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Criteria of didactic suitability as a formative device with prospective mathematics teachers: contributions to the mobilization of pedagogical reasoning

Criterios de idoneidad didáctica como dispositivo de formación con futuros docentes de matemáticas: contribuciones para la movilización del razonamiento pedagógico

Critères d'idonéité didactique comme dispositif de formation avec de futurs enseignants de mathématiques : contributions à la mobilisation du raisonnement pédagogique

Critérios de idoneidade didática como dispositivo de formação com futuros professores de matemática: contribuições para a mobilização do raciocínio pedagógico

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Abstract

The present article aims to investigate the possible contributions of didactic suitability criteria to the mobilization of pedagogical reasoning in formative spaces involving future mathematics teachers. To this end, we used the assumptions of research-based training to structure a formative proposal that addressed the use of Project-Based Learning (PBL) in Mathematics classes. Five stages were proposed: presentation of the PBL perspective, planning a PBL proposal, evaluation of the proposal using didactic suitability as a tool, reformulation of the proposal, and implementation in the school context. For data production, we considered the following instruments: audio and video recordings; observation; field diary; participant records (developed PBL proposal) and focus group. After organizing the data from the analysis of the "Relations" component of Epistemic Suitability, we assumed comprehension, reflection, transformation, and new forms of comprehension as analytical categories. The analysis highlighted the contribution of didactic suitability to processes of reflection, new forms of comprehension, and transformation.

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Keywords: Didactic-mathematical knowledge, Didactic suitability, Pedagogical action and reasoning, Initial education of mathematics teachers, Project-based learning.

Resumen

El presente artículo tiene como objetivo investigar las posibles contribuciones de los criterios de idoneidad didáctica para la movilización del razonamiento pedagógico en espacios formativos que involucran a futuros profesores de matemáticas. Para ello, utilizamos los supuestos de la investigación-formación para estructurar una propuesta formativa que abordó el uso del Aprendizaje Basado en Proyectos (ABP) en las clases de Matemáticas. Se propusieron cinco etapas: presentación de la perspectiva del ABP, planificación de una propuesta de ABP, evaluación de la propuesta utilizando la idoneidad didáctica como herramienta, reformulación de la propuesta e implementación en el contexto escolar. Para la producción de datos, consideramos los siguientes instrumentos: grabaciones de audio y video; observación; diario de campo; registros de los participantes (propuesta de ABP elaborada) y grupo focal. Después de organizar los datos del análisis del componente "Relaciones" de la Idoneidad Epistémica, asumimos la comprensión, reflexión, transformación y nuevas formas de comprensión como categorías analíticas. El análisis destacó la contribución de la idoneidad didáctica a los procesos de reflexión, nuevas formas de comprensión y transformación.

Palabras clave: Conocimiento didáctico-matemático, Idoneidad didáctica, Acción y razonamiento pedagógico, Formación inicial de profesores de matemáticas, Aprendizaje basado en proyectos.

Résumé

Cet article vise à étudier les contributions possibles des critères de pertinence didactique à la mobilisation du raisonnement pédagogique dans les espaces de formation impliquant les futurs professeurs de mathématiques. À cette fin, nous avons utilisé les postulats de la recherche-formation pour structurer une proposition de formation qui abordait l'utilisation de l'Apprentissage par Projet (ABP) dans les cours de mathématiques. Cinq étapes ont été proposées : présentation de la perspective de l'ABP, planification d'une proposition ABP, évaluation de la proposition en utilisant la pertinence didactique comme outil, reformulation de la proposition et mise en œuvre dans le contexte scolaire. Pour la production de données, nous avons considéré les instruments suivants : enregistrements audio et vidéo ; observation ; journal de terrain ; enregistrements des participants (proposition ABP élaborée) et groupe de discussion. Après avoir organisé les données de l'analyse du composant "Relations" de la

Pertinence Épistémique, nous avons adopté la compréhension, la réflexion, la transformation et les nouvelles formes de compréhension comme catégories analytiques. L'analyse a souligné la contribution de la pertinence didactique aux processus de réflexion, de nouvelles formes de compréhension et de transformation.

Mots-clés : Connaissance didactico-mathématique, Pertinence didactique. Action et raisonnement pédagogique, Formation initiale des enseignants de mathématiques, Apprentissage basé sur les projets.

Resumo

O presente artigo objetiva investigar as possíveis contribuições dos critérios de idoneidade didática para a mobilização do raciocínio pedagógico em espaços formativos que envolvem futuros professores de matemática. Para tanto, utilizaram-se os pressupostos da pesquisa-formação para estruturar uma proposta formativa que abordou o uso da Aprendizagem Baseada em Projetos (ABP) nas aulas de matemática. Foram propostas cinco etapas: apresentação da perspectiva da ABP; planejamento de uma proposta ABP; avaliação da proposta utilizando a idoneidade didática como ferramenta; reformulação da proposta; e implementação no contexto escolar. Para a produção dos dados consideraram-se os seguintes instrumentos: gravações em áudio e vídeo; observação; diário de campo; registro dos participantes (proposta ABP elaborada); e grupo focal. Após o processo de organização dos dados oriundos da análise do componente "Relações" da idoneidade epistêmica, assumiram-se como categorias analíticas as etapas de compreensão, reflexão, transformação e novas formas de compreensão. A análise evidenciou a contribuição da idoneidade didática para os processos de reflexão, novas formas de compreensão e transformação.

Palavras-chave: Conhecimento didático-matemático, Idoneidade didática, Ação e raciocínio pedagógico, Formação inicial de professores de matemática, Aprendizagem baseada em projetos.

Criteria of didactic suitability as a formative device for prospective mathematics teachers: contributions to the mobilization of pedagogical reasoning

Over the years, the debate about teacher knowledge has been driven by the notion proposed by Shulman (1986) of "*pedagogical content knowledge*," given its contribution to discussing this investigative focus in different areas of knowledge. In the context of Brazilian mathematics education, researchers in the field of teacher education have sought to understand, through different theoretical lenses, the knowledge of the teacher who teaches mathematics. Among these lenses, we can highlight some perspectives: mathematics teachers' specialized knowledge (MTSK), proposed by Carrillo et al. (2013); mathematical knowledge for teaching (MKT), proposed by Ball et al. (2008); and didactic-mathematical knowledge (DMK), proposed by Godino (2009).

