

Didactic macro-decisions: analysis of a lesson plan from the point of view of the development of algebraic thinking

Macrodecisiones didácticas: análisis de un plan de clase desde el punto de vista del desarrollo del pensamiento algebraico

Macro-décisions didactiques : analyse d'un plan de cours du point de vue du développement de la pensée algébrique

Macrodecisões didáticas: análise de um planejamento de aula sob o ponto de vista do desenvolvimento do pensamento algébrico

Lívia Elaine da Silva Santos¹

Universidade Federal de Pernambuco (UFPE-CAA)

Mestrado em Educação em Ciências e Matemática

<https://orcid.org/0000-0001-7501-0797>

Fernando Emílio Leite de Almeida²

Instituto Federal de Pernambuco (IFPE)

Doutorado em Ensino das Ciências

<https://orcid.org/0000-0001-7059-8050>

Abstract

This work is part of a master's research and aims to analyze the mathematics teacher's macro-decisions for the development of algebraic thinking in students of the seventh grade of Elementary School. We take as a theoretical reference the French-influenced Mathematics Didactics, particularly the Teacher's Activity Levels Model. It starts from the premise of how the participant, through his didactic decisions, can contribute to the development of this way of thinking. Our study had the participation of a mathematics teacher who teaches in the final years of basic education, in a state school, located in the city of Pesqueira, municipality belonging to the Agreste region of Pernambuco – Brazil. The data were constructed through the analysis of the lesson plan on the knowledge of first-grade equations, prepared by the participating teacher, and a semi-structured interview. The obtained results show that the notion of algebraic thinking needs to be widely discussed in initial and continuing teacher education, as well as clarified in curricular guidelines on the teaching of algebra. In general, the teacher points out several important teaching strategies for the development of algebraic thinking. We also add that the choices made and the didactic decisions taken by the teacher establish an approximation with this way of thinking.

¹ liviahelaine@hotmail.com

² fernandoemilioleite@yahoo.com.br

Keywords: Teaching algebra, Algebraic thinking, Didactic decisions, Elementary school.

Resumen

Este trabajo forma parte de una investigación de maestría y tiene como objetivo analizar las decisiones macro del profesor de matemáticas para el desarrollo del pensamiento algebraico en estudiantes del 7° año de la Enseñanza Fundamental. Tomamos como referencia teórica la Didáctica de las Matemáticas de origen francés, particularmente el Modelo de Niveles de la Actividad Docente. Parte de la premisa de cómo el participante, a través de sus decisiones didácticas, puede contribuir al desarrollo de esta forma de pensar. Participó en nuestro estudio un profesor de matemáticas que enseña en los últimos años de la educación básica, en una escuela pública, ubicada en la ciudad de Pesqueira, municipio de Agreste de Pernambuco - Brasil. Los datos fueron construidos a través del análisis de la planificación de la lección sobre el conocimiento de la ecuación de 1° grado, elaborado por el profesor participante y una entrevista semiestructurada. Los resultados obtenidos muestran que la noción de pensamiento algebraico necesita ser ampliamente discutida en la formación inicial y continua del profesorado, así como aclarada en las orientaciones curriculares sobre la enseñanza del álgebra. En general, el docente señala varias estrategias didácticas importantes para el desarrollo del pensamiento algebraico. Agregamos también que las elecciones realizadas y las decisiones didácticas tomadas por el docente establecen una aproximación con esta forma de pensar.

Palabras clave: Enseñanza de álgebra, Pensamiento algebraico, Decisiones didácticas, Escuela primaria.

Résumé

Ce travail s'inscrit dans le cadre d'une recherche de maîtrise et vise à analyser les macro-décisions du professeur de mathématiques pour le développement de la pensée algébrique chez les élèves de la 7e année du primaire. Nous prenons comme référence théorique la Didactique des Mathématiques d'origine française, en particulier le Modèle des Niveaux d'Activité de l'Enseignant. Il part du principe que le participant, par ses décisions didactiques, peut contribuer au développement de cette façon de penser. A participé à notre étude un professeur de mathématiques qui enseigne dans les dernières années de l'enseignement de base, dans une école publique, située dans la ville de Pesqueira, municipalité d'Agreste de Pernambuco - Brésil. Les données ont été construites grâce à l'analyse de la planification de la leçon sur la connaissance de l'équation de 1ère année, préparée par l'enseignant participant et un entretien

semi-structuré. Les résultats obtenus montrent que la notion de pensée algébrique doit être largement débattue dans la formation initiale et continue des enseignants, ainsi que précisée dans les orientations pédagogiques sur l'enseignement de l'algèbre. De façon générale, l'enseignant signale plusieurs stratégies d'enseignement importantes pour le développement de la pensée algébrique. Nous ajoutons également que les choix effectués et les décisions didactiques prises par l'enseignant établissent un rapprochement avec cette façon de penser.

Mots-clés : Enseignement de l'algèbre, De la pensée algébrique, Décisions didactiques, École élémentaire.

