EMP Educação Matemática Pesquisa
ISSN 1983-3156
Qualis A1
http://dx.doi.org/10.23925/1983-3156.2026.v28.e72245

# Topological games in science education: interfaces between epistemology, playfulness, and critical formation in the context of contemporary education

*Juegos topológicos en la enseñanza de las ciencias: interfaces entre epistemología, ludicidad y formación crítica en el contexto de la educación contemporánea*

*Jeux topologiques dans l'enseignement des sciences : interfaces entre épistémologie, ludicité et formation critique dans le contexte de l'éducation contemporaine*

*Jogos topológicos no ensino de ciências: interfaces entre epistemologia, ludicidade e formação crítica no contexto da educação contemporânea*

Antonio Luiz de Almeida[1]
Universidade do Estado da Bahia
Doutor em Física
https://orcid.org/0000-0003-4010-4552

Beatriz Oliveira de Almeida Lima [2]
Universidade Federal da Bahia
Doutora em Ensino de Ciências
https://orcid.org/0000-0001-5539-8368

## Abstract

In this theoretical work, we discuss the integration of topological games into Science and Mathematics education as a pedagogical, epistemological, and interdisciplinary strategy. Topology, as a branch of Mathematics that studies properties that remain invariant under continuous deformations, allows for the expansion of pedagogical practices that stimulate spatial visualization, abstract reasoning, and critical thinking. Through simple objects such as Möbius bands, knots, and toroidal surfaces, students are challenged to revisit their geometric conceptions. In this context, topological games foster connections between different areas of knowledge — from Physics to Art — and encourage inclusive, playful, and collaborative teaching practices. In addition to promoting student agency, teamwork, and philosophical reflection on scientific knowledge, these games serve as starting points for discussions on the limits of

[1] Contato: plataformafisica@gmail.com
[2] Contato: biariobahia@gmail.com



Submetido em:30/06/2025 Aceite em: 10/07/2025 Publicado em: 23/03/2026

visualization, the nature of space, and mathematical representation. Based on these theoretical reflections, we understand that their implementation in the classroom requires continuous teacher training, accessible materials, and a learning environment that values experimentation and dialogue in favor of a critical, meaningful education aligned with the challenges of contemporary education.

***Keywords:*** Topological games, Interdisciplinarity, Spatial visualization, Critical thinking, Science education.

## Resumen

En este trabajo teórico, discutimos la integración de los juegos topológicos en la enseñanza de las Ciencias y la Matemática como una estrategia pedagógica, epistemológica e interdisciplinaria. La Topología, como rama de la Matemática que estudia las propiedades que permanecen invariantes ante deformaciones continuas, permite ampliar las prácticas pedagógicas que estimulan la visualización espacial, el razonamiento abstracto y el pensamiento crítico. A través de objetos simples, como bandas de Möbius, nudos y superficies toroides, se desafía a los estudiantes a revisar sus concepciones geométricas. En este contexto, los juegos topológicos favorecen la articulación entre distintas áreas del conocimiento —desde la Física hasta el Arte— y promueven prácticas pedagógicas inclusivas, lúdicas y colaborativas. Además de fomentar el protagonismo estudiantil, el trabajo en equipo y la reflexión filosófica sobre el conocimiento científico, estos juegos se constituyen como puntos de partida para discusiones sobre los límites de la visualización, la naturaleza del espacio y la representación matemática. A partir de estas reflexiones teóricas, comprendemos que su implementación en el aula requiere la formación continua del profesorado, materiales accesibles y un entorno de enseñanza que valore la experimentación y el diálogo, en favor de una formación crítica, significativa y alineada con los desafíos de la educación contemporánea.

***Palabras clave***: Juegos topológicos, Interdisciplinariedad, Visualización espacial, Pensamiento crítico, Educación científica.

## Résumé

Dans ce travail théorique, nous discutons de l’intégration des jeux topologiques dans l’enseignement des Sciences et des Mathématiques en tant que stratégie pédagogique, épistémologique et interdisciplinaire. La topologie, en tant que branche des

Mathématiques qui étudie les propriétés invariantes sous des déformations continues, permet d'élargir les pratiques pédagogiques en stimulant la visualisation spatiale, le raisonnement abstrait et la pensée critique. À travers des objets simples, tels que les bandes de Möbius, les nœuds et les surfaces toroïdales, les élèves sont amenés à revisiter leurs conceptions géométriques. Dans ce contexte, les jeux topologiques favorisent l'articulation entre différentes disciplines — de la Physique à l'Art — et encouragent des pratiques pédagogiques inclusives, ludiques et collaboratives. En plus de promouvoir l'autonomie des élèves, le travail en équipe et la réflexion philosophique sur la connaissance scientifique, ces jeux constituent des points de départ pour aborder les limites de la visualisation, la nature de l'espace et la représentation mathématique. À partir de ces réflexions théoriques, nous comprenons que leur mise en œuvre en classe nécessite une formation continue des enseignants, des matériaux accessibles et un environnement éducatif valorisant l'expérimentation et le dialogue, en faveur d'une formation critique, significative et adaptée aux défis de l'éducation contemporaine.

