

Generalization of pattern sequences by elementary school students: limits and possibilities

Generalización de secuencias de patrón por estudiantes de años iniciales: límites y posibilidades

Généralisation des séquences de motifs par des élèves des premières années : limites et possibilités

Generalização de sequências de padrão por estudantes dos anos iniciais: limites e possibilidades

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Abstract

The study investigated the influence of the type of pattern and the location of the unknown element in a sequence on solving tasks requiring algebraic generalization. First to 5th graders of Elementary School were asked to solve a sequence problem with a repetitive pattern and another with a non-repetitive pattern. Each problem contained three questions regarding the position of the unknown element in the sequence: immediate, near, and distant. In all grades, performance was better in the repetitive pattern

sequence than in the non-repetitive, and when the unknown was immediate, followed by the near, and then the distant. The greatest challenge, even for the 5th graders, was solving non-repetitive pattern problems when the unknown was distant. Aspects of the development of algebraic thinking related to generalization were identified, highlighting educational implications and questions for further research on algebraic generalization and pattern identification.

Keywords: Algebraic generalization, Pattern sequence, Unknown, Elementary education.

Resumen

El estudio investigó la influencia del tipo de secuencia de patrón y de la localización de la incógnita en resolución de tareas que requieren la generalización algebraica. Estudiantes de 1º a 5º año de Educación Primaria resolvieron un problema de patrón repetitivo y otro de patrón no repetitivo. Cada problema incluía tres preguntas relativas a la posición de la incógnita: inmediata, próxima y distante. En todos los años escolares, el desempeño fue mejor con el patrón repetitivo que con el patrón no repetitivo, y cuando la incógnita era inmediata, seguida de la próxima y de la distante. El mayor desafío, aún para 5º año, consistió en la resolución de problemas de patrón no repetitivo cuando la incógnita era distante. Se identificaron aspectos del desarrollo del pensamiento algebraico relacionados con la generalización, señalándose implicaciones educativas y cuestiones a ser investigadas en investigaciones sobre la generalización algebraica y la identificación de patrones.

Palabras-clave: Generalización algebraica; Secuencia de patrón; Incógnita; Educación primaria.

Résumé

L'étude a examiné l'influence du type de séquence de motifs et de la position de l'inconnu sur la résolution de tâches nécessitant une généralisation algébrique. Les élèves du CP au CM2 devaient résoudre un problème de

séquence avec un motif répétitif et un autre avec un motif non répétitif. Chaque problème comportait trois questions relatives à la position de l'inconnu : immédiate, proche et lointaine. Dans tous les niveaux, les performances étaient meilleures dans la séquence de motifs répétitifs que dans la séquence de motifs non répétitifs, et ce, lorsque l'inconnu était immédiat, suivie de la proche, puis de la lointaine. Le plus grand défi, même en CM2, était de résoudre des problèmes de motifs non répétitifs lorsque l'inconnu était lointain. Les aspects du développement de la pensée algébrique liés à la généralisation ont été identifiés, et les implications pédagogiques et les questions pour des recherches ultérieures sur la généralisation algébrique et l'identification de motifs ont été soulignées.

Mots-clés : Généralisation algébrique ; Séquence de motifs ; Inconnu ; École primaire.

Resumo

O estudo investigou a influência do tipo de sequência de padrão e da localização da incógnita na resolução de tarefas que requerem a generalização algébrica. Estudantes do 1º ao 5º ano do Ensino Fundamental foram solicitados a resolver um problema de sequência de padrão repetitivo e outro de padrão não-repetitivo. Em cada problema havia três perguntas relativas à posição da incógnita: posição imediata, próxima e distante. Em todos os anos escolares o desempenho foi melhor na sequência de padrão repetitivo do que na sequência de padrão não-repetitivo e quando a incógnita era imediata, seguida da próxima e da distante. O maior desafio, mesmo no 5º ano, consistiu na resolução de problemas de padrão não-repetitivo quando a incógnita era distante. Aspectos do desenvolvimento do pensamento algébrico relativos à generalização foram identificados, sendo apontadas implicações educacionais e questões a serem investigadas em pesquisas sobre a generalização algébrica e a identificação de padrões.

Palavras-chave: Generalização algébrica, Sequência de padrão, Incógnita, Ensino fundamental.