This movement has contributed to the planning of different formative spaces where teachers can be inserted to mobilize knowledge specific to teaching and, consequently, collaborate with the process of professional teacher development. Among these spaces, we highlight the Pedagogical Residency Program (PRP), created through Ordinance n. 38 of February 28, 2018[3], by the Coordination for the Improvement of Higher Education Personnel (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Capes). The PRP was established with the central objective of supporting educational institutions in developing proposals that encourage the relationship between theory and practice in the education of prospective teachers.

Silva and Tinti (2021) understand that the proposals for mathematics subprojects included in the PRP institutional projects could be planned to enhance knowledge mobilization in prospective teachers. Using the DMK perspective as a basis, the authors exemplify the possible knowledge that could be mobilized in each action foreseen in the PRP notices.

In line with Silva and Tinti's (2021) ideas, we chose the DMK perspective to support the writing of this article. The Center for Studies, Research, and Practices in the Education of Teachers who Teach Mathematics (Núcleo de Estudos, Pesquisas e Práticas de Formação de Professores que Ensinam Matemática - Nepefem)[4] has adopted this perspective to investigate the knowledge of the teacher who teaches mathematics, which also justifies our choice. Furthermore, considering the particularities of the DMK, we chose to direct our attention to didactic suitability, as this is a conceptual tool that allows us to assess the adequacy of

[3] The Ordinance can be accessed at: https://www.gov.br/capes/pt-br/centrais-de-conteudo/01032018-portaria-n-38-de-28-02-2018-residencia-pedagogica-pdf

[4] https://sites.ufop.br/nepefem

pedagogical practices related to educational objectives, student characteristics, and available resources, among others. Thus, through a systematic analysis of the six facets of teaching suitability proposed by Godino (2009), we can identify points for improvement and reflect on the most appropriate strategies to maximize student learning.

Beyond this characteristic, the data from the master's degree research we developed (Carmo, 2024) made it possible to understand that didactic suitability can be assumed as a formative device to mobilize the process of pedagogical action and reasoning proposed by Shulman (1987). This process highlights the importance of continuous reflection on practice, promoting integration between different types of knowledge and constant adaptation to educational needs.

This article investigates the possible contributions of didactic suitability criteria to mobilize pedagogical reasoning with prospective mathematics teachers in formative spaces. To achieve this objective, we chose an action developed within the scope of a PRP mathematics subproject that sought to discuss the use of project-based learning (PBL) in mathematics teaching and learning processes.

The teacher's knowledge

To achieve the objective proposed for this article, we will discuss teacher knowledge from the perspectives of Lee Shulman and Juan Godino. In this sense, we present the perspectives of the pedagogical action and reasoning process and didactic suitability.

Teacher knowledge from Lee Shulman's perspective

From the teacher education perspective, one of the issues that gave rise to research involves the knowledge that teachers must have to teach. Until the 1980s, teachers' knowledge was not a focus in an academic environment. In this context, Shulman (1986) began developing some constructs –one of them is the basic knowledge for teaching– and proposed three categories of teacher knowledge, as shown in Figure 1.

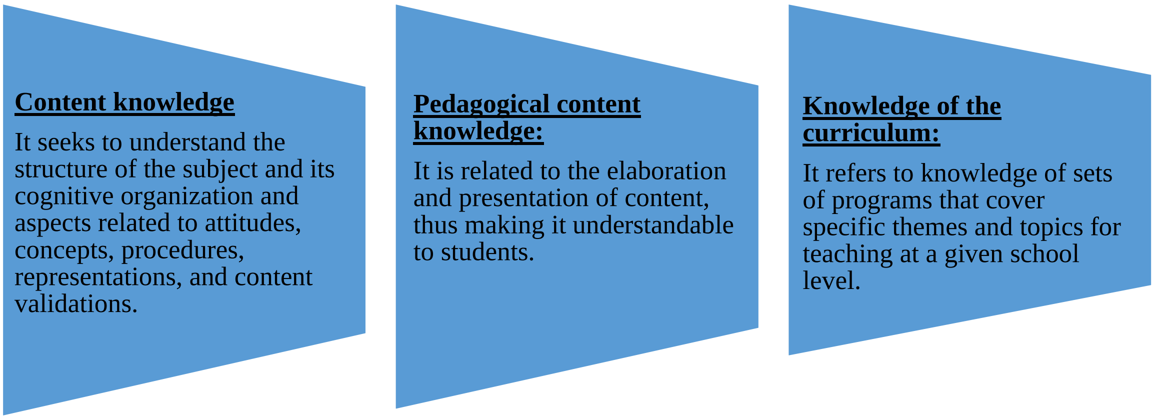

Figure 1.

Basic knowledge for teaching. Adapted from Shulman (1986)

Later, in 1987, when the theoretical framework was revised, four types of knowledge base were added, as shown in Figure 2.

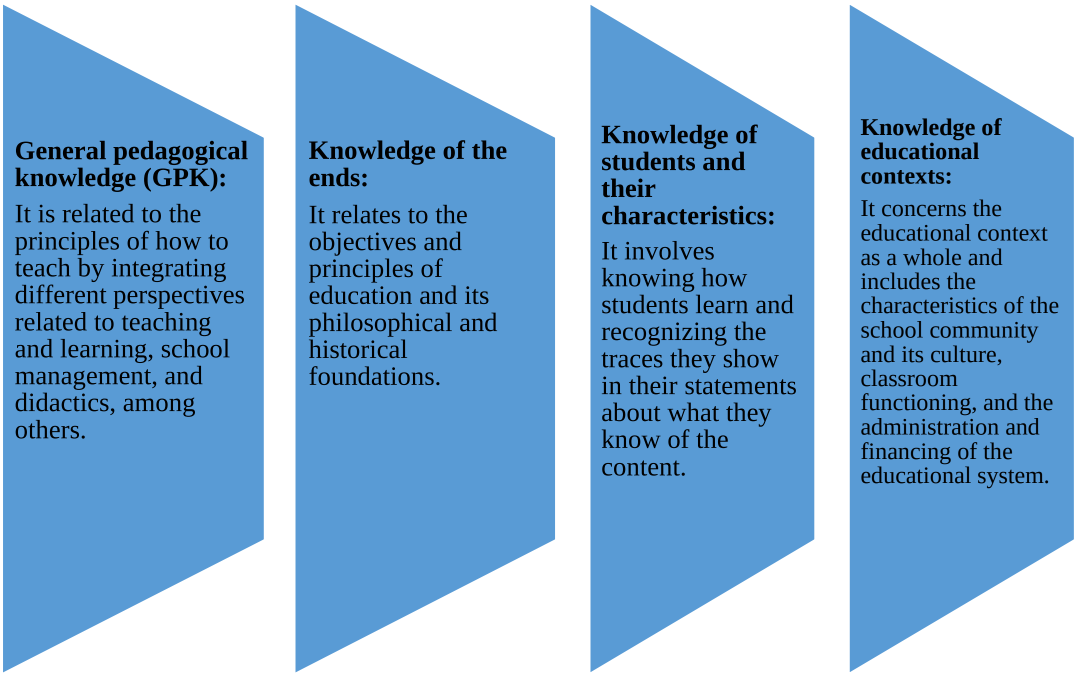

Figure 2.