***Mots-clés*** : Jeux topologiques, Interdisciplinarité, Visualisation spatiale, Pensée critique, Education scientifique.

## Resumo

Neste trabalho teórico, discutimos a integração dos jogos topológicos ao ensino de Ciências e Matemática como uma estratégia pedagógica, epistemológica e interdisciplinar. A Topologia, enquanto ramo da Matemática que estuda propriedades que permanecem invariantes sob deformações contínuas, permite ampliar práticas pedagógicas que estimulam a visualização espacial, o raciocínio abstrato e o pensamento crítico. Por meio de objetos simples, como bandas de Möbius, nós e superfícies toroidais, os estudantes são provocados a revisar suas concepções geométricas. Nesse contexto, os jogos topológicos promovem a articulação entre diferentes áreas do conhecimento — da Física à Arte — e incentivam práticas pedagógicas inclusivas, lúdicas e colaborativas. Além de estimularem o protagonismo estudantil, o trabalho em equipe e a reflexão filosófica sobre o conhecimento científico, esses jogos constituem-se como pontos de partida para discussões sobre os limites da visualização, a natureza do espaço e a representação matemática. Com base nessas reflexões teóricas, compreendemos que sua implementação em sala de aula exige formação continuada de professores, materiais acessíveis e um ambiente de ensino que valorize a experimentação e o diálogo, em favor de uma formação crítica, significativa e alinhada aos desafios da educação contemporânea.

***Palavras-chave:*** Jogos topológicos, Interdisciplinaridade, Visualização espacial, Pensamento crítico, Educação científica.

**Topological Games in Science Education: Interfaces between Epistemology, Playfulness, and Critical Formation in the Context of Contemporary Education**

## Introduction

We live in an era marked by complexity (Morin, 2007), the accelerating pace of technological transformations (Castells, 2010), and the emergence of global challenges (Sachs, 2015) that demand not only technical knowledge (Shulman, 1986) but also critical thinking (Facione, 2015), creativity, and the capacity to act ethically in uncertain contexts (Sternberg, 2006; Noddings, 2003). In this scenario, the teaching of Science (Gleiser, 2013), particularly at the secondary level (Brazil, MEC, 2018), must be reconfigured in its practices, foundations, and purposes. It is necessary to move beyond the mere transmission of ready-made content and the reproduction of fixed truths. Contemporary science education must commit itself to the formation of autonomous (Freire, 1996), reflective (Vygotsky, 1998), and socially engaged individuals (Freire, 1996), capable of understanding and responsibly intervening in the world.

Within this horizon lies the proposal to integrate *Topological Games* (Munkres, 2000) as pedagogical (Bruner, 1961), epistemological (Lakatos, 1976), and philosophical (Piaget, 1977; Poincaré, 1905) tools for the teaching of Science and Mathematics. Topology (Munkres, 2000) — a branch of mathematics concerned with the properties of spaces that remain unchanged under continuous deformations—offers fertile ground for the exploration of sophisticated concepts in a playful and accessible manner.

By engaging with ideas such as continuity (Lee, 2011), topological equivalence (Steenrod, 1999), connectivity (Stillwell, 2010), and invariance (Lee, 2011), topological games challenge conventional geometric intuition, promoting a rupture with the mechanistic and rigid conception of space. They allow students to experience mathematics as a language of transformation (Stillwell, 2010), rather than a static description. Möbius bands (Kauffman, 2001), knots, toroidal surfaces, and other unconventional constructions become not only objects of study but also instruments to provoke abstract thinking, expand cognitive repertoires, and foster investigative curiosity.

However, the use of topological games transcends the mathematical domain. It enables the articulation of diverse forms of knowledge within a truly interdisciplinary approach. The emergent properties of deformable systems (Morowitz, 2002), the unpredictability of certain topological behaviors, and the multiplicity of possible representations of geometric objects create bridges to contemporary physics—such as quantum mechanics (Penrose, 2005) and dynamical systems (Strogatz, 2015)—as well as to biology (in topics such as genetics, morphogenesis, and molecular structures (Gilbert, 2010)), to the philosophy of science (particularly in discussions on models, representation, and the limits of knowledge (Lakatos, 1976; Piaget, 1977)), and to art (through the visual and aesthetic exploration of unconventional surfaces and forms). In this sense, topological games open possibilities for a mode of teaching that integrates science, aesthetics, and ethics, acknowledging both the complexity of reality and the multiplicity of ways of knowing it.

