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Semiotic resources used to produce diagrams by 1st grade elementary school students when developing a mathematical modeling activity

Los recursos semióticos utilizados para producir diagramas por estudiantes de 1º de primaria al desarrollar una actividad de modelación matemática

Les ressources sémiotiques utilisées pour réaliser des schémas par les élèves de 1re année du primaire lors de l'élaboration d'une activité de modélisation mathématique

Os recursos semióticos utilizados para a produção de diagramas por alunos do 1º ano do ensino fundamental ao desenvolverem uma atividade de modelagem matemática

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Abstract

This article aims to understand the knowledge revealed by 1st year Elementary School students in the use of semiotic resources for the production of diagrams during the development of a mathematical modelling activity with the Christmas tree theme. The theoretical framework is supported by Mathematical Modelling in the early years as a pedagogical alternative, in which, based on a problem situation that may be of interest to the students, they focus on presenting a solution to the problem. To do this, they can choose semiotic resources, which are the means used in the production of signs, such as diagrams that organize and externalize what students understand or perceive about a certain object or concept. The qualitative interpretative analysis supported by the triangulation process and the theoretical framework articulated with the audio and video recordings, as well as the written records of a class of 25 students in the 1st year of Elementary School at a school located in the State of Paraná, in the year 2023, in Brazil. The results showed that, by manipulating toilet paper rolls, as well as using other semiotic resources such as speech and gestures, students were able to produce diagrams that naturally revealed knowledge related to content covered in the curriculum at this level of education, such as counting, numerical organization and the multiplicative principle.

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Keywords: Mathematics education, Mathematical modelling, Early years, Prototype, Christmas tree.

Resumen

Este artículo busca comprender los conocimientos adquiridos por estudiantes de 1.er grado de primaria em el uso de recursos semióticos para la elaboración de diagramas durante el desarrollo de una actividad de modelación matemática sobre el tema del árbol de Navidad. El marco teórico se sustenta en la Modelación Matemática en la primera infancia como alternativa pedagógica, donde, a partir de una situación problemática que pueda ser de interés para los estudiantes se centran em presentar una solución. Para ello, pueden elegir recursos semióticos, que son los medios utilizados en la producción de signos, como diagramas que organizan y exteriorizan lo que los estudiantes comprenden o perciben sobre un objeto o concepto determinado. El análisis interpretativo cualitativo, sustentado en una experiencia empírica, se basa em el proceso de triangulación y el marco teórico articulado con grabaciones de audio y video, así como com los registros escritos de una clase de 25 estudiantes del primer año de primaria en una escuela ubicada en el estado de Paraná, en el año 2023, Brasil. Los resultados mostraron que, mediante la manipulación de rollos de papel higiénico, así como otros recursos semióticos como el habla y los gestos, los estudiantes fueron capaces de producir diagramas que revelaron de forma natural conocimientos relacionados con contenidos abordados en el currículo de este nível educativo, como el conteo, la organización numérica y el principio multiplicativo.

Palabras clave: Educación matemática, Modelización matemática, Primera infancia, Prototipo, Árbol de navidad.

Résumé

Cet article vise à comprendre les connaissances révélées par les élèves de 1 ère année du primaire dans l’utilisation de ressources sémiotiques pour la production de diagrammes dans le développement d’une activité de modélisation mathématique avec le thème du sapin de Noël. Le cadre théorique s’appuie sur la modélisation mathématique dès le plus jeune âge, comme alternative pédagogique. À partir d'une situation problématique susceptible d'intéresser les élèves, ils s’attachent à présenter une solution à ce problème. Pour ce faire, ils peuvent choisir des ressources sémiotiques, qui sont les moyens utilisés pour produire des signes, tels que des diagrammes qui organisent et extériorisent ce que les élèves comprennent ou perçoivent d’un objet ou d’un concept donné. L'analyse interprétative qualitative, appuyée par une expérience empirique, repose sur le processus de triangulation et le cadre théorique articules autour des enregistrements audio et vidéo, ainsi que des documents écrits d'une classe de 25 élèves de première année d'école primaire dans l'État du Paraná, en 2023, au Brésil. Les résultats ont montré qu'en manipulant des rouleaux de papier toilette, ainsi qu'en utilisant d'autres ressources sémiotiques telles que la parole et les gestes, les élèves étaient capables de produire des diagrammes qui révélant naturellement des connaissances liées au contenu couvert dans le programme d'études à ce niveau d'enseignement, comme le comptage, l'organisation numérique et le principe multiplicatif.

Mots-clés : Enseignement mathématique, Modélisation mathématique, Petite enfance, Prototype, Arbre de noël.

Resumo

Neste artigo tem-se como objetivo compreender os conhecimentos revelados pelos alunos do 1º ano do Ensino Fundamental no uso de recursos semióticos para a produção de diagramas no desenvolvimento de uma atividade de modelagem matemática com a temática árvore de Natal. O quadro teórico está subsidiado na Modelagem Matemática nos anos iniciais como alternativa pedagógica, em que a partir de uma situação-problema que possa ser do interesse dos alunos estes se debruçam para apresentar uma solução para o problema. Para isso, podem escolher recursos semióticos que são os meios utilizados na produção de signos, como os diagramas que organizam e externalizam o que os alunos entendem ou percebem sobre determinado objeto ou conceito. A análise qualitativa de cunho interpretativo subsidiada em uma experiência empírica é pautada no processo de triangulação e no quadro teórico articulado às gravações de áudio e vídeo, bem como dos registros escritos de uma turma de 25 alunos do 1º ano do Ensino Fundamental de uma escola localizada no Estado do Paraná, no ano de 2023, no Brasil. Os resultados mostraram que, ao manipular rolinhos de papel higiênico, bem como fazer uso de outros recursos semióticos como falas e gestos, permitiram aos alunos produzirem diagramas que revelaram de modo natural conhecimentos relacionados a conteúdos abordados na matriz curricular desse nível de escolaridade, tais como contagem, organização numérica e princípio multiplicativo.

Palavras-chave: Educação Matemática, Modelagem Matemática, Anos iniciais, Protótipo, Árvore de Natal.

The semiotic resources used for the production of diagrams by first-year elementary school students when developing a mathematical modeling activity

Some students find mathematics difficult due to challenges in the teaching and learning process. These difficulties may be related to mathematical content, subjective aspects of the students themselves, or the teacher's approach to imparting knowledge.