Basic knowledge for teaching. Adapted from Shulman (1987)

According to Shulman (1986, 1987), the teacher’s knowledge base consists of different types of knowledge, including content knowledge, pedagogical content knowledge, curriculum knowledge, general pedagogical knowledge, knowledge of ends, knowledge of students and their characteristics, and knowledge of educational contexts. Each type of knowledge is important for teaching practice and can be built gradually over time. Once the seven types of knowledge have been listed, it is necessary to delve deeper into the construction of pedagogical content knowledge, as Shulman (1987) presents an important process for this investigation: the process of pedagogical reasoning and action.

The process of pedagogical reasoning and action

Initially presented by Shulman (1987), the concept of *pedagogical reasoning and action processes* describes a series of steps that occur during teaching practices, mainly aiming to enable teacher-students to learn how to teach different subjects to different students and in different contexts.

This process involves six steps (Figure 3): understanding purposes and content structures; transforming ideas that have been understood into ideas to teach; instruction; assessment; reflection; and new understanding.

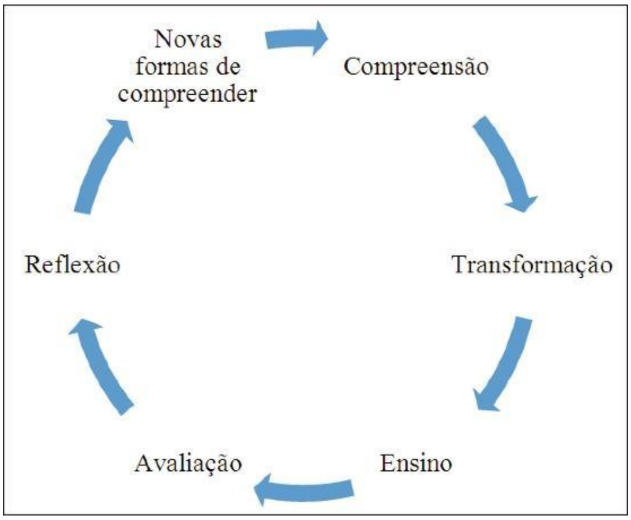

Figure 3.
Phases of the pedagogical reasoning and action model (Backes et al., 2017, p. 6)

Below, we briefly describe the six steps mentioned.

Understanding: It is the teacher's action of understanding the material or theme of his/her class; it involves understanding the process of comprehension, from the first contact with the content to its connection with the subject and curriculum objectives. At this stage, teachers transform the content to make it more transparent so that they can understand the curriculum objectives, the structures of the field of knowledge, and its relationships with the formative and school objectives. Shulman (2014, p. 217) states: “We expect teachers to understand what they teach and, when possible, to understand it in many ways.”

Therefore, understanding aims for prospective teachers to diversify their ways of understanding, knowing, and interpreting the issue as much as possible to transmit this to students.

Transformation: This stage includes weighing up the didactic and pedagogical options selected by the educator, adjusted according to students’ particularities. In the transformation process, the teacher chooses the resources to use, considering content relevance, activity duration, and the most appropriate pedagogical strategy for the class. To enable the transformation of the content knowledge, Shulman (1986) proposes the integration of five distinct subprocesses: (1) Preparation or Critical Interpretation: It can be examined considering both the criteria used in the choice of teaching materials and the judgment of their relevance in relation to the objectives, context, and individual needs of each student; (2) Expression of Conceptions: It involves the use of analyses, illustrations, metaphors, examples, experiments, simulations, dramatizations, music, films, case studies, demonstrations, and other ways of representing the content to students; (3) Selection of Pedagogical Approaches: It encompasses not only the application of conventional teaching methods, but also alternative teaching strategies; (4) Adjustments: All changes that can be introduced in educational practice to facilitate students’ understanding of the content; and (5) Collaborative Adaptations: The challenge consists in considering the individual and distinct characteristics of each student and analyzing them in a joint and integrated manner.

> Those forms of transformation, aspects of the process by which one goes from personal understanding to preparing understanding by others, are the essence of reasoning pedagogically, of teaching as thought, and of –implicitly or explicitly– planning the teaching exercise. (Shulman, 2014, p. 217)

During that stage, the educator decides whether to recommend exploring a chapter of a book, analyzing a text, or watching a video, which can be made available in advance or shown in the classroom, and chooses between teaching a lecture, conducting practical activities or organizing a technical visit, and may or may not assign tasks to be delivered –and this may or may not influence the assessment. This stage involves a considerable process of reflection, as it will likely incorporate learning from previous experiences and draw on knowledge acquired throughout the teacher’s teaching career.

Teaching: It covers the assessment of the visible performance of students and teachers in the application of different teaching approaches, dealing with pedagogical aspects, such as the organization of teaching and learning environments, and involves the use of explanations, descriptions, and demonstrations students can understand clearly and easily. Monitoring, advising students with ideas or challenges, and interacting with them are activities that are based on the reflections and choices made in the previous stage, the transformation phase. Shulman (2014, p. 219) infers that "[The] instruction consequently encompasses management, explanation, discussion, and all observable characteristics of effective direct and heuristic teaching, aspects that are already widely documented in the research literature on effective teaching."

Assessment: It is related to formal procedures for assessing students' progress and learning and obtaining subjective information through interaction with them. That information concerns student performance and learning as assessed by teacher practice, covering both analysis of each student's performance and the performance of the whole class. In this context, the assessment does not focus on the students themselves but rather on the performance and influence of the prospective teacher in achieving educational objectives.

Shulman (2014, p. 221) explains: "Understanding what a student understands requires a deep mastery of the material to teach and the learning processes. This understanding must be specific to each school subject and individual topics within the subject."

Therefore, through direct or indirect, formal or informal feedback, the purpose of the assessment phase in the pedagogical thinking process is to obtain information about teaching practices regarding student learning, and the teacher's ability to adapt his or her knowledge to make it understandable to students.