From an epistemological perspective (Lakatos, 1976), engagement with topological games contributes to the deconstruction of simplifying conceptions of scientific knowledge. When students experience situations in which the outcome of an action contradicts their intuitive expectations—such as cutting a Möbius band in half or attempting to untie a knot that tightens the more it is pulled—they are prompted to question the supposed linearity and stability of knowledge. Science thus reveals itself in its processual, provisional, and contextual nature. The games, as simplified and experimental models, allow students to understand that scientific knowledge is historically constructed, subject to revision, dependent on methodological choices, and imbued with ethical implications. Therefore, while the games serve as support for content learning, they also foster critical reflection on the very foundations of science.

From a pedagogical perspective (Bruner, 1961), topological games foster active, experiential, and collaborative learning. They engage the body, perception, dialogue, and error as constitutive elements of the learning process. In contrast to transmissive (Freire, 1970) and fragmented pedagogies (Moran, 2010) that still prevail in many schools, these games propose an environment of experimentation and discovery in which students become protagonists of their own learning. Within this context, the teacher’s role acquires new contours: it is not about providing answers, but rather about provoking questions, guiding reasoning, and connecting concrete experiences with conceptual abstractions. Through playing, manipulating, observing, and discussing, students develop cognitive skills (such as logical reasoning (Ausubel, 1968), spatial visualization (Stillwell, 2010), and modeling (Strogatz, 2015)); metacognitive skills (Bruner, 1961), such as the ability to evaluate strategies and recognize limitations; and socioemotional skills (Freire, 1996), including cooperation, attentive listening, and respect for diverse perspectives.

Moreover, topological games align with the principles of inclusion (UNESCO, 2017) and educational equity (Darling-Hammond, 2017). As multimodal activities that combine verbal, visual, and tactile languages, they accommodate diverse learning styles and promote the participation of students with varied abilities. The simplicity of the required materials and the possibility of crafting the objects by hand (such as paper bands, strings, or rings) also make these games feasible in contexts with limited technological resources. Thus, while engaging with sophisticated content, topological games prove to be accessible, democratic, and motivating, contributing to a more just and meaningful education.

Nonetheless, it is crucial not to overlook the need for consistent teacher training to ensure the effective implementation of these games in the classroom. Teachers must understand the fundamentals of topology, master the dynamics of playful learning, and be prepared to guide epistemological and interdisciplinary reflections with their students. To this end, continuous professional development programs, the production of contextualized teaching materials, and networks of collaboration among educators and researchers are indispensable. Building a pedagogical culture that values experimentation, playfulness, and critical thinking is a collective process that requires institutional support, public investment, and an ethical commitment to educational transformation.

Finally, it is essential to understand that topological games are not an end in themselves, but rather a powerful strategy within a broader proposal for curricular (Moran, 2010) and didactic renewal (Tobin, 2006). Their integration into science education represents an opportunity to bring scientific knowledge closer to students' lived experiences, to value the complexity of thought, and to cultivate the joy of learning. In a world marked by uncertainty, interdependence, and the urgent need for sustainable solutions, it is increasingly necessary to educate individuals capable of thinking in flexible, systemic, and creative ways. By stimulating these competencies in an engaging and reflective manner, topological games offer a meaningful contribution to one of the great challenges of our time.

## Fundamental Mathematical Concepts for Understanding Topological Games

For topological games to be effectively employed as pedagogical tools in the teaching of Mathematics and Science, it is essential that students grasp certain fundamental mathematical concepts that underpin the logic of these games and their relations to topology and geometry. These concepts not only provide the foundation for the manipulations and transformations involved in the games but also open pathways toward an interdisciplinary understanding of spatial properties and abstract mathematical structures. Among the central concepts are continuity, deformation, topological equivalence, invariance, and the counterintuitive properties of surfaces (Munkres, 2000; Hatcher, 2002).

The concept of **continuity** is fundamental to topology, as it enables the understanding of deformations without ruptures or cuts, characterizing transformations that preserve the intrinsic properties of objects. In topological games, the idea of continuous deformation helps students grasp that figures can change shape without losing their essential characteristics (Lee, 2010). For instance, when manipulating a paper Möbius band, students can twist and join its ends to create a surface with only one side and one boundary. This activity allows them to perceive that, despite the twist, the surface remains continuous and unified, challenging classical geometric intuitions about "sides" and "edges" (Stillwell, 2010).

