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Transversal character of argumentation in the mathematics curriculum

Carácter transversal de la argumentación en el currículum de matemáticas

Caráter transversal da argumentação no currículo de matemática

Caractère transversal de l'argumentation dans le programme de mathématiques

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Abstract

According to the Ontosemiotic Approach to Mathematical Knowledge and Instruction (OSA), arguments are primary mathematical objects, composed of statements that allow the validation or explanation of propositions and procedures. Syllogisms make feasible the classification of mathematical statements, but argumentation is not exclusive to mathematical processes and appears in the curriculum in relation to other areas of knowledge. This study analyses the presence and use of the notions of argumentation in compulsory and pre-university Secondary School curriculum in Spain, with the objective of identifying transversal areas to mathematics, for the design of learning situations that have in their core the notion of argumentation. In the experimental phase, mathematics Teachers in Initial Training (TIT) design potential learning situations in transversal fields based on argumentation. Finally, these designs are analysed to determine which areas of knowledge TITs relate. Moreover, the argumentation used within the learnings situations shows which dimensions of didactic suitability enhance those proposals.

Keywords: Argumentation, Ontosemiotic approach, Mathematics teachers in initial training, Curriculum, Learning situation.

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Resumen

Según el Enfoque ontosemiótico del conocimiento y de la instrucción matemática (EOS), los argumentos son objetos matemáticos primarios, compuestos por enunciados que permiten validar o explicar proposiciones y procedimientos. Los enunciados matemáticos se clasifican en función de un silogismo, pero la argumentación no es exclusiva de los procesos matemáticos, y aparece en el currículum en relación con otras áreas de conocimiento. En este trabajo, se analiza la presencia y la utilización de las nociones de argumentación en el currículum obligatorio y preuniversitario de la Educación Secundaria en España, con el objetivo de identificar ámbitos de transversalidad con las matemáticas, para el diseño de situaciones de aprendizaje que tengan como eje vertebrador la argumentación. En la fase experimental, Docentes de matemáticas de secundaria en Formación Inicial (DFI) diseñan situaciones de aprendizaje en ámbitos transversales basadas en la argumentación. Finalmente, estos diseños se analizan para determinar qué áreas de conocimiento relacionan y conocer sobre qué dimensiones de la idoneidad didáctica pivotan las argumentaciones presentes en las propuestas.

Palabras clave: Argumentación, Enfoque ontosemiótico, Docentes de matemáticas en formación inicial, Currículum, Situación de aprendizaje.

Resumo

Segundo o Enfoque Ontossemiótico do Conhecimento e da Instrução Matemática (EOS), os argumentos são objetos matemáticos primários, compostos por enunciados que permitem validar ou explicar proposições e procedimentos. Os enunciados matemáticos são classificados com base num silogismo, mas a argumentação não é exclusiva dos processos matemáticos, surgindo no currículo em relação a outras áreas do conhecimento. Neste trabalho, analisa-se a presença e a utilização das noções de argumentação no currículo obrigatório e pré-universitário do Ensino Secundário em Espanha, com o objetivo de identificar domínios de transversalidade com a matemática, para o desenho de situações de aprendizagem cujo eixo estruturador seja a argumentação. Na fase experimental, Docentes de matemática do ensino secundário em Formação Inicial (DFI) concebem situações de aprendizagem em domínios transversais baseadas na argumentação. Por fim, estas propostas são analisadas para determinar como conectam diferentes áreas do conhecimento e compreender sobre que dimensões da idoneidade didática se centram as argumentações presentes nas propostas.

Palavras-chave: Argumentação, Enfoque ontosemiótico, Docentes de matemática em formação inicial, Currículo, Situação de aprendizagem

Résumé

Selon l'Approche onto-sémiotique de la connaissance et de l'instruction mathématique (AOS), les *arguments* sont des objets mathématiques primaires, composés d'énoncés qui permettent de valider ou d'expliquer des propositions et des procédures. Les énoncés mathématiques sont classés en fonction d'un syllogisme, mais l'argumentation n'est pas exclusive aux processus mathématiques, et apparaît aussi dans d'autres domaines de connaissance du programme de l'enseignement. Dans ce travail, on analyse la présence et l'utilisation des notions *d'argumentation* dans le programme d'enseignement obligatoire et préuniversitaire secondaire en Espagne, dans le but d'identifier des domaines transversaux avec les mathématiques, pour la conception de situations d'apprentissage qui ont comme axe principal *l'argumentation*. Dans la phase expérimentale, Enseignants de mathématiques en Formation Initiale (EFI) conçoivent situations d'apprentissage potentielles en domaines transversales basés sur la notion d'argumentation. Finalement, les designs sont analysés afin de déterminer quels domaines sont transversaux. En plus, ils permettent d'identifier sur quelles dimensions d'*adéquation didactique* se disposent les notions d'argumentation de la situation d'apprentissage.

Mots-clés : Argumentation, Approche onto-sémiotique, Enseignants de mathématique en formation initiale, Programme, Situation d'apprentissage.

Transversal character of argumentation in the mathematics curriculum

Within the framework of the Ontosemiotic Approach (OSA), arguments are statements whose objective is to explain, verify, justify or validate propositions or procedures (Godino, 2024; Godino et al, 2016). Primary mathematical objects are considered and, with the use of language, concepts, procedures and situations, the management of operative, discursive and regulatory practices in mathematics is feasible. We resort to them due to their usefulness and interest in a context for the solution to a problem or the analysis of a situation without an epistemological analysis of its nature being required.

While arguments are classified, in general, as inductive and deductive, the contexts of use for argumentation in teaching and learning situations exceed this classification in practice. Evidence of this is the existence of arguments of the abductive, analogical or metaphorical type in investigations on mathematics teaching (Manrique & Soler-Alvarez, 2014; Molina, Font & Pino-Fan, 2019; Rondero & Font, 2015). Authors such as Viana & Almouloud (2013) highlight the importance of involving students in argumentative processes that meet the norms of mathematics from an early age. These same authors favour the material resource GeoGebra (a dynamic geometry software) as a tool that facilitates communicative and argumentative processes.