Reflection: It investigates how the pedagogical practice the teacher selected has developed to re-evaluate it if it has not been successful, identifying when it failed and promoting changes to ensure that this approach can meet students' needs. According to Shulman (2014, p. 221), "This is what a teacher does when he or she looks at the teaching and learning that has just occurred and reconstructs, reenacts, and/or recaptures the events, emotions, and accomplishments." It also allows us to reflect on what worked well, considering whether it should be kept or changed.

New ways of understanding: This stage is symbolized as the end of a cycle, marking the beginning of a new reflection. As highlighted by Shulman (2014, p. 222),

> Thus, we arrive at a new beginning, hoping that, through thoughtful and logical teaching acts, the teacher will reach a new understanding of both the purposes and content to teach and the students and the teaching processes.

As mentioned by the author, this new beginning is linked to teachers' new understandings of their teaching practices and methods, resulting from the experiences they accumulated during the cycle. In this context, reflections on pedagogical practices reveal that certain knowledge can be applied to resolve a specific problem situation at that moment or in subsequent moments, which stems from the understanding that a new apprehension of previously acquired knowledge will result in a new perspective that will be a basis for future teachings.

Although the process of pedagogical reasoning and action proposed by Shulman (1987, 2014) is presented in six steps, he and other authors emphasize that its application in pedagogical practices does not need to be sequential or hierarchical (Altet, 2001; Salazar, 2005; Tardif & Raymond, 2000). Another important point to be highlighted –as emphasized by Behets and Vergauwen (2006), Ennis (1994), and Park and Oliver (2008)– is that the dynamic, unstable, and unpredictable nature of pedagogical practices prevents their prior organization and structuring, placing the prospective teacher before the challenge of dealing with these issues simultaneously to achieve their objectives and promote student learning.

The pedagogical reasoning and action process is essential in constructing pedagogical content knowledge and requires a combination of different types of knowledge. Thus, from Shulman's perspective (1987), teacher knowledge is a theoretical construction that includes different types of knowledge that form the basis of teaching. However, this knowledge is activated, related, and constructed through the process of pedagogical reasoning and action.

Didactic-mathematical knowledge

Godino (2009) and Godino et al. (2017) suggest a set of analysis categories of the teacher's mathematical and didactic knowledge that are complemented and elaborated with resources from the onto-semiotic approach to mathematical knowledge and instruction (OSA).

The OSA has been presented as an evolving theoretical construct, and its genesis is rooted in the studies of the research group Theory and Methodology of Research in Mathematics Education (Teoría y Metodología de Investigación en Educación Matemática) at the University of Granada, in Spain. Thus, through multiple perspectives and a scientific basis on mathematical knowledge, its teaching, and learning, Godino and collaborators show a theoretical instrument called DMK.

Figure 4 shows the characteristics and levels of teacher knowledge, which, according to Godino (2009, p. 21, our translation), "is a 'polyhedral' model whose representation in plan indicates the various facets to be considered in a study process and the elevation indicates four levels of analysis on which attention can be focused.”

Figure 4.

Facets and levels of teacher knowledge (Godino, 2009, p. 21)

Godino (2009) suggests analyzing DMK in six facets from this perspective, as shown in Table 1.

Table 1

Facets and levels of teacher knowledge. Adapted from Godino (2009)

Facet	Description
Epistemic	Mathematical knowledge related to the institutional context in which the study process takes place and the distribution of the different contents in the curriculum.
Cognitive	Students’ personal knowledge and learning progress.
Affective	Affective states (attitudes, emotions, beliefs, values) of each student toward mathematical objects and the following study process.
Mediational	Technological resources and time allocation to different actions and processes.
Interactional	Patterns of the relationship between teacher and student and their sequencing oriented toward the fixation and negotiation of meanings.
Ecological	System of relationships with the social, political, and economic environment, ... that supports and conditions the study process.

Regarding the facets, Godino (2009) proposes the levels of didactic investigation shown in Table 2.

Table 2

Levels of didactic analysis. Adapted from Godino (2011, 2017, 2021) and Andrade (2014)

Levels	Description
Mathematical and didactic practices	Description of the actions to solve the proposed mathematical tasks to contextualize the contents and promote learning. The general lines of action of the teacher and students are also described.

Object and process configurations (mathematical and didactic)	Description of the mathematical objects and processes involved in carrying out the practices, as well as those that emerge from them. The objective of this level is to describe the complexity of the objects and meanings of mathematical and didactic practices as an explanatory factor of conflicts in their implementation and the progression of learning.
Didactic configurations	It contemplates the interactions between teacher and student, aiming to identify and describe those interactions, relating them to the student's learning (cognitive trajectory).
Norms and metanorms	Identification of the web of rules, habits, and norms that condition and enable the study process and affect each facet and its interactions.
Didactic suitability	Identification of possible improvements in the study process that increase didactic suitability.

Considering the objective of this article, we will discuss the perspective of didactic suitability below.

Didactic suitability

According to Breda et al. (2018), didactic suitability is:

> (...) the degree to which a teaching-learning process (or part of it) brings together specific characteristics that allow it to be qualified as optimal or adequate to achieve adaptation between the personal meanings achieved by students (learning) and the institutional meanings intended or implemented (teaching), taking into account the circumstances and resources available (environment). (p. 268, our translation)

Figure 5 elucidates how didactic suitability is symbolized and what its proportions are.

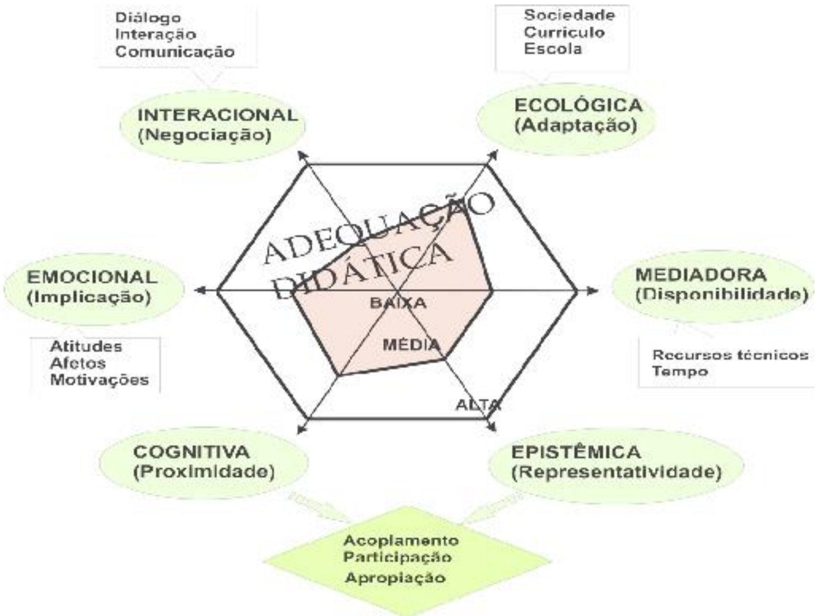

Figure 5.
Representation of didactic suitability and its facets (Godino et al., 2008, p. 24)

Thus, Godino (2009, p. 24, our translation) states: “Didactic suitability occurs when there is a coherent and systemic articulation of these six facets,” which we can understand as presented in Table 3.