**Topological equivalence** refers to the idea that two objects can be considered "the same" from a topological perspective if one can be continuously deformed into the other. This introduces a new way of thinking about shape and structure, distinct from traditional Euclidean geometry (Thurston, 1997). In this sense, when comparing objects such as a circle and an ellipse made of flexible wire, students explore how these objects are topologically equivalent, as one can be deformed into the other without cutting. In contrast, a sphere and a torus (doughnut shape) are not equivalent, since the torus possesses a "hole" that the sphere does not—a topologically invariant characteristic (Ghrist, 2014).

The concept of **invariance** highlights properties that remain constant under continuous deformations, such as the number of connections, holes, or knots, which are central to topological study. These invariants are powerful tools for classifying and understanding different surfaces and spaces (Crowell & Fox, 1999).

For example, games that involve creating and manipulating knots with ropes allow students to experience how certain characteristics of knots remain unchanged even after manipulations such as stretching or pulling. This concrete experience helps internalize the concept of topological invariance, illustrating that some properties are preserved regardless of the transformations applied (Crowell & Fox, 1999).

Moreover, topology challenges classical spatial intuition (Bell, 1937), promoting abstract thinking (Papert, 1980) and epistemological reflection (Piaget, 1977) on the nature of space, form, and mathematical representation. Topological games, therefore, become tools for discussing the limits of visual knowledge (Nelsen, 2006) and the complexity of mathematical concepts, encouraging a critical and investigative stance (Artigue, 2002).

From a pedagogical perspective, the appropriation of these concepts through topological games fosters the development of skills such as spatial reasoning (Lee, 2010), abstraction (Papert, 1980), logical argumentation (Artigue, 2002), and collaborative work (Vygotsky, 1998). It also cultivates an epistemological attitude that values experimentation, dialogue, and the collective construction of knowledge—essential elements for a richer and more meaningful mathematics education (Artigue, 2002; Vygotsky, 1998).

## Theoretical Foundation

Topology, as a fundamental branch of mathematics, provides the essential conceptual foundation for understanding topological games and their pedagogical applications. This field investigates properties of objects that remain invariant under continuous deformations, breaking away from the rigidity of traditional geometry and opening space for a more flexible and abstract perspective on space and form (Munkres, 2000; Hatcher, 2002). Such an approach stimulates non-intuitive spatial thinking and promotes an epistemological reconfiguration of what it means to understand "space" and "form" in science (Piaget, 1977; Lakatos, 1976).

Topological equivalence between objects, such as transforming a sphere into a torus, exemplifies how structural and invariant properties—like the number of "holes"—underpin the classification and differentiation of spaces (Ghrist, 2014). This concept proves essential for developing abstract reasoning in games, where students manipulate concrete models to internalize complex mathematical notions such as continuity, boundaries, connectedness, and invariance (Lee, 2010; Stillwell, 2010).

Topology also intersects with other fields of knowledge, including theoretical physics, where topological spaces model phenomena such as quantum entanglement (Nielsen & Chuang, 2010) and properties of condensed matter (Hasan & Kane, 2010), and the visual arts (Crowe, 1994), which explore the deformation and transformation of forms. This interdisciplinarity reinforces the significance of topological games as didactic tools that transcend formal mathematics, broadening students' epistemological repertoire.

Moreover, topology challenges common sense and geometric intuition, fostering a critical epistemological stance that recognizes the complexity and relativity of spatial representation (Bell, 1937). Through hands-on experimentation, topological games promote the development of critical thinking, creativity, and argumentation—fundamental aspects of contemporary scientific education (Artigue, 2002; Vygotsky, 1998).

Regarding the use of topological games in education, it is essential to establish a philosophical-pedagogical foundation that values abstraction as a tool for knowledge and encourages active student engagement in the learning process. Drawing on the epistemology of abstraction, this approach acknowledges that scientific knowledge is constructed through the articulation between the concrete and the abstract, enabling students to navigate between physically manipulated objects and symbolic or conceptual thinking.

From a pedagogical perspective, constructivism emphasizes learning as an active and meaningful process, in which students build understanding through interaction with objects, peers, and teachers. Topological games, being manipulative and collaborative, foster effective participation, dialogue, and problem-solving—central elements for cognitive and social development.

In parallel, contemporary educational theories—such as active learning and critical pedagogy—emphasize the importance of student agency and epistemological reflection. Topological games, by challenging traditional intuitions about space and form, stimulate the questioning of certainties and foster dialectical thinking, enabling students to recognize the limits of knowledge and the historicity of scientific constructions.

The symbolic and social mediation of learning, as proposed by historical-cultural psychology, is enhanced through the use of games in collaborative contexts that promote the exchange of ideas, the negotiation of meanings, and the collective construction of knowledge. This dynamic strengthens both socio-emotional and cognitive skills, preparing students to operate effectively in diverse and complex contexts.

Thus, the theoretical and pedagogical foundation of topological games integrates the construction of abstract mathematical knowledge with active and reflective methodologies, constituting an innovative educational approach aligned with the contemporary challenges of scientific education.

