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The study of the technological dimension of a didactic problem: a theoretical-methodological essay

El estudio de la dimensión tecnológica de un problema didáctico: Un ensayo teórico-metodológico

L'étude de la dimension technologique d'un problème didactique : essai théorico-méthodologique

O estudo da dimensão tecnológica de um problema didático: Ensaio teórico-metodológico

Naum de Jesus Serra[1]
Doutorando em Educação em Ciências e Matemática
Universidade Federal do Pará – Pará – Brasil
Mestre em Educação Cientifica e Matemática
https://orcid.org/0000-0003-2204-2810

José Messildo Viana Nunes[2]
Universidade Federal do Pará-Brasil
Doutor em Educação Matemática
https://orcid.org/0000-0001-9492-4914

Saddo Ag Almouloud[3]
Universidade Federal do Pará-Brasil
Doutor em Matemática e Aplicações
https://orcid.org/0000-0002-8391-7054

Abstract

It is reasonable to think of digital technologies as entities intrinsically linked to our daily lives and increasingly present in the classroom, prompting several authors to address the theme and, as far as possible, propose models that enhance the teaching and learning of disciplinary content. When we examine the theories of didactics of mathematics (DM), we see that several works strongly appeal to the learning of mathematical objects mediated by these technologies. Because we understand the importance of this theme, we present in this research the main developments and

[1] nahumserra@gmail.com
[2] messildo@ufpa.br
[3] saddoag@gmail.com

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discussions that allowed the constitution of the 5th dimension model of a didactic problem. This is an excerpt from the doctoral thesis of the first author, who proposes such a construction based on the three initial dimensions (epistemological, economic-institutional, and ecological) discussed by Gascón and the 4th dimension (language) addressed by Brandão. The 5th dimension, also called the technological dimension, aims to subsidize inquiries that deal with modeling mathematical objects with the help of digital technologies, therefore, it refers to a theoretical-methodological model that, together with the other dimensions, will enable the understanding of the didactic problem in question and will allow a decision making in the course of a research that involves the 5th dimension of a didactic problem.

Keywords: Didactics of mathematics, Anthropological theory of didactics, Dimensions of a didactic problem, Digital technology applied to the teaching of mathematics.

Resumen

Es razonable pensar en las tecnologías digitales como entidades intrínsecamente vinculadas a nuestra vida cotidiana y cada vez más presentes en las aulas, impulsando a varios autores a abordar el tema y, en la medida de lo posible, proponer modelos que potencien la enseñanza y el aprendizaje de contenidos disciplinares. Cuando observamos las teorías de la didáctica de las matemáticas (DM), vemos que varios trabajos aportan un fuerte atractivo al aprendizaje de objetos matemáticos mediados por estas tecnologías. Por entender la importancia de este tema, presentamos en esta investigación los principales desarrollos y discusiones que permitieron la constitución del modelo de 5ª dimensión de un problema didáctico. Este es un extracto de la tesis doctoral del primer autor, que propone tal construcción a partir de las tres dimensiones iniciales (epistemológica, económico-institucional y ecológica) discutidas por Gascón y la 4ª dimensión (lenguaje) abordada por Brandão. La 5ª dimensión, también llamada dimensión tecnológica, tiene como objetivo principal, subsidiar las investigaciones que se ocupan de modelar objetos matemáticos con la ayuda de las tecnologías digitales, por lo tanto, se refiere a un modelo teórico-metodológico que junto con las demás dimensiones permitirá la comprensión del problema didáctico en cuestión y permitirá una toma de decisiones en el curso de una investigación que involucre la 5ª dimensión de un problema.

***Palabras clave*:** Didáctica de las matemáticas, Teoría antropológica de lo didáctico, Dimensiones de un problema didáctico, Tecnología digital aplicada a la enseñanza de las matemáticas.

Résumé

Il est raisonnable de considérer les technologies numériques comme des éléments intrinsèquement liés à notre quotidien et de plus en plus présentes dans les salles de classe, ce qui a poussé plusieurs auteurs à aborder le sujet et, dans la mesure du possible, à proposer des modèles qui optimisent l'enseignement et l'apprentissage des contenus disciplinaires. Lorsque nous observons les théories de la didactique des mathématiques (DM), nous constatons que plusieurs travaux font fortement appel à l'apprentissage d'objets mathématiques médiatisés par ces technologies. Conscients de l'importance de ce thème, nous présentons dans cette recherche les principaux développements et discussions qui ont permis la constitution du modèle de la 5e dimension d'un problème didactique. Il s'agit d'un extrait de la thèse de doctorat du premier auteur, qui propose cette construction à partir des trois dimensions initiales (épistémologique, économique-institutionnelle et écologique) discutées par Gascón et de la 4e dimension (langage) abordée par Brandão. La 5e dimension, également appelée dimension technologique, a pour objectif principal de soutenir les recherches qui traitent de la modélisation d'objets mathématiques à l'aide des technologies numériques. Elle fait donc référence à un modèle théorique et méthodologique qui, associé aux autres dimensions, permettra de comprendre le problème didactique en question et de prendre des décisions au cours d'une recherche impliquant la 5e dimension d'un problème didactique.

Mots-clés : Didactique des mathématiques, Théorie anthropologique de la didactique, Dimensions d'un problème didactique, Technologie numérique appliquée à l'enseignement des mathématiques.

Resumo

É razoável pensar nas tecnologias digitais como entes intrinsecamente ligados ao nosso dia a dia e que têm se mostrado cada vez mais presentes em sala de aula, forçando diversos autores a abordar a temática e, na medida do possível, propor modelos que potencializem o ensino e a aprendizagem de conteúdos disciplinares. Quando observamos as teorias da didática da matemática (DM), vemos que diversos trabalhos trazem um forte apelo à aprendizagem de objetos matemáticos mediados por essas tecnologias. Por entendermos a importância dessa temática, apresentamos nesta pesquisa os principais desdobramentos e discussões que permitiram a constituição do modelo da 5ª dimensão de um problema didático. Trata-se de um recorte de tese doutoral do primeiro autor, que propõe tal construção a partir das três dimensões iniciais (epistemológica, econômico-institucional e ecológica) discutidas por Gascón e da 4ª dimensão (linguagem) abordada por Brandão. A 5ª dimensão, também chamada de dimensão tecnológica, tem por objetivo principal, subsidiar inquirições que tratam de modelizações de objetos matemáticos com o auxílio de tecnologias digitais, portanto, refere-se a um modelo teórico-metodológico que juntamente com as outras dimensões possibilitarão a compreensão do problema didático em questão e permitirá uma tomada de decisão no decorrer de uma pesquisa que envolve a 5ª dimensão de um problema didático.

Palavras-chave: Didática da matemática, Teoria antropológica do didático, Dimensões de um problema didático, Tecnologia digital aplicada ao ensino da matemática*.*

The study of the technological dimension of a didactic problem: A theoretical-methodological essay

Introduction

Over the last few decades, the use of digital technologies in education has been gaining ground and gradually winning the favor of teachers and researchers, who see technology as a pedagogical tool capable of improving and supporting school teaching. Digital tools have recently become prominent, mainly due to the rapid development of educational platforms and the emergence of applications and software based on active methodologies and *gamification*, which encourage their use, especially in more developed countries (Serra, 2022). Nowadays, a term widely used in the context of this technological evolution is digital information and communication technology (DICT), which encompasses all digital tools, such as games, software, applications, and others that can compose or be part of virtual microworlds.

The UNESCO General Report (2023) warns that we should focus on learning outcomes rather than digital tools, because although digital technology has brought several changes to teaching and learning, it is debatable whether it has significantly transformed education. Unlike in more technologically developed countries, in developing countries, computers and devices are not used on a large scale in classrooms; moreover, the evidence is contradictory regarding the impact of such technologies, even in countries with greater technological dominance.

In Brazil, the justifications for the use of digital technologies appear in official documents. The Law of Guidelines and Bases of National Education (LDBEN) (Brasil, 1996) and the National Common Curricular Base (BNCC) (Brasil, 2017) indicate that basic education teachers must use methodological strategies that enable quality teaching and learning; hence, the texts incentivize the use of ICTs combined with good pedagogical proposals. According to the BNCC:

> Stimulating creative, logical, and critical thinking, through building and strengthening the ability to ask questions and evaluate answers, to argue, to interact with diverse cultural productions, and to use information and communication technologies, enables students to broaden the understanding of themselves, of the natural and social world, and of human-with-human and human-with-nature relationships (Brasil, 2017, p. 56).