Generalization of pattern sequences by elementary school students: limits and possibilities

Introduction

Traditionally, Mathematics in the early years of schooling has focused on mastering Arithmetic and computational fluency. However, the teaching of Mathematics in early Childhood Education and the initial years of Elementary School has undergone significant changes in recent years. One of these changes concerns the inclusion of algebra as a thematic unit. Both in Brazil, as officially established in the National Common Curriculum Base (Brasil, 2018), and abroad (Nctm, 2000), the development of algebraic thinking has gained space in these segments of schooling that traditionally focused exclusively on the development of arithmetic thinking. One alternative to operationalize this proposal would be to anticipate the teaching of Algebra, reduce the time dedicated to Arithmetic, and teach Algebra after students have mastered arithmetic procedures. Nevertheless, besides being simplistic, this alternative is inadequate because it disregards the possibility of integrating these two fields of mathematical knowledge. For some authors, it is necessary not only to establish a relationship between them, but to emphasize this relationship that already exists, highlighting the algebraic nature of elementary Mathematics (e.g., Carraher & Schliemann, 2018; Schliemann, Carraher & Brizuela, 2007; Vergnaud, 1988). At first, one might think that this articulation consists of a didactic strategy; however, in reality, this articulation is conceptual since it is possible to algebraize Arithmetic.

Mason (2018) titles one of his chapters with a challenging question: How early is too early to teach Mathematics algebraically? The author's answer is that it is never too early, stating that learning Arithmetic requires algebraic thinking, even without the use of symbols (letters and numbers). According to Mason, Arithmetic without generalization is a ritual, while Arithmetic that requires children to understand generalization is mathematics. This chapter presents and discusses a series of activities and

problem-solving methods adopted by early Childhood Education children, used by researchers to investigate and develop algebraic thinking. Through these examples, the author argues in favor of the relationship between Algebra and Arithmetic.

This perspective is based on the results of studies that, in an articulated and integrated way, gave rise to what the literature calls *Early Algebra*, which refers to algebraic thinking related to initial notions presented by children. Blanton and Kaput (2011) point out the importance of developing algebraic reasoning from the early years of Elementary School. However, as Carraher, Schliemann, and Schwartz (2017) clarify, *Early Algebra* is not the same as teaching Algebra earlier, but rather developing mathematical reasoning in a way that involves a more organized and powerful understanding of mathematical structures, in which language and other forms of representation play an important role, not just numerical computation. In this direction, Lins and Kaput (2004) argue that starting to teach Algebra earlier does not mean teaching the same Algebra usually taught to older children, but rather presenting new ways of thinking algebraically and immersing them in the culture of Algebra.

According to Blanton et al. (2007), although difficult to define due to variations among researchers, there is agreement that *Early Algebra* involves two central instances: (i) generalizing, or identifying, expressing, and justifying mathematical structure, properties, and relationships; (ii) reasoning and acting from generalizations. In view of this, the authors state that research on this topic tends to focus on the use of arithmetic as a domain for expressing and formalizing generalizations (called generalized arithmetic), describing functional relationships (functional thinking), and generalizing patterns (numerical, quantitative, geometric). Considered a way of reasoning, *Early Algebra* brings meaning and depth to the understanding of already taught concepts, making it possible to generalize relationships (such as understanding the effect of operations on numbers and the covariation between factors) and mathematical properties (commutativity, for example). Its relevance is also associated with the

development of an advanced understanding of complex concepts necessary for success in subsequent school grades.

One aspect that is constantly emphasized when discussing algebraic thinking is generalization. In fact, the ability to generalize is essential for the development of mathematical knowledge in a broad sense (Krutetskii, 1976), and its acquisition is considered one of the central objectives of Mathematics Education (NCTM, 2000). The relevance of this notion to algebraic thinking is recognized by scholars in the field (e.g., Carraher, Martinez & Schliemann, 2008; Kaput, 1999; Mason, 1996).

Researchers state that algebraic thinking is the process by which generalizations are established, regularities are identified, and predictions are made that go beyond the concreteness of situations, allowing one to identify what is constant or what varies in a sequence of patterns (e.g., Blanton & Kaput, 2005; Canavarro, 2007; Kaput, 2008; Schliemann, Carraher & Brizuela, 2007; Radford, 2010).