## Topological games and their epistemological conceptions

Topological games in the classroom extend far beyond mere recreational activities or auxiliary didactic resources. By engaging students directly with counterintuitive phenomena that challenge ordinary spatial perception, these games provide a powerful epistemological field for investigating the nature of mathematics, the construction of knowledge, and the limits of sensory experience. Topology, by addressing properties that remain invariant under continuous deformations, raises a fundamental question: What is essential in a mathematical object? What distinguishes appearance from structure?

By exploring concepts such as the non-orientability of the Möbius strip, the continuous structure of the torus, or the equivalence among different knots, students are encouraged to move beyond a Euclidean and static conception of space, adopting instead a more fluid, relational, and abstract perspective. In this context, mathematics ceases to appear as a closed system of formulas and algorithms and emerges as an investigative practice that demands intuition, experimentation, argumentation, and the revision of hypotheses.

Through these artifacts, it becomes possible to develop an understanding of science that breaks with the notion of absolute certainty, emphasizing instead a progressive construction of conceptual structures validated by internal coherence and consistency with experience. In dialogue with Henri Poincaré—one of the founders of modern topology—mathematics can be seen as a creation of the human mind, guided by intuition yet validated through logic. For Poincaré, topological concepts are "pure forms" that reveal the deep structure of space, independent of its physical manifestations. Within the games, this idea becomes evident: the essential properties are not measurements, but relationships—how many edges, how many holes, how many twists.

Imre Lakatos conceived mathematics as an evolving science, grounded in conjectures, refutations, and reformulations. Topological games offer fertile ground for this perspective: students test hypotheses ("what happens if I cut here?"), confront counterexamples, and revise their ideas. Through play, they experience a process akin to what Lakatos termed *proofs and refutations*, realizing that mathematics is not a static body of established truths, but a dynamic field of ongoing inquiry.

For Jean Piaget, knowledge is constructed through stages of cognitive development in which the child progressively forms more complex mental structures. Topology emerges in his studies as one of the earliest forms of spatial organization in childhood—prior even to the understanding of measures and angles—when the child perceives continuity, interior and exterior relations, separations, and connections. Thus, topological games resonate with deep cognitive structures, supporting the transition toward more abstract forms of reasoning.

From an epistemological standpoint, the analysis of topological games reveals that, far from being merely playful or motivational tools, they can function as powerful catalysts for philosophical and metacognitive reflection. They enable students not only to learn mathematics but also to reflect upon the very act of learning mathematics—on modes of reasoning, the limits of perception, and the criteria for the validation of knowledge. In this sense, such games contribute to the cultivation of a more critical, reflective, and autonomous mindset, aligning with the central goals of a scientific and humanistic education.

## Reflecting on inclusion, playfulness, and interdisciplinarity

By combining spatial reasoning, concrete manipulation, and conceptual exploration, topological games present significant inclusive and formative potential within contemporary educational contexts. By breaking with the rigidity of purely expository and content-centered methods, they create a learning environment that is more responsive to individual differences, more open to experimentation, and more conducive to student agency. Moreover, they enable mathematical content to be accessed through multiple modalities—tactile, visual, collaborative, and creative—thereby reinforcing education as a universal right.

The use of topological games fosters the active participation of all students, regardless of their prior performance in mathematics. The emphasis on exploration, on error as an integral part of the process, and on collaboration helps to lower emotional barriers such as fear of failure, which often contribute to the subjective exclusion of students with low academic self-esteem.

The playful and manipulative nature of these games stimulates natural curiosity and experimentation, valuing intuition and creativity without requiring complex initial formalizations. It promotes social interaction and dialogue, reinforcing peer learning. Furthermore, it allows students to become authors of hypotheses, strategies, and discoveries rather than mere recipients of pre-established procedures.

In this sense, such an inclusive environment promotes a symbolic redistribution of power within the classroom, where students traditionally labeled as "weak" in mathematics may stand out for their abilities in observation, manipulation, or reasoning. This represents a concrete means of fostering student protagonism and cognitive equity.

It is also worth noting that topological games are particularly suitable for students with specific educational needs, as they allow for accessible adaptations without compromising the conceptual richness of the activity. For example, in the case of students with visual impairments, topology—concerned with properties such as continuity, connectedness, and the number of edges or holes, rather than with color, size, or angles—can be explored through three-dimensional tactile models (bands, tori, rope knots). Blind students can identify, manipulate, and compare these structures through touch.

For students with motor impairments, the games can be adapted using ergonomic supports, larger pieces, and low-resistance materials, allowing for easier manipulation or the possibility of collaboration with peers, thereby promoting cooperation and respect for individual limitations.