In the early years of elementary school, it is important “to be committed to developing mathematical literacy, which is defined as the ability to reason, represent, communicate, and argue mathematically” (Brazil, 2018, p. 266). Education is important because it develops children's thinking, values, and skills, transforming them into conscious, critical individuals prepared for life. This is why it is essential to use different teaching methods and educational practices that contribute to students' development while taking their interests and motivations into account. These methods should contribute to quality education, helping children become autonomous, critical, participatory, and active citizens in society (Brazil, 1997).

According to English (2010, p. 288), implementing mathematical modeling in the classroom "provides rich opportunities for children to experience complex data within challenging yet meaningful contexts." Mathematical modeling is a pedagogical alternative that can be used in the early years with diverse themes that contribute to student learning since students become active participants in developing the activity and constructing new knowledge using what they already know.

The literature contains various research covering topics that can be explored with early-years students. For example, Santos (2022) addressed the topic of "Pets" with a fifth grade class. The objective was to investigate the cost of pets. Pelaquim (2023) developed a “Rockets” activity in which fifth-grade students built a rocket prototype and aimed to determine the optimal angle for launching the rocket to achieve the greatest horizontal distance. Furthermore, Alsina and Salgado (2021) presented an activity with the theme "Apples," in which 5- to 6-year-old children had to observe and classify apples' different characteristics for potential buyers. English (2022) described and analyzed an activity regarding "Volcanoes," in which fifth grade students built a cone-shaped volcano prototype and explored the flow time of three simulated lava flows of different viscosities descending a volcano's slope. The mathematical modeling activities in the aforementioned research produced mathematical models that provided answers to the studied problems. Generally, in the early years, these models can be presented through graphs, tables, and other schemes. In Santos (2022), the model produced by the students was implicit in natural language speech; students in the early years of Pelaquim's research (2023) used a written report in which they represented, through drawings, the geometric figures that make up a rocket, as well as representing the distances reached in a table, depending on the angle of launch; in Alsina and Salgado (2021), the mathematical model was represented through drawings and manipulable materials produced by the students; English (2022) used column graphs and line graphs based on data produced in the simulation of the eruption of a prototype volcano.

A mathematical model can be represented in different ways if mathematical structures support it. These structures are abstract models formed by one or more sets accompanied by operations or relationships that obey certain rules. They assist in the study and organization of mathematical ideas in a logical and systematic way by producing signs that correspond to representations of mathematical objects. According to Mavers (2004), resources that are chosen and used effectively in the production of signs are characterized as semiotic resources. These resources can be employed simultaneously to construct meaning. Gestures, speech, manipulable materials, and prototypes of a rocket and a volcano can be considered semiotic resources in modeling activity development, provided they are chosen for producing mathematical signs.

In Brazil, we found research focusing on analyzing semiotic resources in mathematical modeling activities (Araki, 2020; Pessoa, 2024; Goulart, 2020; Almeida et al., 2021). These studies focus on approaches used in the final years of elementary school and in higher education. Therefore, investigating semiotic resources in the early years of elementary school is important, given the lack of research on this topic at this level of education and the significant results of studying the knowledge that semiotic resources can facilitate.

According to Pessoa (2024, p. 37), "for a sign to be produced, there must be a means of production, a resource through which the sign is produced." Thus, semiotic resources can give rise to different signs. Signs are the means by which an object is referred to, represented, or referred to. Peircean semiotics, developed by American philosopher Charles Sanders Peirce, addresses the study of signs. Peirce classified the different types of signs produced by an interpreter following a trichotomy related to the ways in which they are produced. A diagram is a type of sign that reflects how the interpreter organizes and externalizes their understanding or perception of a particular object or concept. Bakker and Hoffmann (2005, p. 353) state that "in Peirce's epistemologically based semiotics, diagrams are not only means of communication but also means of thought, understanding, and reasoning."

In this article, we focus on investigating the question: What knowledge do first-grade elementary school students reveal using semiotic resources to produce diagrams in a mathematical modeling activity? To answer this question, we conducted qualitative and interpretive research based on triangulation, collecting data from a class of 25 first-grade students at a municipal school in Paraná, Brazil. This article considers the theoretical contributions of mathematical modeling in early education and the use of semiotic resources for producing diagrams based on Peircean semiotics, which will be discussed in the next two sections. Next, we present the methodological aspects that guided our investigation, followed by a description and analysis of the mathematical modeling activity developed with the students. We conclude the text with some considerations and suggestions for future research.

Mathematical modeling in the early years

Many researchers at all levels of education have discussed mathematical modeling. In the early years, there are studies at the international level (English & Watters, 2004; Blum & Ferri, 2009; English, 2010; Alsina et al., 2021) and at the national level (Burak, 2004; Luna & Alves, 2007; Tortola, 2016; Biembengut, 2019; Pelaquim, 2023; Veronez & Santos, 2023; Gomes & Silva, 2024).

These studies demonstrate that mathematical modeling can be incorporated into early education. As Bassanezi (2015, pp. 11–12) states, "[...] modeling can be adopted in any educational situation or environment, provided that a context compatible with the students' stage of development is used." Mathematical modeling connects mathematics to the real world. Modeling problems[3] are motivating and lead to better mathematics learning (Aroeira et al., 2024, p. 161).

For our research development, we based ourselves on mathematical modeling as a pedagogical alternative with potential for teaching and learning mathematics. In this approach, we investigate a non-mathematical problem and use mathematics to solve it. Developing a modeling activity involves "translating in both directions between mathematics and the extra-mathematical world" (Blum & Ferri, 2016, p. 65) and provides "opportunities for children to conduct research, debate with peers, and use their representations to construct diverse knowledge" (Veronez & Santos, 2023, p. 172).

When implementing mathematical modeling as a pedagogical practice, different configurations are possible. In the early years, it is important for students to use concepts they already know and learn new ones, thereby expanding their knowledge base (Tortola & Almeida, 2018). When developing a mathematical modeling activity, early-years students "expand their

[3] We understand modeling problems as those that require investigation using mathematical language to represent, understand, and solve a real-world situation.

mathematical skills, becoming adept at problem solving and the modeling process, and are more likely to develop other activities based on real situations with an interdisciplinary approach" (Luna et al., 2009, p. 140).