Relationships between algebraic and arithmetic thinking are frequently pointed out, with many researchers arguing that arithmetic operations could be introduced as functions (e.g., Carraher & Schliemann, 2016; 2018; Schliemann, Carraher & Brizuela, 2007; Carraher et al., 2006; Carraher, Schliemann, Brizuela & Earnest, 2016). Usiskin (1988), for example, conceives of Algebra as generalized Arithmetic. Despite the relationships between algebraic and arithmetic thinking, Radford and Peirce (2006) comment that algebraic generalization requires transforming a given relation into a general rule, which is not necessary in arithmetic generalization. For Radford (2012), algebraic generalization consists of: (a) identifying common characteristics about some elements of a sequence; (b) generalizing these characteristics to all terms in the sequence; and (c) use these common characteristics to deduce an expression (verbal or mathematical) or some type of symbolic representation that allows predicting or calculating any term in the sequence.

Stacey (1989) and Vale (2013) make the important distinction between pattern sequence tasks involving near generalization and distant

generalization. According to the authors, tasks involving near generalization require finding a nearby term in a sequence, which can be achieved through counting or drawing, for example. On the other hand, tasks involving distant generalization require finding the distant term in a sequence, which demands understanding a general rule. Furthermore, Vale and Barbosa (2019) comment that near generalization employs recursive strategies, while distant generalization requires mobilizing functional relationships between the elements of a sequence. Near generalizations demand inductive processes for their solution, while distant generalizations imply the use of more elaborate deductive processes.

Identifying patterns is fundamental to forming a generalization, since understanding the principles governing the pattern is at the heart of the development of algebraic thinking (Erdogan & Ay, 2022; Kaput, 1999; Du Plessis, 2018; Stacey, 1989; 2006; Zazkis & Liljedahl, 2002). Patterns consist of a systematic repetition or a systematic change in a mathematical structure. In repetitive patterns, the elements or terms of a sequence are repeated cyclically (Threlfall, 2005).

Lian and Yew (2011) discuss the different configurations that patterns can assume in a task. For example, repetitive patterns can involve only one attribute (color, shape, size, numbers, letters, etc.) or more than one (shapes of different sizes, shapes of different colors, etc.); as well as involving long sequences (with several terms) or short sequences (with only two terms). The combination of these aspects makes the sequence simpler or more complex. The relationships between the terms of the patterns can be characterized by: (i) being increasing (5, 8, 11, 14, ...) or decreasing (14, 11, 8, 5, ...); (ii) changing in an increasing way (3, 5, 8, 12, 17, ...) or decreasing way (17, 12, 8, 5, 3, ...); and (iii) changing geometrically (3, 9, 27, 81, ...).

Santana and Magina (2025) compared the performance of 4th and 6th graders in solving tasks involving sequences of an increasing repetitive patterns. The sequences consisted of numerical and iconic terms. In general, the data revealed a progression between school years, but whether

the sequence was numerical or iconic was not a determining factor in performance, with the location of the unknown having a more significant impact on the success of students who had difficulties with distant unknown elements.

Amit and Neria (2007) analyzed the generalization strategies, justifications, and notations adopted by 11-13-year-old students, proficient in Mathematics, in solving pattern sequence problems. Three tasks were presented, each involving items related to near and distant generalizations. Through a qualitative analysis of the procedures used by the participants, it was possible to identify two generalization strategies: (i) the global-recursive strategy, of a local and operational nature, allowed for the operational expansion of sequences, but in a limited way because it was effective in cases of near generalization; and (ii) the global-functional strategy, of a conceptual nature, being much more effective than the recursive strategy because it guaranteed correct answers in all cases, including those related to distant generalization, allowing participants to identify the position of any term in the sequence, discovering the variables, constants, and their relationships. In general, participants showed flexibility in transitioning between different forms of representation (pictorial, symbolic, and verbal) and tended to substitute additive procedures for multiplicative ones, which were more effective in solving the tasks. The study reiterated the central role played by generalization in algebraic reasoning.

While Amit and Neria (2007) were interested in investigating students with a good level of mathematical knowledge, other scholars turn to younger children, with a view to examining the emergence of algebraic thinking (e.g., Borralho et al., 2007; Papić, Mulligan & Mitchelmore, 2011; Rodrigues & Serra, 2015).

For example, Rodrigues and Serra (2015) investigated how algebraic thinking emerged in four-year-old children, examining the strategies they adopted to create, identify, analyze, and generalize repetitive patterns. This qualitative study was conducted in a school setting, with data recorded in

audio, video, and photos of the children's productions when solving tasks involving patterns. The resolution procedures were analyzed, as well as the dialogues held between children and the teacher and between the children themselves. The authors highlight the importance of activities that require children not only complete offered sequences of patterns, but also produce repetitive patterns. They considered these activities an approach that facilitated the children's understanding of the concept of patterns and their awareness of the structure of the sequence. According to Borralho et al. (2007), learning patterns in preschool help develop logical thinking, being a way to explore other mathematical content and create a foundation for future algebra learning.