Furthermore, this document reinforces that technological advancements have brought profound change to society, as digital culture has been strengthened by increased internet access and greater availability of technological devices, dynamically integrating students into this culture. However, it is noteworthy that students cannot be treated as mere consumers but as responsible citizens with relevant roles in this process of technological integration, which points to ten general competencies that should be developed throughout basic education. Therefore, it also emphasizes the need for schools to adopt an active pedagogical approach to foster productive and responsible engagement, given that social and cultural changes pose significant challenges for schools and, as a result, play a significant role in the education of new generations.

In line with this thinking, the teacher must take a leading role in mediating between technology and learning, participating in the development of projects that seek relevant relationships between technologies and teaching, proposing methodologies that reflectively corroborate the study of teaching objects, thus contributing to a critical attitude towards teaching and digital media (Serra, 2022). In this context, we infer that the use of digital technologies in mathematics education must involve a methodology that enables the modeling of these objects through a transpositional process that facilitates student learning. Thus, the 5th dimension of a didactic problem can assist the teacher/researcher in constructing teaching models that use digital technology as a tool to build concepts in that subject.

This dimension is spearheaded by transposition processes (didactic transposition (DT)) (Chevallard, 1991) and computer transposition (CT) (Balacheff, 1994). Furthermore, its genesis is related to the dimensions of a didactic problem (Gascón, 2011) and is, therefore, linked to the anthropological theory of didactics (Chevallard, 1999). This dimension arose from our understanding that there is a need to add an extra dimension to the already-existing four dimensions (*epistemological, economic-institutional, and ecological)* developed by Gascón (2011) and the dimension of *language*, established by Brandão (2021).

We understand that digital technologies are now highly relevant in the teaching and learning processes of mathematics, given the advances brought about by ICTs and their impact on the educational landscape; therefore, the 5th dimension emerges within the context of the first author's doctoral research and seeks to support his proposed inquiry, which deals with the modeling of mathematical objects with the aid of digital technology (*Scratch*[4] software). This is a theoretical-methodological model that,

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[4]*Scratch* It is software developed in 2007 by the *Lifelong Kinder Garden Group project in MIT* (*Massachusetts Institute of Technology*), with Mitchel Resnick as its main developer, who was influenced

together with the other dimensions, will seek a better understanding of the didactic problem at hand and enable coherent decision-making throughout the research. Furthermore, this experimental model suggests the possibility of assisting not only this research but also other research on the use of digital technologies (and/or standard technologies) in the context of mathematical modeling.

The following section will present the constitution of the 5th dimension of a didactic problem.

Technological dimension: a new conception of a didactic problem

The teaching work in relation to didactic activities is extremely important for the construction of knowledge in a more dynamic way (Gascón, 2011), therefore we understand that in the current context, the use of technologies can bring significant advantages, if they are associated with methodologies that aim at the teaching and learning of mathematics, since "technological entities can allow new forms of representation of mathematical objects that bring other discussions regarding the content being addressed" (Lima & Silva, 2015 p. 172). In this context, to corroborate future research on these themes, we will add an extra dimension to the dimensions of a 'didactic problem[5]', the 'technological dimension', which will be constituted and implemented from this perspective.

It is worth noting that the term 'technology' to which we refer is close to the idea brought by Lima and Silva (2015), who use this term to define two types of technologies: the "**standard technologies**, such as ruler, compass and set square, and **more advanced technologies**, such as the Internet, educational software, digital videos, etc." (Lima & Silva, 2015, p. 166). Therefore, in our context, 'technologies' correspond to digital technologies (software, educational applications, artificial intelligence, etc.) and 'standard technologies' (concrete materials - abacuses, base-ten blocks, geoboards, Cuisenaire rods, and others).

Gascón (2011) made a significant contribution to the anthropological theory of didactics (ATD) and, consequently, to mathematics didactics, by differentiating a teaching problem from a didactic problem (or research problem), highlighting that a teaching problem can be transformed into a didactic problem by incorporating at least

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by Seymour Papert[4] and the Constructionist theory made programming language popular and accessible, expanding the concept of block language to the school environment of basic education (Serra, 2024).

[5] According to Gascón (2011), a didactic problem is one that is identified as a problem related to mathematics teaching and learning.

three fundamental dimensions: the ***epistemological dimension**,* which places the mathematical context at the heart of the problem, ***the economic-institutional dimension***, which depersonalizes the didactic problem and determines the minimum unit of analysis of the study processes; the ***ecological dimension***, which focuses on the conditions necessary for the institutionalized study of mathematics, pointing out solutions to the restrictions that affect educational institutions.

In a way, this author highlights that a didactic problem arises from the questions: What to teach? Why teach? And how to teach?, being necessary to incorporate these three fundamental dimensions into teachers' problems. In this context, schematically, the heuristic pattern (Gascón, 2011) provides an overview of the development of a didactic problem, and is therefore defined as:

$$\left\{\left[(P_0 \oplus P_1) \hookrightarrow P_2\right] \hookrightarrow P_3\right\} \hookrightarrow P_\delta$$

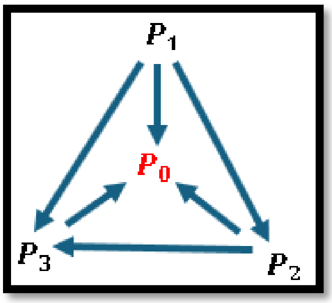
In the expression above, P_1, P_2, and P_3 represent the fundamental dimensions of a didactic problem, and P_0 represents an initial formulation and therefore plays a special role in the process. The symbol $\oplus$, called "additive disjunction," conveys the idea that P_0 is an incomplete didactic problem, lacking at least the initial addition of the **epistemological dimension P_1** for this to be considered a research problem. Furthermore, the **economic-institutional dimension P_2** and finally*,* **the ecological dimension P_3** (Gascón, 2011). These dimensions together are called a didactic problem, represented byP_δ, which contains the three fundamental dimensions (P_1, P_2, and P_3), the relationships with each other, and some new questions that may arise during the course of the research, and that do not appear in any of them. In this model, the symbol $\hookrightarrow$ does not represent inclusion, but rather a movement in which each of the dimensions P_i precedes the dimension P_{i+1}.

The epistemological dimension will enable the study of the highlighted mathematical objects, which, in turn, will allow the economic-institutional dimension to conduct analyses on official documents, manuals, textbooks, etc. Furthermore, it will facilitate the study of the ecological dimension, highlighting the "reason for being" of the mathematical objects in these institutions, in order to understand why they are the way they are and not otherwise, and to identify ways to improve them. In this context, the answers found to these and other questions will facilitate the making of certain inferences and the identification of possible solutions to the problems at hand.

In light of this, we represent in Figure 1 the heuristic model of Gascón (2011) and call it the 'triangle of a didactic problem':

Figure 1

Triangle of a didactic problem (developed by the authors)



Similar to the heuristic model, the 'triangle of a didactic problem' also conveys the idea that the epistemological dimension guides and enables the construction of the economic-institutional and ecological dimensions; therefore, the double arrow starts from (P1) to (P_2) and (P_3). Furthermore, the meaning of (P_2) for (P_3) represents a certain logic that, in general, causes the didactic triangle to move in the direction $P_1 \rightarrow P_2 \rightarrow P_3$, and both connect to P_0, this correlation being the key point for the 'teaching problem (P_0)' to become a 'didactic problem P_δ'.

It is worth noting that, in our specific case, P0 is represented by the initial problem of teaching and learning geometry in the early years of schooling. The epistemological dimension will deal with the study of magnitudes and measurements in the context of plane geometry, which will allow us to study the economic-institutional dimension and, as far as it is concerned, it will allow for analyses of official documents: BNCC (National Common Core Curriculum), textbooks and teaching manuals for the early years, sillabi and others, in order to understand what practices exist in the institutions. The ecological dimension will help us understand the rationale behind the existing praxeologies in the institutions involved, make inferences, and suggest new praxeological knowings for them.

Therefore, since our teaching problem is linked to the teaching and learning of geometry, the question is 'What to teach?' It relates to the teaching and learning of quantities and measurements in a geometric context for the early years, using *Scratch*. Regarding the question 'Why teach?', we infer that it is to enable prospective teachers to add new knowings to their praxeological equipment (PE), broaden their view of magnitudes and measurements in the context of plane geometry, and understand these objects more clearly through digital technology. Regarding 'How to teach?', we suggest implementing a teaching approach mediated by a reference epistemological model (REM) using *Scratch*, thereby enabling prospective teachers to improve their practices and minimize potential gaps in the teaching of geometric quantities and measurements.

Furthermore, since our theme is compatible with digital technologies in mathematics education, based on Gascón's (2011) concepts, we will go beyond the existing relationships between the three initial dimensions discussed by this author, and raise new questions that do not appear in any of those already described, thus requiring us to add an extra dimension to the didactic problem, 'the technological dimension'.