Mathematical modeling can facilitate changes in mathematics education in early years by contributing to more dialogic and dynamic teaching methods that align with children's interests (Martens & Klüber, 2016). According to Parra-Zapata and Villa-Ochoa (2015, p. 236):

> working with mathematical modeling from an early age is something that can happen naturally out of conviction and stimulates children's creativity and imagination. From there, it is possible to generate interest in various activities related to mathematics and numbers.

The mathematical model in the early years requires a different approach than that covered in the final years of elementary or high school. In an early elementary school mathematical modeling activity, Tortola and Silva (2021, p. 4) argue that "the mathematical model corresponds to a representation external to the minds of the subjects, whose statements are in mathematical terms." Thus, mathematical models can be represented by graphs, schemes, and drawings. These models are used to describe, explain, and/or predict aspects of the problem situation (Tortola, 2016).

When developing a mathematical modeling activity, students can use physical materials to deduce a mathematical model. This model can be represented as "a prototype of some part of reality or the result of a mathematization process following experimentation on a prototype" (Carreira & Baioa, 2018, p. 204). A prototype can be an approximation or simplification of a real-world system, enabling the exploration and experimentation of its principles and modes of operation. The model is obtained through manipulation and experimentation using the prototype, and it is developed through mathematization to arrive at an answer to a given problem. According to Carreira and Baioa:

> The model and the prototype may belong to the same class of entities or to different classes. The model can be an image, a drawing, or a diagram in the same way as a system of equations or a set of postulates. The goal is the same, i.e., to control the system within certain limits of approximation (Carreira & Baioa, 2018, p. 204).

When developing a rocket or volcano prototype, students could base their approach on a realistic scenario. The results obtained depended on the launch angle for the rocket and the time it took for the lava to flow for the volcano. To a certain extent, constructing each prototype highlighted mathematical aspects related to the problem situation and provided a solution to the problem being investigated. Moving from a problem to be investigated to a defined problem solution generally involves configuring phases or stages consisting of the set of actions required for developing mathematical modeling activities. Stillman et al. (2007) present a seven-step mathematical modeling cycle for solving modeling problems. Starting with a real-world problem, the cycle continues until the mathematical model solution is accepted. If not, the problem is revisited, as shown in Figure 1.

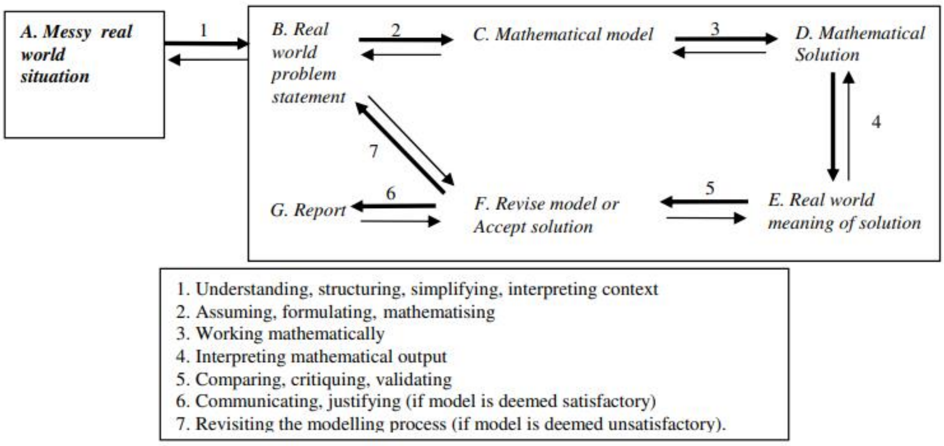


Figure 1.

Modelling Process (Stillman et al., 2007, p. 690).

The stages of developing a mathematical modeling activity are represented by the letters A through G, and the thicker arrows indicate transitions between stages. The lighter arrows pointing in the opposite direction of the modeling cycle emphasize that this procedure is not linear (Stillman et al., 2007). The modeling cycle begins in the upper left corner (stage A), where the real-world situation is located. Next, a real-world problem is presented, and the appropriate mathematical model for the situation is sought. After defining the mathematical model, the next step is to find a mathematical solution and interpret it, verifying whether the model is valid. If necessary, the model is revised. Otherwise, the model is accepted, and a report on the situation is prepared using mathematics.

According to studies based on Peircean semiotics, we consider a mathematical model to be valid if it is supported by mathematical structures and involves the production of signs that correspond to representations of mathematical objects.

Peircean semiotics

Semiotics is the study of signs and language. It values the analysis of signs and their functionality to reveal the characteristics of objects. According to Peirce (1972, p. 27), a sign is "anything that admits an 'interpretant'—that is, anything capable of giving rise to other signs." In this sense, Peirce asserts that:

> A sign, or representamen, is something that represents something to someone in a certain aspect or mode. A sign is directed at someone; it creates an equivalent or more developed sign in that person's mind. I call this newly created sign the interpretant of the original sign. The sign represents something—its object. However, it does not represent the object in all its aspects but rather a type of idea that Peirce sometimes called the "ground of the representamen" (Peirce, 2005, p. 45).

Based on the work of scholars who follow Peirce's semiotic theory, we understand that signs can be considered "means of thought, understanding, reasoning, and learning" (D'Amore et al., 2015, p. 59). Thus, through signs that refer to, represent, or symbolize objects, we can infer the knowledge of interpreters.

Peirce offers several characterizations of signs, and to understand them, one must analyze their relationship with the object and the interpretant. According to Peirce (2005, p. 160):

> A sign is a cognoscible that is, on the one hand, determined by something apart from itself—called its object—while, on the other hand, it determines a concrete or potential mind. I call this determination the interpretant created by the sign. Thus, the object indirectly determines the interpreting mind.

In Peircean semiotics, signs are divided according to three dichotomies, one of which considers the relationship between the sign and its object, in which the sign can be called an icon, index, or symbol. For Peirce (2005), an icon is a sign that refers to the object by similarity, an index is a sign that refers to the object by proximity, and a symbol is a sign that refers to the object by virtue of a law.

According to Peirce (2005, p. 64), “the only way to communicate an idea directly is through an icon; and every method of indirect communication of an idea must depend, to be established, on the use of an icon.” An icon can be classified in Peircean semiotics as images, diagrams, and metaphors, and, according to Peirce:

> Those that participate in simple qualities or first primacy are images; those that represent relations, mainly dyadic ones, or those that are considered as such, of the parts of a thing through analogous relations in their parts are diagrams; and those that represent the representative character of a representamen through the representation of a parallelism with something else are metaphors (Peirce, 2005, p. 64).