Moreover, about students with cognitive disabilities, the concrete and visual dimensions of topological games provide crucial perceptual anchors for learners with attention deficits or difficulties in abstraction. Activities involving experimentation and collective discovery foster sustained focus, learning through repetition, and the internalization of logical structures without premature symbolic overload. These possibilities demonstrate that topological games are highly versatile and democratizing tools, aligned with a pedagogy centered on the learner and on diversity.

It is important to emphasize that playfulness, in this context, is not synonymous with distraction but rather functions as a critical epistemological strategy. The game, by creating a problem situation that challenges conventional logic, compels students to question their prior assumptions, observe, compare, reflect, revise their ideas, negotiate meanings with peers, and construct arguments.

In this sense, play becomes a privileged space for activating critical thinking and metacognition. The surprise experienced, for instance, when cutting a Möbius strip and obtaining a single interlinked loop, generates a cognitive rupture that opens the way for deeper questioning of what is considered "normal" or "expected" in mathematics—

an exercise in reflection that transcends content and reaches the very logic of knowledge itself.

Thus, the implementation of topological games in mathematics education not only broadens opportunities for conceptual understanding but also reconfigures the conditions of access to and participation in mathematical knowledge. By combining accessibility, playfulness, and conceptual complexity, these games constitute a powerful tool for fostering a more equitable, humanizing, and transformative mathematics education—one that aligns with the principles of inclusive education and the holistic development of learners.

From these perspectives, topological games offer a rich opportunity for articulation across different fields of knowledge, fostering interdisciplinary, meaningful, and contextually grounded learning. By stimulating spatial visualization, object manipulation, and the resolution of complex problems, these games become points of convergence between formal knowledge and everyday experience, promoting curricular integration in multiple directions.

In dialogue with the arts, the most evident connection arises between topology and geometry — particularly spatial and descriptive geometry. Concepts such as continuity, deformation, surface, and boundary become tangible through the exploration of objects like Möbius strips, tori, and knots. The distinction between metric properties (such as length and angle) and topological properties (such as the number of holes and connectedness) enables a refined discussion about what remains invariant and what transforms when an object is deformed.

Moreover, topology maintains a profound relationship with the visual arts. The works of M.C. Escher, for instance, explore topological paradoxes and spatial illusions that challenge both perception and conventional logic. Students can be invited to create graphic representations of topological surfaces, construct sculptures inspired by topological objects, and study symmetrical patterns and continuous transformations. This aesthetic approach stimulates creativity, visual perception, and abstract reasoning, while simultaneously revealing the pervasive presence of mathematics in culture and art.

In the fields of Physics, Astronomy, and the Natural Sciences, topology plays an increasingly significant role, particularly within modern physics. It is present in studies of spacetime, in the investigation of topological states of matter (materials with properties invariant under deformation), and even in cosmology, where hypotheses are proposed regarding the global shape of the universe (such as universes with toroidal topology). Games and experiments involving Möbius strips or tori can serve as conceptual models for physical phenomena, including continuous and unidirectional flows (as in electric currents or magnetic fields), non-orientable surfaces (which evoke analogies to antimatter and quantum symmetries), and the notions of curved space and connectivity in the study of the universe. These connections allow students to move fluidly between different domains of knowledge, understanding that abstract mathematical concepts can have concrete and explanatory applications in the physical world.

In the fields of Language and Philosophy, topological games provide fertile ground for reflection on the nature of knowledge, form, and representation. Discussing what is meant by terms such as "space," "boundary," or "orientation" enables the integration of mathematical activity with themes from the philosophy of language, phenomenology, and metaphysics.

Thus, working with topological games directly contributes to the development of several *competências gerais* (general competencies) outlined in the *Base Nacional Comum Curricular* (BNCC), while also articulating *competências específicas* in the field of Mathematics—particularly those related to the understanding of geometric forms and their properties (EF09MA13), the use of models and varied representations, and the cultivation of intellectual autonomy.

The integration of topological games into pedagogical practice extends beyond the teaching of mathematical concepts. It promotes epistemological and methodological exchanges that bring school knowledge closer to the complexity of the real world. This interdisciplinary and contextually grounded approach, aligned with contemporary educational challenges, reinforces the role of the school as a space for critical formation — sensitive to diversity and committed to an integrated and transformative education.

## Modes of play: Playful and Epistemological Experience in the Educational Environment

Topological Games can be developed in various formats that involve concrete manipulation, digital simulation, collaborative problem-solving, and either competitive or cooperative dynamics. Each modality promotes the development of cognitive and social skills while deepening students' understanding of topological concepts, stimulating abstract and spatial reasoning, fostering discussions on fundamental notions of topology—such as continuity, deformation, and equivalence—and integrating mathematical, scientific, and philosophical knowledge.