Table 3

Didactic suitability. Adapted from Godino (2011, 2017) and Andrade (2014)

Facets	Description
Epistemic	It refers to the degree of representativeness of the implemented (intended) institutional meanings with respect to a reference meaning.
Cognitive	It expresses the degree to which the intended/implemented meanings are in the students’ zone of potential development, as well as the proximity of the achieved personal meanings to the intended/implemented meanings.
Affective	Degree of involvement (interest, motivation) of students in the study process. Affective suitability is related to factors that depend on the institution and those that basically depend on the student and their previous academic history.
Mediational	Degree of availability and adequacy of material resources and time necessary for the development of the teaching-learning process.
Interactional	Refers to evaluating whether interactions resolve students’ doubts and difficulties.
Ecological	The degree to which the study process fits into the design of the educational center, school, and society and the conditioning of the environment in which it takes place.

According to Breda et al. (2015), the six facets of didactic suitability are likely to occur at high, medium, or low levels. The regular hexagon, which symbolizes didactic suitability and its proportions, shows the maximum suitability of a process or research, while the internal irregular hexagon shows what has been fully achieved (Figure 7).

Although the six facets differ, they are not analyzed separately since, when carrying out a mathematical activity, the teacher uses different meanings (epistemic facet), adapting the various processes to the knowledge, capabilities, contexts, and attention of their students (interactional, cognitive, ecological, and affective facets) and managing material and temporal resources (mediational facet).

In this sense, according to the epistemic, cognitive, interactional, mediational, affective, and ecological facets, didactic suitability is a tool that supports reflection on didactic practice, allowing the perception of didactic adequacies in the teaching and learning process.

Next, we will show the grouping of observable indicators that construct each of the six facets of didactic suitability, enabling the assessment of the degree of adequacy of each element of the process or desired research. According to Godino (2011), components and indicators are proposed to be applied in different contexts for each facet, such as in the development of a teaching unit, in a class, in a course, in a curriculum proposal, or even in the planning of content. Furthermore, we can use these strategies to study partial aspects of teaching material, students’ responses, and specific activities. Thus, the criteria of didactic suitability can assist, *a priori*, in instructing the mathematics teaching and learning processes and *a posteriori*, in evaluating the results achieved since its implementation.

Thus, the theoretical model developed by Godino and collaborators articulates the notions of didactic analysis skills and the teacher's didactic-mathematical knowledge, presenting methodologies to support reflection on didactic practice, its valorization, and progression.

Didactic suitability thus allows the development of analytical skills that have the potential to contribute to the development of prospective teachers in their initial education, given that they construct and reconstruct their knowledge through the experiences and practices experienced in their formative process.

Godino and Batanero (2009) say that guidance processes (guided reflection) are not only about the reflection that arises from prospective teachers' practice but must also be present in the activities carried out in the academic formative processes.

Providing licensing students with opportunities to learn about and put into practice – and reflect on– the different approaches and methodologies in mathematics teaching and learning and the knowledge they articulate in these processes can influence their future classroom practices.

Methodological trajectory

To investigate the possible contributions of didactic suitability criteria to mobilize pedagogical reasoning in formative spaces involving prospective mathematics teachers, we chose to develop qualitative research. Sandín Esteban (2010) states that qualitative research has several uses and meanings that have been transformed throughout history. For her, "qualitative research is a systematic activity aimed at an in-depth understanding of educational and social phenomena, the transformation of socio-educational practices and scenarios, decision-making, and also the discovery and development of an organized body of knowledge" (Sandín Esteban, 2010, p. 127).

Within the field of qualitative research, we chose to adopt the principles of research-education where, according to Fog (1995, p. 15), "The person is simultaneously the object and subject of education." The choice for this type of research is justified because it is the most appropriate for research-related and education-related goals, since we establish a mediation relationship between the researcher, residents, and preceptors, contemplating the possibility of changes in the practices of the subjects in training, including the authors of this article.

To respond to the proposed objective, we sought support in data from a master's research (Carmo, 2024) that assumed the PRP as a formative space. Among the actions of a mathematics subproject, we proposed studying the PBL's perspective. Thus, the following stages were structured for the following formative proposal: i) Presentation of the PBL's perspective; ii) Preparation of a teaching plan from the PBL perspective; iii) Reflection and evaluation of the proposal prepared taking didactic suitability as a formative device; and vi) Reformulation of the proposal. After the reformulation, participants could implement the proposal in the school context and then reflect on the results and adjustments that would still be necessary. We followed this process but cannot report the data in this article because it requires the Ethics Committee's[5] approval.

The formative proposal was carried out with 11 students –who participated in the PRP mathematics subproject and were called residents– from the mathematics teaching degree course at a federal higher education institution in Minas Gerais. Three mathematics teachers who work in basic education, called preceptors, also participated in the study.

Participants were organized into groups in the second stage of the formative proposal to develop proposals for implementing PBL in field schools. The data we selected for this article are specific to Subgroup 1, formed by a scholarship preceptor and a scholarship resident. All participants agreed to participate in the research and signed the Free and Informed Consent Form (FICF).

The instruments considered for data production were audio and video recordings, observation, field diary, participant records (PBL proposal developed), and focus group. The focus group assumed didactic suitability as a formative device.

During data organization in the first moment, we made transcriptions of the formative proposal meetings. Subsequently, we sought to triangulate the data considering the six dimensions of didactic suitability. Finally, as this study is an excerpt from a completed master's degree research project –and due to the page limit–, we chose to focus on data relating to epistemic suitability, focusing on indicators of the "Relationships" component.

Having defined this data set, we sought to analyze them in light of the pedagogical reasoning and action process proposed by Shulman (1987), assuming four stages as analytical categories.

