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Editorial – special issue: theoretical references for discussing mathematical knowledge and knowing

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Éditorial – numéro spécial : références théoriques pour discuter des connaissances et des savoirs mathématiques

Editorial – número especial: referenciais teóricos para discutir conhecimentos e saberes matemáticos

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As guest editors, we would like to thank everyone who submitted articles for the thematic issue *Theoretical frameworks for discussing mathematical knowledge and knowings mobilized and produced by teachers who teach mathematics in middle school and high school*, the second issue of Volume 27. We are immensely grateful to all the reviewers and authors who contributed to the implementation of this editorial proposal. Despite the challenges faced throughout the process, we completed this work cycle and are making this issue available to the interested community, bringing together 18 articles that discuss the proposed theme. We especially acknowledge *Educação Matemática Pesquisa (EMP)* journal, which welcomed and enabled the publication of this thematic issue, expanding its reach among researchers and mathematics educators.

We would also like to thank our partners and collaborators who dedicated some of their time to carrying out a broader research project linked to Working Group 7 —Education of Teachers who Teach Mathematics— of the Brazilian Society of Mathematics Education

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(SBEM). This investigation had the participation of the following researchers: Dr. Eliane Matesco Cristovão (Federal University of Itajubá, Unifei), Dr. Enio Freire de Paula (Federal Institute of Sao Paulo, IFSP), Ms. Lana Thaís Santos Silva (Federal University of Sergipe, UFS, Municipal School Dr. Lourival Baptista), Dr. Henrique Rizek Elias (Federal Technological University of Paraná, UTFPR), Dr. Lya Raquel Oliveira dos Santos (Federal University of Piauí, UFPI), Dr. Marta Elid Amorim Mateus (UFS), Ms. Mayara de Miranda Santos (Federal Institute of Piauí, IFPI), Dr. Marlova Estela Caldatto (*in memoriam*, UTFPR), Dr. Sabrina Bobsin Salazar (Federal University of Pelotas, UFPel), Ms. Silvânia da Silva Costa (College of Application of the Federal University of Sergipe, Codap/UFS), Dr. Vânia Cristina da Silva Rodrigues (Federal University of Triângulo Mineiro, UFTM) and Ms. Vânia Batista Flose Jardim (Federal Institute of Education, Science and Tecnology of São Paulo, IFSP).

Initially conceived during the VII International Seminar on Research in Mathematics Education (Sipem), held in November 2018 in Foz do Iguaçu, Paraná, Brazil, the broader research began to be consolidated at the VIII Sipem, held online in November 2021 in Uberlândia, Minas Gerais, Brazil. Three years later, we initiated the process of planning and structuring the investigations linked to this theme, which culminated in the publication of this thematic issue in 2024 and the disclosure of the results produced by this collective research movement, focusing on theoretical frameworks for discussing mathematical knowledge and knowings, in 2025.

For some decades now, the debate about a specific mathematics of teaching practice — distinct from that practiced by professionals such as engineers, architects, economists, and even academic mathematicians— has been gaining ground, both nationally and internationally. Based on Lee Shulman's initial propositions, different theoretical models were configured to account for this specificity. An example of this is the studies developed by Deborah Ball and collaborators, who called it mathematical knowledge for teaching. In a complementary direction, but based on a sociological approach, Jill Adler and Zelda Davis investigated not only how this mathematics for teaching is constituted in and through teaching practice, but also which principles legitimize it as professional knowing.

More recently, Brent Davis and Moshe Renert proposed the concept of mathematics for teaching as a participatory disposition, cultivated in collaborative environments, in which deep understandings of mathematics emerge from practice. Such understandings qualify the teacher's knowledge and also subsidize and guide their performance in the classroom, emphasizing the collective and dynamic nature of the constitution of professional teaching knowledge.

Several countries have centered on in-depth research on what to teach in mathematics and how to do it, recognizing the complexity involved in professional teaching knowledge. Brazil is part of this international movement and expresses this commitment to scientific production. Therefore, we invite readers to carefully and critically examine the texts that comprise this volume, understanding that, depending on the theoretical lenses adopted, the mathematics inherent to teaching can assume different configurations.