In fact, the choice of the material resource influences the uses of argumentation. For example, argumentation by means of a dynamic geometry software surpasses the inductive character of analysis of the specific examples of a mathematical property, since it can show an infinite number of examples in a short period of time, while the absence of counter-examples when using the dynamic model is enough evidence for a student of the veracity of a property (Lasa & Wilhelmi, 2013). Additionally, there is the progressive introduction of developments in the *automatic demonstration of theorems* which allows one to demonstrate a proposition if it meets a finite number of cases (Botana et al., 2015).

These argumentative schemes result in *dialogical didactic configurations* in which the instructor fits the student's argument into an implicit theory of reference (Lasa, Wilhelmi & Abaurrea, 2017). Hence, although a property can be verified for a reduced number of cases (those exemplified in class), the argumentation is not merely inductive and is supported by an implicit theory that gives support to the argumentation, according to the following syllogism:

$$A \subset B, \forall x \in A, P(x) \rightarrow \forall x \in B, P(x)$$

The use of argumentation in teaching and learning situations in mathematics should be adjusted, therefore, to its use according to the institutional norm in effect (GN, 2022a, 2022b).

In this sense, one should look for methodologies founded on collective argumentation that allow, besides conveying mathematical knowledge, the development of skills such as investigation, argumentation and reasoning (Coelho & Alvarenga), 2024). In the context of an educational curriculum, as an additional circumstance, *argumentation* is also expressed in the areas of pure, experimental, human, social and artistic sciences. In these cases, a situation of transversal learning in two fields, there should be awareness of the institutional meanings of argumentation in both disciplinary fields.

The first aim of this study is, therefore, to analyze the presence of concepts related to argumentation in the mandatory and pre-university curriculum of secondary education (GN, 2022a, 2022b). With this, it is our intention to compare the *institutional meanings* of argumentation in different subjects in order to identify areas of transversality with mathematics. In this sense, the planning of learning situations with a mathematical component connected to other extra-mathematical content is facilitated. It is the intention that the main axis of transversality constitute the very notion of argumentation, regardless of the curricular content dealt with.

The second aim is related to the experimental phase, in which secondary school math teachers in initial training (TIT) are requested to plan a transversal learning situation based on argumentation. With this, the idea is to find out what aspect of didactic suitability (Godino, 2024) most TITs evoke in the argumentations that emerge in the elaboration of a transversal learning situation.

The OSA determines six aspects to evaluate the didactic suitability of educational-instructional processes: *epistemic* (the knowledge and articulation of the system of institutional meanings of math content), *ecological* (the adequacy of a formative action to learn mathematics in a given environment), *mediational* (the availability of adequate resources for the optimal development of the teaching-learning process), *interactional* (patterns of interaction to develop communicative competencies and promote learning autonomy), *cognitive* (the adequacy of learning objectives so they are attainable to students as well as the consistency between the personal meanings achieved and the institutional ones planned), and *affective* (student involvement, interest, motivation, self-esteem and disposition). Hence, the aim is to identify which aspect the TITs resort to when planning their transversal proposals based on argumentation.

Argumentation in the secondary education curriculum

In the curriculum (GN, 2022a, 2022b), the concepts of argumentation and reasoning come up both in scientific and technological areas, classified as STEM (table 1), as well as in humanistic (language and literature), social (geography and history) and artistic (visual and musical arts) areas.

STEM descriptors define the competencies in science and technology that a student should acquire at the end of each educational level. Basic Education in Spain³, composed of Primary (7- 12 years old) and Secondary (13-16 years old) Education, is followed by the pre-university level called Baccalaureate (17-18). Among these descriptors is mathematical reasoning in the context of the resolution of problems. The first descriptor states that all students, when the mandatory level has ended, must "[use] *inductive* and *deductive* methods" (GN, 2022a, p.27) in different contexts of familiar situations. At the end of the pre-university level, the descriptor evolves, and the students must not only use these methods but also "choose" the one that is the most appropriate in each case (GN, 2022b, -.21).

In the areas of Biology and Geology, the idea of *argumentation* is seen connected to the application of the scientific method in the planning and development of investigative projects. In other cases, it is associated with the resolution of problems and computational thinking. For example, specific competency (SC) 4 explains that every student must:

Use reason and computational thinking, critically analyzing the answers and solutions and reformulating the procedure, if needed, to solve problems or explain daily life processes with the help of biology and geology (GN, 2022a, p.35).

In this sense, argumentation should be based on scientific data and facilitate coherent decision-making. Furthermore, communicative interactions should be constructive, respectful and flexible. *Computational thinking* is considered in contexts of problem resolution, the use of *logical reasoning* and mathematics for the development of mathematical models that validate the empirical character of biology and geology, or the use of mechanisms that are a shield from pseudoscience. When evaluating these competencies, the application of basic principles in

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³ The educational system in Spain states that it is the responsibility of the Autonomous Communities to plan and adjust the educational curriculum to the setting in which it will be applied. Hence, state curriculum is adjusted to the territorial reality of each community which can, for example, have official languages aside from Spanish. This adjustment is carried out through the realization of specific competencies, evaluation criteria and basic knowledge. In this sense, this study's reference is the Foral Decrees of application in the Foral Community of Navarra, a community that retains a broad margin of self-government.

biology and geology on the part of the student is made clear, in a context that must necessarily be guided by the teacher.

In the separate subjects of Biology and Geology at the Baccalaureate level, argumentation is kept up in similar contexts, but the guided nature of the investigation gives way to investigative contexts that are potentially transversal, which imply "raising or solving issues, locating and using rigorous sources of information, contrasting and justifying data and coming to conclusions" (GN, 2022b, p.38). The context in which notions of biology, geology and environmental sciences are applied resembles real, current-day issues related to the environment (consumerism, responsible use, etc.) in which argumentation on concrete examples should lead to objective conclusions.

In the areas of Physics and Chemistry, all *argumentation* should be based on the observance of the norms of communication (IUPAC, etc.) and international agreements (social setting, environment, etc.) adopted by the scientific community and which have a universal and interdisciplinary initiative. Thus, for example, SC2 indicates that there should be:

The expression of observations made by students in the form of questions, formulating hypotheses to explain them and demonstrating these hypotheses through scientific experimentation, questioning, and the search for evidence, in order to develop individual reasoning skills and improve abilities in the use of scientific methodologies. (GN, 2022a, p. 92).