[5] CAE Process: 66728222.5.0000.5150

Data analysis and discussion

Considering data from the analysis of the “Relationships” component of epistemic suitability, we organized the analysis to highlight didactic suitability as a formative device that contributes to the process of pedagogical reasoning and action. Thus, we will seek to reflect and give examples of the stages of understanding, reflection, transformation, and new forms of understanding. Given the limitations imposed by the Ethics Committee, we will not present data on the teaching and assessment processes, but they will be taken into account for inferences in the final considerations. However, we are discussing a cyclical and non-hierarchical process. Thus, the fragmentation we propose here is for analytical purposes.

Comprehension

In the Understanding phase, we present how the four stages of the investigation occurred. It consists of a presentation of the PBL perspective, participants’ preparation of the PBL proposal, participants’ evaluation of the PBL proposal using suitability as a tool, and participants’ implementation of the proposal and the formative proposal Subgroup 1 developed.

In the first stage, the researcher presented the PBL perspective to the participants, emphasizing its relevance and applicability in the educational context. The objectives of the PBL were shown, and a discussion was held about how this approach can contribute to the teaching and learning processes and promote students’ engagement. This process aligns with Shulman’s (2014) ideas when addressing the perspective of knowledge bases for teaching.

In the second stage, participants were divided into small groups, which promotes collaboration. Each group had to discuss and plan a project proposal based on PBL, respecting the characteristics of their respective educational contexts. In addition, each group chose a specific theme for their project, including the learning objectives, activities, assessments, and resources needed to implement it. This collaborative approach not only reinforces understanding of the content among participants but also simulates the dynamic reality of classrooms, in which, according to Shulman (2014), understanding must be constantly adapted and applied in a practical way.

Figure 6 below exhibits the proposal prepared by Subgroup 1.

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Primeiro passo é fazer controle de tudo aquilo que você paga (despesas) e recebe como renda (salários e rendimentos) para conseguir planejar a sua vida.

Available at: https://g1.globo.com/pr/parana/economia/educacao-financeira-no-parana/noticia/2022/11/11/planejamento-financeiro-e-aliado-para-ter-uma-vida-mais-tranquila-e-conquistar-sonhos-e-objetivos.ghtml

DRIVING QUESTION:

How much are you investing in yourself and your future?

RESOURCES:

Computer lab, calculator, paper, pen, pencil, eraser, ruler.

Research: What are fixed and variable expenses (assets and liabilities)? Give examples. Make a presentation (debate).

APPLICATION TIME

Four classes (one class to present the anchor, the driving question, and the project; one class for research; one class to prepare the table; and one class for debate)

A one-week interval to organize the data to fill in the table.

ARTIFACT:

Preparation of a table with expenses (fixed and variable) and income (fixed and variable) and a general analysis. Present the final analysis (debate).

EVALUATION:

Development in the classroom (including research and debate), construction of the table, and presentation of conclusions.

Figure 6

The first version of the project prepared by Subgroup 1 (Carmo, 2024, p. 106)

In the third stage, after preparing the proposals, the participants used a matrix of didactic suitability criteria to evaluate their proposals in accordance with Godino (2009), including content relevance, suitability of learning activities, and alignment with educational objectives. The feedback was collected and discussed in groups, allowing participants to revise their proposals based on the evaluations and suggestions received. This critical evaluation and feedback process is fundamental in pedagogy, as indicated by Shulman (2014), as it allows educators to reflect on their understanding of the content and teaching strategies, ensuring that approaches are relevant and productive.

Finally, in the fourth stage, the developed projects were implemented in class, allowing participants to observe and analyze the fruitfulness of their PBL approaches in practice. The researcher monitored the implementation of the proposals, made observations, and recorded the results in field diaries. This stage was crucial to collect data on students' interaction with PBL and observe the impacts on learning.

Reflection

In the Reflection stage, didactic suitability was used as a tool. Considering the data scope produced in the research (Carmo, 2024), in this article, we present data relating to epistemic suitability, which includes five components: problem situations, languages, rules, arguments, and relationships. Each component has its indicators, which will be detailed later in this study. Given this scope, we focused on the "Relationships" component.

Epistemic suitability addresses the degree of representativeness of the institutional meanings implemented (intended) on a sense of basis. Therefore, observing how the objects in the analysis process were produced, such as problem situations, languages, rules, arguments, and relationships, is essential. Thus, Godino (2011) suggests five elements as components and parameters of epistemic suitability, as presented in Table 4.

Table 4

Components and indicators of epistemic suitability (Godino, 2011, p. 9, our translation)

Components	Indicators
Problem situations	- A representative and articulated sample of contextualization, exercise, and application situations is presented. - Problem-generating situations are proposed (problematization).
Languages	- Different ways of expressing mathematics (verbal, graphic, symbolic), translations, and conversions between them are used. - The language level is appropriate to the students at which it is aimed. - Situations for expressing and interpreting mathematics are proposed.
Rules (definitions, propositions, procedures)	- The definitions and procedures are clear and coherent and are adapted to the educational level they are aimed at. - Fundamental statements and procedures for the theme at the given educational level are presented. - Situations in which students have to generalize definitions, propositions, and procedures are proposed.
Arguments	- The explanations, proofs, and demonstrations are appropriate to the educational level they are aimed at. - Situations in which students must present arguments are promoted.
Relationships	- Mathematical objects (problems, definitions, propositions, etc.) are related and connected to each other. - The different meanings of objects that influence mathematical practice are identified and articulated.

Using this perspective as an analytical tool, the participants and the researcher reached the levels –shown in Table 5– for each component.

Table 5

Summary of the analysis of the levels of epistemic suitability evidenced in the analysis of the lesson plan prepared by the resident and the preceptor (Subgroup 1). Prepared by the researcher (2023)

Components	Level measured	
	By the researcher	By the participants
Problem situations	Low	Medium
Languages	Low	High
Rules (definitions, propositions, procedures)	Low	Medium
Arguments	Medium	Medium
Relationships	Low	Low

In our view, the potential of this tool is not in the fact that it measures a level but rather in the process involved in reaching this measurement. Taking as a basis, the process of the pedagogical reasoning and action proposed by Shulman (1987), we can see that it can trigger a new understanding and mobilize the teacher to reformulate the teaching proposal to offer their students different learning opportunities.

When reflecting on the data presented in Figure 11, we can observe that the researcher and the participants measured the "Relationships" component as low. Thus – to exemplify the potential of didactic suitability as a formative device and mobilizer of reflections in favor of improving the quality of teaching –, we chose to focus on it.