We hope that the studies gathered here inspire new perspectives and in-depth analysis, encourage questions and reconstructions, and outline new investigative paths, thereby fueling the ongoing movement of research in mathematics education. Given this scenario, we invite the scientific community to read the collection of articles, briefly presented below.

The theoretical essay entitled “On the specific mathematics of teachers,” written by Prof. Dr. Jonei Cerqueira Barbosa, guest author, opens the discussions on the topic of this thematic issue by analyzing the specificity of teachers’ mathematics and the limitations of the mathematical knowledge for teaching (MKT), mathematics teacher’s specialized knowledge (MTSK), and mathematics for teaching (MfT) models in capturing the situated and controlled nature of teachers' doing. The author proposes the distinction between mathematics in teaching (*matemática no ensino* - MnE), which occurs in pedagogical interaction, and mathematics to teach (*matemática para ensinar* - MpE), which guides this interaction. He argues that both are articulated recursively, being influenced by socio-institutional factors, such as public policies and curricula. The study suggests that teaching mathematics is not reduced to a set of knowledge but is related to teaching and the school context.

The second article, authored by Caroline Silva, Sandra Menezes, and Miguel Ribeiro, “Mathematics teachers' specialized knowledge and interpretative knowledge: Weaving theoretical relationships within the scope of isometric geometric transformation rotation,” discusses contributions to discussions on the specificities of the teacher's mathematical knowledge. This discussion is theoretically based on two articulated conceptualizations: the mathematics teacher’s specialized knowledge (MTSK) and the interpretive knowledge. The MTSK offers a detailed modeling of the teacher’s knowledge, covering mathematical and pedagogical domains, with emphasis on *mathematical knowledge* (MK) and its categories. The *interpretive knowledge*, in turn, is based on a hermeneutic listening of the students' mathematical thinking, allowing the teacher to interpret their productions, including unconventional ones, and make informed pedagogical decisions.

Henrique Rizek Elias, Sabrina Bobsin Salazar, and Vânia Batista Flose Jardim present to the community the third article, “Mathematical knowledge for teaching: Limits and

possibilities for Brazilian research." The main theoretical reference is the MKT model, proposed by Deborah Ball and collaborators. Based on this model, the authors carry out a critical analysis of the use of MKT in Brazilian research, pointing out that, in many cases, its subdomains are used as fixed categories, disregarding the national context. The study also takes into account recent productions from Deborah Ball's group, indicating a continuous revision of the model. They defend the need for critical adaptation of MKT to the Brazilian context, proposing that its subdomains be investigated, expanded, or reformulated based on local realities.

The fourth article, by Katiane de Moraes Rocha and Aparecida Santana de Souza Chiari, "Trajectory, resources, and knowledge for teaching mathematics at the beginning of teaching," contemplates a theoretical framework that combines different approaches to analyze mathematics teachers' education and the knowledge necessary for teaching. The author draws on studies by Deborah Ball and collaborators and presents discussions that put forward an organization of teaching knowledge that includes common content knowledge (CCK), specialized content knowledge (SCK), and pedagogical content knowledge (PCK), emphasizing that mathematics teaching requires more than mastery of academic content. In general, the study shows that initial education has been based on the construction of knowledge centered on academic mathematics.

The fifth article, "Technological transformations in mathematics teaching: A multifaceted study on the development of technological pedagogical content knowledge", authored by Lya Raquel Oliveira dos Santos, Lana Thaís Santos Silva, and Mayara de Miranda Santos, adopts the technological pedagogical content knowledge model (TPACK) proposed by Punya Mishra and Matthew J. Koehler as a theoretical framework. With roots in Lee Shulman's studies, the authors seek to integrate knowledge of pedagogy, technology, and content. The TPACK is advocated as a powerful conceptual framework for analyzing how teachers use technologies, going beyond merely instrumental use. The theory thus supports the importance of contextualized, critical, and transformative practices, based on a broad understanding of teaching knowledge.

Marília Lidiane Chaves da Costa Alcantara and Claudianny Amorim Noronha, authors of the sixth article, "Mathematical knowledge for teaching: Dialogue on the teaching work of mathematics teachers who work in basic education," take as a starting point the concept of pedagogical content knowledge (PCK), proposed by Lee Shulman, deepening the discussions based on the studies of Deborah Ball and collaborators. When identifying limitations in the appropriation of the PCK, the authors develop their reflections based on the MKT model,

anchored in teaching practice and focused on the real demands of mathematics teaching. For them, the theoretical model of Ball and collaborators shifts the focus from the teacher to the demands of teaching, representing a significant inflection in research in the area of mathematics education.