The evaluation of these competencies assesses how the application and communication of these laws, theories and principles are argued in contrast to pseudoscience. Argumentation is also part of the content and basic knowledge (BK) within the planning of experiments and investigative projects, in topics such as energy or dynamics (GN, 2022a, pp.96-97). In these cases, logical-mathematical reasoning should allow the validation of the experimental and the inquisitive nature that are inherent to the scientific method.

In the separate subjects of Physics and Chemistry in the Baccalaureate, logical-mathematical reasoning takes on a technical character, at the same level of experimental work, research or the search for evidence. Scientific reasoning should lead to "the formation of a higher order of thinking", allowing for the understanding of scientific laws and theories (GN, 2022b, p.150).

In mathematics, the idea of *argumentation* and *reasoning* has an important role from the very preamble of the curriculum in which it is specified that "reasoning, argumentation, modelling, knowledge of space and time, decision-making, predicting and controlling

uncertainty or the correct use of digital technology are all characteristics of mathematics" (GN, 2022a, p.184).

The formulation of conjectures is addressed, but also the establishment of intra and extra mathematical connections among mathematical elements and other subjects. Reasoning and argumentation lead to the creation of new knowledge in the context of validating conjectures (SC3, p.187), reformulating new conjectures or resolving problems (SC1, p.185). This is made explicit in terms of *analytical thinking*, *demonstration* or *reflection*. As content (BK), it is associated with *numerical meaning* and *spatial meaning* in contexts of proportionality or geometric visualization and modelling (GN, 2022a, p.193).

In the separate subject of Mathematics in the Baccalaureate, the notion of reasoning and argumentation continue to be associated with modelling, computational thinking and problem solving. The process of reasoning and validation is also associated with the determination of mathematical notions and knowledge so that reasoning and argumentation should also serve the purpose of "giving meaning and structuring mathematical learning" (GN, 2022b, p. 314) with the support of digital tools. The notion of creativity appears in connection with processes of *reasoning* and *proof*. Logical reasoning appears as content (BK) within the meaning of space and algebra: "Visualization, reasoning, and geometrical modelling [...]. Comparing alternative algorithms for the same problem through logical reasoning" (GN, 2022b, p. 319).

The curriculum also establishes a field of integrated science for basic grade formative cycles, called Applied Science. *Argumentation* is used here in contexts of embracing healthy habits, in interpreting and modelling, in scientific terms, of problems and situations of daily life and work, in problem-solving and global challenges. *Inductive* and *deductive* reasoning are specified, as well as in terms of *computational thinking* (automation, algorithmic thinking, breaking down into parts, etc.). The evaluation of these competencies is carried out through the reasoned application of the scientific method to explain phenomena or carry out predictions. The results obtained in investigative projects are also argued with the use of mathematical and technological tools.

The field of General Sciences is also part of the Baccalaureate, in which reasoning is seen as "an essential tool for scientific investigation" and, along with it, logical-mathematical reasoning" (GN, 2022b, pp.54-57.

In Technical Drawing, as well as in drawing applied to visual arts, the curriculum contemplates argumentation in problem solving that requires the use of spatial vision through procedures that are precise, clear and well executed, even while contemplating the use of a computer software to assist in the drawing. In other words, "the solving of mathematical-

graphical problems with the application of inductive, deductive and logical reasoning" (GN, 2022b, p.83).

Table 1.

Presence of argumentation in STEM areas (GN, 2022a, 2022b)

| Subject | PRE | SC | EVC | BK |
|---|-----|----|-----|----|
| STEM Descriptors | 2 | 2 | 0 | 0 |
| Biology, Geology and Environmental Sciences | 1 | 16 | 20 | 1 |
| Physics and Chemistry | 1 | 6 | 14 | 6 |
| Mathematics, General mathematics and | | | 17 | 6 |
| Mathematics applied to social sciences | | | | |
| Applied Science and General Science | 0 | 6 | 5 | 4 |
| Technical drawing, technical drawing applied to visual arts and to design | 3 | 5 | 6 | 1 |
| Technology and engineering | 0 | 0 | 2 | 0 |

PRE: Preamble; SC: Specific competencies; EVC: Evaluation Criteria; BK: Basic knowledge.

The concepts of argumentation and reasoning appear as well in the subjects of human, social and artistic sciences (Table 2). In the same way as with the STEM descriptors, there are specific descriptors in the curriculum to determine the linguistic competence level a student needs to reach at the end of the mandatory school stage.

In the mandatory curriculum of Secondary Education, the subjects Language and Literature share the specific competencies of communication and reading. In the case of communities with more than one official language, the study of languages contemplates the state language (Spanish), the community language (in the case of the Foral Community of Navarra, Euskera) and foreign languages (English and, in this case, French as well). In all cases, the systematic study of the language is promoted with its use in a variety of contexts, the production of oral and written texts or the acquisition of strategies to write simple arguments. In this manner, by means of SC9, language learning aims at improving reasoning and argumentation capacities:

To mobilize knowledge about the structure of the language and its uses and to reflect in a progressively independent manner on linguistic and discursive choices, with the right terminology, in order to develop linguistic awareness, increase the communicative repertoire and improve abilities both in oral and written production and understanding as well as in critical interpretation. (GN, 2022a, p. 140).

The evaluation also contemplates argumentation in its criteria. Specifically, students must be able to explain and argue the interpretation of works that have been read, orally express simple texts with the aim of arguing and informing, and also, within a metalinguistic dimension, "compare and argue the similarities and differences among different languages" (GN, 2022a, p. 164). In a more general sense, students must be able to "explain and argue the interrelation between the communicative proposal and the linguistic choices of the sender as well as the effect on the receiver, using explicit knowledge of the language and a specific metalanguage" (GN, 2022a, p. 141).

At the Baccalaureate level, aspects of argumentation connected to written texts are expanded, as well as the oral justification of opinions, with special importance being given to aspects of mediation, with growing extension and complexity. For example, SC2 stipulates that there should be the production of "original texts [...] to express ideas and arguments creatively, adequately and coherently, according to concrete communicative proposals" (GN, 2022b, p.261).

Argumentative reading is far from mere entertainment and intends to move towards an independent reading corpus that also encourages argumentative oral deliberation, especially in relation to universal literary works. The purpose is to also "fight against, in an argumentative form, linguistic biases and stereotypes" (GN, 2022b, p.246) about dialectal and language varieties.

