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An approach from the curriculum dimension of vector algebra: ecological designs in the contexts of secondary and higher education

Uma abordagem a partir da dimensão curricular da álgebra vetorial: delineamentos ecológicos nos contextos dos ensinos médio e superior

Una aproximación desde la dimensión curricular del álgebra vectorial: diseños ecológicos en los contextos de la educación secundaria y superior

Une approche à partir de la dimension curriculaire de l'algèbre vectorielle : conceptions écologiques dans les contextes de l'enseignement secondaire et supérieur

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Abstract

This article focuses attention on aspects inherent to the curriculum dimension of the mathematical object vectors from the perspective of the secondary and higher education levels. The theoretical and methodological foundation was outlined through some assumptions of the anthropological theory of the didactic, specifically, from the ecological perspective of what is institutionally outlined in the official documents that govern, in transpositive terms, the knowing to teach vectors and how this aspect reverberates in terms of personal relationships

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with this object in the context of a mathematics teaching degree course at a public university in Bahia. In this context, the following question arose: What do the official documents reveal about the vector approach in secondary school, and how do these voices resonate within the scope of a mathematics teaching degree course? To understand this transpositive process, the objective chosen was to identify dialogues between the curriculum movements involved in this context, particularly interactions between conditions and restrictions that affect the referenced process. In conclusive terms, it was possible to perceive noise in the communication between the vectorial diffusion of what regulates the official documents and what reverberates in the institutional scenarios in which this topic is addressed. Given this finding, it is necessary to invest in actions that promote the reestablishment of harmony between the transpositive stages that live on the border of institutional transition between secondary and higher education.

Keywords: Curriculum, Vectors, Anthropological theory of didactics, Ecology, Teaching degree in mathematics.

Resumen

Este artículo dirige la atención a aspectos inherentes a la dimensión curricular del vector objeto matemático desde la perspectiva de los niveles de educación secundaria y superior. El fundamento teórico y metodológico fue delineado a través de algunos supuestos de la teoría antropológica de la didáctica, específicamente, desde la perspectiva ecológica de lo que se afirma institucionalmente en los documentos oficiales que rigen, en términos transpositivos, el saber para enseñar vectores y cómo este aspecto repercute en las relaciones personales con ese objeto en el contexto de una licenciatura en matemáticas en una universidad pública de Bahía, Brasil. En estos términos, surgió la siguiente pregunta: ¿Qué nos dicen los documentos oficiales sobre el enfoque vectorial en la escuela secundaria y cómo resuenan esas voces en el ámbito de una licenciatura en matemáticas? Para comprender este proceso transpositivo, el objetivo fue identificar diálogos entre los movimientos curriculares involucrados en este contexto, particularmente interacciones entre condiciones y restricciones que afectan el proceso referenciado. En términos concluyentes, fue posible percibir ruido en la comunicación entre el vector de difusión de lo que regula los documentos oficiales y lo que repercute en los escenarios institucionales en los que se aborda este tema. Ante este hallazgo, es necesario invertir en acciones que promuevan el restablecimiento de la armonía entre las etapas transposicionales que viven en la frontera de transición institucional entre la educación secundaria y la superior.

Palabras clave: Currículo, Vectores, Teoría antropológica de la didáctica, Ecología, Curso de licenciatura en matemáticas.

Résumé

Cet article attire l’attention sur les aspects inhérents à la dimension curriculaire de l´objet mathématique vecteur du point de vue des niveaux d’enseignement moyen et supérieur. Le fondement théorique et méthodologique a été esquissé à travers certaines hypothèses de la Théorie Anthropologique du Didactique, spécifiquement, du point de vue écologique de ce qui est institutionnellement affirmé dans les documents officiels qui régissent, en termes transpositifs, la Connaissance à Enseigner le Vecteur et comment cet aspect se répercute sur les relations personnelles avec cet objet dans le contexte d'un cursus de « *Licenciatura en Mathématiques*[5] » dans une université publique de Bahia. En ces termes, la question suivante s'est posée : que nous disent les documents officiels sur l'approche vectorielle au lycée et comment ces voix résonnent-elles dans le cadre d'une licence de mathématiques ? Pour comprendre ce processus transpositif, l’objectif était d’identifier les dialogues entre les mouvements curriculaires impliqués dans ce contexte, notamment les interactions entre conditions et restrictions qui affectent le processus référencé. En termes concluants, il a été possible de percevoir du bruit dans la communication entre la diffusion vectorielle de ce qui régule les documents officiels et ce qui se répercute dans les cadres institutionnels dans lesquels ce sujet est abordé. Face à ce constat, il est nécessaire d'investir dans des actions qui favorisent le rétablissement de l'harmonie entre les étapes de transposition qui vivent à la frontière institutionnelle de transition entre l'enseignement secondaire et supérieur.

Mots-clés : Curriculum, Vecteurs, Théorie anthropologique du didactique, Écologie, Cours de licenciatura en mathématiques.

Resumo

Este artigo direciona a atenção para aspectos inerentes à dimensão curricular do objeto matemático vetor na perspectiva dos níveis médio e superior. A fundamentação teórica e metodológica foi delineada por meio de alguns pressupostos da teoria antropológica do didático, especificamente, a partir do recorte ecológico do que está institucionalmente posto nos documentos oficiais que regem, em termos transpositivos, o saber a ensinar vetores e como esse aspecto reverbera no que se refere as relações pessoais com esse objeto no contexto de um curso de licenciatura em matemática de uma universidade pública baiana. Nestes termos, emergiu o seguinte questionamento: o que nos *dizem* os documentos oficiais acerca da

[5] Est un cours de d´université d´une durée de quatre ans, axé sur la formation des enseignants pour enseigner les mathématiques dans les écoles primaires et secondaires. Le cours fournit une base solide en contenu mathématique, combinée à des connaissances pédagogiques et didactiques pour travailler en classe

abordagem vetorial no ensino médio e o como essas *vozes ecoam* no âmbito de um curso de licenciatura em matemática? Para compreender esse processo transpositivo, elegeu-se como objetivo identificar diálogos entre os movimentos curriculares envolvidos nesse contexto, particularmente, interações entre condições e restrições que incidem no processo referenciado. Em termos conclusivos, foi possível perceber ruídos na comunicação entre a difusão vetorial do que regulamenta os documentos oficiais e o que reverbera nos cenários institucionais em que esse tema é abordado. Frente a essa constatação, faz-se necessário investir em ações que promovam o restabelecimento da sintonia entre as etapas transpositivas que vivem na fronteira de transição institucional entre o ensino médio e superior.

Palavras-chave: Currículo, Vetores, Teoria antropológica do didático, Ecologia, Curso de licenciatura em matemática.