The seventh article, “Problematizing approaches for mobilizing knowledge of mathematics teachers: Contributions, limitations, and challenges”, written by Vânia Cristina da Silva Rodrigues, Eliane Matesco Cristovão, and Enio Freire de Paula, presents connections between theories on teaching knowledge/knowings and those focused on problematizing approaches in mathematics teaching. The study highlights links between these approaches and theoretical models, such as the mathematical knowledge for teaching (MKT), proposed by Deborah Ball and collaborators, mathematics teachers’ specialized knowledge (MTSK), and didactic knowledge. The centrality of exploratory and investigative tasks stands out as vectors for developing pedagogical knowledge of the content, contributing to teacher education and teaching practice.

“Mathematics for teaching the concept of polynomials based on recontextualizations in textbooks” completes the selection. Authors Alana Santiago Oliveira and Jaqueline de Souza Pereira Grilo develop an investigation based on Basil Bernstein's theory of codes, focusing on different forms of communication, in dialogue with the assumptions of mathematics for teaching, understood as the set of strategies and forms used to communicate mathematics, as proposed by Brent Davis and Moshe Renert. The concept of polynomials was the object of study, and the proposed analysis revealed six panoramas of realization of the concept — among them, generalization, algebraic structure, and geometric structure—, which, to a greater or lesser extent, express different classifications and frameworks.

Article number nine, written by Marta Élid Amorim, Ruy Cesar Pietropaolo, and Silvânia da Silva Costa, is titled “Knowledge for the education of teachers who teach statistics in basic education.” The research is based on the theoretical frameworks of Lee Shulman and Maurice Tardif. While Shulman contributes to conceptualizing the different types of knowledge necessary for teaching, content knowledge, pedagogical content knowledge, and curriculum knowledge, Maurice Tardif expands the discussion by treating teaching knowledge as constructions arising from both academic education and professional experience, marked by interactions, practices, and socialization processes. The study also draws on the references of Iddo Gal, when conceptualizing statistical literacy, and of Frances Curcio, when discussing graphic understanding, thereby composing a robust panorama on the teaching and learning of statistics in basic education.

Alana Nunes Pereira and Samira Zaidan present to the community the tenth article, "Mathematical knowledge for teaching flat and spatial figures in middle school: Visualization in focus." The research is based on references that discuss specific knowledge for mathematics teaching, with emphasis on geometry and visualization as central components. It dialogues with Plínio Cavalcanti Moreira and Maria Manuela M. S. David when treating school mathematics as knowledge constructed in teaching practice, and with Deborah Ball and collaborators when understanding the teachers' knowledge as multifaceted and situated. The research highlights visualization as specific knowledge for teaching flat and spatial figures, and not just as a teaching strategy.

The article "Specialized knowledge in teaching exponential functions through problem solving in the context of the Institutional Program of Teaching Initiation Grants (Pibid)," authored by Caleb Campelo and Marcelo Carlos de Proença, is based on the theoretical model of mathematics teachers' specialized knowledge (MTSK), proposed by José Carrillo-Yañez and collaborators, with roots in the work of Lee Shulman and Deborah Ball and her team. The MTSK is structured into two domains, mathematical knowledge (MK) and pedagogical content knowledge (PCK), and six subdomains that range from specific knowledge of mathematical topics to teaching and learning practices.

In the 12th article, Mikaelle Barboza Cardoso and Marcilia Chagas Barreto reveal another theoretical perspective through the work called "Teaching knowledge mobilized and reworked by mathematics teachers from the perspective of lesson study and semiotic representations," in which they analyze the reworking of mathematical knowledge for teaching affine functions, through the experience of a formative process based on lesson study (LS), with the contribution of the theory of registers of semiotic representation (TRSR). The experience with LS highlighted the different phases of class organization and implementation, as well as the importance of collaboration in building innovative practices. We noted that the formative process not only strengthened content knowledge but also improved pedagogical skills, promoting teachers' continuing education and the mobilization, elaboration, and re-elaboration of mathematical knowledge for teaching with the contribution of LS and TRSR.