In a mathematical context, for instance, a set of circles of various sizes may represent a sequence of mathematical objects. The interpreter who chose to represent the object with circles of different sizes may not have used a scale to indicate the sequence of figures, but the representation may be sufficient for analyzing the repetition of the sequence. If more detail is needed, the interpreter may need to refine the diagram.

A diagram is a sign that establishes relationships between the object and the interpreter. According to Peirce (2005, p. 66), "Many diagrams do not resemble their objects in appearance at all; the similarity between them consists only in the relationship between their parts," as is the case with the sequence of circles representing a recurrence.

The resources chosen and used effectively in the production of signs are characterized as semiotic resources (Mavers, 2004). Leeuwen defines semiotic resources.

> as the actions and artifacts we use to communicate, whether they are produced physiologically—with our vocal apparatus; with the muscles we use to create facial expressions and gestures, etc.—or through technologies – with pen, ink, and paper; with computer hardware and software; with fabrics, scissors, and sewing machines, etc. (Leeuwen, 2005, p. 3).

In the case of a sequence defined by a set of circles, the semiotic resources chosen may be pen and ink on paper, pencil and paper, or chalk and a blackboard if the interpreter is in a classroom.

When producing signs, it is essential to highlight not only which semiotic resources are employed but also how they are used in a given context. Depending on the activity, the most appropriate resources should be chosen. While there are no fixed rules for using semiotic resources, they must make sense in the context. For example, it would be useless to use a drawing to express something during an audio call because this resource would not be accessible to the other participants.

According to Mavers (2004, p. 59)

> [...] an individual must select appropriate semiotic resources according to their suitability for the task at hand. From the repertoire of what might be selected, choice (and that which was not chosen) represents what was deemed the most apt means of representation.

The choice and combination of semiotic resources depend on the representational needs and ability to adapt them of the interpreter (student). When modeling activities with students in the early years, various semiotic resources can be used, including gestures, speech, drawings, and manipulatives. The choice of resources depends on the student's representation needs and the most appropriate way to express their interests.

Researchers such as Goulart (2020), Pessoa (2024), and Araki (2020) have investigated the use and function of semiotic resources in mathematical modeling activities developed by higher education and elementary school students, respectively. Goulart (2020) argued that combining different resources associated with different semiotic systems increases students' involvement in developing mathematical modeling activities. Pessoa (2024) emphasized that various mathematical knowledge was mobilized with the help of semiotic resources chosen to refer to the phenomenon under study, producing data for the development of the modeling activity. Araki (2020) showed that the semiotic resources used contributed to attributing meanings to mathematical objects by associating investigative, experimental activities with the modeling cycle.

Considering these points, we analyzed how students in a first-grade elementary school class used semiotic resources to produce diagrams when developing a modeling activity, as presented in the following topic.

Methodological aspects

In this article, we present partial results from the first author's master's research, conducted under the guidance of the second author. The study implemented modeling activities in the first year of elementary school. This research yielded an educational product that provides suggestions for modeling practices in the first year (Gomes & Silva, 2025).

We analyzed an activity developed with a class of 25 students, aged 6 to 7, from a municipal school in Arapongas, Paraná. Authorization was requested from the school administration to develop the activity, and a free and informed consent form was sent to parents to be completed and signed. This form authorized the development of the activities and the use of the collected data, including written records, photos, audio recordings, and videos made by the teacher. The images and dialogues presented in this article were selected from the collected material based on their relevance and ability to reflect the students' perceptions and knowledge when solving the problem. In the body of the text, students are referred to by fictitious names to maintain anonymity, and the teacher is referred to only as "professor."

In 2023, the class teacher (one of the article's authors) taught first-year mathematics classes three days a week for one hour and forty-five minutes each day. During that time, students developed six mathematical modeling activities with different themes. This allowed them to become familiar with the procedures for defining a problem to be investigated mathematically, collecting data, and developing a mathematical model. Thus, they became autonomous in each phase of the modeling cycle.

This article analyzes the sixth activity, "Christmas Tree," which was developed between November 30 and December 6, 2023, in four 1-hour-and-45-minute classes. This was the last activity developed with the students, and producing signs occurred more naturally with less professor intervention.

The professor chose the Christmas Tree theme because of the proximity to the end-of-year holidays. During this time, it is common to work with students on commemorative dates in a pedagogical and engaging way. We decided to make a Christmas tree with toilet paper rolls after seeing an image on a website (Figure 2). Students were asked in advance to collect toilet paper rolls and bring them to class to carry out the activity. All students participated in collecting the material, totaling 340 rolls.

Figure 2.

Christmas tree model to be built (Authors, 2023)

To address the research question, "What knowledge do first-grade elementary school students reveal through the use of semiotic resources to produce diagrams in a mathematical modeling activity involving the construction of a Christmas tree?" We based our analysis on students' written records, photos taken by the professor, and audio and video recordings produced during the activity.

We took a qualitative and interpretive approach to our analysis, based on the triangulation process that considers the subject, the object, and the phenomenon (Tuzzo & Braga, 2016). This type of analysis uses qualitative data, which consists of ideas and texts focused on interpretation and the process's meaning. In our study, the subjects were the 25 first-grade students, the object was the semiotic resources they produced in the diagrams used in the mathematical modeling activity, and the phenomenon was the knowledge revealed by the students during the activity. The metaphenomenon corresponds to the analytical movement articulated within the theoretical framework of mathematical modeling and Peircean semiotics, which we present in the next section.

Analysis of modeling activity on building Christmas trees

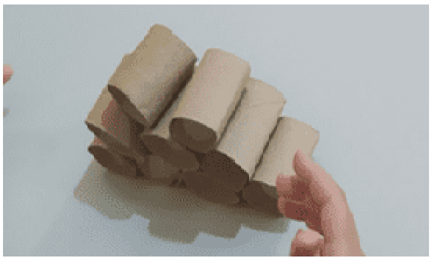
The modeling activity began on November 30 with the professor addressing the Christmas theme due to its proximity. She then discussed using recyclable materials, such as toilet paper rolls, to build a Christmas tree and showed the students an image from the Internet (Figure 2) using her notebook as a resource. Through this technological resource, the professor aimed to influence the students' minds so that they would associate the structure of the Christmas tree with its construction. The students were expected to produce signs so that they would associate the structure of the Christmas tree with possibilities for its construction.