An approach from the curriculum dimension of vector algebra: Ecological designs in the contexts of secondary and higher education

The investigative scope emerged from concerns governed by elements that fuel the high failure rates and consequent grade retention in curriculum components that have Vector Algebra at their core. In this context, it is pertinent to note that this issue can be examined from several perspectives, including curriculum design. That said, it is possible to focus on an ecological problem raised through questions that involve the transpositive context between the vector knowings that *live* or *should live* within the scope of secondary and higher education, which can be preliminarily described as follows: *Where are the vectors located in the secondary school curriculum structure? What role do they play within the scope of mathematics teaching degree courses?* Goulart (2021) presents arguments that partially answer the second question:

> (...) Vectors are integrated not only into the mathematics teaching degree and research degree courses in Brazil, but also into courses within the scope of exact sciences at higher education levels. Its institutional scope is reflected in the content present in subjects such as Physics, Calculus, Linear Algebra, and Geometry. (Goulart, 2021, p. 27)

Tacitly, the author emphasizes the institutional scope of this mathematical entity. However, this aspect contrasts with a high grade-retention rate and consequent failure of students in the curriculum component that contains the vectors in its syllabus. It is essential to note that other courses in the exact sciences area are not immune to this issue. Similarly, they are affected, as Rosa et al. (2019, p. 417) indicate in an analysis of the academic records of some first-year students in the Statistics course at the Federal University of Goiás, finding that in "Analytical Geometry, of the 47 novice students, five were approved (10.6%), three with an average below 7.0, and two with an average between 7.0 and 7.9."

Similarly, Di Pinto (2000), Passos et al. (2007), and Silva et al. (2016) highlighted gaps indicated by failure rates. At the State University of Campinas (UNICAMP) and the University of São Paulo (USP), for example, the failure rate in Analytical Geometry reaches 35%. Richit (2005) noted that in many higher education institutions (HEIs), this subject is considered a problem. Another example cited by the author is UNESP-RC, which had an average failure rate in the subject of around 39% in 2004. Even after two decades, we note that this problem persists and is particularly highlighted by Santos (2024), who leverages this weakness to justify the theme of his end-of-course work.

> The inspiration for this work came from the difficulties my classmates and I encountered in the second semester of the mathematics teaching degree course, specifically in the

> subject of Vectors and Analytical Geometry. I observed that the significant gap in our school background contributed to this difficulty. Students reaffirm that, if they had seen vectors in high school, this subject would certainly not be so "scary," and the difficulties faced by the class in the math teaching degree would be not so significant or almost non-existent, from understanding the concept of vectors to the interpretation and resolution of the exercises and activities proposed in the classroom by the teacher. (Santos, 2024, p. 11)

This aspect has already been proposed by Rodrigues (2015), who highlighted the need to repatriate vectors to the domain of mathematics in secondary education. The studies by Assemany (2011), Assemany et al. (2013), and Azevedo (2013) reinforce the relevance of the vector base in the curriculum of secondary education, arguing that it constitutes a leveling up for students in the transition to higher education. This perspective is reinforced by the statement:

> The introduction of the notion of vectors in R^2 in the 1st grade of high school aims to employ an important and practical tool in the study of the contents presented in sequence, but mainly trigonometry and affine functions, thereby reducing unnecessary calculations that these topics resort to when taught in isolation. The organization of structured content based on vectors aims to lead the student to geometric interpretations of algebraic facts. (Assemany, 2011, p. 130)

In this sense, the author exemplifies the strength of the applicability of vector concepts in the first stage of high school, a fact that was ratified by Dorier (1995), when signaling that the development of the concept of this mathematical entity occurred in the search for understanding of algebraic results, since this magnitude provides contributions to geometry through direction and sense and not only length (scalar magnitude), thus allowing the idea of movement, which, according to Venturi (2015), is intrinsic to the etymological root of the word vector, which comes from the verb *vehere*, which means to transport, to take.

From this etymological perspective, it is possible to establish metaphorical associations between the structuring of the transposition process and the concept of vector. In this sense, Barquero et al. (2010, p. 240, our translation from Spanish to Portuguese[6]) describe the transpositive movement between knowings from the:

> (...) ecological dimension of the problem of mathematical modeling, which means assuming that there are restrictions that result from subjugations imposed on them, regardless of the will of the subjects, which affect the institutional life of mathematical modeling.

[6] (...) dimensión ecológica del problema de la modelización matemática significa assumir la existência de restricciones que, com independência de la voluntad de los sujetos, inciden sobre la vida institucional de la modelización matemática.

The structural outline undertaken by the transpositive process reveals characteristics of the curriculum scope, which can be defined as a set of works (contents) described by Chevallard (2009) as a series of monuments, which are fed by the assumptions of the paradigm of visiting works (PVW). In other words,

> The curriculum, the state of the education system at a given moment, is not entirely defined by official programs. These establish a framework that imposes itself as a system of explicit restrictions on the process of didactic transposition, but which cannot be determined with precision. More importantly, however, (and also largely neglected or even ignored) are the permanent didactic restrictions that exert their effects quite frequently – in the absence of a minimum of in-depth didactic analysis –without the agents of the education system being aware of them. (Chevallard & Almouloud, 2023, p. 565)

In this sense, conditions and restrictions fed the methodological path of the text, especially those that exist in official documents such as the National Common Curriculum Base (BNCC, 2018), the National Curriculum Guidelines for Secondary Education (DCN, 2006) and the National Textbook Program of Mathematics (PNLD, 2018) in comparison with the Pedagogical Political Project (PPP) of a teaching degree course in mathematics at a public university in Bahia. Thus, it is evident that the concept of curriculum is embedded in the process of didactic transposition, specifically in the *Mathematical Organization to Teach*, which will be presented throughout this study.

Faced with this curriculum scenario, in which vectors are minoritized and silenced, an investigative approach is presented based on the research question that emerges through preliminary inquiries: *What do official documents reveal about the vector approach in high school, and how is this knowledge applied in a mathematics teaching degree course?* The answers will be constructed through theoretical and methodological guiding threads based on ecological assumptions arising from the anthropological theory of didactics (ATD).

Thus, the article is organized into four interconnected sections. Initially, the reader will be led to identify some nuances that refer to the epistemological dimension of the concept of vector. In the second topic, we focus on identifying the level of protagonism of vectors in official documents that govern secondary education, guided by the question: What do the official documents that regulate secondary education tell us about vectors? In the third section, the features of the vector praxeological curriculum model for a mathematics teaching degree course will be described through the lens of transpositive bias. And finally, the fourth topic aims to present an interpretive reading of the personal relationships among a group of students and vectors within a mathematics teaching degree course, incorporating final considerations and references.