Within the study of classical languages, the study of Latin also promotes reasoning and argumentation. Hence, SC4 determines that "translation [...] boosts memory and potentializes the habit of discipline in studying to promote sharpness in reasoning and learning" (GN, 2022a, p.127). At the Baccalaureate level, the study of Latin and Greek focuses on the logical interpretation of original or adapted texts, while translation continues to be the tool through which "logical reasoning" and the "capacity for analysis and synthesis" are developed (GN, 2022a, p.189).

In Philosophy, the use and identification of arguments within the practice of conversation is demanded, as well as a cognizant mastery of the procedures of argumentation. This is, therefore, a transversal competency which also requires honesty. In this sense, it is required that one (SC3):

Use and value, in an adequate manner, arguments and argumental structures based on both their formal and informal analyses to rigorously produce and appraise different types of discourse and avoid dogmatic, fallacious and biased forms of asserting opinions and hypothesis. (GN, 2022b, p. 134).

Going on with the subjects pertaining to the humanities, Geography and History use argumentation, along with questioning, as a tool to create products that lead to the development of critical thinking. The evaluation of this competency is applied to current topics and happenings, with special attention given to the use of trustworthy sources of information, such as historical proof, upon which a student should argue. Specifically, argumentation should focus on social issues of vital importance, such as real equality among men and women, against any discriminatory attitude, or the need to preserve the environment:

To argue the need for defensive, protective and conservative actions and the improvement of our surroundings (natural, rural and urban) through proposals and initiatives that result in commitments and conducts that favor sustainability and the fair and humanitarian distribution of resources. (GN, 2022a, p.113).

Following with other social disciplines, we can find references to argumentation in Education on civic and ethical values, such as content (BK, B.1) regarding society, justice and democracy. Here, "the virtues of communication and argumentative norms" (GN, 2022a, p.83) are mentioned as an instrument to develop empathy for other people. Indeed, in SC1, on personal and professional formation and guidance, it says:

[...] It is important that students know, on the one hand, about the neuro-scientific findings that allow one to understand the process of reasoning, decision-making and problem solving [...] It is important to know about the impact of emotions on the processes of motivation, reasoning, learning and behavior [...]. (GN, 2022a, p.102).

At the Baccalaureate level, the subject expands into several other subjects that continue to focus on argumentation based on trustworthy, accessible and contrasting data. In the study of Geography, rigorous justifications and the application of scientific methods are the aim, as well as the description of social phenomena such as wealth distribution, environmental phenomena such as sustainable lifestyles, and social-work phenomena to diagnose problems and offer logical solutions.

Therefore, for example, SC4 stipulates that "the ICT [information and communication technologies] should use their potential to achieve these goals with the use of interactive maps and resources that facilitate arguments to justify the extension of each phenomenon" (GN, 2022b, p.168). The same occurs in subjects related to national and contemporary History in which evaluation criteria (EVC) such as 7.1 is found, according to which students must "come up with argumentative opinions, debate and exchange ideas and knowledge concerning the role

that thoughts and ideologies have had in transforming reality (based on documents)" (GN, 2022b, p.232).

Similarly, concepts related to argumentation can be seen in the History of Philosophy, History of Music and Dance or Art History, in which, in the same manner, artistic works must be evaluated and argued about, favoring in this case, dialogical methodologies. Specifically, besides developing reasoned historical and artistic commentaries regarding works, students should "develop their own arguments about the idea of beauty, comparing canons and different types of works" (GN, 2022b, p.222).

Lastly, references to argumentation can also be seen in artistic disciplines. For example, in Plastic, visual and audiovisual Education, EVC3.2, for the third year, specifies that students should:

Debate the pleasure produced by art reception in all its forms and expressions, both as an individual and social observation and as the creative formulation of the answers given in each era to universal questions raised by human beings, sharing impressions and emotions respectfully and expressing one's personal opinion openly. (GN, 2022a, p. 72).

In artistic disciplines, argumentation is always based on the search for information from well-founded, trustworthy sources, and every argument should be based on the use of the terminology proper to that subject. For example, in Musical Analysis, EVC3.3 indicates that students should "justify their personal opinions about the works analyzed, investigating and selecting the most pertinent information based on analogical and digital means" (GN, 2022b, p.28).

Similar entries are found for Performing Arts (the argued identification of the target audience of a specific work), Choir and Vocal Techniques (reasoned analysis of vocal pieces), Audiovisual Culture (arguments based on aesthetic criteria) and Artistic Foundations (the construction of an argued discourse). Artistic Drawing goes a step further and in its EVC1.3 states that students must "defend the importance of freedom of expression for cultural and artistic plurality through a reasoned and argued discourse in an active, engaged and respectful manner" (CN, 2022b, p.75).

Table 2.

Presence of argumentation in the human, social and artistic areas (GN, 2022a, 2022b)

| Subject | PRE | SC | EVC | BK |
|--|-----|----|-----|----|
| Language and literature (Spanish, Euskera and foreign; dramatic, universal literature) | 4 | 6 | 30 | 15 |

| Classical languages (Latin, Greek) | 2 | 2 | 5 | 2 |
|--|---|----|----|---|
| Philosophy | 1 | 5 | 4 | 2 |
| Geography and History (Geography, History, History of Spain, History of Philosophy, History of music and dance, Art history, Contemporary world history. | 4 | 17 | 13 | 0 |
| Economy, Business and business model plans | 1 | 2 | 3 | 1 |
| Artistic subjects (Musical analysis, Performing Arts, Choir and vocal training, Audiovisual culture, Artistic drawing, Artistic projects, Cultural and artistic movements, Graphic-practical techniques of expression, Volume) | 4 | 7 | 28 | 0 |

PRE: Preamble; SC: Specific competencies; EVC: Evaluation Criteria; BK: Basic knowledge.

Planning of potential learning situations: experiment and results

After the analysis of the current curriculum was carried out through the perspective of argumentative transversality, the TITs in the Secondary Education Teaching Graduate Program were asked to plan potential learning situations that connect Mathematics to another subject in Secondary Education. The connecting link between the two subjects needed to be *argumentation*. The sample is made up of 11 TITs who previously had graduated in scientific-technological programs in Mathematics, Science or Engineering (Table 3). Subsequently, the learning situations proposed by these TITs were analyzed through an analytical model based on the observation and examination of these situations whose discussion is focused on the position the concept of argumentation had as a curricular transversal link.