Thus, the implementation of topological games in educational contexts requires practical proposals that combine playfulness with conceptual rigor, while also accommodating diverse educational levels and learning styles. Among the classical examples of physical manipulation games is the Möbius strip, constructed from a strip of paper or plastic that is twisted 180° before being joined at its ends. This non-orientable surface, characterized by having only one side and one edge, invites students to explore topological phenomena through hands-on experimentation. In this activity, learners cut along the center of the Möbius strip and observe the formation of a single twisted loop, challenging their geometric intuitions. The evaluative process may include written or video records of students' observations and explanations of the topological properties they discovered.

The *torus* represents another instructive example—a continuous surface obtained by deforming a cylinder with toroidal junctions. Although difficult to model physically, it can be simulated using rubber materials, modeling clay, or 3D software. Through these tools, students can investigate how curves and closed paths behave on the surface of the torus, as well as compare their observations with topological maps in video games (such as looping worlds).

In the realm of digital games, software such as *KnotPlot*, *Geomview*, and other simulators designed for topological visualization can be employed to enable virtual manipulation, allowing students to explore continuous deformations that preserve essential properties or to identify equivalent knots. Assessment in these activities may be based on creative outputs such as video recordings or screenshots that document the manipulations performed, accompanied by conceptual explanations.

Whether through physical manipulation games or digital simulations, educators can adopt a problem-solving approach by organizing students into small groups to tackle specific challenges. The instructor may, for instance, propose tasks such as determining whether two knots are equivalent or constructing surfaces with specific properties. From there, groups engage in debate and experimentation, testing knot equivalence while recording the steps of their manipulations and the reasoning behind their conclusions. Presentations may be conducted orally, allowing students to discuss their strategies and results, thereby fostering scientific argumentation, critical thinking, and collaborative inquiry.

## Conclusion

The present study discussed the integration of topological games into the teaching of Science and Mathematics as a pedagogical, epistemological, and interdisciplinary strategy — presenting them not merely as playful tools, but as powerful epistemological, educational, and intellectually transformative instruments. Throughout the investigation, it became evident that topological games transcend the simple manipulation of objects or the solving of mathematical puzzles; rather, they invite us to reconsider the very nature of space, form, knowledge, and learning.

Topology, as a branch of mathematics concerned with properties that remain invariant under continuous deformations, offers a unique lens through which to understand the world around us. It reveals that what we perceive intuitively often does not correspond to the structural essence of phenomena. This insight alone constitutes an epistemological revolution, as it challenges immediate thought and fosters the development of abstract, critical, and reflective reasoning—competencies that are fundamental to contemporary scientific education.

By materializing such abstractions, topological games bring students closer to a complex and fascinating mathematical universe that is both challenging and accessible. Through the exploration of Möbius strips, tori, knots, and surfaces with varied topologies, students are encouraged to experiment, make mistakes, question, and—above all—to cultivate a flexible understanding of geometry and space. This process is not merely a technical learning experience, but also an aesthetic, cognitive, and ethical journey that promotes creativity, collaboration, and student agency.

From an epistemological standpoint, topological games foster a rich dialogue with the ideas of major thinkers such as Poincaré, Lakatos, and Piaget, who remind us of the boundaries of mathematical knowledge and the importance of the social and historical construction of understanding. Reflecting on the limits of visualization and empirical knowledge reinforces the notion that mathematics is not a fixed set of absolute truths, but rather a dynamic and evolving field, constantly expanding through models, experimentation, and critical debate.

Moreover, the use of topological games is situated within a deeply inclusive and humanizing educational perspective. These games promote accessibility and participation among students with diverse abilities and needs, broadening horizons and challenging exclusion. In this context, playfulness emerges not as mere entertainment, but as a critical strategy — a privileged means for the holistic development of the learner and for the advancement of a democratic and plural scientific culture.

Interdisciplinarity, approached in this work as a structuring principle, demonstrates the remarkable capacity of topological games to interconnect mathematics, art, physics, philosophy, and culture, weaving a network of knowledge that mirrors the complexity of the real world. This convergence of disciplines, aligned with the competencies outlined in the *Base Nacional Comum Curricular* (BNCC), highlights the potential of these games to foster meaningful, contextualized education that responds to the contemporary challenges of society.

However, the effective implementation of this proposal demands more than the mere inclusion of games in classroom activities. It requires qualified teacher training, appropriate pedagogical materials, and environments conducive to experimentation and dialogue, as well as a school culture that values innovation and critical reflection. Only under such conditions will it be possible to overcome fragmented practices and develop an educational process that genuinely engages with the real needs of students and society.