Resumo

Este trabalho é um recorte de uma pesquisa de mestrado e tem como objetivo analisar as macrodecisões do professor de matemática para o desenvolvimento do pensamento algébrico de estudantes do 7º ano do Ensino Fundamental. Tomamos como referencial teórico a Didática da Matemática de origem francesa, particularmente o Modelo de Níveis da Atividade do Professor. Parte-se da premissa de como o participante através de suas decisões didáticas pode contribuir para o desenvolvimento dessa forma de pensar. Participou do nosso estudo um professor de matemática que leciona nos anos finais da educação básica, de uma escola da rede estadual de ensino, localizada na cidade de Pesqueira, município do Agreste de Pernambuco – Brasil. Os dados foram construídos por meio da análise do planejamento de aula sobre o saber equação do 1º grau, elaborado pelo professor participante e de uma entrevista semiestruturada. Os resultados obtidos mostram que a noção de pensamento algébrico precisa ser amplamente discutida na formação inicial e continuada de professores, bem como clarificada nas orientações curriculares sobre o ensino de álgebra. De modo geral, o professor aponta várias estratégias de ensino importantes para o desenvolvimento do pensamento algébrico. Acrescentamos ainda, que as escolhas feitas e as decisões didáticas tomadas pelo professor estabelecem aproximação com essa forma de pensar.

Palavras-chave: Ensino de álgebra, Pensamento algébrico, Decisões didáticas, Ensino fundamental.

Didactic macro-decisions: analysis of a lesson plan from the point of view of the development of algebraic thinking

In the initial period of teacher training, some research indicates that several concerns arise that usually become strong research questions in the construction of teacher identity. (Pimenta, 2012). Particularly, the concerns existing in the initial training about the teaching of algebra were strengthened in professional practice and became motivation for research, in particular, the interest in the central issues for the development of algebraic thinking in the final years of Elementary School, on which the idea of constructing meaning for algebra emerges.

Accordingly, developing algebraic thinking is to construct meaning both for language and for a given algebraic object, such as equations, systems of equations, inequations, among others. Thus, it is something that goes beyond the simple manipulation of symbols. Therefore, it requires the understanding of concepts and of the algebraic language itself.

The investigation also suggests that, in didactic situations that provide the mobilization of this way of thinking, students construct more knowledge. In this sense, the teacher's role is essential in several aspects, among which we highlight is the way in which he/she plans his/her classes and the activities to be developed in the classroom. It happens in a way that they can provide the student with the opportunity to understand the use of letters in mathematics, showing the function of algebraic language in a meaningful way.

In this context, it was also found that one of the main problems related to the teaching of algebra is strongly linked to the lack of relationship between thought and language. In turn, the mechanical manipulation of symbols has generated the use of algebraic language without understanding by students and the absence of a work focused on the purpose of algebra in basic education – the development of algebraic thinking (Brasil, 2017).

In view of this, we can observe, in general, the relevance of a discussion that relates the purpose of algebra and the teacher's didactic decisions, in particular, those made in the preparation of the class. This is because the resource system of a mathematics teacher and its relationship with the teaching of a certain content are factors that influence and imply in the creation of an environment that favors student learning. Accordingly, we understand that it is important to look at the moment when students will experience the teaching situations in which algebraic language will be introduced and, in this bias, try to understand the way in which this language is used for the teaching of algebraic objects that are also worked on in this phase.

In this sense, most of the time, there is no dialectical relationship between thought and algebraic language in approaches to algebra. In addition, another aspect that we can highlight is that from the final years of Elementary School, extending to the other stages of basic

education, students will deal with this language in various teaching situations, whether in the algebraic field or in other mathematics fields.

Therefore, the way in which algebra has been treated over the years has not generated positive results in terms of student learning; on the contrary, in this field of mathematics, the difficulties presented by them in the teaching and learning process have been discussed. We can observe such negative results in external evaluations, through tests applied by the Basic Education Evaluation System (SAEB, as per its Portuguese acronym), which is an evaluation process carried out by the National Institute of Educational Studies and Research Anísio Teixeira (INEP, as per its Portuguese acronym), in several regions of the country, as well as the Basic Education Evaluation System of Pernambuco (SAEPE, as per its Portuguese acronym), at the state level, which reveal that the students' rate of correct answers in the items related to algebra is very low in these evaluations (Almeida, 2019).

It is perceived that students present a picture of aversion to mathematics in the final years of Elementary School, a moment when they traditionally come across symbolic algebraic language. In general, students are unmotivated, which is why we believe that these facts are linked to superficial approaches with emphasis on meaningless algebraic manipulations performed by teachers.

Considering that the teaching of algebra encompasses a large part of the contents in the aforementioned phase until the advancement of basic education, we feel the need to understand the teacher's role in the promotion of algebraic thinking from the point of view of the didactic decisions to be made by him/her to improve the learning of students in terms of algebra.

However, in this work, we will focus on the following question: how can the mathematics teacher, through his/her didactic decisions, contribute to the development of algebraic thinking of students in the 7th grade of Elementary School in the solvability of first-degree equations?

In this sense, in order to have a more current perspective of the teaching of algebra in the educational process in basic education, it is not enough to look at the symbols, that is, at the algebraic language and thus develop the work in the classroom, but rather to take into account a look through these symbols, understanding their different meanings and application in the various teaching situations (Blanton & Kaput, 2005; Radford, 2009).