Table 3. *Transversal subjects and argumentation in the experiment*

| Level | Transversal subject to mathematics | Authors |
|----------------------------------|------------------------------------|-------------------|
| Mandatory Secondary Education | Biology and Geology | TIT1 |
| Mandatory Secondary Education | Physics and Chemistry | TIT 2 |
| Mandatory Secondary Education | Geography and History | TIT 3, TIT4, TIT5 |
| Mandatory Secondary Education | Music | TIT6 |
| Baccalaureate | Language and Literature | TIT7, TIT8 |
| Baccalaureate in Sciences | Biology | TIT9 |
| Baccalaureate in Sciences | Technical Drawing | TIT10 |
| Baccalaureate in Social Sciences | Economy, World History | TIT11 |

In the sample, 6 TITs developed proposals for the mandatory level (2 proposals in sciences, 3 in Social Sciences and 1 in Art). Five other proposals focused on the Baccalaureate level (2 proposals in the Humanistic area, 1 in sciences, 1 in the technical category and 1 for Social Science). Therefore, there are 3 proposals that seek to relate the mathematics curriculum with other natural and experimental sciences, both in the mandatory and the pre-university level (Table 4).

Table 4.

Transversality of mathematics with natural and experimental science

| Subject | SC | EVC | ВК |
|------------------------|---------------------------|---|--|
| Mathemtatics | 1, 2, 3, 6, 7 and 8 | 1.1, 2.1, 2.2, 3.1, 6.1, 6.2, 6.3, 8.1, 8.2 | A3, B2, B2.1, C1.1, C3, C3.2, C3.3, C4, D2, D2.1, D2.2, D3.1, D3.2, D4, D4.1, D4.2, D4.3, D5.1, D5.2, F1.1, F1.3 |
| Biology and Geology | 1 and 4 | | (Phases of planning a scientific experiment) |
| Physics and Chemistry | 1, 2, 3, 5 and 6 | 1.1, 1.2, 2.1, 2.2, 3.1, 5.1, 6.1 | D1, D2, D3, D4. |
| Technical Drawing | 1, 2 and 4 | 1.1, 2.1, 2.3, 4.2 | A1, A5, A7 |

SC: Specific competencies; EVC: Evaluation criteria; BK: Basic knowledge.

In the first (TIT1), a learning situation was planned to search for patterns in nature (for example, in honeycombs, sunflower seeds, nautilus shells or the phyllotaxis of leaves), focused on Mandatory Secondary Education. By means of a project based on research, number sense (operations in contextualized situations), geometric sense and algebraic sense (the search for patterns in nature) are related. In this case, argumentation allows students to observe that these mathematical patterns in nature are not random but a response to the principles of biological *efficiency* and *optimization*. Each structure follows mathematical rules that allow the elements in nature to *maximize* resources, grow in an *optimal* fashion and ensure their *survival*. The concepts of *reasoning* and *argumentation* lead to the rigorous understanding of the process of scientific experimentation.

The second proposal for a learning situation (TIT9) is also focused on mathematical patterns in biological contexts, but at the more advanced level of the Baccalaureate. The characteristics of this level allow that the activity be centered on aspects of modelling, using algebraic and functional concepts with the use of digital tools. Hence, concepts related to

virology and epidemiology are studied, such as transmission, infection curves or containment measures.

The third proposal in the scientific area (TIT2) consists of a learning situation to study the *looping* of roller coasters in amusement parks. The development of this proposal is once more based on a process of inquest, starting with the historical evolution of looping, the comparison of different models and their physical properties. Experimentation through the scientific method is conducted with the use of models, marbles and chronometers, modelling with a software, modelling with functional and algebraic expressions and their physical meaning, with the aim of arguing which model is better and why.

There is a fourth proposal of the technical type (TIT10) focused on a transversal learning situation between Mathematics and Technical Drawing at the mandatory level. Here, a learning sequence of isometric transformations and the tessellations of the plan are projected. It starts with specific examples from daily life, such as tiles, roofs or mosaics, and the resolution of graphical-mathematical problems are worked out through *inductive* (properties that emerge from concrete cases in architectural patterns that are repeated, the search for rules, the construction of categories, etc.) and *deductive* procedures (composition and decomposition of complex tiles based on an already validated rule). Throughout the entire process, special attention is given to precision in technical drawing.

Two proposals that aim at relating the mathematical curriculum with humanistic subjects were identified in the generic block of the pre-university level, related to language and literature. Both learning situations can be applied to the vehicular languages Euskera or Spanish.

The first learning situation (TIT7) connects the discussion about an educational system with the writing and analysis of a text explaining and giving examples of an algebraic model that needs to be solved. Each student writes an individual text and subsequently this text is the object of study to analyze its structure and coherence. The final objective of this learning situation is to develop criteria for the classification of texts that lead us to determine in which educational systems described there will be one solution, several solutions or none.

The second transversal proposal in the human sciences (TIT8) is focused on the logical analysis of language to differentiate logically correct arguments from argumentative fallacies (formal or informal). Mathematical topics are dealt with from the language construction perspective in small groups and at moments of content micro-institutionalization (Brousseau, 1998).

Table 5.

Transversality of mathematics and human sciences

| Subject | SC | EVC | ВК |
|-------------------------|----------------------|--------------------------------------|--|
| Mathematics | 1, 2, 3, 8 and 9 | 1.1, 1.2, 3.1, 8.1, 9.1, 9.2, 9.3 | A1.1, A1.2, C1.1, C3.2, C3.3, D1.1, D2.2, D2.3, D3.2, D5.1, F1.1, F1.2, F3.1 |
| Language and literature | 2, 3, 5, 8 and 10 | 3.1, 3.2, 5.1, 5.2, 10.2 | B1.1, B2.1, B2.2, B3.1, B3.3, B3.4, B3.5, B3.6, B4.2, B4.3, B4.4, B4.5, B4.6 |

SC: Specific competencies; EVC: Evaluation criteria; BK: Basic knowledge.

Four learning situation proposals that aimed at relating the mathematics curriculum with several social sciences were identified, both at the mandatory and the pre-university level.