Nuances of the epistemological dimension of the concept of vector from a transpositive perspective

Research in the field of the didactics of mathematics has as one of its theoretical anchors the epistemological studies of objects that it proposes to investigate. Traces of this aspect are inserted in the transpositive process, which refers to the scope of the institutional relativity of mathematical knowing. Given this situation, it is possible to adopt *Wise Knowing* as a starting point, conceived as a knowing of reference constructed throughout historical-epistemological development. In short, according to Almouloud (2011, p. 194), "wise knowing is constructed and is part of the researcher's cultural heritage." In other words, this aspect directs attention to the genesis of the objects of study, which highlights "(...) the '*raison d'être'* or rationale that gives sense to the performed mathematical activity. And it also contains institutional restrictions that provide and limit the application of the corresponding mathematical knowledge" (Bosch et al., 2006, pp. 2-3).

Based on these assumptions, and considering that "a knowing does not exist in a vacuum, in a social void. All knowledge appears, at a given moment, in a given society, anchored in one or more institutions" (Chevallard, 1989, p. 32), it is possible to project the look at some elements that belong to the genesis of the concept of vector. In this sense, we must note that many historical records associate vector conceptualization with a multifaceted format according to the domains and areas of knowledge to which it belongs.

Through this approach, Dorier (2000) points out the existence of three "ecological habitats[7]": the geometric vector, the algebraic vector, and the physical vector. This fact reinforces the multiple characteristics that accompany the concept of this object, as Chevallard (1994, p. 22, our translation from French into Portuguese[8]) puts it, situating "(...) a knowing that some institutions recognize to live in them." Thus, it is possible to understand the conceptual multiplicity of the vector, which can even represent an obstacle to learning. As Brousseau (1976) points out, all conceptions can, at some point, become obstacles to future acquisitions.

[7] Term linked to the ecology of knowings that indicate the functions that the object of knowing performs in interaction with other objects, as well as situating the object in terms of institutional location. (Chevallard, 1996).
[8] "(…) Soit un savoir S dont un certain nombre d'institutions reconnaissent qu'il vit en leur sein".

Given this direction, Táboas (2010, p.1) emphasizes that “the word ‘vector’ has been used with a more up-to-date algebraic sense, i.e., as a difference between two points in space, based on Sir William R. Hamilton’s (1805-1865) study with quaternions” (generalization of complex numbers), since,

> (...) He set up an elaborate terminology by using such words and combinations as vector, vehend, vection, vectum, revector, revectum, revehend, revection, revectum, provector, ... transvector, ... factor, ... profactor, ... versor, ... and quadrantal versor, which is of course a semi-inversor (...) (Hamilton, apud Crowe, 1994, p. 36)

These terminological derivations also refer to the characteristics associated with physics, given that Ba and Dorier (2006) pointed out that:

> The word vector refers to astronomers who used to speak of the eddy vector to designate the movement of a planet, and the radius vector to designate the segment that joins the focus of the conic describing the planet’s trajectory to a position in its orbit. The use of the term radius vector then became widespread in geometry, but ultimately has nothing to do with the vector as it is currently understood. (Ba & Dorier, 2006, p. 18)

From what has been presented, we identify traces of the conceptual multiplicity of this object, which integrates the modeling of reality associated with physical phenomena. However, one should not overlook the existence of geometric approaches that support physical problems. As highlighted by Crowe (1994):

> The early history of vectorial analysis is most appropriately viewed within the context of two broad traditions in the history of science. One of these traditions relates to mathematics, the other to physical science.The first tradition, that within the history of mathematics, extends from the time of the Egyptians and Babylonians to the present and consists in the progressive broadening of the concept of number. Throughout time the concept of number has been expanded so as to include not only positive integers, bus negative numbers, fractions, and algebraic and transcendental irrationals. Eventually complex and higher complex numbers (including vectors) were introduced. The activities of some of the figures in the history of vector analysis may be viewed as belonging to this tradition. (Crowe, 1994, p. 1)

Currently, this dialogical process between physics and mathematics continues, even in the face of curriculum silencing, which will be presented throughout the text. From this context, it is feasible to establish associations between the niche and habitat of the vectors, since, according to Chevallard (1994):

> Ecologists distinguish, in terms of an organism, its habitat and its niche. To put it in deliberately anthropomorphic language, the habitat is, in a sense, the address, the place of residence of the organization. The niche is the functions that the organization fulfills:

> it is, in some way, the profession that is practiced there. (Chevallard, 1994, p. 142, our translation from French into Portuguese)[9]

Through metaphorical language, the author signals that mathematical objects can live in different institutions and can assume varied functions, which means, in Matheron's (2000, p. 52) words, that:

> The didactic organization allows for studying the same mathematical notion designated by the same name, but with mathematical organizations of various natures, if developed within different institutions.

Based on this conception, it is possible to affirm that the epistemological trajectory of vectors has undergone modifications in their representations, as:

> This transformation consisted in the shift in emphasis from such scalar quantities as position and weight to such vectorial quantities as velocity, force, momentum, and acceleration. The transition was neither abrupt nor was it confined to the seventeenth century. Later developments in electricity, magnetism, and optics acted further to transform the space of mathematical physics into a space filled with vectors. (Crowe, 1994, p. 1)

In this context, mathematics and physics converged in various periods of history; one of these periods was the 19th century, marked by the development of vector methods. This perspective is reinforced by Dorier (1997):

> The synthetic approach, through vector calculus, to the geometric world has shown its advantages over the analytical method inherited from Descartes. The introduction of geometric language into functional analysis shifts this debate to an infinite dimension, thereby favoring the abandonment of analytical representation in favor of an approach that treats the function as an object independent of its representations. (Dorier, 1997, p. 34, our translation from French into Portuguese[10])

Through this epistemological scope, we can observe inherent traits in the characterization of vectors in different contexts, informed by multiple representations that encourage us to consider the role that vectors play in the high school curriculum.

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[9] "Les écologistes distinguent, s'agissant d'un organisme, son habitat et sa niche. Pour le dire en un langage volontairement anthropomorphe, l'habitat, c'est en quelque sorte l'adresse, le lieu de résidence de l'organisme. La niche, ce sont les fonctions que l'organisme y remplit: c'est en quelque façon la profession qu'il y exerce".

[10] L'approche synthétique par le calcul vectoriel du monde géométrique a montré ses avantages par rapport à la méthode analytique héritée de Descartes. L'introduction du langage géométrique en analyse fonctionnelle déplace ce débat en dimension infinie favorisant ainsi l'abandon de la représentation analytique pour une approche traitant la fonction comme un objet indépendant de ses représentations.

What do the official documents that regulate secondary education *say* to us about vectors?