Gascón (2011) reports not only the possibility of altering the order in which the study of the basic dimensions of a didactic problem can occur, but also points out the existence of other secondary dimensions, stating that:

> The historical development of a didactic investigation does not always strictly adhere to and respect the heuristic pattern, since, in real history, the order in which the basic dimensions of a didactic problem are studied can change. Furthermore, secondary dimensions—such as cognitive, personal, ostensive, instrumental dimensions, etc.—can be valued as primary and priority dimensions and, therefore, remain implicit; on the other hand, they may not even be a dimension that the ATD considers basic or fundamental, such as the epistemological one (Gascón, 2011, p. 206, our translation).

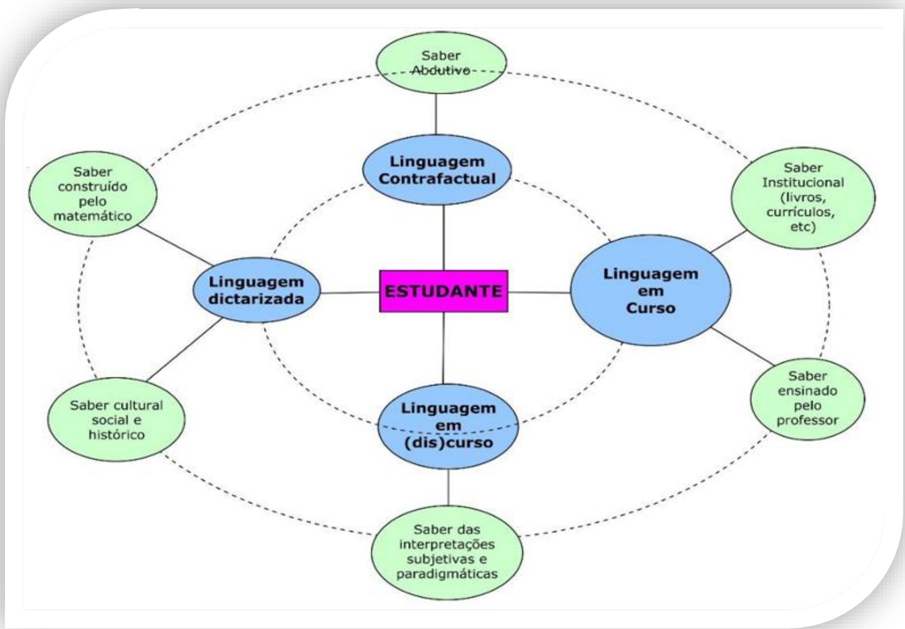
In this context, the author's words suggest the existence of extra dimensions, which motivates us to add the technological dimension, since it can, in many cases, support the researcher in choosing and using tools to model mathematical objects. However, before we continue the study of this dimension itself, it is worth emphasizing that the 4th dimension of the didactic problem corresponds to the dimension of language and was highlighted in Brandão's thesis (2021), and subsequently in Brandão, Silva, and Almouloud (2024). In the cited works, the authors "presented the 4th dimension as a theoretical construct for analyzing the didactic problem, situating it as "a fundamental dimension, with the same degree of relevance attributed to the epistemological, economic, and ecological dimensions" (Brandão et al., 2024, p. 367), highlighting it as a comprehensive field that permeates all other dimensions. Moreover, the authors describe it as both internal and external to the didactic problem, since it acts as a "boundary dimension in which movements are dynamic, expanding, or in a zone of interrelations between different media" (p. 368).

For them, the dimension of language is formed by four axes, namely: 1) The 'dictarized language' —characteristic of mathematics, formed by symbols, rules, and procedures inherent to the field of mathematics, composed of techniques, technologies, and theories of the area itself; 2) The 'counterfactual language', linked to abductive knowledge (spontaneous, insights) whose role is essential in sensory perception, attributed to a transcendental vision that emerges from the creative process; 3) The 'language in course', linked to the didactic transposition for teachers, evidenced by textbooks and the teacher's lesson plan, containing their individual interpretations of the mathematical object. This last axis revolves around the mathematical and didactic organizations that aim to support the teaching and learning of students in the classroom. According to the authors, "The language currently being used is based on the repetition of textbooks and teaching methodologies guided by the 'visit to the works' paradigm (Brandão et al., 2024, p. 372). Finally, 4) The 'language in (dis)course', establishing the knowing of subjective and pragmatic interpretations to construct an individual and critical interpretation by both the student and the teacher. It positions itself in relation to the power relations embedded in discourses and institutions.

In summary, Figure 2 presents the four language types proposed by the authors, highlighting some of the knowings involved in each.

Figure 2

Knowings constituting the dimension of language (Brandão et al., 2021, p. 104)



The authors report an inverse relationship between the four languages, such that the more the dictated language is used, the less the counterfactual language is constructed; analogously, the greater the language in (dis)course, the less the language in course will be used. Furthermore, the less dictated language is used, the more elevated the language in (dis)course will be, and the less counterfactual language there is, the more language in course will be. According to these authors, the balance point would be the ideal form and would occur with the use of the four languages at a point "where the levels of proportionality reach an ideal level of interweaving for the teaching and learning of mathematics" (Brandão et al., 2024, p. 374).

That said, with the constitution of this new dimension, the heuristic model (Gascón, 2011) changes and comes to consist of four dimensions, namely, the epistemological (P_1), the economic-institutional (P_2), the ecological (P_3), and the dimension of language (P_4):

$$\left\{\left[\left[\left[(P_0 \oplus P_1) \hookrightarrow P_2\right] \hookrightarrow P_3\right] \hookrightarrow P_4\right]\right\} \hookrightarrow P_\delta$$

Under these conditions, (P_4) emerges as an entity that permeates the other dimensions and reinforces the teaching and learning of mathematical objects by creating possibilities for interpretation and analysis of these objects within a didactic system (Brandão 2021; Brandão et al., 2024).

Returning to the study of the 5th dimension of a didactic problem, we can say that the genesis of this 'technological dimension' goes back to Almouloud (2005), because, although this author did not use this term per se, he presented indications and pointed to the existence of an important and inseparable dimension, even before Gascón (2011) announced his contribution to the ATD involving the study of the three dimensions of the didactic problem (which would happen years later). In this context, Almouloud (2005), when questioning the use of computers, digital technologies, and teachers' roles concerning existing paradigms in teaching mediated by such tools, sought to analyze digital technologies in the context of didactic transposition (DT) and information transposition (IT), which leads us today to theorize and establish a new dimension to pre-existing dimensions.

As seen, Gascón (2011) highlights that the study of the three dimensions of a didactic problem directly emphasizes the mathematical objects that undergo in-depth study, detailing their historical and epistemological character, as well as their representation in textbooks, manuals, and official documents. Furthermore, it seeks to understand how these objects coexist within educational institutions.

On the other hand, Brandão (2021), using two theoretical perspectives, the ATD (Chevallard, 2015) and the semiotic theory (Pierce, 2005), highlights the teaching and learning of mathematical objects (double integral, for calculating the measure of the volume of a quadric surface, the hyperbolic paraboloid). The author based her work on the ATD and the vision of changing the paradigm from 'visiting the works' to the paradigm of 'questioning the world', and on Peirce's semiotic theory (2005), which expands the space for the analysis of mathematics by "attributing to mathematical objects meanings and senses that emerge from the reading of culturally established signs, from the various languages that allow us to understand the world, and from the contexts of use of those signs" (Brandão 2021, p. 106).

Therefore, we understand that none of these authors (Gascón, 2011; Brandão, 2021; Brandão et al., 2024) aimed at incorporating technologies to support mathematical modeling in the dimensions described. In light of this, in this section, we will construct the technological dimension—which includes 'digital' technologies—and although it is not the focus of this work, we emphasize that the model also encompasses and can be used from the perspective of 'standard technologies'. Our goal here is to address digital technologies as tools for mathematical modeling; therefore, we will not discuss standard (non-digital) technologies.