The "Relationships" component includes two essential indicators that were not met by participants and the researcher alike. The first indicator proposes a reflection on the relationships between mathematical objects. When reflecting on this indicator, Resident 1 concluded that his proposal did not contemplate this characteristic: "*I think it's not explicit, but especially if someone has a loan issue, something could come up. But that would be a very atypical situation*" (Record of the speech of participant Resident 1 at the meeting held on March 31, 2023).

To improve the proposal to contemplate this indicator, it could include practical activities that encourage solving financial education problems, such as interest or discount calculations, demonstrating the interconnection between different mathematical aspects. This movement aligns with the ideas discussed by Shulman (2014), who emphasizes the importance of a “new beginning” based on thoughtful and logical reflections on teaching.

The second indicator of epistemic suitability concerns the identification of the different meanings of mathematical objects and the articulation between them. About this indicator, Godino (2011) emphasizes that it is crucial to identify and articulate the different meanings of the objects that intervene in mathematical practices.

The reflection around this indicator showed participants the need to reformulate the proposal to highlight this articulation between the different mathematical objects that are explicit and implicit in the proposal. This perception is apparent in the following excerpt:

> I think it can be beneficial, but at least when we planned it, we didn’t have this idea of using it this way. However, I think it can be beneficial because just like now that I’m seeing, for example, the statistics part and such, which I’m starting to study, for example, the pie chart that Preceptor 1 even mentioned. We haven’t thought of it yet, but we can also use these other forms to identify the percentage in table format for calculations, or geographically, or something like that. But honestly, it wasn’t in the project when we did it. (Recording of the speech of Resident 1, a participant in the GF, during the meeting held on March 31, 2023)

The reflective process allowed participants to think about including activities that favored integration with other mathematical concepts, such as percentages. During the reflection, participants assessed that this approach could enrich students’ understanding of how percentages are applied in real financial contexts, such as interest calculations, discounts, and analysis of percentage variations in budgets and expenses. By directly relating financial education to the practical study of percentages, students could develop a broader understanding of the mathematical concepts in the PBL proposal and demonstrate their applicability in real-life situations.

This movement of reflection that enables new ways of understanding and, consequently, reformulating teaching proposals is in line with Shulman’s perspective (2014) when highlighting that teaching acts must be thought through and should lead teachers to achieve a new understanding of the purposes and the content taught.

As we highlighted previously, the “Relationships” component was classified as low due to the lack of adequate connection and articulation between mathematical objects and their meanings. By using it as a reflection tool, participants could see that including activities that favored integration and a broader apprehension of mathematical concepts in financial education would favor a broader and more contextualized understanding. Thus, the reformulation would make possible a higher degree of suitability and improve the teaching and learning processes resulting from the PBL proposal that would be implemented.

New ways of understanding

As we have sought to demonstrate so far, throughout the formative proposal, participants could learn about and develop a PBL proposal to be implemented in the school context (Understanding). After this stage, a formative moment was mobilized to evaluate the proposal, assuming the criteria of didactic suitability as a formative device (Reflection). This reflective process favored, among other things, **new ways of understanding**, which culminated in the reformulation of the initial PBL proposal. So, at this point, we will try to illustrate this movement.

The transition from the first to the second version presents several modifications and refinements, contributing to the better-structured project. This movement converges on the importance of understanding and pedagogical reasoning in the teaching process, as highlighted by Shulman (2014). Overall, the changes included a greater emphasis on questions, tasks, and practical problems that can benefit student learning.

The new version presents significant changes in terms of disciplinary specificity, objects of knowledge, target audience, skills, interdisciplinary contextualizations, material resources, proposed activities and artifacts, application time, evaluation, inclusion of technology, and graph preparation, reflecting an interdisciplinary approach. These changes were only possible after a reflective movement that assumed didactic suitability as a formative device.

Considering that didactic suitability has six dimensions, and given the limitations imposed by the length of this article, we will focus on the changes that emerged from the reflection based on epistemic suitability, specifically in the "Relationships" component. Thus, we will look at the indicators presented in Figure 10, namely: i) "Mathematical objects (problems, definitions, propositions, etc.) are related and interconnected"; and ii) "The different meanings of the objects that influence mathematical practice are identified and articulated."

The first indicator leads us to reflect on the integration of mathematical concepts with other areas, such as geography. In the second version of the project, the integration of mathematical concepts with geography themes can promote conceptual expansion in the classroom and highlight the connections between mathematics and different areas of knowledge. This approach agrees with Shulman's (2014) idea of the continuous cycle of learning and teaching, in which each new cycle brings the opportunity for new understanding. Among other aspects, prospective teachers mobilize knowledge of interdisciplinarity by proposing to approach mathematical concepts together with geography concepts.

Furthermore, using spreadsheet software to create tables and graphs demonstrates how technological tools can be used to connect abstract mathematical concepts with visual and practical representations. These activities allow students to explore and understand the relationships between numerical data and their graphical representations, highlighting the interconnection between mathematical objects.

In turn, the second indicator leads us to reflect on the mathematical concepts' relationships within the area. When analyzing the reformulated plan, we can see the inclusion of activities that involve using the concept of percentage in financial planning, demonstrating a concern in showing students that mathematical objects can be applied in everyday contexts.

Transformation

At this stage, Figure 12 shows the improved version of the project prepared by the participants and, subsequently, the discussion on didactic suitability.

SUBJECT: Mathematics **THEMATIC UNIT:** Numbers and Algebra **OBJECT OF KNOWLEDGE:** Percentage **TARGET AUDIENCE:** 3rd grade of high school – YAE modality **PROJECT ANCHOR:** To introduce the project, a video about financial planning, available on Globoplay, will be used as the project's anchor. In addition, students will be presented with a text on the same subject, available on Globo.com's G1, for reading.

Diário TV 2ª Edição

Planejamento financeiro auxilia famílias a controlarem gastos mensais - 12/01/2023

5 min

Além disso, planejar as finanças pode ajudar a guardar dinheiro para gastar ou investir.

Available at: https://globoplay.globo.com/v/11275483/

Planejamento financeiro é aliado para ter uma vida mais tranquila e conquistar sonhos e objetivos

Primeiro passo é fazer controle de tudo aquilo que você paga (despesas) e recebe como renda (salários e rendimentos) para conseguir planejar a sua vida.