The discourses that permeate official documents and currently regulate secondary education point to its interdisciplinary character, i.e., the aim is to interconnect content, a fact that "proposes overcoming the radically disciplinary fragmentation of knowledge" (BNCC, 2017, p. 15). However, in contrast, the vectors that carry within themselves aspects inherent to interdisciplinarity do not have *voice* in the mathematics curriculum in this context. Thus, it is important to highlight that even assuming a prominent role as "[...] an important and practical instrument in the study of content [...]" (Assemany, 2011, p. 130) and with the potential to reduce calculations, the vector approach has been weakened and has been silenced, as highlighted by Nasser et al. (2021):

> The concept of vector is not part of the high school mathematics curriculum, nor is it included in the National Common Curriculum Base (BNCC). When entering higher education, some students bring a notion of vectors from high school physics, but an analytical geometry approach could help represent and visualize situations [...] (Nasser et al., 2021, p. 96)

The absence of a geometric approach supported by vector input in the mathematical context was and continues to be a topic that has fueled concerns. In these terms, Souza (2015) emphasizes that,

> The concept of vector in elementary school, and mainly in high school, is only seen in the subject of Physics, giving the impression that this very important entity is an exclusive object of this subject; its role in this context is undeniable. However, a vector is a mathematical object, so it can be included in the mathematics curriculum, considering its geometric and algebraic aspects, along with its specific characteristics and properties. Vectors are tools that simplify calculations in problem-solving and demonstrating important results. They appear in areas such as analytical geometry, linear algebra, and calculus, and are associated with complex numbers and their operations. (Souza, 2015, p. 18)

In a similar sense, Vaz et al. (2017) also note that:

> When observing the difficulties presented by students upon arriving at university, the group began to question what is taught about vectors and straight lines in high school. To our surprise, it turned out that "vectors" is a topic that is generally not part of the high school mathematics curriculum. In some cases, it appears in the Physics curriculum, but it is not clear whether students can apply this knowledge to use vectors in Mathematics. (Vaz et al., 2017, p. 57)

Emphatically, Elon Lages Lima et al. (2001, p. 62), after analyzing several high school mathematics books, express their discontent when referring to how scarcely vectors appear:

> For some obscure reason, or no reason at all, the important mathematical concept of vector [...] is absent from this and other Brazilian textbooks, being used only by physics teachers.
>
> The author reinforces his opinion by stating that:
>
> The book manages to overcome the difficulty generated by the fact that most school programs do not include vectors, by discreetly constructing a "little vector calculation" of addition and product by number to be used in this context. [...] One of the flaws of this book and all the high school mathematics books on the market is that vectors are totally left out. Interestingly, vectors are typically taught in physics books, rather than in mathematics books. (Lima et al., 2001, p. 130)

The considerations, in this sense, gain more and more strength when Lima et al. (2001) go on reviewing the process of book analysis:

> The first chapter explores the analytical geometry of the straight line, beginning with the concept of algebraic measurement of an oriented segment (although the definition of an oriented segment is never provided) [...] (Lima et al., p. 156)
>
> Not presenting the notion of vector (in the book, no sense is attributed to $x_1 - x_2$, only to $|x_1 - x_2|$) complicates the deduction of the formula for the coordinates of a point on a mid-segment. (Ibid., p. 210)
>
> [...] It is worth reflecting on the content of analytical geometry in national secondary education books. Why not talk about vectors? The student needs to know what a vector is. Physics uses it, and analytical geometry becomes much richer with this tool, simplifying demonstrations and enabling better solutions to problems. (Ibid., p. 259)

Likewise, the curriculum guidelines for secondary education recommended, not unintentionally, that mathematics teachers include the concept of vectors in classes, stating that,

> It is also desirable that the mathematics teacher addresses the concept of vectors with their students, both from a geometric perspective (a collection of oriented segments of the same length, direction, and sense) and from an algebraic perspective (characterized by its coordinates). In particular, it is essential to relate the operations performed on coordinates (addition, multiplication by a scalar) to their geometric meaning. The inclusion of the notion of vector in the topics covered in mathematics classes would correct the distortion caused by the fact that it is an important mathematical topic, but is only present in high school Physics classes. (Brasil, 2006, p. 77)

However, this perspective does not have a clear impact on documents such as the National Mathematics Textbook Program Guide (PNLD, 2018), in which only one topic alluding to vectors is found: "[...] the study of complexes must be a privileged opportunity for articulation with topics such as vectors and geometry on the plane, with trigonometry and with algebraic equations" (Brasil, 2017, p. 24). In the current versions of the guide, we found no reference to the mathematical object vector.

Similarly, the BNCC (2018) implicitly dedicates a small space to vectors in the area dedicated to mathematics and its technologies, when addressing movement and position in skill EF05MA15: "Interpret, describe, and represent the location or movement of objects in the Cartesian plane (1st quadrant), using Cartesian coordinates, indicating changes in *direction and sense* and spins." (Brasil, 2018, p. 297, our emphasis)

In light of what has been presented, it is possible to perceive the existence of contradictions, since vectors are, in essence, mathematical objects that represent physical vector quantities such as speed, acceleration, and force. Thus, the existence of inhibitions that hinder the consolidation of this knowledge becomes tacit; that is, this dissociation has restricted and impacted the development of teaching and learning processes regarding the mathematical vector approach.

Upon noticing this curriculum problem, Goulart (2021) sought to examine the traits of the personal relationships of some students in a mathematics teaching degree course. As a guideline, some questions were drawn up:

> 1- Tell us about your relationship with vectors:
> a) Have you had contact with the subject in high school? If so, in which subject(s)?
> b) If you answered yes to the previous question, describe how you approached this content.
> c) Do you consider it an easy topic to learn? Please, justify. (Goulart, 2021, p. 129)

With the answers in hand, the author could identify the level of belonging of this object in each cognitive universe[11], which can be described as a set of relations generated by multiple institutional subjections, i.e., "When an object *o* exists for a person *x*, we also say that *x* knows *o*, the ratio *R (x; o)* specifying how *x* knows *o*" (Chevallard, 2009, p. 1). The author also clarifies:

> It is important to emphasize that the adjective cognitive is not taken here in its common intellectualist sense: I have a personal relationship with my toothbrush, the coffee machine at the coffee shop, the brake pedal of my car, etc., all objects that are part of my cognitive universe, in the same way that, for example, the notion of a quadratic equation or a derivative are part of it. (Chevallard, 2009, pp. 1-2)

[11] We then call the cognitive universe x the set $UC(x) = \{(o, R(x ; o)) / R(x ; o) \neq \emptyset\}$

An excerpt from the records of the students’ reports reveals the lack of personal relationships between *x* (students) and the object *o* (vectors), that is, $R\ (x;\ o) = \emptyset$.