In addition, we must take into account that the knowledge and conceptions of the mathematics teacher are factors that influence the choices and didactic decisions made from the moment of preparation of the class until its execution, in particular, in the teaching of algebra will take place with an emphasis on algebraic language or algebraic thinking. Thus, the role

performed by the mathematics teacher in the teaching and learning process can be the apex for the creation of an environment of interest and learning for students.

In this sense, there is a desire to focus specifically on the teacher's didactic decisions, in particular the macro-decisions, which are those taken at the time of construction of the lesson plan on the teaching of the concept of first-degree equations, with a view to promoting the development of algebraic thinking.

We believe that the lesson plan developed by the teacher participating in this study will give us the opportunity to analyze and understand aspects related to his teaching intentions, such as the organization of the teaching sequence, the way in which he plans to develop it, as well as the choices of activities and resources. In addition, if these measures are likely to lead students to develop algebraic thinking.

From this perspective, we took for our study the field of investigation of French-influenced Mathematics Didactics, an area of knowledge that allows, among other aspects, to understand the determining elements of the didactic situations that institute the didactic system, where the interactions among the teacher, the student and the mathematical knowledge in an environment are established.

In this sense, we consider that the structuring of the environment developed by Brousseau (1986, 1998) and later expanded by Margolinas (2002, 2005) is a fundamental part of modeling the teacher's didactic decisions, as it allows us to consider both the didactic situation and the activities outside this situation (Brasset, 2017).

According to Brousseau (2008), we can identify that the teacher assumes two distinct positions in the structure of the environment: the first concerns the teacher who prepares the class and the second the teacher who teaches. Thus, we will use more precisely the first position assumed by the teacher, in order to understand the way in which he/she organizes the environment for passing on knowledge related to first-degree equation, especially if the lesson plan enables the development of algebraic thinking by students in the 7th grade of the final years of Elementary School.

According to Lima, Faria and Martins (2017), teaching can be seen as a sequence of decision-making by the teacher, and the authors has drawn attention to the fact that creating and organizing a class are not easy tasks, even for experienced teachers, because, in order to be successful in class, teachers need to make choices and create a situation with good didactic intentionality. Thus, for this moment of preparation, he/she has several possibilities of choices, keeping in mind some questions: "What is the best way to approach content? What problems to choose?" (Lima, Faria & Martins, 2017, p. 76).

Based on the perspective of algebraic thinking, which is underpinned by the idea that the student thinks about mathematical knowledge and, consequently, constructs meanings for both the algebraic object and the language, it is understood that, in this perspective, learning is obtained through the construction of a means that favors the development of students' algebraic thinking skills.

Accordingly, the structuring of the environment to the detriment of didactic decisions and algebraic thinking lead us to question the existing relationship between both, as well as the factors that influence such decisions. In this regard, the model of levels of teacher activity, developed by Margolinas (2005) makes it possible to look at the teacher's activity at different levels, whether outside the classroom (macro-decisions) or inside the classroom (micro-decisions). This model envisions the teacher's role in the didactic relationship that is established among the teacher, the student and knowledge in an environment. In order to better explain these levels, in the following table, we will present:

Table 1.

Teacher's Activity Levels Model (Margolinas, 2005, p. 11)

Levels	Description
Level +3: values and conceptions about teaching and learning.	Educational project: educational values, teaching and learning conceptions.
Level +2: theme construction.	Overall didactic construction in which the class is included: notions to be studied and learning to be achieved.
Level +1: lesson plan.	Specific teaching project for a lesson: planning the work.
Level 0: teaching situation.	Executing the class, interacting with the students, making decisions in action.
Level -1: observation of student activity.	Perception of student activity, regulation of work assigned to students.

That said, in this work, our main interest is the teacher's activity that consists of planning (level +1), seeking to analyze the decision-making for the preparation of a didactic situation from the point of view of the development of students' algebraic thinking about the teaching of first-degree equations.

Thus, the model presented above, as much as it advocates the existence of levels in a certain order, does not advocate that such levels are totally independent of each other, and can often be worked on concomitantly. Accordingly, such a model allows to guide the teacher's activity, enabling him/her to self-evaluate at each level. In addition, this categorization also helps the teacher to identify the choices to be prioritized during the preparation of the class, so

that it is possible to make the decisions that best respond to the pedagogical practice and the individual needs of his/her students.

According to Margolinas (2005), all levels interact with each other, because when a teacher plans his/her class (level +1), he/she also interacts with what is beyond the classroom, since he/she considers what he/she can accomplish and observe during the class.

Corroborating this discussion, the authors Espíndola, Júnior and Silva (2018) point out that “the class project that will be constructed is conditioned to the choices made at the level of the construction of the theme, and finally to the didactic situation to be experienced, this, [which] in turn, is largely determined by the previous choices” (Espíndola, Júnior e Silva, 2018, p. 38).

In this sense, based on the aforementioned levels of the model, we will focus particularly on those that correspond to macro-decisions, depending on the decisions made by the teacher at the time of preparing the class. Accordingly, our analysis will be carried out from top to bottom: level +3 (values and conceptions about teaching and learning), level +2 (theme construction) and level +1 (lesson plan).