Finally, this work opens pathways for future research, particularly in the areas of learning assessment and success indicators—fields that require deeper exploration in order to consolidate and expand the impact of topological games in education. Assessing not only conceptual mastery but also the development of critical, ethical, social, and creative skills will be the next essential step toward ensuring that this pedagogical approach reaches its full transformative potential.

In sum, topological games are not merely a didactic innovation; they constitute a genuine bridge between the abstract and the concrete, between scientific rigor and human experience, between teaching and holistic formation. Their study and application represent a valuable contribution to building an education that is more critical, inclusive, and committed to shaping individuals capable of understanding and engaging with an increasingly complex, dynamic, and interconnected world.

## References


Artigue, M. (2002). *Teaching and learning mathematics in a digital environment: The evolution of technologies and their impact on mathematics education*. Educational Studies in Mathematics, 47(1), 1-15.

Ausubel, D. P. (1968). *Educational Psychology: A Cognitive View*. Holt, Rinehart and Winston.

Bell, E. T. (1937). *Men of Mathematics*. Simon and Schuster.

Brasil, Ministério da Educação (MEC). (2018). *Base Nacional Comum Curricular (BNCC) - Ensino Médio*. Brasília: MEC.

Bruner, J. S. (1961). *The Act of Discovery*. Harvard Educational Review, 31(1), 21-32.

Castells, M. (2010). *The Rise of the Network Society*. Wiley-Blackwell.

Crowell, R., & Fox, R. H. (1999). *Introduction to Knot Theory*. Springer.

Darling-Hammond, L. (2017). *The Right to Learn: A Blueprint for Creating Schools that Work*. Jossey-Bass.

Facione, P. A. (2015). *Critical Thinking: What It Is and Why It Counts*. Insight Assessment.

Freire, P. (1970). *Pedagogia do Oprimido*. Paz e Terra.

Freire, P. (1996). *Pedagogia da Autonomia*. Paz e Terra.

Ghrist, R. (2014). *Elementary Applied Topology*. Createspace.

Gleiser, M. (2013). *Ciência, Fé e Religião: Conflitos e Diálogos*. Vozes.

Gilbert, S. F. (2010). *Developmental Biology*. Sinauer Associates.

Hasan, M. Z., & Kane, C. L. (2010). Colloquium: Topological insulators. *Reviews of Modern Physics*, 82(4), 3045.

Hatcher, A. (2002). *Algebraic Topology*. Cambridge University Press.

Kauffman, L. H. (2001). *Knots and Physics*. World Scientific.

Lakatos, I. (1976). *Proofs and Refutations: The Logic of Mathematical Discovery*. Cambridge University Press.

Lee, J. M. (2011). *Introduction to Topological Manifolds*. Springer.

Morin, E. (2007). *Introdução ao Pensamento Complexo*. Sulina.

Morowitz, H. J. (2002). *The Emergence of Everything: How the World Became Complex*. Oxford University Press.

Moran, J. (2010). *Metodologias Ativas para uma Educação Inovadora*. Papirus.

Munkres, J. R. (2000). *Topology*. Prentice Hall.

Nelsen, R. B. (2006). *Proofs Without Words: Exercises in Visual Thinking*. Mathematical Association of America.

Nielsen, M. A., & Chuang, I. L. (2010). *Quantum Computation and Quantum Information*. Cambridge University Press.

Noddings, N. (2003). *Caring: A Feminine Approach to Ethics and Moral Education*. University of California Press.

Papert, S. (1980). *Mindstorms: Children, Computers, and Powerful Ideas*. Basic Books.

Penrose, R. (2005). *The Road to Reality: A Complete Guide to the Laws of the Universe*. Jonathan Cape.

Piaget, J. (1977). *The Development of Thought: Equilibration of Cognitive Structures*. Viking.

Sachs, J. D. (2015). *The Age of Sustainable Development*. Columbia University Press.

Shulman, L. S. (1986). *Those Who Understand: Knowledge Growth in Teaching*. Educational Researcher, 15(2), 4-14.

Strogatz, S. H. (2015). *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*. Westview Press.

Stillwell, J. (2010). *Mathematics and Its History*. Springer.

Steenrod, N. (1999). *The Topology of Fibre Bundles*. Princeton University Press.

Sternberg, R. J. (2006). *Critical Thinking in Psychology*. Cambridge University Press.

Stillwell, J. (2010). *Mathematics and Its History*. Springer.

Thurston, W. P. (1997). *Three-Dimensional Geometry and Topology*. Princeton University Press.

Tobin, K. (2006). *Teaching and Learning Science: A Handbook*. Praeger.

UNESCO. (2017). *Education for Sustainable Development Goals: Learning Objectives*. UNESCO Publishing.

Vygotsky, L. S. (1998). *Mind in Society: The Development of Higher Psychological Processes*. Harvard University Press.