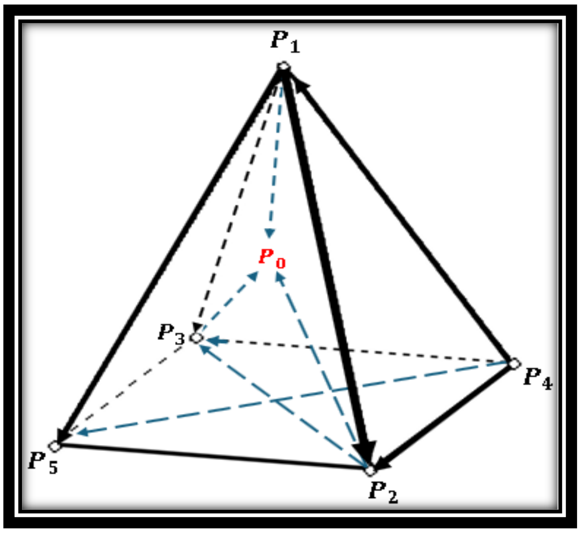
Thus, in the context of the ATD, based on the concepts of Gascón (2011) and Almouloud (2005), we dare to announce the insertion of the 'technological dimension': the '5th dimension of the didactic problem', which expands and modifies Gascón's (2011) initial heuristic model by adding an extra dimension that brings modifications, so that a new symbolic model emerges, which we call the 'expanded heuristic model' (EHM).

$$\left\{\left[\left[\left[(P_0 \oplus P_1) \hookrightarrow P_2\right] \hookrightarrow P_3\right] \hookrightarrow P_4\right] \hookrightarrow P_5\right\} \hookrightarrow P_\delta$$

We emphasize that in EHM, P_0 is an initial teacher problem, the symbol $\oplus$ brings the idea that P_0 is an incomplete didactic problem, requiring the inclusion of, at least, the epistemological dimension P_1. Furthermore, P_2 represents the economic-institutional dimension, P_3 the ecological dimension, P_4 the language dimension, and, finally, P_5 the technological dimension. We emphasize that while Gascón's (2011) heuristic model is geometrically represented by a triangle (Figure 1), we conjecture that the EHM will be geometrically represented by a pentahedron (Figure 3), which we call the 'pentahedron of a didactic problem'.

Figure 3

Pentahedron of a didactic problem (developed by the authors)



Just as the EHM suggests a sequential and ordered movement of dimensions, its geometric representation is constructed in a similar way. However, it is worth remembering that the *language dimension* (P_4) connects with all other dimensions. The *epistemological dimension* (P_1)*,* by placing the mathematical context at the heart of the didactic problem (Gascón, 2011), interconnects with and allows for the constitution of the economic-institutional dimensions (P_2ecological (P_3), influencing them directly, allowing (in P_2) the analysis of textbooks and teaching manuals to verify how praxeologies exist within institutions, also highlighting (in P_3) the' reason for being' of these objects in order to know why they behave in a certain way and not differently. Furthermore, these three dimensions (P_1, P_2, P_3) are linked to the teaching problem (P_0), so that, in this way, a research problem is constituted (P_δ). The epistemological dimension also makes possible (along with P_5) a systematic study of the relationship between mathematical objects and technological tools, enabling a proper understanding and a coherent selection of these tools for modeling.

In the next section, we will discuss the concepts that will allow the construction of the '5th dimension model of a didactic problem'.

The construction of the 5th dimension model of a didactic problem

First, we want to emphasize that the term 'conception' described here is used in a philosophical sense, referring to the idea of the creation and construction of concepts; thus, "a conception is also the product of someone's intelligence, and often contributes to the formation of various theories"[6]. That being said, the concepts presented in Almouloud (2005) will be the driving force for the formulation of the 5th because the author, at that time, was analyzing the effects of didactic transposition (DT) and computer transposition (CT) in mathematics teaching mediated by digital technologies, raising questions about the use of computational environments and software in education, as well as the consequences of these tools on student learning. From then on, questions arose, such as: What mathematical knowing or expertise do we want to teach? And how do we teach it? Regarding this, the author reveals that:

> If the computing resource is the teaching aid chosen by the teacher, they should know that computers and educational software are work tools that require a pedagogical strategy. It is the teacher who must define this strategy, based on the teaching objectives and the tools available to them (Almouloud 2005, p. 55).

In this context, the author highlighted the importance of teacher training linked to didactic aspects involving issues related to this type of technology, given that teaching and learning go beyond mathematical knowledge, and that in a teaching context supported by digital technologies, the teacher must possess knowledge that enables them to handle technological tools (computers, languages, graphic tools, educational software, etc.). Thus, for Almouloud (2005), it is essential to have training that takes into account the new educational paradigms introduced by digital technologies in teaching, in order to contribute to student learning effectively, that is, "The objective of using computer environments in education is to provide students with favorable conditions for acquiring knowledge and overcoming teaching-learning difficulties" (Almouloud 2005, p. 55).

That being said, we infer that the theories of transposition (didactic and computational) are the guiding threads of the technological dimension, as they will enable, at the appropriate time, the modeling of activities through technologies. We emphasize that although the author's questions focused on digital technologies, they also relate to 'standard technologies'. Therefore, in the context of the technological dimension, we emphasize that the model to be constructed can be used in both perspectives; however, we will describe these two technologies generically as 'technological tools or technological objects'. In this sense, the 5th dimension should support the teacher/researcher in the search for answers to specific questions, so that

[6] https://www.significados.com.br/concepcao/

the didactic analysis of the answers to the chosen technological tool helps construct problem situations that meet the objectives of teaching and learning.

The questions Almouloud (2005) elicits are relevant and should be fundamental points for teachers/researchers regarding the technological tool that can be used in the classroom. To achieve our goals satisfactorily, we made minor adjustments to some of these issues. In that direction, we present new inquiries based on Almouloud (2005) that allow us to question the following, regarding the chosen technological object:

a) Does it allow for creating situations in which the variables are controllable?
b) Does it allow for identifying and interpreting errors and the conditions under which they emerge?
c) Can models of erroneous processes be constructed?
d) Can didactic situations be created in which these processes are unbalanced?
e) Will the use of this technology help achieve the teacher's educational objectives?

These questions are essential to the 5th dimension and will be part of the 'technological conception' (TC). These are a priori analyses that the teacher must perform, since these questions will enable a didactic and epistemological analysis of the effects of the knowledge that students can learn in a teaching environment mediated by technological tools.

Similarly, other arguments highlighted in Almouloud (2005) underwent minor modifications, with the same aim of being more comprehensive and enabling an analysis of the use of technological objects in the classroom. These are essential topics for the teacher/researcher to consider. Namely:

f) What are the limitations (or restrictions, conditions) that technologies impose on the user?
g) What behaviors do they induce, and what type of teaching and learning do they effectively enable?
h) What are the effects of teaching and learning with educational software (or with concrete materials) on the knowledge built in the classroom?
i) What are the effects of computer-based and didactic transposition of mathematical knowledge on the knowledge constructed by the student in interaction with the technological tool?

Questions (f, g, h, and i) guide the 5th dimension, since the answers will enable the construction of actions, activities, or models through a systematized study that can lead the teacher/researcher to an understanding of the themes supported by technological tools in teaching. The main objective of the technological dimension is to support research into a didactic problem involving the use of these tools, ultimately to infer and make strategic suggestions for addressing the obstacles and gaps in teaching mediated by these technological objects.

This dimension brings to its formation, in addition to technological concepts **(TC)** (comprising the questions described (a, b, c, d, e, f, g, h, i), the historical and epistemological study of the technological object **(ETO)** (applications, educational software, or concrete materials) to be used. In its composition, it also includes an integrative literature review **(ILR)** and the mathematical concept **(MC)**. These four conceptions, mediated by conceptions of language **(CL)** and the conceptions of the professor/researcher **(C. Subject)**, will allow for a global perspective and a more in-depth understanding of the technological tools to be used in the teaching modeling process.

It is worth highlighting that in inquiry, the teacher/researcher is an agent of action and mediation, and is therefore a key player in the development of scientific work, leaving their own impressions throughout the process, which, in a way, has some similarity to the movement carried out by the teacher in the context of the CL. However, while in the 4th dimension the teacher will seek a balance between the four languages through classroom action, primarily using languages in course and in (dis)course, within the 5th dimension, the researcher's interaction occurs from the perspective of the C. Subject.

We understand that in a teaching context, the teacher should be an active subject who brings their social and cultural experiences, as well as their epistemology, formed throughout their academic training and life, incorporating into the 'text of knowings' the institutional and epistemological variables and their respective values that shape its didactic *milieu* (Silva, 2019). Although in scientific research the researcher's interaction with the mathematical object bring in nuances of the language in course governed by the internal didactic transposition process, it occurs under a different dynamic, because, from the perspective of the 5th dimension, the researcher must carry out an in-depth epistemological study of these objects (in accordance with P_1) and technological tools (through ETO), as well as the possible relationships between them. Therefore, the conceptions of the subject (C. Subject) go beyond the vision on which the language in course is based, systematized by the 'paradigm of visiting the works'.