In this regard, regarding the teacher’s didactic decisions at the higher levels of the presented model, particularly about the moment of planning, Lima and Trgalová (2010) consider that:

When preparing the plan, the teacher tries to anticipate what will happen at the time when he/she is interacting with his/her students. He/she then determines the teaching objectives and the means required to achieve these objectives. Among other aspects, he/she organizes future actions in terms of choosing problems and means required to carry out teaching; determines the time to experience each planned action and organizes the students’ work. In addition, it is at this moment when the teacher must choose the evaluation instruments that will allow him/her to observe, in a pertinent way, whether there has been learning on the part of the student (Lima & Trgalová, 2010, p. 55).

In fact, when the teacher prepares a teaching sequence, he/she is subjected to demands from different sources, in addition to factors that influence his/her choices, such as, for example, the types of knowledge on which he/she relies for decision-making, his/her relationship with a certain knowledge, resources, etc.

According to Margolinas (2002), when the teacher reflects on the didactic situations, which are transformed in the didactic means and assumes the position of preparing his/her class, he/she is in a situation that requires the review of the decisions made, analysis of his classes, as well as studying the behavior of students through actions, knowledge and specific skills. We understand that such decisions and interactions with different media can cause transformations

in the teacher's own knowledge and produce learning on his/her part.

This conception translates what is of interest in this study about the teaching of algebra, about which we do not seek to look at the teacher only as an actor during the class. On the contrary, it is important to rethink teaching practices in the light of macro-decisions, in other words, we seek to study the teacher's activity at a level outside the classroom, that is, when he/she is not in interaction with students and knowledge.

We understand that the act of deciding, both in macro-decisions and in micro-decisions, which is in accordance with Margolinas (2005), represents an important moment in teaching practice. Accordingly, didactic decisions should be guided by the intention of creating a learning environment that favors the development of algebraic thinking and the different ways of expressing it, through opportunities for contact, use and exploration of the algebraic language that is proposed to students in the final years of Elementary School.

It is worth underlining that we do not intend to characterize in this study the different conceptions that the participating teacher mobilized when planning his class, but rather if his teaching objective is to develop the students' algebraic thinking or is to teach them to solve equations. Likewise, we do not intend to explore all the possibilities of using the model in question, its advantages and limitations, nor the methods of mastering Mathematics Didactics as the most effective way to teach mathematics.

In this context, when we undertake our study on the teaching of algebra from the point of view of the development of algebraic thinking, we seek to discuss algebra in a reflective way, through the deepening of some central aspects that enable the identification of the knowledge to be mobilized by the teacher participating in the research, as well as the resources used by him, and linking them to our theoretical framework.

Algebraic thinking

The first studies focused on discussing the teaching of algebra date from the late 80s and early 90s of the XX century. From the 90s, a series of studies aimed at analyzing this field of knowledge emerged; however, according to Miguel, Fiorentini and Miorim (1992, p. 52), one of the challenges about these works would be to carry them out in such a way that they could "make explicit the specificity of algebra and the role that it plays in the history of human thought, particularly in the history of scientific and mathematical thought". Other studies also announced the need for a reflection on the teaching of algebra in basic education, such as Kaput (1999).

In a way, this concern is still part of a set of issues related to the teaching of algebra, and permeates investigations over the years, especially contemporary ones inside and outside the Brazilian setting. In order to start the discussion, we find the following pertinent questions: What is algebra in Elementary School? What is algebraic thinking?

For Câmara dos Santos (2017), working with algebra in Elementary School implies new perspectives, such as the development of a type of thinking fundamentally linked to the abstract relationships with which this field works more directly, designated as a central element for the construction of meaning in algebra, namely, algebraic thinking. On the other hand, if this didactic approach does not intend to develop the student's act of thinking, this construction of meaning about algebra will hardly be achieved.

Booth (1995) considers algebra as a symbolic language, used to assign values to unknown quantities. In turn, Usiskin (1995) points out four conceptions about algebra, generalized arithmetic, the study of relationships, algebra as a structure and algebra as a means of solving problems. Nevertheless, talking about algebra in Elementary School and algebraic thinking, more specifically, in the final years of this stage of schooling, implies considering the conceptions and knowledge developed by teachers.

In this sense, the formal work of the algebraic language starts in the second stage of Elementary School, when most students have already had contact with the use of letters in the teaching of mathematics, for example, the standard measure of length, symbolically represented by the letter "m" (meters). Nonetheless, the letter in this type of didactic approach is not used to designate unknown numerical values, but rather to represent measurements, a denomination used on a large scale in arithmetic works. Accordingly, the use of letters in the final years of Elementary School, especially in the algebraic field, starts to have new meanings.

In our case study, it is worth emphasizing that the students are in the seventh grade, which means that they are in the first year of studying algebra using alphanumeric language. Nevertheless, we focus on the double conception between manipulating symbols, through techniques and procedures essential to solving an equation, and thinking algebraically, a process in which the student shows that he/she is capable of establishing relationships, operating with the unknown term in an analytical way, making generalizations and modeling problems in natural language for symbolic language, among other characteristic aspects of this way of thinking (Almeida, 2016).

Thus, with regard to algebraic thinking, we understand that its development is subject to strong links between the teacher's didactic intentions and the instruction of the activity, since it is not only the activity that leads to thinking algebraically, but the way in which this activity

is worked, as this can influence the knowledge that the student will invest at the time of his/her activity.