The first three proposals focus on the mandatory level and deal with issues related to ancient history or demographics. The first proposal (TIT3) addresses argumentation from the perspective of ancient history, taking mathematics to the study of ancient Egypt. Students needed to solve numerical and geometric tasks by developing arguments regarding the technological tools used at that time. The second and third proposals (TIT4 and TIT5) respectively focus on the reading of population pyramid graphs and the evolution of the world population. In these cases, argumentation is connected both to the development of graphs with the use of trustworthy sources of information and the process of debating the interpretation of these graphs.

The fourth proposal in the area of social sciences (TIT11) describes a learning situation that focuses on analyzing wealth distribution around the world. Argumentation is used to validate the strength of the solutions found for the problem in terms of equity or in defending eco-socially responsible initiatives. Students must debate their choices with arguments based on economical, social and environmental aspects, using the mathematical meaning of the main economic indices or indicators (GDP, GDP per capita, Gini index, etc.). Transversality with history also facilitates documented evidence of the historical origin of these inequalities, encouraging critical thinking.

Table 6.

Transversality of mathematics and social sciences

| Subject | SC | EVC | BK |
|-------------|----------------|-------------------------------|---|
| Mathematics | 2, 3, 6 and 7 | | C5.1, C5.2, D2.4, E1.2, E1.3, E1.4, E2.2., E3.1, E3.2 |
| Geography | and 1, 2, 3, 5 | 2.1, 2.2, 2.4, 3.1, 3.4, 3.5, | A5, A6, B1, B6 |

| History | and 8 | 5.2 | |
|---------|-------|----------|------------|
| Economy | 5, 6 | 5.1, 6.2 | C2, C3, E1 |

SC: Specific competencies; EVC: Evaluation criteria; BK: Basic knowledge.

A last proposal (TIT6) aims at relating the mathematics curriculum to musical artistic subjects at the mandatory level. In this case, the proposal focuses on transversal contents of proportionality. It is observed that proportionality is a content that is part both of mathematics and of sound in musical instruments. In artistic contexts, the concept of argumentation is connected to "the pleasure of sound" and, in this case, mathematical argumentation is associated with the description of sound with functional graphs.

Table 7. *Transversality of mathematics and arts*

| Subject | SC | EVC | BK |
|-------------|------------|-----|--------------------|
| Mathematics | 1, 2 and 8 | 8.2 | A5.3, D5.2, D5.3 |
| Music | | | A1, A2, A5, A8, A9 |

SC: Specific competencies; EVC: Evaluation criteria; BK: Basic knowledge.

Results Analysis

The curriculum of the Baccalaureate level (GN, 2022b) establishes the subjects students must take or that they can choose for different paths. The curriculum describes each of them and which specific competencies and basic knowledge students should acquire, as well as the evaluation criteria teachers should implement to assess each competency.

Of the 44 subjects in the curriculum (GN, 2022b), 41 explicitly mention the need for argumentation, reasoning and justification. The only exceptions are Economy, Business and business activity; Physical education; and Language and musical practice, which do not include these aspects. The subjects that most frequently mention these abilities are part of the sciences, such as Mathematics, Biology and Geology. However, Chemistry, Physics and Technology and Engineering mention them less frequently, although they are part of the STEM area. Philosophy and Literature of languages (Spanish and Basque) also extensively require argumentation, reasoning and justification. In contrast, subjects related to the Arts and Economy include these abilities less frequently.

Emphasis on argumentation is found mainly in EVCs, more than in the SCs and BKs. In some cases, the SCs do not directly mention argumentation, although it is found in their description. For example, in Biology, SC3 establishes the "analysis of studies of investigation or dissemination related to the biological sciences, verifying with a critical view their veracity or if they have followed the steps required by scientific methods to evaluate the trustworthiness of their conclusions" (GN, 2022b, p.40).

Nevertheless, their description highlights:

[...] not only critical thinking but also communicative and digital skills and *logical reasoning* [...] preparing students to recognize fallacies, hoaxes and pseudoscientific information and to form their own opinion based on *reasoning* and evidence, hence contributing positively to their personal and professional integration and participation in a democratic society. (GN, 2022b, p.40).

In Philosophy, SC2 does not mention argumentation at the beginning:

To search, manage, interpret, produce and transmit information related to philosophical issues correctly, based on the comparative and safe use of sources, the rigorous use and analysis of these sources and the handling of basic investigative and communicative procedures to develop an inquisitive, independent and creative attitude in the field of philosophical reflection. (GN, 2022b, p. 134).

However, its description emphasizes that "the objective is that students, genuinely inspired by questions and philosophical issues, and once they reach a basic and informed understanding of the main philosophical theses and concepts through *argumentation* and discussion, go on to..." (GN, 2022b, p.134).

Thus, when analyzing, verifying with a critical view or communicating reflections, an implicit reference is made to argumentation.

Furthermore, there are subjects in which argumentation is not mentioned explicitly in the SC but is seen in their EVCs. For example, in Contemporary World History, although SC5, SC6 and SC7 do not include argumentation, some of their EVCs do. For example:

7.1 To develop opinions with argumentation, to debate and transfer ideas and knowledge about the role that thought and ideologies have had in transforming reality, from the beginning of the Contemporary Era until today, understanding and contextualizing this phenomenon through work done with historical and historiographical texts and literary sources from the movies and other audiovisual documents. (GN, 2022b, p.232).

In these cases, the SCs refer to critically evaluating, comparing, debating or analyzing. The same is seen in Universal Literature where there is no mention of argumentation in its SC but it is found in four EVCs that correspond to SC1, SC2 and SC3. For example, EVC1.1 indicates that students should:

Explain and use argumentation for the interpretation of works they read based on the analysis of the internal relations of their constitutive elements and the meaning of the work and the external relations of the text with its social-historical context and the literary tradition, with the use of a specific metalanguage, incorporating value judgements associated with the aesthetical appreciation of the works. (GN, 2022b, p.305).

This idea is reflected implicitly in the specific competency with the expression "interpret and assess classics of universal literature" (GN, 2022b, p.304).

We can definitively say that, as a first result, the concepts of *reasoning* and *argumentation* appear irregularly in the SCs, EVCs and BKs throughout the curriculum of the Baccalaureate due to the role each subject or discipline takes on.