This perspective is reiterated by Papic, Mulligan, and Mitchelmore (2011), who emphasize that exploring repetitive patterns in early Childhood Education is an appropriate approach for this school stage, as evidenced in the intervention conducted by these authors. In this study, the children in the experimental group demonstrated a greater understanding of the unit of repetition of the patterns than those in the control group, both in the post-test and in follow-up assessments one year after the intervention.

Warren (2005) examined the teacher's actions aimed at stimulating nine-year-old students to consider repetitive patterns as a covariation relationship. The results revealed that explicit instruction about this relationship helped students identify the patterns and express this relationship abstractly. The conclusion was that pattern problems are very effective in revealing the ability to generalize and symbolize, and promoting the development of algebraic thinking.

Objectives and purpose of the study

The studies presented highlight the importance of identifying patterns and understanding their logic in the production of a generalization, which is central to algebraic thinking. This relevance is reiterated in research conducted with preschoolers and Elementary School students, in which

repetitive pattern sequences are presented to the children. Some authors describe the different configurations repetitive patterns can have in a task, while others discuss the location of the unknown element, which is the term that must be discovered in a sequence, making the important distinction between tasks involving near generalization and distant generalization. These points, taken together, reveal the need to investigate algebraic generalization through the association between different types of pattern sequences and the location of the unknown in Elementary School students. A study with this configuration can reveal the limits and possibilities of children's reasoning in relation to their ability to make algebraic generalizations and contribute to educational implications.

Based on these considerations, this research aimed to examine the influence of the type of pattern sequence and the location of the unknown element in a sequence on the performance of students from the 1st to the 5th grade of Elementary School in solving tasks that require algebraic generalization. To this end, the following aspects were considered:

(i) the nature of the sequence, investigating sequences with repetitive and non-repetitive patterns. The hypothesis is that children perform better in the first type of sequence than in the second. While sequences with repetitive patterns are widely addressed in research in this area (see Lian & Yew, 2011), sequences with non-repetitive patterns are less considered. In the present study, a non-repetitive pattern would involve a constant in all terms of the sequence; that is, each term of the sequence involves a relationship between the constant and another aspect that varies (example: one black ball and two white balls in the first position, one black ball and four white balls in the second position, one black ball and six white balls in the third position, ...). This sequence is associated with an affine function.

(ii) the location of the unknown in each of the presented sequences: immediate, near, and distant. The immediate unknown is an aspect that has been poorly examined in pattern sequence tasks, but it is considered relevant and deserves to be investigated, as pointed out by Merlini, Spinillo, and Magina (2025). These locations represent different levels of generalization required for solving algebraic problems. The hypothesis is that children perform better when the unknown is immediate than when the unknown is near or distant.

(iii) the participants' educational level: students from the 1st to the 5th grade of Elementary School. The hypothesis is that performance improves with increasing grade level. The prediction is that students in the 1st to 3rd grades will have great difficulty with non-repetitive pattern sequence problems where the unknown is distant, but this difficulty will be overcome by students in the 4th and 5th grades.

Method

This study is a descriptive and comparative empirical research of a quantitative nature. It sought to examine how students' performance on pattern sequence tasks varies with their level of schooling. The data analysis focused on the performance of a large number of participants rather than the problem-solving processes they adopted. This justifies the choice of a quantitative approach, which allows one to map progress and limitations in relation to the presented tasks.

The study included 985 children of both sexes, students from the 1st to the 5th grade of Elementary School in public schools in five Brazilian cities from three regions of the country: south (Rio Grande and Pelotas), southeast (Rio de Janeiro), and northeast (Ilhéus and Feira de Santana). The participants were distributed as follows: 130 students from the 1st grade, 149 from the 2nd grade, 214 from the 3rd grade, 222 from the 4th

grade, and 270 from the 5th grade. The children did not present sensory limitations or any type of neurodevelopmental disorder; these aspects were considered exclusion factors for participation in the study. Participants voluntarily signed the Informed Assent Form (TALE-LÚDICO), and their guardians signed the Informed Consent Form (TCLE). The project was approved by the Research Ethics Committee (CAAE 58016222.9.0000.5526), meeting all legal and ethical requirements in the field of Human Sciences.