Therefore, the teacher can play an important role so that the student can think and attribute meaning to the studied concept. In addition, the promotion of algebraic thinking should not be linked to the solvability of activities with a merely algebraic structure, but rather to the development of skills related to this thinking through a set of decisions made by the teacher. This is in line with Blanton and Kaput (2005), among other factors they highlight that:

Constructing a practice that develops children's algebraic reasoning requires a significant process of change for Elementary School teachers, who are often educated in different arithmetic ways of doing mathematics. In summary, Elementary School teachers should develop "eyes and ears" algebra as a new way of looking at the mathematics with which they are dealing, teaching, and listening to students' thoughts about it (Blanton & Kaput, 2005, p. 443).

We share this perception, considering that the teaching of algebra for a long time was seen as the introduction of symbols (letters) to represent, most of the time, indeterminate quantities, whose main focus is the manipulation of these symbols through the application of rules and procedures used in the solvability of equations, for example, "the number that passes to the second term changes sign" and vice versa.

Nevertheless, today, it is known that it is not only required to use the algebraic language for meaningful learning in algebra. This is because the development of algebraic thinking is placed in the curricular guidelines of the Brazilian National Common Curriculum Base (BNCC, as per its Portuguese acronym), as a center for the teaching of the thematic unit related to algebra from the early years.

Therefore, one of the main objectives for the teaching of algebra in basic education is to develop a special type of thinking, as guided by one of the most current curricular proposals. In this regard, the BNCC provides the following guidance:

In turn, the thematic unit related to algebra, aims to develop a special type of thinking – algebraic thinking – which is essential to use mathematical models in the understanding, representation and analysis of quantitative relationships of quantities and also of mathematical situations and structures, making use of letters and other symbols (Brasil, 2017, p. 270).

In fact, the purpose of teaching algebra defended by the document in question is related to the evidence that algebra and algebraic thinking are intrinsically related, as well as should be present throughout the educational process of basic education. These questions have permeated research on the second theme in particular for decades and cause concern to scholars in this

area, as this relationship is mostly ignored in approaches to algebra. In this respect, the work with algebra in the different stages of Elementary School is distinguished by the fact that the document recommends the use of symbolic representation only from the second phase, respectively, the seventh grade of Elementary School.

In the midst of discussions among researchers who focus on algebraic thinking, it is noted that the debate on algebra was essentially symbolic in nature for a long time. The reflections of the emphasis given to language caused the students to present an aversion to mathematics, which can be seen through the negative results, as mentioned before.

Thus, if we think about the teaching of algebra from the point of view of its purpose based on the literature and text of the BNCC and as an indispensable condition for learning with meaning and understanding by students, we see that algebraic language, by itself, is not enough to promote and demonstrate success in terms of teaching.

In view of this, we call attention to the range of choices that the teacher has to make decisions about his/her teaching project, the lesson plan, on which he/she relies to create a means, which, among other things, have to favor the thinking, communication and autonomy of students in the classroom and, in this bias, overcome the teaching model that was constituted on algebra.

In this sense, we understand that algebraic thinking does not develop spontaneously, that is, it does not emerge naturally. This requires a greater interest on the part of the mathematics teacher in the final years of Elementary School in terms of assisting the student in the acquisition of fundamental mathematical notions of algebra, which allow the use of letters to represent numbers, model a natural language problem for algebraic through first-degree equations, operate with the unknown (letters) as if they were numbers. From this perspective, the didactic approach to algebraic language should stimulate the development of skills that contemplate several dimensions, such as interaction in the classroom, since algebraic thinking can be expressed through different languages, including gestural languages, depending on the student's level of experience with teaching situations that promote skills inherent to this way of thinking, as Blanton and Kaput (2005) point out, and autonomy by allowing the student to construct his/her own knowledge..

According to Soares (2018), we must rethink the way in which we view the teaching of algebra, since:

To deal with the development of algebraic thinking is to have a new look at the teaching of algebra. It is to consider that learning algebra goes far beyond manipulating algebraic terms in expressions and equations. It also means considering that algebra permeates

different branches of mathematics, which can favor the construction of relationships and favor the understanding of the studied mathematical objects (Soares, 2018, p. 195).

Thus, it is assumed that the choices and decisions made by the teacher before the didactic situation developed within the classroom, especially the factors that influence him/her in terms of making these decisions, are intended to lead students to learn algebraic concepts with meaning. For this, based on the role of symbols in the teaching of mathematics, the letters used in algebra promote a new meaning for its teaching, since, when worked with a focus on the construction of meaning, it helps the student to develop a more analytical thinking.

In this line of thought, we consider algebra to be that worked in basic education (or should be), with the objective of developing algebraic thinking through teaching situations, and that, by developing it, the alphanumeric language will also be known and understood by students, as Almeida defends (2016).

In the context of the perspectives of algebraic thinking, it is noted that the discussions are somewhat complex because there is no single definition among researchers in the area for the concept of this way of thinking mathematically; however, there is a broad consensus on the importance of this thinking being mobilized by students in the process of teaching and learning algebra. For this reason, several authors who focus on the theme present some central aspects of thinking algebraically, called characterizations, categories/strands or ways of thinking. We understand that these classifications can help the teacher's work, as they help in the identification of strategies and the level of algebraic thinking at which the student (more developed) is in terms of solving problems.