Having said that, this uneven presence does not limit the potential of the concept as a main axis for transversal learning situations for other subjects that appear in areas connected to other disciplines in science and technology, social sciences, human sciences and the arts.

The proposals of transversal learning situations with other scientific disciplines (TIT1, TIT2 and TIT9) are marked by trying to naturally find a structure for a project that allows the articulation of the phases of an experiment or the implementation of a methodology through research. Mathematics provides algebraic, geometric and functional elements in this context for the modelling of a biological or physical process. A frequent phrase adds the adjective "scientific" to the term "argumentation", especially in contexts of the "formulation among equals". The aim of the *scientific argumentation among equals* consists of developing critical thinking and the capacity to justify invariably in a school context that requires the *explanation* of what is being done *rigorously*.

Outside STEM contexts there are also opportunities to link the communication of arguments used to humanistic and social contexts, strengthening the inter-relational dimensions of the process of studying. In this sense, in the learning situation proposals, it is seen that the TITs bestow a great deal of importance to a student's argument due to its *inter-relational suitability*. Hence, argumentation is seen as an element that encourages exchanges and communication among students, as a tool to convince their peers and to find a consensus, or as a strategy to encourage inclusiveness. In the Social Science proposals, the processes of the

rigorous search for information, as well as the development of graphic supports and their communication, also foster the *inter-relational suitability* of the learning situation.

Specifically, the concept of argumentation is connected to moments of *action* and *formulation* (Brousseau, 1998) in the development of a project or task that needs to be solved, i.e., to discussions in small work groups in which students give their opinions more frequently. When the time comes to *communicate results*, argumentation continues to have the character of a *formulation*, now associated with the *validation* of the argument (Brousseau, 1998) in a large group discussion. In this sense, argumentation encourages the process of building learning, hence strengthening cognitive appropriateness along the way. Meanwhile, in written communication processes, the writing of coherent texts that represent a mathematical model also inspire the development of these mathematical learnings.

To a smaller degree, the adjective "validate" appears with the term "argumentation", an expression that refers to the *value of truth* of the argument used. In their proposals for learning situations, the TITs only mention the validity of the mathematical arguments used, or the adequacy of the proof and demonstrations used, educationally. In humanistic and social contexts, the TITs found opportunities to give significance to the validity of an argument. For example, in proposals TIT7 and TIT8, in the linguistic and humanistic areas, the object of differentiating between true and false arguments emerges, which evokes the epistemic appropriateness of the argument, in this case, from the point of view of language use.

The arguments used in the context of the Social Sciences, when sensitive topics such as wealth distribution are dealt with, allow for the optimization of the process of studying mathematics in contexts to which students feel connected. In these proposals, the timeliness and closeness of the topics proposed incorporate elements of *affective suitability*. It is in the musical context (TIT6) that argumentative concepts appear most clearly, connected to taste and pleasure. In this sense, the affective suitability of the proposal is greater than in the other proposals studied. The context is conducive for working with the language of functions and graphs and the role of the main axis of the concept of proportionality is respected.

Lastly, it is observed that when the proposal has a technical character (TIT10), the concept of *argumentation* is closer to the meaning of *procedure*. In this case, the OAS differentiates between what *arguments* (inductive, deductive or other types) and *procedures* (algorithms or techniques) are among primary mathematical objects. Nevertheless, the curriculum treats these in the same way when giving a procedure a "formal and rigorous argumentative" character.

Conclusions

Reasoning, argumentation and justification are not always clarified in the specific competencies of the curriculum. However, argumentation is doubtlessly fundamental in the evaluation of students in all subjects and appears to a large extent in the evaluation criteria of the curriculum. In this sense, it presents itself as an effective element when articulating connections between different subjects, a fact that facilitates transversality.

In the potential learning situations proposed by the TITs, it is observed that the meanings of *argumentation* revolve around the *cognitive*, *inter-relational* and *epistemic* dimensions of didactic appropriateness and surpass *in practice* the classification of arguments as inductive and deductive:

- Cognitive suitability. The formulation of initial conjectures and their development through questioning allows for the construction of an argument that answers the questions raised in the learning situation. During this process, students differentiate between the group's consensual and non-consensual arguments.
- *Inter-relational suitability*. The process of communicating a result implicitly requires rhetorical elements to convince as well. Hence, argumentation includes expository abilities, participation in debates or the use of digital media to give support to explanations.
- *Epistemic suitability*. The validity of an argument is proven to be true due to its worthiness as truth insomuch as it is a logical syllogism.

Nevertheless, it is observed that there is a divergent interpretation of the use of the terms argument and procedure within didactic theory on mathematics and in the official curriculum. As *primary mathematical objects*, arguments would be enunciations (inductive, deductive or any other) that are good justifications for a proposition (for example, a theorem) or a procedure (for example, an algorithm). However, in the technical areas of the curriculum, it is established that a "rigorous" procedure that requires both formal precision and practical execution have the character of an argument in itself.

Transversality with the artistic disciplines creates a schism between mathematics as a useful tool for scientific production and the pleasure of doing math as an object in itself. Recent studies (Albizu, 2024) assert precisely the importance of encouraging the pleasure of doing math in opposition to the tendency fostered by the STEM of focusing on teaching mathematics

and sciences from a point of view of "productivism", to a certain degree, as an element that fosters progress, competitiveness or production. In this sense, the proposals of transversal learning situations have the potential to encourage affective suitability in the human, social and artistic aspects of mathematics.

