teaching –specifically, the concept of limit– for several decades. This concept is the cornerstone of calculus and mathematical analysis; however, it has been the source of recurrent and persistent difficulties in understanding and learning worldwide. Therefore, theoretical productions on teaching and learning limits are essential for mathematics education. This *Educação Matemática Pesquisa* issue aims to meet this objective. It is supported by a CNPq Universal Project coordinated by Sônia Maria Monteiro da Silva Burigato, a UFMS researcher. In this Project, Pierre Job, PhD in didactics of mathematics and professor at ICHEC Brussels Management School, Brussels, Belgium, coordinated different activities focused on building a reference epistemological model for limit. The participation of the PUC-SP in the project included, among other activities, the publication of this *Educação Matemática Pesquisa* special issue. The result was quite rich, with 23 articles.

Below, we briefly present the texts comprising this third issue of volume 26 of the 2024 *Educação Matemática Pesquisa* journal.

The first article is authored by Ana Karine Caires Brandão, Maria José Ferreira da Silva, and Saddo Ag Almouloud, *Double integral, quadric surfaces and the works of Antoní Gaudi: Possibility of developing a reference epistemological model.* The researchers propose a REM for teaching developed for the construction of a device called study and research path (SRP) involving quadric surfaces, more specifically, double integrals.

Marcio Vieira Almeida and Sonia Barbosa Camargo Igliori are authors of the article *David Tall's contributions to the development of a reference epistemological model (REM) for understanding the concept of derivatives.* The authors bring arguments guided by the inclusion of theoretical constructs, such as those developed by Tall for teaching derivatives, since, for

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them, these constructs add cognitive and didactic contributions to learners and teachers, reinforcing the importance of vigilance over the dominant epistemology of the concept of derivatives for teaching.

The third article, written by Cheick Oumar Doumbia, Saddo Ag Almouloud, and Luiz Marcio Santos Farias, is called *Un modèle épistémologique de référence pour la limite*. The authors intend to present a REM on the notion of limit based on the anthropological theory of the didactic.

Tainá Taiza de Araujo and André Luis Trevisan authored *Epistemological elements for teaching density and mass: Exploratory tasks through integrals of one and more variables*. In this article, the authors propose a study of epistemological elements of density and mass knowledge through multivariational integrals, since integrals are essential knowledge for the exact sciences. Expansive generalization was used to expand procedural questions of calculating an integral, just as reconstructive generalization was used to understand structural aspects of the Riemann integral of more than one variable.

Epistemological obstacles in learning the limit of real functions of a real variable was written by Emili Boniecki Carneiro, Maria Ivete Basniak, and Dion Ross Pasievitch Boni Alves. Based on productions from the Capes Catalog of Theses and Dissertations (CTD), with filters for academic the last ten-year productions of master's and doctorate degrees, a theoretical framework that categorizes epistemological obstacles regarding limits was developed.

The sixth article is by Pierre Job and Kevin Balhan and is called *Quelques considérations relatives à la notion de modèle épistémologique de référence (MER)*. For them, a central characteristic of didactics is questioning knowledge, which can be done mainly through a reference epistemological model (REM). They explain this notion and present a REM of the calculus and analysis.

A proposal for a reference epistemological model for the study of limits through dialogue via attention mechanisms is an essay by Edmo Fernandes Carvalho, Vinicius Souza Bittencourt, and Laerte da Silva Fonseca, aiming to offer proposals for tasks and praxeological analyses in a REM for teaching differential and integral calculus, redefining the diffusion of the notion of the limit of a function through definition.

Elisangela Pavanelo and Maria Aparecida Viggiani Bicudo are the authors of the tenth article, entitled *Hermeneutics and the doing of the mathematics teacher: A possibility of work in Differential and Integral Calculus classes*. The authors present a method of intervention in a Differential and Integral Calculus class of a degree course in mathematics when working on the

intermediate value theorem based on hermeneutic work with mathematics texts in the classroom.

The ninth article is entitled *An epistemological reference model in calculation and kinetics of chemical reactions*, authored by Jose Vieira do Nascimento Junior and Geciara da Silva Carvalho. They discuss the application of differential and integral calculus in chemical kinetics teaching in teacher education based on the reference and dominant epistemological models around the object laws of the velocity of a chemical reaction.

Karina de Oliveira Castro and Antonio Sales, in *Reference epistemological model in function teaching in its basics: A praxeological conception*, take as a starting point a study of the elements that underpin the REM through the anthropological theory of the didactic (ATD). The justification lies in the field of teaching calculus and in the attempt to contribute not only to the study of mathematical content itself but, mainly, to provide an alternative analysis through a praxeological model constructed for this purpose.

Renato Silva Ignacio, Valdir Bezerra dos Santos Junior, and Marlene Alves Dias are the authors of *The reference epistemological model as a hypothesis of a didactic problem and an example*. They brought an excerpt from a doctoral study that investigated the limits and possibilities of the didactic methodology study and research path (SRP) of the anthropological theory of the didactic (ATD) as a teaching alternative for basic education in Brazil.