As previously exposed, the structuring of the environment allows us to consider the teacher's didactic situation, that is, the teaching activity. In addition to the phenomena that can be observed in the classroom, resulting from teaching strategies, the activities of the teacher outside this situation also deserve to be investigated, such as his/her didactic knowledge related to a certain mathematical content and his/her lesson plan, since a large part of what happens during the didactic situation depends on the environment that he/she has organized.

Nevertheless, armed with this discussion, we believe that the development of thought is a way to help the student develop a new mathematical language, the algebraic language, and, consequently, understand concepts from the algebraic field, such as equations. Therefore, regarding algebra and its teaching, we consider it important to discuss the process of constructing the lesson plan inherent to the didactic macro-decisions addressed in our study.

Methodology

With regard to the methodology, we will divide the approach into two main segments, namely: methods and main activities. As for the first set, we will use an approach that starts with a qualitative approach, which, according to Minayo (2012), focuses on the act of understanding the object of study from the perspective of those who are in the opposite situation to that of the researcher, immersing himself/herself in his/her sensitivity. Along with this approach, the inductive method will also be added and, finally, we will use the descriptive method as a paradigm.

Accordingly, as we have as direct sources for the construction of the data a virtual environment, a participating mathematics teacher who has practical experience with the researched problem and a researcher who assumes the role of explaining and interpreting the reason for the facts or phenomena that occur, without the intention of quantifying values, we can say that our research is constituted with the logical bases of qualitative research. In view of this approach, as well as the described general objective, we consider that our investigation consists of a case study.

It is worth highlighting that, due to the pandemic moment experienced during this study, field research in the school environment, which we intended at the beginning of this study, became unfeasible. In view of this, we opted for the virtual environment, through an online meeting held by the Google Meet platform, which, in turn, ensured greater security for the participants, with regard to the social context. Here, videography appears as a fundamental resource to capture all data, both said and unsaid, in the most legitimate way possible.

After a first moment of acclimatization and the establishment of a conversation with the participant, primary data will be collected from a semi-structured interview with the mentioned mathematics teacher (BONI; QUARESMA, 2005). To that end, the teacher was personally invited to participate in the study individually in the school environment where he teaches. Soon after, an e-mail was sent to him with the attachment of the Free and Informed Consent Form about the research, in order to provide the participating candidate with the main information. In addition, we suggest the dates and times of the meetings and are open to any proposal, so that there are no possible interruptions. Furthermore, the confirmation of acceptance with the signature of this document was also made via e-mail by the teacher before the construction of the data, as well as, at the beginning of the first online meeting with the participant, the term was read and clarified, in order to resolve any doubts.

In order to carry out the construction of the data, we used a notebook with Wi-Fi internet access for the stages that were experienced with the teacher, fundamentally through two meetings: the first for a conversation about the research and the second for a lesson plan on the

concept of first-degree equations, through a semi-structured interview. To that end, we made use of the Google Meet tool, which allows us to record through audio and video and to save the meeting in a computer folder. This planning was prepared and sent via e-mail by the teacher before the interview. In addition to a notebook, the researcher had a cell phone to be used as a support, if required, during the interview with the teacher. For our records, in addition to the audio and video recordings, a logbook was used for notes that the researcher considered important, as well as the transcription of the interview. Regarding the use of data construction instruments, we emphasize that the participating teacher was informed in advance of their use.

Given the instruments and procedures for constructing data, we believe that there may be some risks for the research volunteer, such as invasion of privacy about his teaching practice, discomfort when answering some questions and breach of confidentiality and anonymity. Therefore, in order to avoid and/or reduce adverse conditions that could cause the aforementioned risks to the participant, we chose the semi-structured interview because it provides open questions in the form of an interview guide. Thus, the researcher was attentive to the signs of discomfort in relation to the answers given, in order to, if required, expand the questions. In addition, the confidentiality and privacy of the data were ensured, excluding the identity of the participating teacher or any other indications that could identify him from scientific publications, as well as ensuring the protection of his image.

Based on the above, the research was developed from four main stages: (1) Online meeting with the teacher about the research; (2) Online meeting with the teacher and semi-structured interview; (3) Transcription of the constructed data and (4) Analysis and discussion of the data.

The first stage consisted of an online meeting, through the Google Meet platform, without recording. The main objective of this meeting was to ask the teacher to prepare a lesson plan for the teaching of the concept of first-degree equations.

In the second stage, we conducted a semi-structured interview with the teacher, in order to know the conceptions and knowledge that he mobilized for the lesson plan, in addition to his influences on the didactic choices and decisions made at the time of preparation of the teaching sequence (level +1), depending on the level of the teacher's activity that precedes the class, considered as macro-decisions. At that time, we sought to resume some of the elements that were presented in the lesson plan, with the objective of comparing the teacher's answers with the aforementioned plan. Thus, we asked the teacher to provide information about the plan and its teaching objectives, so that he could justify the choices and decisions that he considered important and indispensable, especially in relation to the choices of activities proposed for the

students, material resources used by him, class organization, support material for the construction of the theme, among others.