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Anexo A (Presencia de la argumentación en el currículum obligatorio)

| Disciplinas STEM | Ubicación (GN, 2022a) |
|---------------------|---|
| Descriptores STEM | Preámbulo |
| | Descriptor STEM1 |
| Biología y Geología | Competencias específicas 1, 3, 4 y 6 |
| | Criterio de evaluación 4.1 (cursos 1, 2 y 4) |
| Física y Química | Competencia específica 3 |
| | Criterios de evaluación: 1.1 (curso 2), 2.1 (cursos 2 y 3), 2.2. (curso 4). |
| | Saberes básicos: A.2 (cursos 2 y 3), C.3, D.1 |
| Matemáticas | Preámbulo |
| | Competencias específicas 1 y 3 |
| | Saberes básicos: C.4 |
| Ciencias aplicadas | Competencias específicas 2 y 7 |
| | Criterios de evaluación: 3.1, 3.3 |
| | Saberes básicos: I.5 |

| Disciplinas humanas | Ubicación (GN, 2022a) | |
|-----------------------------------|---|--|
| Lengua y literatura | Competencias específicas 2 y 9 | |
| (castellana, euskera, extranjera) | Criterios de evaluación: 2.1 (cursos 3 y 4), 5.1 (curso 4), 8.1 (cursos 1, 2, 3 y 4), 9.2 (cursos 2, 3 y 4) | |
| Latín | Competencia específica 4 | |
| Geografía e | Competencia específica 2 | |
| Historia | Criterios de evaluación: 1.2 (cursos 1 y 2), 2.2 (cursos 1 y 2), 4.2. (curso 2), 6.4 (curso 4) | |

| Disciplinas sociales | Ubicación (GN, 2022a) |
|--|--------------------------|
| Educación en valores cívicos y éticos | Saberes básicos: B1 |
| Formación y orientación personal y profesional | Competencia específica 1 |
| Disciplinas artísticas U | Ubicación (GN, 2022a) |

Educación plástica, visual y audiovisual Criterio de evaluación 3.2 (Curso 3)

Anexo B (Presencia de la argumentación en el currículum preuniversitario)

| Disciplinas STEM | Ubicación (GN, 2022b) |
|--|---|
| Descriptores STEM | Preámbulo |
| | Descriptor STEM1 |
| Biología | Preámbulo |
| Geología | Competencias específicas 1, 2, 3, 4, 5 y 6 |
| Ciencias Ambientales | Criterios de evaluación: 1.2, 1.3, 2.2., 2.3, 3.2, 3.4, 4.1, 4.2, 5.1, 5.2, 6.1 |
| | Saberes básicos: C4. |
| Física y Química | Preámbulo |
| | Competencias específicas 1, 2, 4, 5 y 6 |
| | Criterios de evaluación: 1.2, 2.1, 2.3, 3.3, 4.2, 5.2, 5.3, 6.1 |
| Matemáticas | Preámbulo |
| Matemáticas aplicadas a las ciencias sociales Matemáticas generales | Competencias específicas 1, 2, 3, 5 y 7 |
| | Criterios de evaluación: 2.1, 2.2, 3.1, 7.1, 9.2, 9.3 |
| | Saberes básicos: A4.1, C1, C3, D5.1, D5.2 |
| Ciencias generales | Preámbulo |
| | Competencias específicas 3 y 4 |
| | Criterio de evaluación: 4.1 |
| | Saberes básicos: A2, A4 |
| Dibujo técnico | Preámbulo |
| Dibujo técnico aplicado a las artes plásticas y al diseño | Competencia específica 2 |
| | Saberes básicos: A7 |
| Tecnología e ingeniería | Criterio de evaluación: 1.3 |

| Disciplinas sociales | Ubicación (GN, 2022b) |
|--|---|
| Economía | Preámbulo |
| Empresa y diseño de modelos de negocio | Competencias específicas 4 y 5 |
| | Criterio de evaluación: 1.2, 5.3, 6.1 |
| | Saberes básicos: D1 |
| Geografía | Competencias específicas 1, 4, 5, 6 y 7 |
| | Criterios de evaluación: 1.1, 4.1, 5.1, 6.1, 6.2, 7.1 |

| Ubicación (GN, 2022b) |
|---|
| Preámbulo |
| Competencias específicas 2, 3, 5, 8 y 9 |
| Criterios de evaluación: 3.1, 3.3, 5.2, 8.1 |
| Saberes básicos: A1.3, B1.2 |
| Preámbulo |
| Competencias específicas 2, 7 y 8 |
| Criterios de evaluación: 2.1, 2.2, 3.1, 5.1, 7.1, 7.2, 8.1, 8.2, 9.1, 9.2 |
| Saberes básicos: A4, B3.3, C1.3, C2.5, D4 |
| Preámbulo |
| Criterios de evaluación: 1.1, 2.1, 3.1, 3.2 |
| Saberes básicos: A2.6, A3.6, B4 |
| Preámbulo |
| Competencia específica 1 |
| Criterios de evaluación: 1.2, 1.3, 4.2 |
| Saberes básicos: A2.7 |
| Preámbulo |
| Competencias específicas 2 y 5 |
| Criterios de evaluación: 5.2 |
| Preámbulo |
| Competencias específicas 2 y 4 |
| Criterios de evaluación: 2.2, 4.2, 5.2, 6.1, 6.2, 7.1 |
| Saberes básicos: A1 |
| Competencias específicas 2, 3 y 4 |
| Criterios de evaluación: 2.1, 2.2 |
| Preámbulo |
| Competencia específica 5 |
| Criterios de evaluación: 5.2 |
| Preámbulo |
| Criterios de evaluación: 5.1, 7.1 |
| |

| Disciplinas artísticas | Ubicación (GN, 2022b) |
|--|--|
| Análisis musical | Competencia específica 3 |
| | Criterios de evaluación: 3.1, 3.3 |
| Artes escénicas | Criterios de evaluación: 4.1, 4.2, 5.1 |
| Coro y técnica vocal | Criterio de evaluación: 1.1 |
| Cultura audiovisual | Competencia específica 1 |
| | Criterio de evaluación: 1.3, 3.2, 4.1 |
| Dibujo artístico | Competencias específicas 1 y 2 |
| | Criterios de evaluación: 1.3, 2.2, 7.1, 9.4 |
| Fundamentos artísticos | Preámbulo |
| | Competencia específica 1 |
| | Criterios de evaluación: 1.2, 3.1, 7.1 |
| Movimientos culturales y artísticos | Preámbulo |
| | Competencia específica 4 |
| | Criterios de evaluación: 3.3 |
| Proyectos artísticos | Criterios de evaluación: 1.2, 3.2, 3.3, 4.2, 5.1 |
| Volumen | Competencia específica 2 |
| | Criterios de evaluación: 2.2, 4.3, 6.2, 6.4 |
| Diseño | Competencias específicas 5 y 6 |
| | Criterios de evaluación: 2.2, 3.2, 4.1 |
| Técnicas de expresión gráfico-plástica | Preámbulo |
| | Competencia específica 5 |
| | Criterios de evaluación: 3.1, 5.1 |