In the technological dimension, the researcher brings balance to the inquiry through the C. Subject, which, supported by the paradigm of 'questioning the world', represents a relationship R (î, E), in which **î** designates a person or an institutional position and **E**, the knowing (in Greek, ἐπιστήμη, episteme). In the context of the ATD, the 'C. Subject' is analyzed from the perspective of the R (î, E) relationship; therefore, the teacher/researcher (î) is a subject who must be aligned with the five attitudes (Chevallard, 2013; Chevallard & Strømskag, 2022), these being:

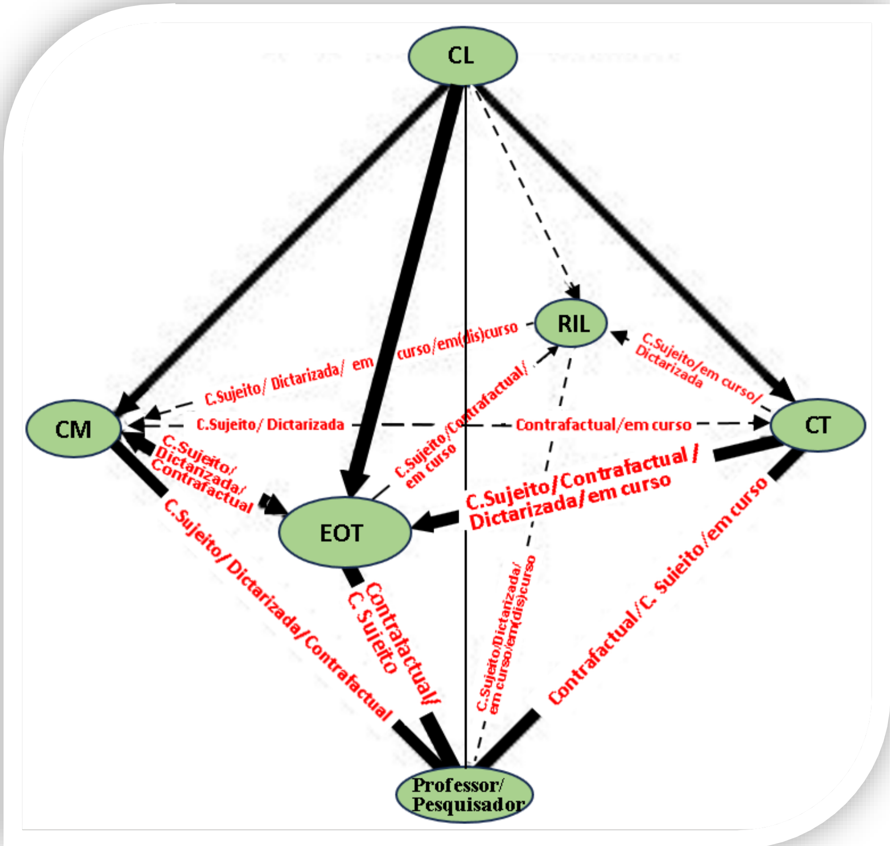
- The problematizing ($\hbar_1$, as it involves recognizing the 'problematic' aspects of lived or observed situations, that is, asking questions about them. This is, obviously, an essential attitude, from which both the generating question q of the inquiry, and the generated questions qk emerge
- Herbartian ($\hbar_2$), which consists of not avoiding any question q as such (denying it, ignoring it, repressing it) and, specifically, in dedicating oneself to its study here and now or, at least, in putting its study on hold.
- Procognitive ($\hbar_3$), which contrasts with the retrocognitive attitude typical of the 'visiting the works' paradigm, which makes us 'look back' at the knowledge acquired so far, which is sometimes doubtful.
- The esoteric ($\hbar_4$), which opposes the esoteric illusion of those who think they know everything (at least in a given area). In contrast, the esoteric attitude consists of always seeing oneself as having to study in order to learn more, or even to verify or question what one thinks one knows.
- The common encyclopedist ($\hbar_5$), which consists of seeing oneself as not alien to the set of possible praxeological fields, even if it has a 'degree of exotericity' close to zero, while constantly striving to increase that degree of exotericity as much as is useful.

In this context, C. Subject seeks to align the researcher with the five attitudes so that they are problematizing, Herbartian, procognitive, esoteric, and common encyclopedist (Chevallard & Strømskag, 2022, p.23).

That said, the 5th dimension, aligned with the other dimensions of the didactic problem, will allow us to understand the 'reason for being' of these technological tools, how they exist and move within educational institutions (formal or non-formal), and how they integrate with mathematical objects. Furthermore, the technological dimension, in conjunction with the ecological dimension, can outline guidelines and point to possible solutions, since it aims to corroborate the (re)construction of mathematical concepts through actions or activities integrated with technological tools. Therefore, due to its relevance and because we understand that this dimension is inseparable from a didactic problem focused on the use of technological tools, we emphasize that it has the same degree of relevance as the previous dimensions. In this sense, we highlight the model that represents the 5th dimension of a didactic problem, as illustrated in Figure 4.

Figure 4

5th Dimension model of a didactic problem (developed by the authors)



This model is composed of four main pillars that, associated with the dimension of language (CL) and the conceptions of the teacher/researcher (C. Subject), aim at a coherent choice and a systematic study of the technological tool to be used for modeling mathematical objects.

In this context, the four pillars we are referring to are:

1. The epistemology of the technological object (ETO) — corresponds to the historical/technical study of the chosen object, in which 'dictating' and 'counterfactual' languages prevail, as they can mobilize ideas of diagrams,

symbols, and programming, in addition to abductive thinking in which the researcher's creativity is revealed for the construction of learning objects (LO);

2. The integrative literature review (ILR) — provides comprehensive information about other studies that have used technological tools in education. An ILR collaborates with a new vision and new knowings relevant to strengthening the praxeological equipment (PE) of the teacher/researcher. It introduces modulations of a dictated language because it can highlight mathematical rules, symbols, and representations in studies. Furthermore, it highlights nuances of language 'in course' (with possibilities of language in (dis)course), represented by the LOs.
3. The mathematical conception (MC) — brings together the perspectives of the epistemological study of the mathematical object (epistemological dimension, P1) and of mathematical thinking, encompassing the knowings necessary for its articulation with the technological tools to be used in the modeling. In this case, the 'dictated language' stands out.
4. Technological conception (TC) — aims to assist research in the search for an appropriate choice of digital technological tool, is aligned with questions (a, b, c, d, e, f, g, h, i), and is mediated primarily by C. Subject, linked to the teacher/researcher, which, in turn, along with the language being used, may enable mobilization of the counterfactual language, important in the creation of learning objects through transpositional means.

It is worth noting that the technological conception (TC) guides the 5th dimension model and, ideally, this conception should be the first to be observed, since it provides the a priori analysis (supported by questions: a, b, c, d, e), which is fundamental for the clarity about the technological object to be used in the modeling, so as to make it possible to analytically observe whether it is capable of achieving the didactic objectives set by the teacher. This model then suggests that the study of the technological object should be emphasized through the epistemology of the technological object (ETO), thereby allowing the teacher/researcher greater depth of understanding and enabling the discovery of properties that may not yet have been revealed to them. Therefore, the 'arrow' is directed from the TC to the ETO and then to the integrative literature review (ILR), in such a way that the latter can allow a comprehensive view of national and/or foreign works, helping the researcher to observe and understand the mathematical conceptions (CM) and the technological conceptions (TC) present in the analyzed projects, as well as conditions and/or restrictions and possible relationships between them.

When dealing with studies of mathematical concepts supported by technology, there is a strong dialectic between mathematical conceptions (MC) and technological conceptions (TC), since an in-depth view of the former, combined with questions from the latter, may allow for an adequate modeling of mathematical objects mediated by technological tools. Similarly, the 'double arrow' between the MCs and ETO indicates the close relationship that should exist between these two conceptions, as their convergences offer several possibilities for the development of activities.

The logic behind the movement of each arrow in the '5th dimension model' is not fixed, as it may change depending on the specifics or needs of each research project. However, the conceptions described in this model are constitutive of this dimension and should therefore not be ignored.

The model also highlights the main nuances between the 4th and 5th dimensions of a didactic problem, both from multiple interactions between the conceptions of the subject (C. Subject) with the other conceptions (CL, TC, MC, ILR, ETO), as well as in their mutual relationships. Under these conditions, the teacher/researcher, when interacting with the MCs, mobilizes the 'C. Subject', also highlighting mainly the conceptions of dictated language. In its interaction with ETO, the C. Subject, the dictated language, and possibly the counterfactual language can occur, linked to creativity and abductive reasoning. In their relationship with the TCs, the C. Subject and the language in course occur, although counterfactual language may also appear. In contact with the ILR, C. Subject and nuances of the dictated and in course languages prevail (with chances of observing language in (dis)course).