Although the participants came from cities in different regions of the country, this fact was not analyzed in this investigation since the objectives were not related to the role of possible regional differences in the performance of the tasks. While the regional variable is relevant and could be investigated in future research, the purpose of including participants from different regions in this study was to obtain a broader sample.

In a single session, with free time, participants were asked to perform a task consisting of two items: one involving a repetitive pattern sequence (Item 1) and the other a non-repetitive pattern sequence (Item 2). Each item had three questions relating to the location of the unknown element: immediate (Question A), near (Question B), and distant (Question C), as shown in Figure 1.

The immediate unknown is the unknown term positioned immediately after the last known term presented in the item. The relationship between the position and the term found there can be easily established from the representation supports presented. The near unknown is the unknown term whose position is further away from the last known term presented in the item. The relationship between the position and the term found there can be established without necessarily involving generalization, through the production of unknown terms until the term to be known is reached. The distant unknown is the unknown term whose position is significantly distant

from the last known term presented in the item. The relationship between the position and the term to be known can only be established through generalization, which allows one to deduce any unknown term without producing all the terms to reach it.

Figure 1

Item 1 (repetitive pattern sequence), Item 2 (non-repetitive pattern sequence)

João has many marbles. He always arranges them like this: first comes the blue one, then the red one, and then the yellow one. And then he continues arranging his marbles, always following this order of colors.



Question A (immediate): "Draw the marble that will be in position 6. Color it, following the order of João's colors."

Question B (near): "Draw a marble and, following the order of João's colors, color the marble with the color it will have in position 8."

Question C (distant): "And if it were position 18? What would the color of the marble be? Draw it and color it with the correct color."

Observe the sequence of figures formed by the marbles that Beto made.



Question A (immediate): "What is the next figure in this sequence?"

Question B (near): "What is the figure in the 10th position?"

Question C (distant): "What is the figure in the 20th position?"

The task was performed individually and administered collectively in the classroom, in the presence of the examiner and the classroom teacher. Each participant received a pencil, an eraser, and a small booklet. Each page of the booklet contained one of the items, its respective questions, and space to answer. The examiner read each question aloud, and the items were solved one at a time in a fixed order. The child followed along with the reading, and explanations and clarifications were provided for each item and its questions.

Results

In total, 5,910 responses were analyzed. The data were analyzed in two ways: based on the number of correct answers and according to the participants' performance profile. Both forms of analysis took into account school grades, item types (repetitive and non-repetitive pattern sequences), and question types related to the location of the unknown (immediate, near, and distant).

There was a gradual increase in the percentage of correct answers in each schooling level: 23.9% in the 1st grade, 38.03% in the 2nd grade, 53.71% in the 3rd grade, 63.42% in the 4th grade, and 70.19% in the 5th grade. In general, students in the first two grades answered less than 50% of the questions correctly, while those in the last two grades answered more than 60% of the questions correctly. The children in the 3rd grade, in turn, presented an intermediate percentage of correct answers.

As shown in Table 1, and confirmed by the Wilcoxon test, the pattern of results was the same in all school grades, since students systematically performed better on items with repetitive sequence patterns than on items with non-repetitive sequence patterns (p -values < 0.001).

Table 1

Percentage of correct answers in both types of sequences in each school grade.

	Repetitive pattern sequence	Non-repetitive pattern sequence
1st grade	39.28	8.53
2nd grade	44.07	31.99
3rd grade	59.56	47.85

4th grade	62.81	48.02
5th grade	73.95	66.66

The Kruskal-Wallis test evidenced significant differences between school grades in both repetitive pattern sequences ($\chi^2 = 200.84$, $df = 4$; $p < 0.001$) and non-repetitive pattern sequences ($\chi^2 = 258.27$, $df = 4$; $p < 0.001$). According to Dunn's test, this occurred because significant differences were identified between school grades in both types of sequences ($p < 0.001$), except for the 3rd and 4th grades, where the percentage of correct answers was similar in both repetitive (59.56% and 62.81%, respectively) and non-repetitive (47.85% and 48.02%, respectively) sequences. These results suggest that performance was similar in the 3rd and 4th grades.

In Table 1, the highest performance was in the fifth grade for the repetitive pattern sequence (73.95%), and the lowest performance was in the first grade for the non-repetitive pattern sequence (8.53%). Additionally, as students advanced in their schooling, they showed progress in their performance, even in the non-repetitive pattern sequence, which was the most difficult.