The students were organized into groups of four and given a number of toilet paper rolls so that they could manipulate them and experiment with building trees with any number of rolls at the base, thereby determining the number of rolls needed to build a Christmas tree for each student in the classroom. At that point, the professor began a dialogue about the construction, as transcribed below:

> Teacher: How are we going to assemble our tree?
>
> Carolina: By placing one next to the other [shows how], one next to the other, and I'll keep doing that. (Dialogue between teacher and students, 2023).

When Carolina suggested placing them side by side, she arranged the rolls in a row, overlapping them with another row, as shown in Figure 3. At this point, the image of the Christmas tree (Figure 2) and the rolls served as a semiotic resource that supported the visualization, manipulation (Figure 3a), and schematization of the prototype's structure in production (Figure 3b). These rows, schematized by the semiotic resource of the rolls, conveyed an idea and stimulated "a meaning in the mind of an interpreter" (Ribeiro, 2021, p. 263). In other words, the scheme in Figure 3 represented a diagram of the tree in production.

(a)



(b)

Figure 3.

Carolina manipulating the rollers (Authors, 2023)

The paper rolls demonstrated that the size and shape of the tree depended on the number of rolls used at the base. With this information, we can identify an approach to the situation that is "amenable to mathematical analysis" (Stillman, 2015, p. 42). This is evident in the students' statements transcribed in the dialogue below:

> Teacher: Letícia, how many rolls did you put in the first row?
> Letícia: I put in four rolls.
> Teacher: In the second row, how many did you put?
> Letícia: Three.
> Carlos: In the first row, there could be ten, and then there have to be nine, eight, seven, six, five, four, three, two, and one.
> Teacher: Why does it have to be like that, Carlos?
> Carlos: Because if they're all the same size, it won't look like a tree. They have to get smaller.
> Teacher: Letícia, after three, what comes next?
> Letícia: Two, and then one.
> Teacher: So Letícia started to assemble the first row with four, and what happened next?
> Alícia: Then three, two, and one.
> Teacher: So what happens?
> Renata: It gets smaller.
> Carlos: To form a tree, it has to get smaller; otherwise, it won't work. (Dialogue between teacher and students, 2023).

During the construction process, Carlos, one of the students, realized that one roll needed to be removed from each row to form the tree. This revealed the students' knowledge of descending order as well as their understanding of the tree scheme format. The activity enabled students to establish “a connection between the mathematical and real worlds” (Alsina et al., 2021, p. 92). If we keep the same number of rolls at the base in the other rows, however, a decreasing structure will not form when observing the construction of the tree from the base to the top. Carlos's statement revealed his understanding of the mathematical content, as transcribed below:

> Carlos: I know what this task is teaching; it's order of growth and decrease, so this task is teaching excellent things, and also with the numbers three, two, one, nine, eight, seven, eleven, ten, ah, all the numbers, growth, and decrease. (Dialogue between teacher and students, 2023).

Clearly, for Carlos, the role of the rolls as the semiotic resource for producing the Christmas tree diagram revealed the mathematical concept of order, whether ascending or descending, indicating his understanding that mathematics was involved in creating the scheme.

This action promoted the skills of "reasoning, representing, communicating, and arguing mathematically" (Brasil, 2018, p. 266).

While the students were handling the rolls, the professor asked questions to assess their knowledge of how many rolls could be used to build the tree, as indicated in the following dialogue:

> Teacher: And if we use just one roll, can we make a tree?
> Bruno: No, we need more.
> Carlos: We need two more, three more, four more.
> Teacher: So to start a tree, how many rolls do I need at least? Carlos: You need four, three, two, one.
> Teacher: So let's try with two rolls.
> Marcos: It doesn't work either; you need one more.
> Teacher: So can you do it with three?
> Marcos: It will be a little smaller, but you can do it.
> Renata: I did it with three; it works, but it's small.
> (Dialogue between teacher and students, 2023).

The student Renata observed, while handling the rolls, that it is possible to use three in the construction of the tree. This was revealed when viewing the diagram produced with the rolls lined up (Figure 4a) and also when using the semiotic resource of gesture, as shown in Figure 4b. However, the student reported that the tree would be small.

(a) (b)

Figure 4.

Student setting up the tree and gesturing (Authors, 2023)

The assembly made by the student with the row of rolls and the finger gestures indicating the quantity of three rolls generated "a process in which something was being made more mathematical than it was before" (Almeida, 2018, p. 21). In fact, what was revealed was that the student associated the toilet roll resource, producing the diagram, with the number 3 represented by the semiotic gesture (with her fingers) in association with the speech – I did it with three [...]. The diagram produced with the semiotic resource – toilet paper rolls – was configured as Renata's "means of thought, of understanding, and of reasoning" (Bakker &

Hoffmann, 2005, p. 353) when she presented her impressions on the use of three rolls: “[...] it works, but it's small.

After the student reported that the Christmas tree could be built with three rolls, the professor continued the dialogue as follows:

> Teacher: Can we do it with four rolls? Shall we try?
> Carlos: Four will work.
> Letícia: No, it won't work.
> Teacher: Why won't it work, Letícia?
> Letícia: We need more.
> Bianca: Five will work.
> Teacher: What about six rolls?
> Carolina: Yes.
> Teacher: Can we build a tree with any number of rolls?
> Letícia: No, because they will get smaller, and we won't have enough to make the top of the tree.
> (Dialogue between teacher and students, 2023).

Through modeling activities, students could “express their thoughts, raise and test hypotheses, and investigate concepts” (Tortola, 2016, p. 58). Testing the hypotheses involved observing and experimenting with the diagrams (rolls) to reflect on the quantities needed to build the tree and how to create and organize the rolls to compose the structure (Figure 5a). This is evident in the student's gesture when she realized a roll was missing from the top of the tree (Figure 5b).

(a) (b)

Figure 5.

Letícia conducting an experiment and pointing out the lack of a roller at the top of the tree (Authors, 2023)

The diagram constructed by manipulating toilet paper rolls enabled effective relationship building and allowed Letícia to recognize the need to increase the quantity because there would not be enough at the top of the tree. Thus, manipulating the rolls allowed “students to visualize, test, and examine their existing incorrect mathematics” (Yoon & Miskell, 2016, p. 90). While exploring the number of rolls needed to build the tree, students used speech, gestures, and diagram to report their estimates. They also discussed related content, such as comparison, greater than, and less than.