Available at: https://g1.globo.com/pr/parana/economia/educacao-financeira-no-parana/noticia/2022/11/11/planejamento-financeiro-e-aliado-para-ter-uma-vida-mais-tranquila-e-conquistar-sonhos-e-objetivos.ghtml

DRIVING QUESTION: As a motivator after the presentation of the video and text, the following question will be asked: How much are you investing in yourself and your future?

SKILLS:

- (EF05MA06) Associate the representations 10%, 25%, 50%, 75%, and 100% with the tenth part, quarter part, half, three quarters, and a whole, respectively, to calculate percentages using personal strategies, mental calculation, and a calculator in financial education contexts, among others.
- (EF06MA13) Solve and develop problems involving percentages, based on the idea of proportionality, without using the "rule of three," using personal strategies, mental calculation, and a calculator in financial education contexts, among others.
- (EF08MA04) Solve and develop problems involving percentage calculations also using digital technologies.

INTERDISCIPLINARY CONTEXTUALIZATIONS:

In addition to the study of percentages, it is possible to draw a parallel between the context of students' work and Brazilian socioeconomic data, integrating mathematics with geography. In this sense, the use of pie charts is a good tool and is present in the following skills:

- (EF07MA37) Interpret and analyze data presented in pie charts published by the media and understand when its use is possible or convenient.
- (EF07GE10) Create and interpret bar graphs, pie charts, and histograms, based on socioeconomic data from Brazilian regions.
- (EF09GE14) Create and interpret bar graphs and pie charts, thematic and schematic maps (sketches), and geographic anamorphoses to analyze, synthesize, and present data and information on global sociopolitical and geopolitical diversity, differences, and inequalities.

MATERIAL RESOURCES:

- Computer lab with access to software to work with spreadsheets and graphs (excel, libre office, etc.).
- Calculator.
- Paper, pen, pencil, eraser, ruler.
- Table model (spreadsheet) to be prepared.

NOTE: the table to be prepared must contain data on family expenses and income and the percentage of each expense in relation to total income.

PROPOSED ACTIVITIES AND ARTIFACT:

Initially, students should research and record what fixed and variable expenses are and cite some examples. After the research, they should form a circle and discuss the researched content, explaining what they understood and highlighting examples found and doubts.

As the final products of the work, students must prepare a table with their expenses and income (fixed

and variable), itemizing them. Next, based on the data collected, they should build a pie chart to represent them.
At the end, there will be a moment for a final analysis of the work, inciting a debate in which students must explain and argue their conclusions about the personal project carried out. At this moment of argumentation, if the student answers straighforwardly, they will be asked questions to direct and guide them toward an argument aligned with their project.

APPLICATION TIME

The project will take place throughout four 50-minute classes spread over two different weeks:
1st week:
- One class will focus on the presentation of the anchor, the driving question, and the project.
- One class will focus on the research to be proposed.

2nd week:
- One class will focus on preparing the table (spreadsheet).
- One class will focus on debate, presentation of considerations, and conclusions about the work.

Additionally, during the break between weeks, students will be required to collect data necessary to fill out and prepare the table (spreadsheet).

ASSESSMENT:

Students will be assessed on the development of classroom activities (research and discussion), the construction of the table (spreadsheet) of expenses and income, the preparation of the pie chart, and the arguments presented based on their projects.

Figure 7.
Project restructured by participants (Carmo, 2024, pp. 189-191)

When reflecting on the Transformation stage, based on epistemic suitability, the analyzed data showed that the first version of the project employed –albeit implicitly– mathematics in a financial planning context, connecting mathematical objects such as percentages, sector graphs, and proportion calculations to real-life situations. In the second version, specific skills from the National Common Curriculum Base (BNCC) were incorporated, relating percentages to mental calculation and technological strategies. This reformulation movement based on the perspective of the prescribed curriculum may be related to the reflective movement proposed by ecological suitability.

In identifying and articulating the different meanings of mathematical objects, we could see that mathematics was used in the first version of the project to help students manage personal finances, addressing concepts of fixed and variable expenses. In the second version, we observed an expansion of this perspective through the applicability of the concepts studied through interdisciplinarity, i.e., discussing socioeconomic data of the Brazilian population.

Following the concept of transformation described by Shulman (2014), the previous practice illustrates an application of the subprocesses of pedagogical transformation, especially regarding the "Expression of Conceptions" and the "Selection of Pedagogical Approaches." By employing real data in an interdisciplinary context, the educator not only chooses pedagogical strategies that facilitate students' conceptual understanding, but also adjusts these strategies to engage students, reflecting on how school subjects apply outside the classroom walls.

Final considerations

We present this article as a contribution to research that investigates the knowledge of teachers who teach mathematics and, more specifically, the importance of reflecting on how initial education spaces can contribute to the mobilization of knowledge.

The data that support the analyses emerged from a research-education process with PRP mathematics residents and preceptors. They emerged from a formative proposal that sought to problematize using PBL in mathematics teaching and learning processes. Thus, the first stage provided participants with an initial understanding of the PBL perspective, which supported the development of a teaching plan.

When considering didactic suitability as a formative device for the assessment process of the developed teaching plan, the data allowed us to perceive the potential of this analytical tool for the processes of reflection and new forms of understanding. Considering the different components and indicators of didactic suitability, participants could reflect on whether or not the proposal was suitable for implementation in the school context. This reflective process allowed new forms of understanding to emerge, which were adopted by the participants to reformulate the plan initially drawn up (transformation). With this plan in hand, they could implement and assess the proposal with basic education students. Since we are discussing a cyclical process, it is important to highlight that these processes are not hierarchical and may not occur as linearly as the writing suggests. This fact may have limited our analysis, but it was how we could highlight the possible contributions of the didactic suitability criteria to mobilizing pedagogical reasoning in formative spaces with prospective mathematics teachers.

Thus, based on the data, we could see that the choice of didactic suitability to promote reflection on the adjustments that would be necessary in the teaching plan was important. The different components and suitability indicators allow for a broader proposal evaluation. Indeed, if we had used another theoretical perspective, the results would have been different. However, would carrying out a reflection without a conceptual parameter favor new forms of understanding? When we compare this experience with experiences we have had in other formative spaces, we can infer that it is not. From our point of view, the absence of a reference to guide reflection would disregard other aspects, such as interdisciplinarity and the connections between mathematical objects emphasized in epistemic suitability.

Given the above, we understand that the results presented here align with the relevance of teacher educators planning formative actions intending to mobilize didactic-mathematical knowledge, as advocated by Silva and Tinti (2021).

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