We can conjecture that, at the interface between the epistemology of the technological object (ETO) and the integrative literature review (ILR), the C. Subject may prevail, allied with the dictated and in-course languages, driven by the existing relationships between programming languages, software, applications, and others (with the possibility of the emergence of counterfactual language through insights, in addition to language in (dis)course). Between IRL and the mathematical conceptions (MC), the following stand out: C. Subject, dictated language and language in course, evoked by the perceptions of other researchers who bring new knowings about the teaching of mathematics and the construction of learning objects (LOs) modeled by technological tools. It is reasonable to assume that various works present activities carried out by students who, attracted by the knowledge of digital culture (programming, computers, cell phones, applications, and others), leave their impressions, which the researcher can access by observing nuances of language in (dis)course.

In the region between mathematical conceptions (MC) and technological conceptions (TC), the following stand out: C. Subject, dictated language, language in course, and possibly counterfactual language, attracted by computational thinking (CT) and mathematical thinking (MT). This interface is influenced by the epistemology of the teacher/researcher, their work philosophy, and their cultural influences, which, combined with manuals, textbooks, and studies mediated by 'questioning the world' (Chevallard, 2022), allow for a "new face" and enable the construction of learning objects mediated by technological objects. Between TCs and the ETO, the C. Subject, the dictated and the in-course languages essentially prevail, with the viability of counterfactual language.

On the border between the ETO and the MCs, the C. Subject and the dictated language prevail, with the opportunity of appearance of the counterfactual language, driven by the chosen digital tools and the mathematical knowings that these can mobilize. At the interface between the ILR and the TCs, C, Subject, the dictated language and the language in course emerge, stimulated by the analysis of the LOs, evidenced in the research and in their relationship with the mathematical topics to be mobilized by the teacher/researcher. Presumably, it is also possible to observe nuances of language in (dis)course promoted by the work of the students, who leave their mark on the activities.

Let us note that the language dimension requires a balance between the four areas of knowings that comprise it; otherwise, it can jeopardize the quality of teaching and learning in the classroom. Therefore, the teacher's role is to mediate and seek a balance between these knowings. Similarly, when these conceptions of language are combined with those of the 5th dimension, the entire system must maintain a certain balance for the research to be sound, and in this case, this balance must be mediated by the C. Subject, as described earlier.

The fifth dimension has a strong connection with transpositional phenomena, since it constitutes the scenario of a didactic problem, which, in turn, operates within the context of the ATD and, consequently, of the DT and CT. Therefore, this dimension aligns with the praxeologies that directly drive the modeling of the chosen mathematical objects. In our case, since we are dealing with digital tools, the TC has a significant presence, because the investigative *milieu* will be linked to the ideas and concepts of this theory, since the transformations of the knowing to teach with the mediation of digital objects complement and integrate with the DT (Balacheff, 1994). In this case, the teacher/researcher must participate in the entire didactic transposition process integrated with the computer dimension, so that the interactions and transformations of mathematical knowledge in computer representation are associated and combined in a complex way with those of didactic transposition (Balacheff, 1994).

The technological model primarily concerns scientific research, but beyond mere inquiry, the study, mediated by the concepts of the 5th dimension model, allows the modeling of mathematical objects using technological tools (digital or traditional) in different contexts (e.g., teacher training and classroom teaching). Regardless of the circumstances, the models take into account the existence of a dialectic at play (S-MO-TO) such that:

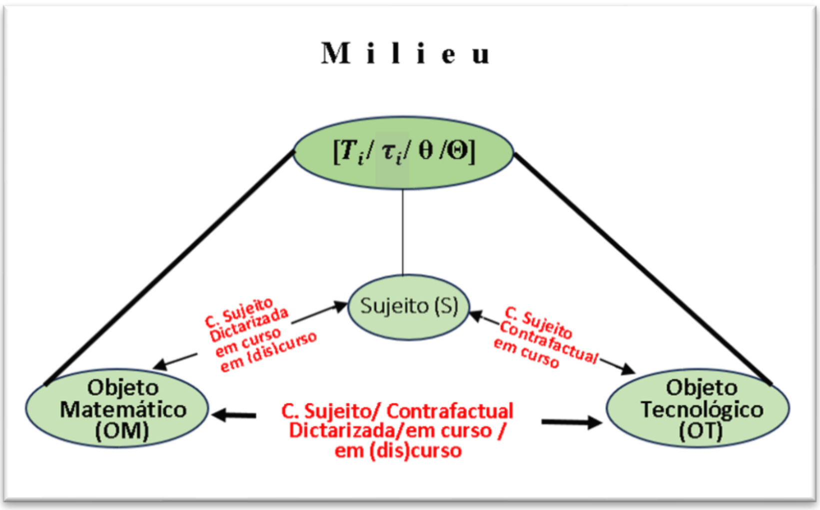
The subject (S) — represents a teacher/researcher, a student, or a group of people involved with the use of the technological tool.

The mathematical object (MO) — consisting of the mathematical topic(s) related to the teaching problem and which will later form part of the research problem.

The technological object (TO) — represents the technological tool that will be used by the subject to model the chosen mathematical object(s) supported by transpositional processes (computer and/or didactic). In this regard, we highlight the dynamics of this dialectic (Figure 5), which, in our case, will be mediated by a TC-supported investigative *milieu*, to enable modeling work.

Figure 5

Dialectics: S-MO-TO (Prepared by the authors)



In that investigative *milieu*, the dialectic (S-MO-TO) will be conducted by the praxeological blocks [T_i/ τ_i/ θ /Θ] (Chevallard, 1991), integrated into the computational dimension; and both will be aligned with the four pillars of the 5th dimension (TC, MC, ILR, ETO), as well as with the knowledge of the language dimension and the conceptions of the subject (C. Subject). Thus, in the development of a reference epistemological model (REM), for example, the types of tasks (T_i), technique (τ_i), technology (θ) and theory (Θ) have a fundamental role, as they will, among other things, enable a dynamic relationship among the knowings internal to the 5th dimension, and foster a dialogue with the other dimensions of a didactic problem (epistemological, economic-institutional, and ecological).

In the context of modeling, the relationship between S-TO highlights the C. Subject that integrates into the languages in course (or (dis)course), with the possibility that counterfactual language appears in this dynamic. Between S-MO, the C. Subject, the dictated language and the language in course may prevail. On the other hand, in the MO-TO relationship, the dictated language, the C. Subject and the language in course (or (dis)course) stand out, with the possibility of the emergence of counterfactual language. In the next section, we will present some authors who have addressed the use of digital technologies in education and proposed inquiry models for the modeling of mathematical objects.

Research and models that have addressed digital technologies in education

It is worth remembering that several authors have already addressed the topic of 'digital technologies in education' (Mishra & Koehler, 2006; Resnick, 2007; Papert, 1980; Wing, 2006, 2011; Balacheff, 1994; Lima & Silva, 2015; Santos, 2016). Mishra and Koehler (2006), for example, used design experiments as a research methodology and developed a model called technological pedagogical content knowledge (TPCK), based on Shulman (1996), to integrate the complexity and interaction between three main components of learning environments: content, pedagogy, and technology.

Beyond Shulman's (1996) ideas, the authors incorporated digital technology across the theoretical, pedagogical, and methodological levels. They argued that the appearance of digital technologies has drastically changed routines and practices in most areas of human work. However, in the educational sphere, reality has lagged far behind the vision due to a strong tendency to focus only on the technology, not on how it is used. Thus, Mishra and Koehler (2006) argue that simply introducing 'new technologies’ into education is not enough; teachers must properly incorporate technology into teaching.

Lima and Silva (2015) addressed ‘didactic knowledge’ related to the didactics of mathematics (DM) and added other groups of knowledge to those constructed by Mishra and Koehler (2006). Furthermore, they deepened the discussions on the differences between didactic and pedagogical knowledge, based on Bailleul and Batalle (2014), highlighting didactics as a science that takes into account the systematic way in which disciplinary content is treated, unlike pedagogy.

Santos (2016) coined the term’ didactic-computer engineering’ based on software engineering with the theoretical and methodological contributions of didactic engineering (DE) by Artigue (1996), highlighting that the DE phases involve the development of didactic sequences with characteristics common to those of software processes. According to Santos (2016), this association became necessary due to the perceived absence of elements for the development of products that address the needs of both mathematics teaching and learning, as well as technologies (didactic, cognitive, epistemological, technological aspects, among others).