> **Student 4** a) No. I studied at a public school and studied teaching in high school. I didn’t have access to many math subjects. I’m good with percentages and some quadratic equations.
> **Student 7** a) More or less, it was just an introduction to a class and then never again. b) It was an introduction, but there was no exercise or anything like that for real learning. c) I don’t think so, not much.
> **Student 3** a) Yes, in the subject of Physics. b) It was a simple approach. The content addressed was Coulomb’s law, which depended on basic knowledge of vectors. We saw the sum and difference between vectors in the same direction and in various directions. We also studied the parallelogram rule. c) I found the part I saw in high school easy.
> **Student 9** a) No! b) c) No. Maybe because I didn’t have any contact with geometry in basic school. (Goulart, 2021, p. 129, author’s emphasis)

Through what the answers ratified, we can note that the *voices* of official documents reflect and echo in the final stage of the transpositive process within the scope of the learned knowing. This aspect highlights the lack of personal relationships with the mathematical vector entity. In didactic terms, a problem of an ecological nature is materialized, which was identified repeatedly by members who make up the *Wise Knowing*, but miniaturized by the group that makes up the knowing to teach, possibly because it has a greater degree of influence in decision-making about what is characterized as relevant to integrate the praxeological equipment[12] of high school students. This aspect can be described at the upper levels of the hierarchical scale of didactic codetermination (Chevallard, 2002), as illustrated in Figure 1.

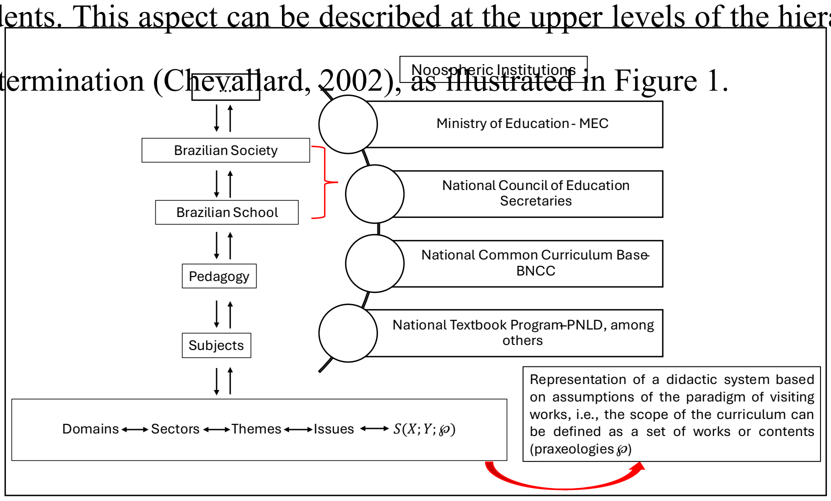

Figure 1

Levels of didactic codetermination (adapted by Bosch 2011 and Chevallard 2002, pp. 9-10)

[12] All the praxeologies that people have at their disposal. See Chevallard (2009).

Thus, it is essential to note that the Brazilian curriculum structure is based on the assumptions of the paradigm of visiting works or the paradigm of visiting monuments, which, according to Chevallard (2012), is limited to listening to a report or narrative recited by the teacher about the monument visited. Regarding the ecological approach, Bosch and Chevallard (1999, p. 4, our translation from French into Portuguese[13]) highlight that "the ecological problem will expand the field of analysis and address the restrictions created among the various objects of knowledge to be taught." The above means that vectors do not have a habitat[14] in the high school mathematics curriculum. This fact has repercussions for the teaching and learning processes of Vector Algebra within the syllabi of higher education courses related to the field of exact sciences.

Traces of a vectorial praxeological curriculum model from the transpositive bias within the scope of a mathematics teaching degree course

The theory of didactic transposition (TDT), designed by Chevallard (1991; 1994), emerged from the expression "didactic transposition" introduced by Verret (1975), who, in his work entitled *Le temps des études* [Study time], added the distribution of time in the execution of school activities. In didactics, this term inspired and promoted a configuration which was disseminated among researchers in didactics of mathematics, thus assuming a central position in the theoretical design of the TDT, an aspect revealed by the statement: "I introduced the theme of didactic transposition in the French community of mathematical didactics in the early 1980s." (Chevallard, 2002, p. 1, our translation from French into Portuguese[15]).

It is also important to highlight that this theoretical construct was based on the origins of the didactic systems (DS), which integrate teacher, student, and knowledge as advocated in the theory of didactic situations (Brousseau, 1997), meaning that there is a minimum system, which establishes associations between the conception of teaching and the organization of knowledge communicated by the teacher, in which students are involved in the process of

[13] "Le problème écologique élargira le champ d'analyse et traitera les contraintes créées entre les différents objets de connaissance à enseigner".
[14] For Chevallard (1994, p. 142), ecologists distinguish, in terms of an organism, its habitat and its niche. To put it in deliberately anthropomorphic language, the habitat is, in a sense, the address, the place of residence of the organization.
[15] "J'ai introduit le thème de la transposition didactique dans la communauté française des didacticiens des mathématiques au tout début des années 1980".

acquiring a specific knowledge. In these terms, these didactic relations are situated in a communicational information scheme because, according to Brousseau (2008, p. 16), "This scheme is usually associated with a teaching conception in which the teacher organizes the knowledge to be transmitted in a series of messages, from which students take for themselves what they must acquire." From this organizational perspective of knowledge, Almouloud (2011) presents some directions and questions that corroborate the investigative scope, given that,

> Teaching scientific concepts at a given level of education requires making it accessible to students. Therefore, it needs to be transformed based on a knowing of reference, which is generally the knowings of experts in the subject (wise knowing). In this work, we present theoretical tools from the didactics of mathematics to answer the following questions: *How to analyze the curriculum of a specific level of education? For a given notion, which aspects are privileged in teaching, in curriculum proposals, and in classroom practices? What are the important aspects of the notion that are missing in the transformation processes?* What didactic choices can be made? (Almouloud, 2011, p. 193, our emphasis)

Thus, this scenario of questions and guidelines encourages the outlining of some transpositive chains, which were defined, designed, and reflected upon, culminating in an expansion of this theoretical framework, denoted by Chevallard (1996) as anthropological theory of the didactic (ATD), which began to house the TDT, through an ecological configuration, which, from Artaud's (1999) perspective, consists of the ecology of knowings, which questions reality. In other words,

> What exists, and why? But also, what does not exist and why? Or could it exist? Under what conditions? Or conversely, given a set of conditions, which objects are forced to live, or conversely, which are prevented from living under these conditions? These are the characteristic questions of ecological problems. (Artaud, 1999, p.101, our translation from the French to Portuguese[16])

Does this perspective encourage conjecture about what exists and what should exist in the transpositive scope of *Vector Knowing* taught and learned in a mathematics teaching degree course at a public university? As a premise, it is worth highlighting that at the heart of didactics,

> Teaching and learning mathematics is not considered as teaching and learning mathematical ideas, notions or concepts, but as teaching and learning a situated human

[16] Qu'est-ce qui existe, et pourquoi ? Mais aussi, qu'est-ce qui n'existe pas, et pourquoi ? Et qu'est-ce qui pourrait exister ? Sous quelles conditions ? Inversement, étant donné un ensemble de conditions, quels objets sont-ils poussés à vivre, ou au contraire sont-ils empêchés de vivre dans ces conditions ? Telles sont les questions caractéristiques de la problématique écologique.

activity performed in concrete institutions. Moreover, a situation includes the "raison d'être" or rationale that gives sense to the performed mathematical activity. And it also contains institutional restrictions that provide and limit the application of the corresponding mathematical knowledge. (Bosch et al., 2006, pp. 2-3, authors' emphasis)

From this perspective, it is possible to consider the structural contours and subsequent questions that enable vector knowledge organizations to *live* within a mathematics teaching degree course. In this way, it becomes feasible to identify the distances between the *Vector Mathematical Organizations to Teach* and the *Vector Mathematical Organizations that are Taught and Learned*. This perspective closely aligns with the schematic scope represented in Figure 2.