We call attention to this type of interview, which allows the researcher to present open questions, establishing a conversation between the researcher and the volunteer, allowing the interviewee to feel free to speak, expose his/her points of view and give explanations. It should be emphasized that this interview was recorded in audio and video, through one of the functions available by the used platform, being saved in cloud sharing and accessed later, which we deem indispensable for the next stage of the research, which corresponded to the transcription of the interview.

After completing the data construction, we started the third stage, with the transcription of the semi-structured interview with the teacher. Continuing, the fourth stage of this research was carried out, which consisted of the qualitative analysis of the data and the construction of possible results. Such analysis was based on our theoretical framework, where we cast our gaze at the moment experienced with the teacher from the point of view of his lesson plan, prepared for a class of the seventh grade of Elementary School on the knowledge related to first-degree equations, as well as his speeches about the teaching of algebra in the context discussed here.

Data analysis and discussion

In this topic, we briefly present what was observed during the data construction stage. In this topic. In this aspect, we were able to draw some considerations about the data constructed from the lesson plan and the semi-structured interview with the participating teacher and, therefore, to discuss, through this parameter, the possible approximations and distancing of the teacher's didactic decisions for the development of the algebraic thinking of his students.

The first point of our conversation during the interview focused on knowing the teacher's academic and professional profile, his teaching experience and his point of view on some fundamentally relevant themes addressed in this study, as well as his relationship with the algebraic field since basic education. Thus, initially, we aimed to ask some more open questions, in order to get to know the research participant in advance.

Seeking to enter the conversation about algebra, from this first moment of conversation with the teacher, we move on to more specific questions in the framework of our study. To that end, we initially seek to know what is its relationship with the research problem – the teaching

of algebra and, consequently, on the lesson plan on mathematical knowledge related to first-degree equations.

Regarding the teaching of algebra, the teacher emphasizes what we portrayed at the beginning of this study, which we call “aversion to mathematics”, a moment when students in the final years of Elementary School are faced with the use of algebraic language in their teaching and, consequently, feel discouraged by the lack of understanding. The teacher makes this mention referring to his student experience and as a teacher. Nonetheless, with experience in the classroom and outside it, his difficulties and initial conceptions regarding algebra were overcome. Thus, it is assumed that the teacher has a good relationship with the knowledge related to first-degree equations.

According to the planning provided by the teacher, the didactic situation was organized as follows: first, in the computer laboratory, a virtual game was proposed to the students using the two-plate scale. Subsequently, an individual activity with problem-situations also involving the use of the scale. After that, in small groups of three or four students, a second activity involved concepts of the geometric field, where a student representing each group should present to the class the strategies and solutions found. Finally, a didactic game inspired by a domino (with sentences in everyday language and others with algebraic and numerical sentences) was presented, where everyone could participate independently.

We can observe that the teacher brings strong indications that he will not start the study of algebra with an explanation of exercises, whose nature is not relevant to the content to be worked on – first-degree equations. We have that the work with algebra will be started through the problematization of a game, followed by the exploration of problem-situations, which, in our understanding, will represent the construction of a certain meaning by the students for the deepening of the knowledge to be taught.

Based on the model used in this study, we will be able to understand how the teacher created and organized the environment (didactic situation), through the exploration of the higher levels of the model considering the knowledge mobilized during the lesson plan, as well as his/her teaching intention (Margolinas, 2005).

With regard to level +3, the values and conceptions about teaching and learning were more visible at the beginning of our conversation, although such conceptions were also expressed in other moments of his speech.

Level +2 establishes a strong connection with epistemic factors reflected in the mobilization of knowledge that concerns its relationship with the subject, content and knowledge. We note that he considers the curricular guidelines important, particularly the Pernambuco Curriculum, not only the use of the didactic book used by the students, the consultation of support materials (articles and dissertations), technological resources, for example.

At level +1, the teacher took into account the knowledge and conceptions described in the previous levels that are part of the decision process for the preparation of the plan. At this point, decision-making happens in a more objective way.

Through the information provided by the teacher, the factors related to the didactic history, which concern the teacher-student relationship, prevail in the decisions made for the preparation of the plan. We can observe that he strongly considers his knowledge about the reality of the class, in terms of situations related to daily life, taking into account the specificities of the class, as well as the particular characteristics of each student, for example, his/her difficulties, as well as the establishment of teacher-student relationships regarding the object to be taught. These relationships are woven over time and tend to evolve (Brasset, 2017).

From the point of view of the teacher's didactic decisions about the resources to be used, which were indicated in the lesson plan, such as blackboard, brush, computer, multimedia projector, slide presentation, computer laboratory, cardboard and printed material, it is perceived that the teacher combined different teaching strategies instead of an isolated strategy. These decisions establish links to epistemic factors, which refer to the teacher's personal relationship with the object to be taught and other domains, for example, his knowledge about the subject; what learning is; how to teach; the programs and resources and the community of practice (Espindola, Luberiaga, Tragalova, 2018).

With this, it is evident a conception of teaching that can promote a certain effect, because when carrying out an educational practice in an environment different from the usual one, such

as the computer laboratory, the teacher creates a change of setting, which helps to promote the novelty effect in the students' eyes. In addition, the use of technologies can be seen as a mediating resource for learning and relevant in the school context in which the students were inserted before and during the execution of this study, as already mentioned.