The twelfth article is authored by José Gerardo Piña-Aguirre, Antonio M. Oller-Marcén, and Rosa María Farfán Márquez. With the title *The use of figures related to the complex integral and Cauchy's integral theorem in complex variable textbooks*, the authors addressed the similarities and differences between original mathematical works in complex analysis and contemporary textbooks regarding the use of figures (conceived as two-dimensional images) to address concepts in this branch of mathematics.

José Carlos de Souza Pereira, José Messildo Viana Nunes, and Saddo Ag Almouloud presented ideas linked to some objects of school mathematics that reveal transpositive connections pertinent to the elaboration of reference epistemological models, linking them to the notions of mathematical objects of the differential and integral calculus, in an article entitled *Transpositive connections from the perspective of developing reference epistemological models based on school mathematics objects*.

Conway's notion of real numbers and the principle of complementarity, some contributions to the development of reference epistemological models is the title of the article written by Rogério Ferreira da Fonseca, and Sonia Barbosa Camargo Igliori. In this article, the authors highlight the potential of Conway's theory regarding the classical concept of numbers,

with a view to contributing to the development of a REM for teaching differential and integral calculus.

The fifteenth article, *The* Principia *by Isaac Newton: A proposal for an epistemological model for teaching integral mathematics in undergraduate mathematics* courses, authored by Everaldo Roberto Monteiro dos Santos, Lucélia Valda de Matos Cardoso, and Reginaldo da Silva, proposes an alternative epistemological model (AEM) for teaching calculus in mathematics teaching degree courses, using the *Principia* by Isaac Newton.

Catarina Lucas and Josep Gascón wrote the article *Construction of an epistemological reference model as the foundation of a new didactic paradigm for the study of differential calculus*, supported by the anthropological theory of the didactic (ATD). In this article, they formulate, in a coordinated manner, an outline of a reference epistemological model for functional modeling REM(FM), which gives elementary differential calculus (EDC) a new reason for being.

In the article *Between intuition and formalization of calculus: Applications of derivatives in comics*, Fabiana Alves dos Santos, Fábio Nunes da Silva, Lauriclecio Figueiredo Lopes, and Priscila Santos Ramos present the results of a didactic experiment whose objective was to analyze the potential of comic books in promoting learning in higher education, especially in the Differential Calculus I component, during the remote period resulting from the COVID-19 health emergency.

The eighteenth article, entitled *Prolegomena for the construction of a reference epistemological model for teaching Calculus: What are models? What is Calculus?*, was written by Bartira Fernandes Teixeira and Luiz Marcio Santos Farias, proposing a theoretical reflection on the definition of models and Calculus, pointing out preliminary notions and basic principles so that a REM can be constructed for teaching this subject.

Vera Souza and Ana Nobre Veloso's article *Toward a shared reference epistemological model (REM)?* In this article, the authors discuss the proposal of the Study Group on Calculus in Secondary Education and Higher Education, led by researchers Dr. Pierre Job (ICHEBrussels Management School-Belgium) and Dr. Luiz Márcio Santos Farias (UFBA-Brazil), aiming to create a joint Brazil-Belgium project in search of a reference epistemological model for teaching calculus and analysis.

Mustapha Rachidi and José Luiz Magalhães de Freitas, in the article *Some considerations on the constructions of the set of real numbers: a need for an epistemological model of reference?* present some elements about the set of real numbers and a synthetic view of the rigorous constructions of this set in the 19th century, which became a requirement for the

arithmetization of mathematical analysis with the work of Cauchy and Weierstrass. They analyzed some didactic considerations concerning the teaching of the set of real numbers in high school and at the beginning of university, in order to provide subsidies for the elaboration of epistemological models of reference for studies and research into the contents of functions, limit and continuity, among others.

A discussion on the definition of the limit of a sequence is the title of the article by Sonia Maria Monteiro da Silva Burigato, Claudemir Aniz, and Lilian Milena Ramos Carvalho, bringing a discussion about the concepts involved in the conceptual field of a limit of a sequence, together with the analysis of a proposed situation.

In *The concept and theorem in action as elements for the integration of pragmatic and deductive epistemological models*, Anderson Souza Neves, Sonia Maria Monteiro da Silva Burigato, and Luiz Márcio Santos Farias present a study on the construction of the concept of function limit using two research projects, one completed and another in progress, with different theoretical references, based on the theory of conceptual fields and the anthropological theory of the didactic.

The last article, entitled Subsidies for the development of an epistemological model of reference for understanding the concept of ordinary differential equations, by Celina Aparecida Almeida Pereira Abar and Amábile Jeovana Neiris Mesquita, presents a study on Ordinary Differential Equations, considering some historical elements on the subject, as well as difficulties and advances in the teaching and learning processes, taking in the results of some research. Modeling and the theory of semiotic representation registers made it possible to compose a scenario that highlighted the knowledge necessary for teaching ordinary differential equations.