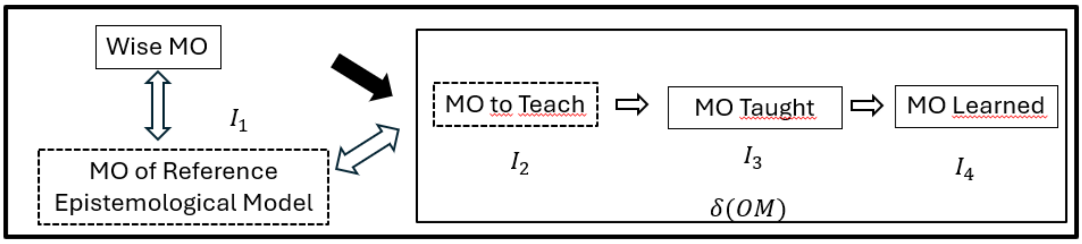

Figure 2

Diagram of Didactic Processes (Bosch & Gascón, 2005)

In this sense, the authors characterize each term presented in Figure 2:

The MO_2 to teach constitutes a praxeological model of the mathematics curriculum. The empirical basis for the development of this model is found in curriculum documents (official programs) and textbooks. Its influence on δ (MO) is central, although neither the teacher nor the educational institution explicitly has this model; they only have more or less well-articulated praxeological materials. [...] this influence cannot be interpreted correctly if we do not have an epistemological point of view. This point of view is provided by a MO of reference, whose description is generally made from the learned MOs that legitimize the teaching process. The MO of reference is the one considered by the researcher for analysis. It does not necessarily coincide with the learned MOs from which it originates (because it includes them in the analysis), but it is expressed in very similar terms. The MO of reference is the one that the researcher puts to the test of contingency and that, therefore, undergoes permanent remodeling. (Bosch & Gascón, 2005, p. 117, our translation from French into Portuguese[17])

[17] L'OM2 à enseigner constitue un modèle praxéologique du curriculum de mathématiques. La base empirique pour élaborer ce modèle se trouve dans les documents curriculaires (programmes officiels) et dans les manuels. Son influence sur δ(OM)3 est centrale bien que ni le professeur ni l'institution scolaire de dispose explicitement de ce modèle mais uniquement de matériaux praxéologiques plus ou moins bien articulés entre eux. Mais cette influence ne peut être adéquatement interprétée si nous ne disposons pas d'un point de vue épistémologique. Ce point de vue est fourni par une OM de référence dont la description se fait généralement à partir des OM savantes légitimant le processus d'enseignement. L'OM de référence est celle que considère le chercheur pour son analyse. Elle ne coïncide pas nécessairement avec les OM savantes d'où elle provient (parce qu'elle les inclut dans

Faced with this argument, attention turns to I_2, as shown in Figure 2, since the investigative focus is situated within an ecological analysis of the knowing-to-teach of the concept of vector; i.e., paraphrasing Bosch and Gascón (2005), it is inserted in the axis of support of the curriculum praxeological model. In other words, Chevallard (1982, p. 8) highlights that I_2 occupies the place where “conflicts develop, negotiations take place, solutions mature.” Furthermore, it is important to highlight that:

> The operational center of the transposition process, which translates the response to the observed imbalance (expressed by mathematicians, parents, and teachers) into facts, is the noosphere. Any conflict between the system and the environment moves there and finds its privileged place of expression. In this sense, the noosphere acts as a buffer. Even in times of crisis, it maintains the autonomy of teaching operations within acceptable limits. (Chevallard, 1982, p. 11, our translation from French into Portuguese[18])

Artaud (1999) points out the existence of ecosystems, among them the noospheric one, in which mathematics is manipulated for transposition purposes. This definition supported Goulart’s (2021) interpretation when characterizing the noosphere as an institution composed of subjects who think about the knowledge that will be disseminated within the scope of the didactic systems (DS), thus mediating dialogues between the spheres of *Wise MOs* and *taught MOs*, in order to provide adequate functioning of DS in terms of conditions and restrictions that are imposed on the teaching and learning process. From this context, among the elements that *live* in I_2, we will focus on some official documents that regulate and institutionalize vector algebra in mathematics teaching degree courses.

In this sense, as already highlighted in Figure 1, it is the responsibility of noospheric institutions to govern and regulate the organization of curriculum in initial teacher education courses. The National Education Council (CNE, in its Portuguese acronym), in resolution CNE/CP n. 4, of May 29, 2024, provides for the national curriculum guidelines for initial higher education training of basic school education teaching professionals (teaching degree, pedagogy

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l’analyse), mais elle se formule dans des termes très proches. L’OM de référence est celle que le chercheur met à l’épreuve de la contingence et qui subir pour cela de permanents remaniement.

[18] Le centre opérationnel du processus de transposition, qui va traduire dans les faits la réponse à apporter au déséquilibre créé et constaté (exprimé par les mathématiciens, les parents, les enseignants eux-mêmes), c’est la noosphère. Tout conflit entre système e environnement s’y déporte, et y trouve son lieu d’expression privilégié. À cet égard, la noosphère joue un rôle de tampon. Même en période de crise, elle maintient dans des limites acceptables l’autonomie du fonctionnement didactique.

training courses for non-licensed students, and degree courses as a second license). Thus, it is possible to highlight the characteristics of the profile of those who graduate from initial education in terms of the sphere of knowledge. As described in Art. 10,

> At the end of the initial training at a college level, the graduate must be able to: I - demonstrate knowledge and understanding of the epistemological organization of the concepts, key ideas, structure of the area(s) and curriculum components for which he/she is being qualified to teach; II - critically understand the normative frameworks that underpin the curriculum organization of each of the stages and modalities of basic education and, in particular, the National Curriculum Guidelines for Basic Education (DCN) and the National Common Curriculum Base (BNCC); (...) VII - demonstrate knowledge of the use of language and logical mathematical thinking in the development of specific teaching content; VIII - demonstrate knowledge of the several ways of presenting the content of the components and curriculum areas for which the teacher is qualified, using this knowledge to select appropriate teaching resources that contemplate access to knowledge for a diverse group of students; (...). (Brasil, p.7, 2004)

In the first section of this article, we could observe that themes associated with vectors do not *live* in the BNCC (2017). However, in the opposite direction, the Pedagogical Political Project (PPP) of a mathematics teaching degree course at a public university located in the state of Bahia, Brazil, highlights the relevance of the vector contribution in the curriculum connections of this course. Thus, we correlate the habitat of vectors at these levels of education, highlighting subjects and curriculum components that include a vector approach in programs and/or syllabi, as highlighted in Figure 3.