From another theoretical perspective, the 13th article, "Possibilities and limitations of micro-paths of study and research in geometry: An experience of continuing education with public school teachers," is written by Cintia Melo dos Santos, José Luiz, and Tatiani Garcia. The research analyzes the development of study and research micropaths (PEP) as a formative strategy in the continuing education of mathematics teachers, with a focus on teaching symmetry. Based on the anthropological theory of the didactic (ATD), the study examines the

praxeologies developed and the impacts of the paradigm of questioning the world, identifying pedagogical practices adopted by the participating teachers. According to the authors, following the principles of the ATD, all human activity can be described through praxeologies, which inseparably articulate know-how and knowledge.

The 14th article, "The emergence of didactic obstacles in the teacher-student-knowledge contractual relationship in probability teaching," by Vitória Farias and Anna Paula de Avelar Brito Lima, examines the relationship between the didactic contract and didactic obstacles in the teaching of probability in middle school. The research is based on the theory of didactic situations (TDS), proposed by Guy Brousseau, to analyze how the didactic contract — understood as the set of implicit rules that guide the interaction between teacher and students — influences the construction of mathematical knowledge. The study demonstrates that this contract can generate didactic obstacles of different natures —ontogenetic, epistemological, and didactic—which make learning probabilistic concepts difficult.

The 15th article, "Mathematical knowledge for teaching in the context of inclusive education: An analysis of descending bifurcations in the teaching of arithmetic mean," authored by Wuallison Firmino dos Santos and Marcus Bessa de Menezes, is based on the TDS, proposed by Guy Brousseau, with an emphasis on the model of structuring the *milieu,* adapted by Claire Margolinas. The research aims to analyze the didactic knowledge mobilized by teachers in inclusive contexts. This model allows us to understand different levels of teaching activity, from planning to classroom interaction, highlighting the decisions and adaptations made considering students' diversity. The study underscores the importance of specific knowledge for inclusive education, defending practices that go beyond specific adaptations and moving towards more systematic proposals that are sensitive to students' singularities.

Next, "The construction of the professional identity of the mathematics teacher: A case study from the perspective of the documentary approach to teaching" is written by Sandra Cristina Martini Rostirola, Elisa Henning, and Ivanete Zuchi Siple. The text analyzes the construction of professional teaching identity from the perspective of the documentary approach to teaching. The research is based on the concept that teaching knowledge is multifaceted and develops throughout initial education and professional practice. The documentary approach to teaching understands that the teacher constantly interacts with various material, symbolic, and institutional resources, transforming them and being transformed by them, in a dialectical process of instrumentation and instrumentalization.

Article number 17, "Didactic suitability in the initial education of mathematics teachers: Reflections on aspects of interaction," authored by José Fernandes da Silva, Alexsandra Braga

Horta and Vicenç Font , discusses three perspectives related to the problem of instructional design in mathematics teaching. The first, based on positivism, defends practices based on scientific evidence and quantitative methods. The second, of an institutional nature, emphasizes compliance with curriculum prescriptions, limiting teaching autonomy. The third, assumed by the author, values critical reflection and teacher autonomy, anchored in the framework of didactic suitability criteria (DSC). The study is based on the anthropological theory of teaching suitability, proposed by Juan Díaz Godino and collaborators, and on the development of the concept of didactic suitability.

From the same theoretical perspective, the 18th article, "Criteria of didactic suitability as a formative device for prospective mathematics teachers: Contributions to the mobilization of pedagogical reasoning," authored by Jean Carlo Francis Wanderley Graciano do Carmo and Douglas da Silva Tinti, investigates how criteria of didactic suitability can contribute to the mobilization of pedagogical reasoning in initial education spaces. Based on the theory of didactic suitability, the authors problematize the concept of didactic-mathematical knowledge (DMK) and discuss the notion of pedagogical action and reasoning, as proposed by Lee Shulman. In this context, didactic suitability criteria are considered a powerful conceptual tool for assessing the adequacy of teaching practices in light of educational objectives, student characteristics, content involved, and available resources.

Inscription

We dedicate this work to Marlova Estela Caldatto (UTFPR), who passed away on November 28, 2022, before the research was completed. Her contributions made a difference. We thank her for all her dedication to mathematics education!