Mishra and Koehler (2006), considering the articulated knowledge that teachers should possess, created a framework emphasizing the connections, interactions, possibilities, and constraints between content (C), pedagogy (P), and technology (T), stating that the articulation between these areas of knowledge is central to the development of new practices and is treated organically. Thus, they emphasize a complex interaction between these three bodies of knowledge. On the other hand, Lima and Silva (2015) focused on an initial training course for mathematics teaching degree students, and sought to identify the knowledge that could be developed in geometry subjects, based on the analysis of teaching materials, observing which elements could provide conditions for students to relate different types of knowledge: content knowledge (CK), didactic knowledge (DK), pedagogical knowledge (PK) and technological knowledge (TK), built throughout the course.

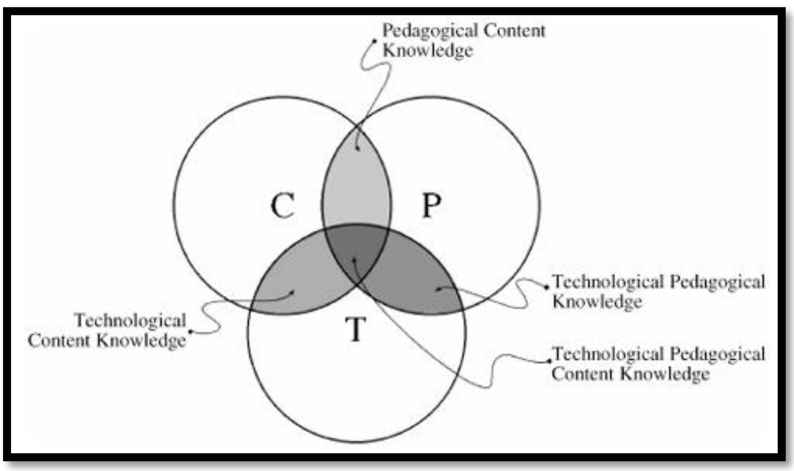
Therefore, Lima and Silva (2015) adopt Mishra and Koehler’s (2006) conception, but, beyond that, they introduce didactic knowledge and recognize it as knowledge of mathematics teaching and learning processes, encompassing “theories, processes, and practices that relate to the teaching and learning of concepts of this science and [...] adaptations, which aim at application in the educational process of mathematics, of general pedagogical notions” (Lima & Silva, 2015, p. 163).

Santos (2016) combined didactics and software engineering because he believed that didactics brings important elements to the conception, development, and analysis of educational software. The central objective of this author's work was to analyze and validate a microworld development process for learning mathematics through these two engineering disciplines. Their methodology consisted of conceiving, creating, and analyzing an educational software development process that integrates technological potential with teaching and learning theories. To achieve this, he relied on a multidisciplinary team of collaborators who assisted him in conceiving, developing, and improving the software process for teaching the rate of change of mathematical functions.

MIshra and Koehler (2006) highlight that "TPCK is the basis of good teaching with technology and requires an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content" (2006, p. 11, our translation). They emphasize that the teacher must not only looks at each of the components in isolation but also look at them in pairs, i.e., at pedagogical content knowledge (PCK), technological content knowledge (TCK), technological pedagogical knowledge (TPK), and all three together as technological pedagogical content knowledge (TPCK), as indicated in the model in Figure 6. This movement is similar to that carried out by Shulman (1986), who considered the relationship between content and pedagogy, which he called pedagogical content knowledge (PCK).

Figure 6

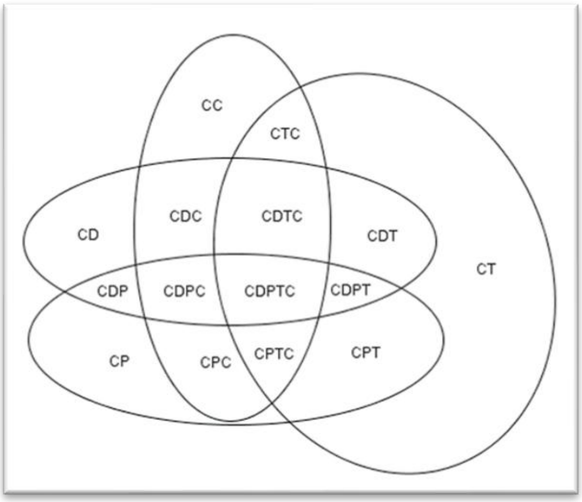
Technological and pedagogical content knowledge. The three circles—content, pedagogy, and technology—overlap to lead to four more types of interrelated knowledge (Mishra & Koehler, 2006, p. 9)



According to Lima and Silva (2015), technological didactic content knowledge (TDCK) enables the teacher to analyze, based on theories of the didactics of mathematics, how various technologies can be used for the teaching and learning of a given mathematical object. Pedagogical content knowledge (PCK) relates general pedagogical knowledge to a specific area, in this case, mathematics. Technological content knowledge (TCK) is the knowledge of how technology and a given content (or field) of mathematics are related. Figure 7 shows their dynamic correlation and highlights the multiple forms of knowings resulting from this interaction.

Figure 7

The several categories of teaching knowledge (Lima & Silva, 2015, p. 165)

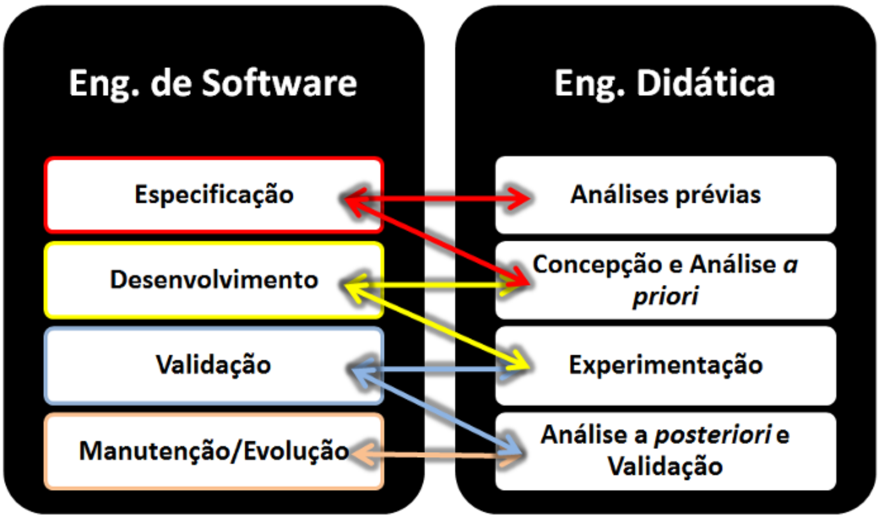


The work carried out by Lima and Silva (2015) enabled the identification of multiple areas of knowledge, relating them dynamically to one another through the study of geometry using the digital tools 'Geogebra' and 'Cabri 3D'. In their analyses, for example, they connected content knowledge (CK) to pedagogical knowledge (PK), proving pedagogical content knowledge (PCK); content knowledge (CK) to didactic knowledge (DK) and technological knowledge (TK), pointing to didactic-technological content knowledge (TDCK); in addition to content knowledge (CK) to technological knowledge (TK), revealing technological content knowledge (TCK).

Santos's research (2016) also highlighted a diagram (Figure 8) that shows the similarities between didactic engineering and software engineering. The author states that the equivalence between the two engineering fields is concerned with the "articulation and relationships of teaching and learning specific content: one with the creation of teaching sequences, the other with the creation of products that aid teaching and learning" (Santos, 2016, p. 47).

Figure 8

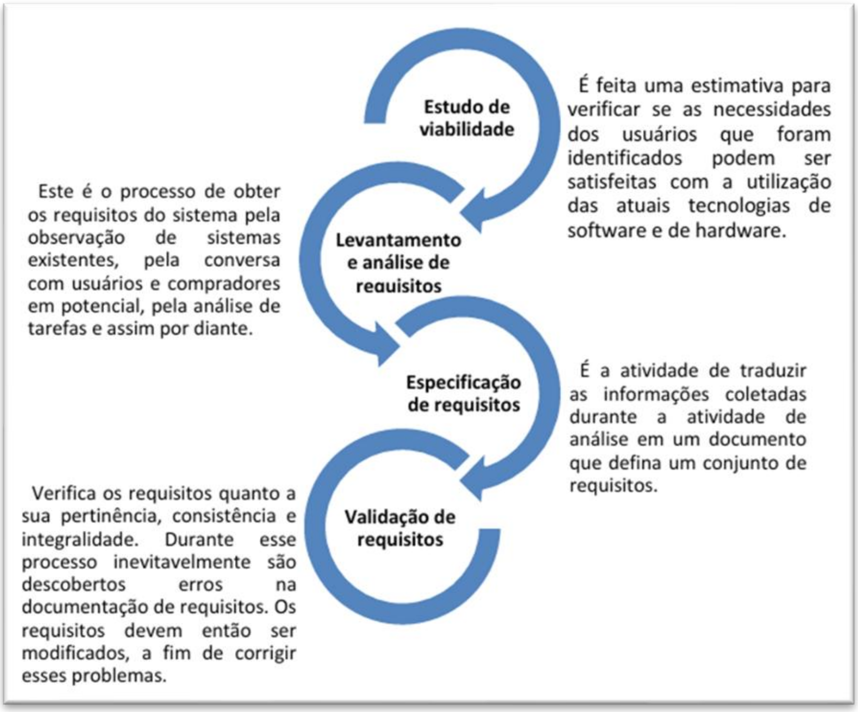
Comparison between the engineering fields (Santos, 2016, p. 46)



According to the author, these similarities enabled the integration of theoretical knowledge about the teaching and learning of mathematical concepts with the technological capabilities that enabled the development of the software and contributed to the specification and implementation of requirements, achieving the specificities of the knowledge in question. For Santos (2016), software engineering (SE) is linked to requirements engineering (RE), the latter being a stage of SE that corresponds to a process for discovering the purpose, through the identification of stakeholders (teacher and students) and their needs, and documenting them in a way that allows for analysis, communication, and subsequent implementation. Figure 9 illustrates the main characteristics of the RE.