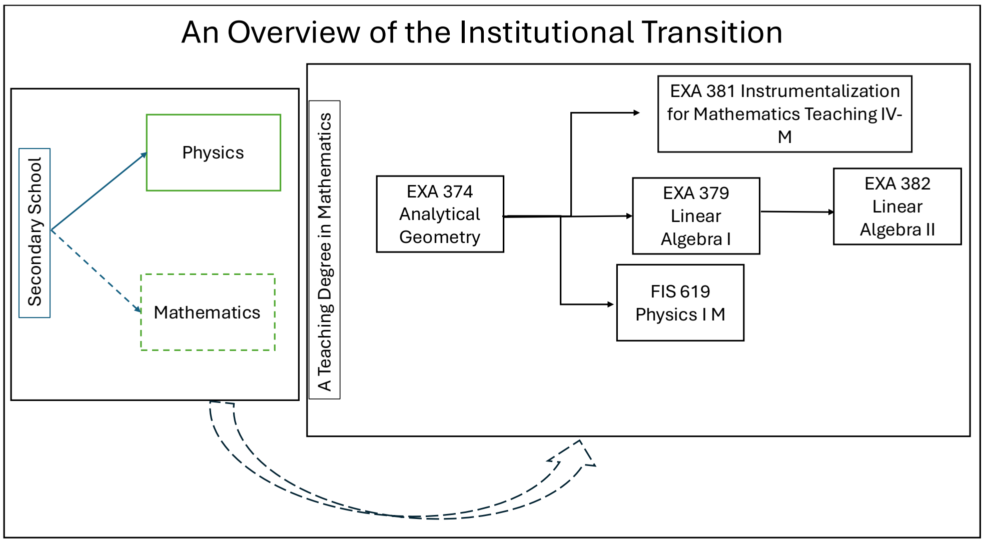

Figure 3.

Comparative diagram of vector habitat in secondary school institutions and a mathematics teaching degree course (authors, 2025)

It is possible to observe variations in the context of mathematics licenciate courses, including differences in the names of curriculum components, subject codes, semester offerings, workloads, bibliographical references, and details of the syllabi. However, in all the PPPs consulted, Analytical Geometry and its derivations are tacit and integrate vector concepts, which are presented to students and, consequently, prospective mathematics teachers.

These aspects were also highlighted in textbooks included in the bibliographic scope of this curriculum component. From this perspective, Chaachoua (2014) indicates that demonstrating the viability of conceiving by studying the books promotes possibilities for identifying traits of institutional relations. Supported by this conception, Goulart (2021) consulted some books on analytical geometry referenced in the PPP, and in all the works analyzed, the core was the vectors, as Souza (2019) highlighted, regarding skills and competencies that the student should develop, since:

> It is expected that after taking the course, the student will have understood: the formal concept of the theory of vectors in the plane and space, operations with vectors in the plane and space including products, and the notion of vector spaces and the various concepts included in this topic, such as linear combination, linear dependence, basis, and dimension. (Souza, 2019, p. 2)

Other expectations are also projected into another work. In this case, the author expects prior contact with elementary geometry when declaring that,

> The first aspect relates to the interconnections established between elementary geometry and analytical geometry, as revealed in some excerpts: The student is familiar with the rectilinear segment as a geometric concept; however, for the purposes of analytical geometry, we extend this notion of a rectilinear segment, including direction or orientation. (...) From the point of view of elementary geometry, the lengths of the oriented rectilinear segments AB and BA are equal; however, in analytical geometry, a distinction must be made between the signs of these lengths. Thus, we arbitrarily specify that a straight line segment with a particular orientation has a positive length, while one that is oppositely oriented will have a negative length. (Lehmann, 1991, pp. 1-2, author's emphasis)

These signs point to what is expected of good institutional subjects, those who present a response that has been stipulated and determined by the institution. Since "when individuals come to occupy such positions, they become subjects of institutions –active subjects who contribute to giving life to institutions by the very fact of being subjected to them" (Chevallard, 1999, p. 94). In this context, the study focuses on the personal relationships of some students in a mathematics teaching degree course with vectors, particularly regarding the subjection to which they are exposed, given that novice students are expected to have prior knowledge of vectors.

An interpretative reading of the personal relationships of a group of students regarding vectors within a teaching degree in mathematics.

From what has been outlined, we can identify gaps that shape a problem of an ecological nature that inhabits scenarios of curriculum transitions and points to the need to foster dialogues and closer links between the education systems (secondary education and higher education), as outlined in Figure 3. It becomes tacit that there are gaps that generate restrictions in the construction of the praxeological baggage of prospective mathematics teachers and consequently inhibit the structuring of the praxeological equipment of these subjects within the scope of vector knowledge, given:

> [...] the genesis of praxeological equipment (and associated institutional reports) agrees with the positions of the student and the teacher during the praxeological construction. The topos (place, in ancient Greek) of the student (respectively, of the teacher) is the part of the student's position (resp. the teacher) that relates to the praxeological entities constructed or under construction in the classroom. (Chevallard, 2009, p. 82, our translation from French into Portuguese[19])

We must also not forget that "a person x is the result of their past and present subjections" (Chevallard, 2018, p. 37). This conception is linked to the fundamental notion of a:

> [...] personal relationship of an individual x with an object O, an expression by which we designate the system, we note R(x, O), of all the interactions that x can have with the object O –whether x manipulates it, uses it, talks about it, dreams about it, etc. We will say that O exists for x if the personal relation of x to O is "non-empty," which is noted as R(x, O) ≠ ∅. (Chevallard, 2002, p. 81, our translation from French into Portuguese[20])

[19] Genèse des équipements praxéologiques (et des rapports institutionnels associés) selon les positions d'élève et de professeur au cours de la construction praxéologique. Le topos (le lieu, en grec ancien) de l'élève (respectivement du professeur) est cette partie de la position d'élève (resp. de professeur) qui a trait aux entités praxéologiques construites ou en cours de construction dans la class.