After manipulating the rolls to determine how many were needed to build the tree, the students were asked to draw on a sheet of paper how they built the tree. At this point, the students produced written records highlighting different quantities (Figure 6). Their written records in the form of schemes showing how they assembled the tree became semiotic signs that generated new diagrams. These drawings indicated the number of rolls in each row, representing the situation under study and allowing for further experimentation with the diagram.

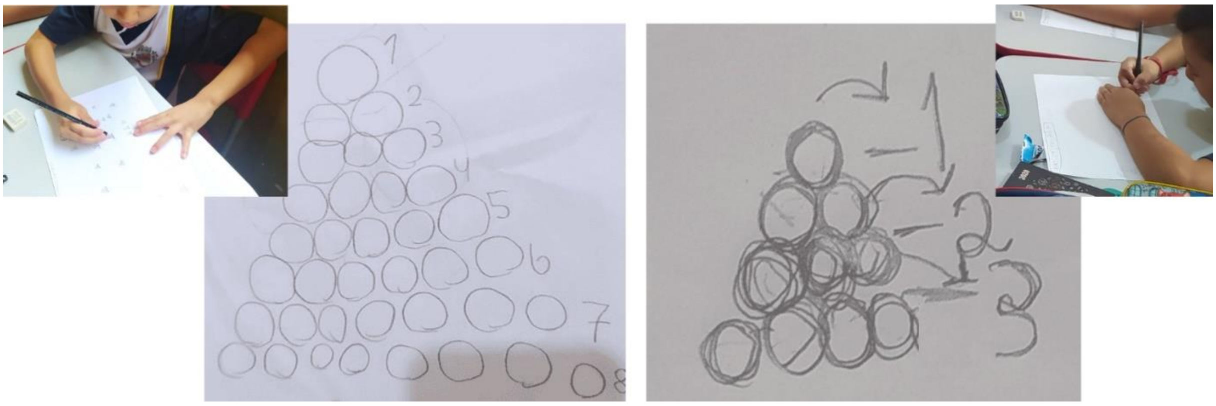
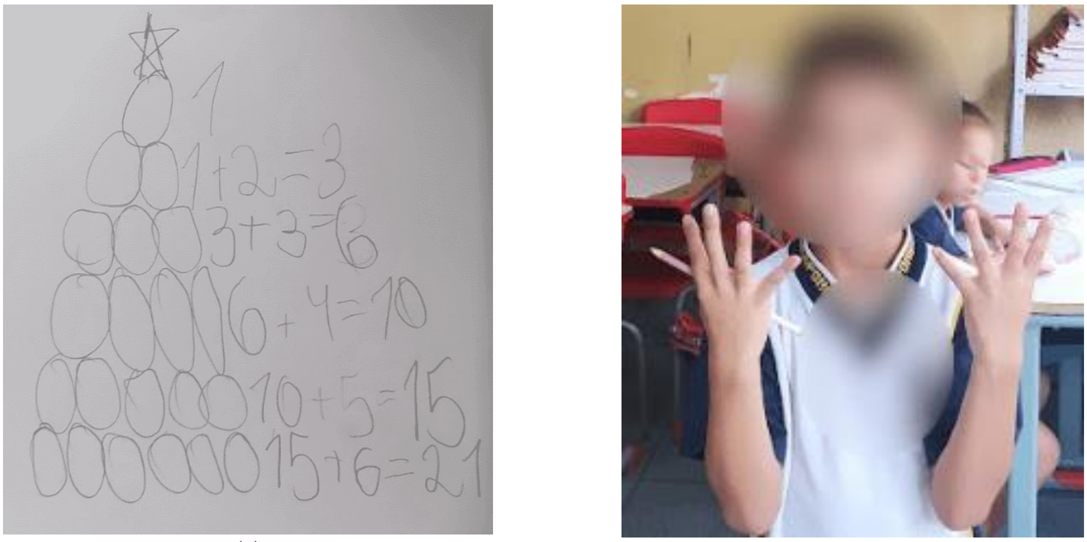


Figure 6.

Record of the tree with different quantities of rolls (Authors, 2023)

The students' schemes took the form of diagrams because "they represented an idea and stimulated meaning in an interpreter's mind" (Ribeiro, 2021, p. 263). By depicting the roles as circles, the records, to a certain extent, illustrated "the relationships between the parts of their object using analogous relationships and their parts" (Santaella, 2012, p. 101) about the shape of the Christmas tree. The mathematical structure revealed in the students' diagrams represented a model derived from a “mathematization process involving experimentation on a prototype” (Carreira & Baioa, 2018, p. 204).

To reveal the students’ knowledge and build new knowledge, the professor continued the activity by asking how many rolls they used to build the tree and having them record the answer on the sheet. At that moment, different semiotic resources emerged to represent the students’ answers: gestures, speech, and written records. Figure 7 shows the written record and gestures used by student Bruno to determine the number of rolls needed for the tree.



(a) (b)

Figure 7.

Written record of students and gestures (Authors, 2023)

When observing Bruno's record (Figure 7a), which showed a recurrence in adding rolls to obtain the sum, according to the quantity used in the base, the professor felt the need to ask all students some questions, as transcribed in the dialogue below:

> Teacher: With two rows, how many rolls do we use?
> Renata: Three.
> Teacher: If there are three rows, how many rolls will we use?
> Renata: Six.
> Teacher: If you use four rolls in the fourth row, how many rolls will there be in total?
> Bruno: Ten.
> Teacher: Why ten, Bruno?
> Bruno: Because, teacher, six plus four is ten.
> Teacher: And if I have five rolls in the fifth row? [Student thinks for a moment and answers].
> Bruno: Fifteen.
> Teacher: And if I have a sixth row?
> Bruno: Twenty-one, which is fifteen plus six.
> Teacher: And if I have a seventh row?
> Marcos: Twenty-seven.
> Bruno: Twenty-eight, because it's seven more, twenty-one plus seven.
> Teacher: How did you know?
> Carolina: Just add them up.
> (Dialogue between teacher and students, 2023).