Regarding the two activities proposed in the plan, it was possible to observe that the choice of the two-plate scale resource was highlighted as the main element in the process of teaching the knowledge related to first-degree equations in the first of them. The teacher made the activity available so that students could discover the weight of objects based on the information contained on the scale. We understand that, depending on the exploration, the scale can be a facilitating resource in the process of the first-degree equation. If he tries to show the students that, by removing or adding the same weights on both sides, the equivalence between the two sides will be maintained. In this aspect, we find a close relationship with Generalized Arithmetic, associated with category B (exploring properties of operations with integers), defined by Blanton and Kaput (2005).

In this perspective, Ponte, Branco and Matos (2009, p. 96) underline that it is important for the teacher to take into account that such a resource “facilitates the understanding of the operation of eliminating the same term from both members and also the operation of multiplying both members by a positive number”, but they emphasize that it is essential that students know this material and its operation, so that the equivalence meaning of the equality sign is not lost, which is very important for understanding the concept of equation. In this regard, the teacher makes clear his knowledge in relation to the class.

In the second activity, it is noted that the teacher tries to illustrate situations that involve mathematics in a broader way in relation to their fields of knowledge. Through the problem-situations, the teacher established relationships between concepts from the field of quantities and measures (area and perimeter), some concepts of geometric figures (triangle and rectangle) and concepts from the algebraic domain, whose main objective would be to represent algebraically some problem-situations through first-degree equations and find their unknown values.

In this regard, it is emphasized that the activity alone is not capable of leading the student to think algebraically, because instruction also plays a relevant role in the student's learning (as much as the activity), especially in the teaching of algebra. Bearing in mind that the teacher must "accentuate the bidirectional value of the equality symbol", that is, make the proper reading of the symbol, for example, $5 + 2$ "is equal to" 7 instead of "it is" 7, according to Booth (1995, p. 29). This can bring the student's thinking closer and further away from the correct meaning of the equality sign, which focuses on the study of equations.

In general, we observed that, during the interview, the teacher's intentions about teaching evolved or became more explicit in relation to the importance of establishing the teaching of algebra as a way of leading the student to think.

Final considerations

In this study, we present a discussion about the didactic macro-decisions of a mathematics teacher based on the analysis of his lesson plan on the knowledge related to first-degree equations from the point of view of the development of algebraic thinking in the final years of elementary school, particularly of students in the seventh grade of Elementary School.

In view of the setting experienced in our study (Covid-19), we undertook our study on the participant teacher's lesson plan, where we explored the higher levels of his activity: level +3 (values and conceptions about teaching and learning), level +2 (theme construction) and level +1 (lesson plan), contemplated in the Teacher's Activity Levels Model (Margolinas, 2005).

When reflecting on the observed results, we believe that the didactic macro-decisions made by the teacher during the planning process, in large part, reveal indications that the teaching strategies presented here can favor the students' cognitive skills. Although this verification brings us to the central idea that we discussed about the teaching of algebra in the final years of Elementary School, we cannot say that the student will develop algebraic thinking only through one point of view, that of the teacher. This is because, in the classroom, the teacher should not be the actor of the didactic relationship, but the student. For this reason, our reflections are around approximation and distancing of this way of thinking.

Regarding the teacher's knowledge, an aspect discussed in this study, but in the global sense for teaching, we want to draw attention to the teacher's knowledge about algebraic thinking, considering that most basic education teachers do not have access to the central issues for their development. We can illustrate this reality, taking as an example, the teacher who has a good relationship with the main Brazilian National Common Curriculum Base (BNCC), which reveals the development of algebraic thinking as the purpose of algebra in basic education. On the other hand, in a certain sense, the document does not define what this way of thinking is, nor does it point to elementary knowledge to develop it, that is, the fundamental notions about algebraic thinking.

In this bias, we consider that perhaps if the results of investigations that contemplate the teaching of algebra from the perspective of the development of algebraic thinking reached the official instructions, the teacher would be better able to develop a work with algebra aimed at understanding symbols through representation, unlike what is usually done with algebra in Elementary School.

As we understand, it seems to be a challenge for the mathematics teacher to find a situation that leads the student to think algebraically, as well as to act autonomously on the problem proposed in the classroom. This, there is a need on the part of the teacher to try to prepare the student for the functioning of a didactic environment by choosing situations that can be accepted by him/her in the sense of assuming the responsibility of solving a given problem, where he/she can think and act, instead of following established paths that satisfy the teacher's will.

In this sense, we consider that the teacher participating in our research sought to ensure that students related realistic problems with the mathematical object in question. With this decision, he expresses that working with algebra is linked to elements of social meaning. Thus, we understand that our study may arouse in the participating teacher reflections on aspects that evidence his role at the level of his activity that precedes the execution of the class, the very plan, which is an important moment in teaching practice for student learning.

Thus, it can be said that the results in general show that the choices made and the didactic decisions made by the teacher in the preparation of the lesson plan on the knowledge related to

first-degree equations establish approximation with the development of algebraic thinking and are determinant for the student's school development.

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