[20] Le notion fondamentale est celle de rapport personnel d'un individu x à un objet o, expression par laquelle on désigne le système, noté R(x, O), de toutes les interactions que x peut avoir avec l'objet o – que x le manipule,

Given this outline, it is feasible to analyze the “fissures” through records of the personal relationships of students in a mathematics teaching degree course, focusing on an object of knowledge, in this case, vectors. As highlighted by Chevallard (1989):

> This personal relationship includes notably everything that we normally think we can say in terms of “knowing,” “knowing how to do,” “conceptions,” “skills,” “mastery,” and “mental images” of “representations,” of “attitudes,” of “fantasies”, etc. ... of X over Os. Everything that can be asserted, whether right or wrong, in conformity or not, must be considered (at best) to be an aspect of X’s personal relationship with Os. (Chevallard, 1989, p. 219, our translation from French into Portuguese[21]).

In association with the highlighted aspects, it is relevant to revisit a section of the research question on which this section is based, namely (...) *How do prospective teachers project this knowledge within the scope of a teaching degree course in mathematics?* The support used to conjecture an answer to this question is based on Goulart’s thesis (2021), as the author adopted one of the investigative biases—conceptions of students in a mathematics teaching degree — about the vector mathematical entity, aiming to identify characteristics of the praxeological background of these subjects. Figure 4 shows an excerpt of these aspects.

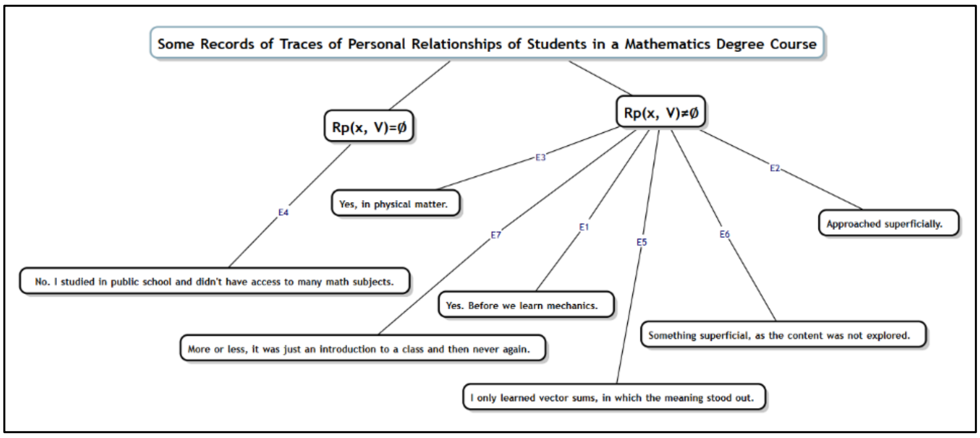

Figure 4

Scope of records of some students’ relationships with the vector object (adapted by Goulart’s 2021 authors)

Specifically, the students’ reports corroborate what was revealed in the preceding sections, namely, that the silence of the vectorial approach in the curriculum design of

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l’utilise, en parle, en rêve, etc. On dira que o existe pour x si le rapport personnel de x à o est « non vide », ce qu’on note R(x, O) ≠ ∅.

[21] De ce rapport personnel relève notamment tout ce qu'on croit ordinairement pouvoir dire - en termes de "savoir", de "savoir-faire", de "conceptions", de "compétences", de "maîtrise", d' "images mentales", de "représentations", d' "attitudes", de "fantasmes", etc…- de X à propos de Os. Tout ce qui peut être énoncé - à tort ou à raison, pertinemment ou non - doit être tenu (au mieux) pour un aspect du rapport personnel de X à Os.

secondary education echoes a structural didactic void within the mathematics curricula of Brazilian basic education, as reflected in the praxeological baggage of prospective mathematics teachers. In this context, Chevallard (1989) emphasizes that knowledge does not exist in a social vacuum, but is linked to at least one institution, and all knowledge of a given society is grounded in one or more institutions. Thus, it is possible to infer that the personal relationship can only be established when the person enters the institution where the object exists. As described by Chevallard (1996),

> An object exists from the moment a person X or an institution I recognizes it as existing (for it). More precisely, we can say that the object O exists for X (respectively, for I if there exists an object that I will denote by R(X, O) (resp. RI (O)), which I will call X's relationship with O (resp. I's institutional relationship with O). In other words, object O exists if it exists for at least one person X or for one institution I; that is, if at least one person or one institution has a relationship with that object (Chevallard, 1996, p. 127).

In this context, one of the interpretative paths that can be followed intersects with the reflections of the mathematics curriculum design on secondary education, which are projected within the scope of higher education, specifically in a teaching degree course in mathematics. Thus, the reading that is done takes into consideration that vector knowledge touches on the absence in students' praxeological baggage, which impacts personal relationships that could be strengthened at the higher education level.

Final considerations

The problems revealed in this text point to the need to (re)visit and, consequently, (re)formulate the elements that make up the curriculum bases, in the scope of mathematics, in the context of secondary education. And specifically, direct attention to aspects intrinsic to the genesis of the vector concept. Based on this observation, the need for dialogues between the levels of didactic codetermination is projected, as illustrated in Figure 1, to repatriate the vectors for the mathematics realm in secondary education. For this to happen, it is necessary to reconnect the guiding threads between the curriculum foundations that govern secondary education and those that regulate higher education in the context of initial education courses for mathematics teachers. As vehemently highlighted by several authors, the vector basis at the secondary level of education constitutes a differential for students in the transition to higher education.

In terms of theoretical foundation, the text presented in this article seeks to ratify the potential of some assumptions of the theory of didactic transposition (TDT) (Chevallard, 1991; 1994), which made it possible to visualize more clearly some points inherent to the transformations imposed on vector knowing through different habitats and niches, as highlighted by Dorier (2000). Likewise, the anthropological theory of the didactic (ATD) (Chevallard, 1992, 1996) adopts the premise that all knowledge belongs to a social entity (institution I), which imposes on the people who belong to I ways of doing and thinking specific to the demands arising from I. In light of this conception, it was possible to identify aspects that overlap with the conceptualization of curriculum as a practice and social construction that structures knowledge and expertise in favor of formative specificities.

Given the above, this article aims to provide directions that promote joint actions in favor of curriculum integration during the institutional transition process. In other words, high school students should be offered the opportunity to equip their praxeological baggage to fulfil the scope of an institutional "good subject," that is, those who present a response as projected by the institution. As Chevallard (1999, p. 4) affirms, "when individuals come to occupy such positions, they become subjects of institutions –active subjects who contribute to giving life to institutions by the very fact of being subjected to them."

In this context, the study focuses on the personal relationships of some students in a mathematics teaching degree course with vectors, particularly regarding the subjection to which they are exposed, given that novice students are expected to have prior knowledge of vectors. In short, we observed some noise in the communication between the vectorial diffusion of what regulates official documents and what reverberates in the institutional scenarios in which this topic is addressed. Given this finding, it is necessary to invest in actions that promote the reestablishment of harmony between the transpositive stages of vector knowing.

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