Bruno usually performed the calculations mentally to arrive at the result. At times, he resorted to using his fingers to count (Figure 7b). In a mathematical context, raising fingers to count refers to quantity; these gestures corresponded to a semiotic resource that indicates quantity. The student's expression of the mathematical approach to the phenomenon under study

— the total number of rolls according to the base — represented the specificities of a mathematical model at this level of schooling (Burak, 2004; Tortola & Almeida, 2018). At this stage, mathematical symbolism was still developing.

After experimenting with the rolls to build the Christmas tree, the next step was for each student to make a prototype tree to take home to highlight the mathematical knowledge that emerged during the activity. Considering the need for each student to have a tree, one of the students — Bruno — suggested, as transcribed in the discussion below:

> Bruno: Professor, today there are 18 students. If we make a Christmas tree for all the students, we'll make this one here with ten rolls.
> Teacher: And how many rows does it have?
> Ricardo: One, two, three, four [counting on his fingers].
> Teacher: If it has four rows, how many rolls do we need?
> Marcos: Ten.
> Teacher: To make one for every student in the class today, how many rolls will we need?
> Bruno: We'll need one hundred and eighty.
> Teacher: How did you figure that out?
> Bruno: Uh, counting by tens… ten, twenty, thirty... [...]
> Teacher: Our tree will have a trunk with two whole rolls. To make them for all the students, we have 24 students in the class. How many rolls will we need?
> Bruno: Forty-eight.
> Teacher: Why?
> Bruno: Because twenty-four plus twenty-four equals forty-eight. Count twenty-one, then four: the first is twenty plus another twenty, which equals forty, then count four, which is four plus four equals eight, which equals forty-eight.
> (Dialogue between teacher and students).

Bruno considered the number of rolls (ten rolls) per student in his search for the total number of rolls needed for all students to build their trees. He calculated this using the multiplicative principle, in which he used the sum of equal parts.

Bruno added tens to tens and units to units to report the sum of the rolls at the base for all students. He used speech, defined by Nöth and Santaella (2017) as "verbal communication that manifests itself through hearing," to explain his method of resolution and establish a new diagram to answer the professor's question. According to Hoffmann (2005), the relationships in the diagram were supported by conventions and exhibited consistent, accurate representations.

To build a small tree that could be taken home, Bruno suggested that each tree have a total of ten toilet paper rolls, i.e., four rolls at the base. Thus, each student received ten rolls to build the tree and two rolls to build the trunk so that it would be stable (Figure 8). For the number of students present when the prototypes were made (18.10 + 18.2), 216 rolls were used to make Christmas tree prototypes. This left 124 rolls to be made available to absent students. Students also received paintbrushes and two colors of gouache paint (green for the leaves and brown for the trunk) as well as various shapes of EVA rubber to decorate their trees.

Figure 8.

Group of students building their Christmas trees (Authors, 2023)

The production of prototypes during the development of a modeling activity "encouraged practical work, cooperative learning, discussion, research, questioning, conjecture, justification, report preparation, and problem-solving" (Baioa & Carreira, 2019, p. 11).

Throughout the activity, different types of semiotic resources were evident, including technology, manipulatives (rollers), speech, gestures, and written records that represented signs for producing various diagrams. The activity also enabled students to visualize the mathematical approaches involved in developing the activity, which contributed to their learning.

Table 1 presents the semiotic resources, diagrams, and knowledge revealed during the activity.

Table 1.

Summary of the Christmas tree activity analysis (Authors, 2023)

Semiotic feature	Diagram	Revealed knowledge
Notebook	Image of the Christmas tree	Association of the Christmas tree structure for its construction
Toilet paper rolls	Queuing scheme	The size and shape of the tree depend on the number of rolls used at the base.
Gestures	Rolling pin alignment	The minimum number of rolls needed to build the tree should be three, and to build a tree with a base of four, 4+3+2+1=10 rolls should be used.
Written record	Drawing	The possibility of constructing the tree with different numbers of rolls, performing the addition operation, and obtaining the sum of the rolls.
Talk	Use of the multiplicative principle	The structure of the multiplication process

In this activity, we emphasized the use of various semiotic resources to produce diagrams. This was evident when students formed rows with the rolls to create the shape of the tree, representing the diagram of the tree. Each student's written record of the tree they built also generated a new diagram: the drawing. This allowed students to experiment with the new diagram.

Figure 9 shows the adapted mathematical modeling cycle (Stilman et al., 2007) for the Christmas tree construction activity. The thick arrows containing numbers 1 to 7 indicate transitions between steps A to G. The resources and diagrams used in the activity are indicated in both the transitions and steps.