Figure 9

Requirements engineering activities according to Sommerville (2004) (Santos, 2016, p. 41)



According to the author, the process that constitutes the RE is fundamental for the construction of educational software, since it considers the development requirements of this type of product and serves as an a priori analysis of SE, which is used before the implementation of the software in question.

In the next section, we will present the conclusions, including analyses of the technological dimension and its possible similarities and differences with the models highlighted so far.

Conclusion

According to UNESCO (2023), although technology promises easier access to education, the reality is that digital divides still exist, to the point that educational inequalities are exorbitant. Furthermore, even if connectivity were universal, it would still be necessary to demonstrate, from a pedagogical perspective, that digital technology offers real added value for effective learning. One way to achieve such learning may be centered on interactive/active work, capable of immersing the student in the world of understanding, enabling them to develop cognitive skills and logical and deductive reasoning (Brasil, 2017).

In this context, studies indicate that digital technology, when related to teaching through a coherent didactic proposal, is capable of enhancing student learning and becoming an important ally of mathematics and other areas of knowledge through the pursuit of studies that highlight the construction of academic concepts (Papert, 1980; Resnick, 2007; Serra, 2022; Santos, 2016; Lima & Silva, 2015; Almouloud, 2005, among others). Papert (2008) suggests that digital technology can change the way children learn mathematics. Win (2011), Papert (1980), and Resnick (2007) emphasize that the use of such supports can actively corroborate the development of cognitive skills, enhancing the learning of mathematical concepts and computational thinking.

Our concern with this topic aims to support research in mathematics education that addresses the use of technological tools (especially digital ones) for modeling mathematical objects. Therefore, the 5th dimension model of a didactic problem seeks to support inquiries into the construction of models using these technological resources for mathematics teaching and learning, given that official documents demand increasingly efficient answers when the subject is the use of such tools in education. The four pillars of the 5th dimension, associated with the language dimension and the C. Subject dimension, systematically seek to ensure that the teacher/researcher makes an assertive choice of the technological object, as well as a modeling mediated by praxeological blocks, which are, therefore, fundamental and mediating tools, showing that the transpositional (didactic/computer) process is the guiding thread in the modeling process of mathematical activities.

It is reasonable to infer that the technological model presents some similarities and differences from the models described in the previous section. By drawing a parallel between the 5th dimension model and ‘didactic-computer’ engineering (Santos, 2016), we can see that the analyses performed in requirements engineering (RE) are similar to the technological conception (TC) presented by the 5th dimension. However, in the context of the technological dimension, this analysis is carried out by the teacher/researcher who, after verifying the potential of the technological object, tests the software and seeks to adapt the application’s utilities to mathematical problems for modeling that will be carried out through transpositional processes, unlike what occurs in Santos (2016), who performs an a priori analysis basically through interested users (teacher and students).

On the other hand, the technological dimension, unlike 'didactic-computer engineering', does not aim to create educational software, but rather to analyze those that have potential for mathematics teaching and learning (whether or not they were created for this purpose), as is the case with *Scratch*, for example, which originated in the field of computer science, designed to encourage the teaching of programming to children from the age of eight, but which also offers opportunities for learning academic concepts.

Beyond the relationship between Santos's (2016) work and the technological dimension, what similarities and/or differences might also exist between the PCK (Shulman, 1986), TPCK (Mishra & Koehler, 2006), and TDCK (Lima & Silva, 2015) models and the 5th dimension model of a didactic problem?

As seen, TPCK and TDCK are based on Shulman's initial ideas (1986); therefore, they share similarities with PCK. The distinguishing feature of Mishra and Koehler (2006) was to add digital technologies to Shulman's (1986) model and to consider four new knowings (TK, TCK, TPK, and TPCK). On the other hand, Lima and Silva (2015), based on the ideas of these latter authors, introduced the didactics of mathematics (DM), understanding that pedagogical knowledge, although important, is more general and, by itself, does not achieve the specific objectives of the subjects, since it deals with processes and practices or methods of teaching and learning in a global way (general pedagogical knowledge) and not specifically related to a particular content or area of knowledge. Furthermore, "pedagogy is assigned the management of the classroom situation, i.e., the interactions between the teaching situation, the teacher, and the students, while the didactics of mathematics deals with the relationships between students, teacher, and knowings" (Lima & Silva, 2015, p. 163).

In this sense, drawing on the theoretical contributions discussed (PCK, TPCK, and TDCK), our study is more closely aligned with TDCK and the ideas by Lima and Silva (2015), since, in the context of the 5th dimension, the theoretical contributions of DM (ATD, DT, and CT) are central and enable the processes of mathematical modeling, in addition to observing the personal and institutional relationships at play (Chevallard, 2022). Furthermore, "technological didactic content knowledge (TDCK) allows the teacher to analyze, based on the theories of the didactics of mathematics, how various technologies can be used for teaching and learning a given mathematical object" (Silva & Lima, 2015, p. 172). Drawing a parallel between the TDCK and the 5th dimension, this process of verifying the use of the digital technological object occurs as the teacher/researcher, guided by the C. Subject, begins the inquiry based on the a priori analysis provided by the technological conception (TC).

Silva and Lima (2015) emphasize that pedagogical content knowledge (PCK) seeks to relate general pedagogical knowledge to the specific area. In the context of the 5th dimension, this relationship is deepened and mediated by the ATD, which, with its anthropological perspective, brings to the research context the cultural and social relations, as well as the historical and epistemological knowledge regarding mathematical objects, in order to prioritize possible 'didactic' choices that will be important for the modeling. These are processes mediated by the conception of the subject (C. Subject) that seek to balance inquiry within the teacher/researcher's *milieu* during the didactic and computer-based transposition process.

Silva and Lima (2015) emphasize that technological content knowledge (TCK) is knowledge about how technology and a specific content (or field) of mathematics are related. In the 5th dimension model, TCK resembles processes involving knowledge of mathematical conception (MC) and knowledge of the study of the technological object (ETO), which, mediated by the C. Subject, allow the teacher/researcher to analyze the best way to model mathematical objects.

In relation to Shulman's work (2006), we can infer that content knowledge (CK) is to PCK as well as the epistemological dimension (P_1) and the mathematical conception (MC) are for the 5th dimension. However, studies of mathematical objects in the technological dimension are systematically conducted, based on the 'paradigm of questioning the world', because, in an inquiry through the C. Subject, the researcher must be problematizing, Herbartian, procognitive, esoteric, and common encyclopedist (Chevallard, 2022).

Furthermore, the knowledge specific to mathematics didactics goes beyond the pedagogical knowledge (PK) of Shulman's model (1996) and its generalist view, because in the context of ATD, the study also shapes the didactic transposition processes mediated by an inquiry-specific *milieu* related to the research of mathematical objects for modeling purposes.

Regarding TPCK, Mishra and Koehler (2006) propose general ideas about the use of technology in teaching, as the intention of this framework is to provide models of didactic possibilities by associating technological, pedagogical, and content knowledge through design methodologies. However, the authors do not specify, for example, how to develop models of mathematical objects with these digital technologies, as occurs in the 5th dimension model, which suggests the mediation of praxeologies as modeling tools for mathematical concepts.

That being said, we infer that the technological dimension brings possibilities and new guidelines to this research and possibly to others that highlight in their context the use of technological objects for the purpose of modeling mathematical objects, since, together with the other dimensions, it will allow the understanding of the didactic problem and enable decision-making during the inquiry.

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