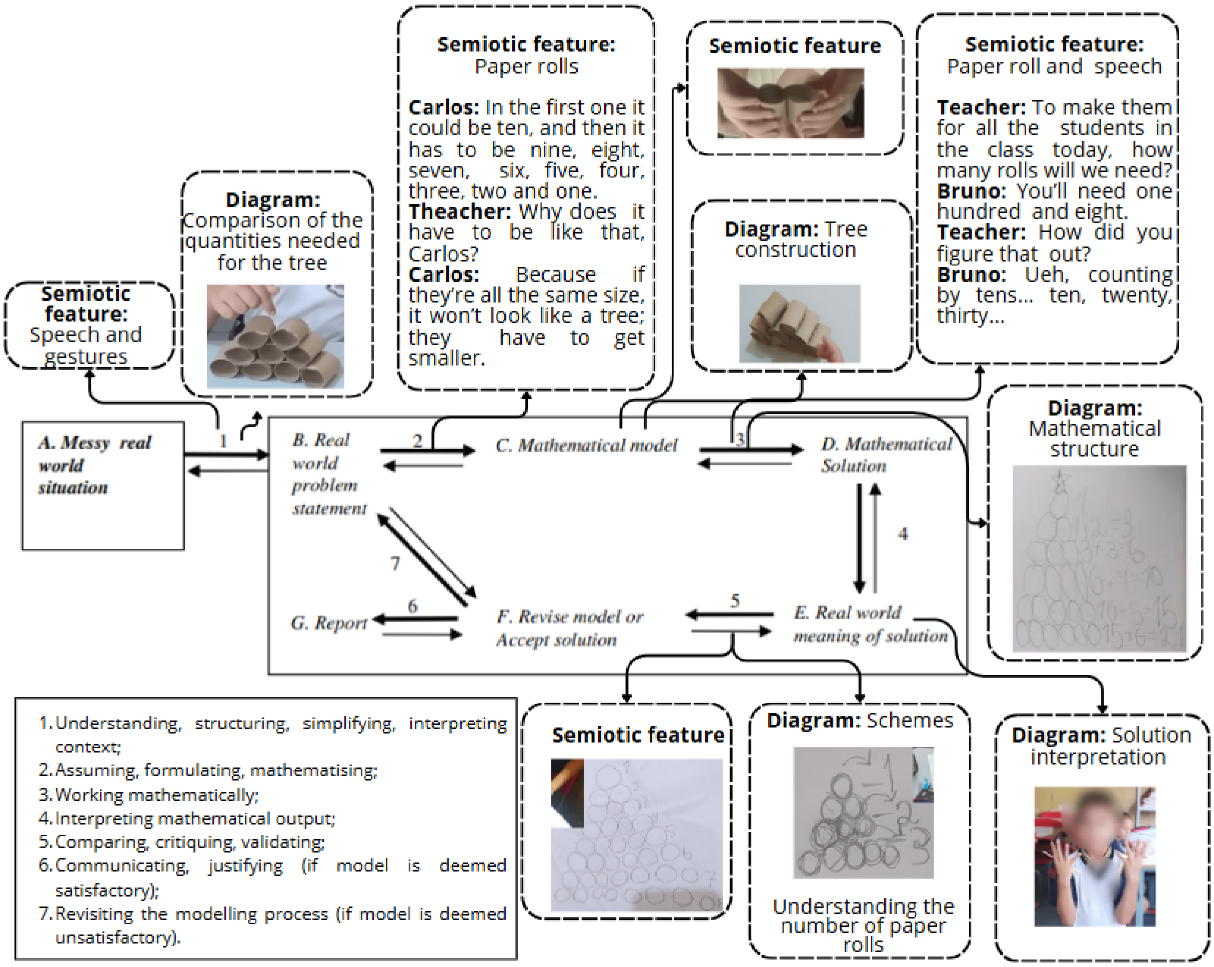


Figure 9.

Modeling cycle for the Christmas tree activity (Authors, 2023)

In step 2, students used paper rolls as a semiotic resource to solve the problem mathematically, as seen in the dialogue excerpt between student Carlos and the professor. Next, as highlighted in step C, a student constructed a mathematical model by manipulating the rolls (a semiotic resource) to generate a diagram of the Christmas tree's construction.

In step 3, the students used speech and paper rolls to work mathematically. One student performed the calculation in his head to determine the number of rolls needed for the entire class. The resulting diagram was the mathematical structure of the tree, organized using a diagram on a sheet of paper. In phase E, the diagram for interpreting the results was produced, and the semiotic resource of gestures was used when the student explained how he performed the calculation. In step 5, the comparison and validation of the model took place. The semiotic resource of drawing generated a diagram of the roll scheme to understand the number of rolls needed.

The Christmas Tree activity development cycle was designed to identify the activity's stages, highlight the semiotic resources and diagrams that emerged, and better visualize the stages of mathematical modeling.

Final considerations

Considering that mathematical modeling is a pedagogical alternative that can be implemented in the early years, addressing real-life situations through diverse themes that contribute to student learning, we focused on analyzing the knowledge revealed by first-grade elementary school students in the use of semiotic resources to produce diagrams in an activity involving the construction of a Christmas tree. To this end, we relied on the theoretical assumptions of Peircean semiotics.

Based on the presentation of a Christmas tree built with toilet paper rolls suggested by a website, which was configured as a sign, the students developed a mathematical modeling activity with the aim of building the tree to analyze the number of rolls that would be adequate for each student to build their prototype. What emerged was that, based on a situation presented through an image, a mathematical approach—the number of rolls—was promoted so that diagrams could be constructed using semiotic resources chosen by the students to solve the problem, thereby demonstrating their knowledge. In other words, the photo of the tree built with toilet paper rolls became “a specific question to be answered mathematically” (Stillman, 2015, p. 47), which was the professor’s intention in implementing the activity in the classroom. And with that, there was “a commitment to the development of mathematical literacy” (Brazil, 2018, p. 266).

During the activity, it was possible to observe the use of various semiotic resources, such as the use of a notebook to access the image of the Christmas tree; toilet paper rolls, gestures, written records, and speech, which enabled the production of diagrams and highlighted the students' mathematical knowledge. The notebook, as a semiotic resource, enabled students to associate the structure of the tree with the image (diagram) presented, revealing its possibility of construction. The toilet paper roll, as a semiotic resource, allowed students to create a queuing scheme, consisting of a diagram for the construction of the tree. This action allowed students to use “their representations and, in this sense, (re)construct diverse knowledge” (Veronez & Santos, 2023, p. 172), enabling them to identify that both the size and shape of the tree depended on the number of rolls used at the base, revealing mathematical knowledge related to number ordering.

During the activity, various semiotic resources were used, such as a notebook to access an image of a Christmas tree, toilet paper rolls, gestures, written records, and speech. These resources enabled the production of diagrams and highlighted the students' mathematical knowledge.

The notebook enabled students to associate the tree's structure with the presented diagram, revealing its construction possibilities. Toilet paper rolls allowed students to create a queuing scheme consisting of a diagram for constructing the tree. This allowed students to use “their representations and, in this sense, (re)construct diverse knowledge” (Veronez & Santos, 2023, p. 172). They identified that the size and shape of the tree depended on the number of rolls used at the base, revealing knowledge related to number ordering.

In 2023, first-year elementary school students at a municipal school in Paraná were presented with six modeling activities. The sixth activity was the construction of a Christmas tree, which we understand to be a progression of solid mathematical skills based on connection with students' real-life contexts (Alsina et al., 2021). During the activity, knowledge of mathematical content from the first-grade curriculum was revealed naturally. These contents included the concepts of greater and lesser, ascending and descending order, quantity and addition, and multiplication. This contributed to students' knowledge construction.

The configuration of modeling activities for early years, in which "mathematics is connected to the real world, with modeling problems being considered motivating and promising for better learning of mathematics" (Aroeira et al., 2024, p. 161), overcomes challenges related to students' difficulties in the classroom. However, we acknowledge that an individual analysis of the knowledge mobilized was not conducted, which limits our analysis due to the collaborative nature of these activities. An interview with each student could provide a means to carry out this analysis and open up possibilities for future research.

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