The relationships between the distance of the unknown (Question A: immediate, Question B: near, and Question C: distant) and school grades are presented in Table 2. As can be seen, the distance of the unknown played a determining role in overall performance, as revealed by the Friedman Test ($\chi^2 = 510.62$, $df = 2$, $p < 0.001$). Students in the 5th grade performed best regarding the nearby unknown, with 85.17% of correct answers, while 1st grade students performed worst regarding the distant unknown, with only 15.51% of answers correct.

Table 2

Percentage of correct answers to questions in each school grade.

	Question A (immediate)	Question B (near)	Question C (distant)
1st grade	30.62	25.59	15.51
2nd grade	49.00	37.92	27.18
3rd grade	70.28	53.93	36.92
4th grade	76.24	63.79	50.23
5th grade	85.17	71.23	54.18

The Wilcoxon test identified that in each school grade, the percentage of correct answers in Question A (immediate unknown) was significantly higher than in the other questions, while the percentage of correct answers in Question C (distant unknown) was invariably lower than that observed in the other questions (p -values < 0.001)

Comparisons between school grades in each question were examined using the Kruskal-Wallis test, which revealed that in each of the three questions there was a significant increase in the percentage of correct answers between school grades (Question A: $\chi^2 = 245.6$, $df = 4$, $p < 0.001$; Question B: $\chi^2 = 211.24$, $df = 4$, $p < 0.001$; Question C: $\chi^2 = 204.12$, $df = 4$, $p < 0.001$).

What can be observed from these data is that the pattern of results is similar across all school grades investigated, and that the type of

sequence and the location of the unknown contribute to this. Therefore, it is interesting to examine in greater detail the students' performance in relation to these two factors, considering all participants together, as shown in Table 3.

Table 3

Percentage of correct answers to questions in each type of sequence

	Repetitive pattern sequence	Non-repetitive pattern sequence
Question A (immediate)	70.84	53.68
Question B (near)	58.27	42.71
Question C (distant)	38.58	35.02

In general, the best performance occurred in Question A (immediate unknown) in the repetitive pattern sequence (70.84%), while the poorest performance was observed in Question C (distant unknown) in the non-repetitive pattern sequence (35.02%). The only significant difference between the type of sequence in each question was observed only in relation to the percentage of correct answers in Question A (immediate unknown), which was higher in the repetitive pattern sequence (70.84%) than in the non-repetitive pattern sequence (53.68%), as revealed by the Wilcoxon test (p -values < 0.01).

Taken together, the results suggest that both the types of sequences and the distance of the unknowns are determining factors in the performance of all schoolchildren. Although there is progression with advancing grades, the pattern of results remains consistent, indicating that the nature of the difficulty remains the same from the 1st to the 5th grade. The greatest challenge for students is dealing with distant unknowns in situations involving non-repetitive sequence patterns.

Performance profiles of participants in each type of sequence

As mentioned, in addition to the number of correct answers, the data were analyzed based on the participants' performance profile in the three questions in each type of sequence. These profiles were defined based on the level of generalization required to correctly answer the three questions, namely: Question A (immediate) did not require generalization, Question B (near) required an intermediate level of generalization, and Question C (distant) required the highest level of generalization.

Through this analysis, we sought to examine whether the participants who correctly answered the items related to the distant unknown (Question C), thus expressing a notion of algebraic generalization, were also the ones who correctly answered the other two questions (Question B: near and Question A: immediate). Taking this into account, four participant profiles were identified:

Profile 1: composed of participants who correctly answer Question C (distant), incorrectly answer Question B (near), and Question A (immediate).

Profile 2: composed of participants who correctly answer Question C (distant unknown) and incorrectly answer Question B (near).

Profile 3: composed of participants who correctly answer Question C (distant unknown) and incorrectly answer Question A (immediate).

Profile 4: composed of participants who correctly answer Question C (distant unknown), correctly answer Question B (near) and Question A (immediate).

Table 4 lists the distribution of these profiles in relation to each type of sequence.

Table 4

Percentage of participants of each profile in each type of sequence.

	Repetitive pattern sequence	Non-repetitive pattern sequence
Profile 1	4.57	1.22
Profile 2	8.52	2.47
Profile 3	12.55	3.60
Profile 4	22.10	26.71

Note: Profile 1: participants who correctly answer Question C (distant), incorrectly answer Question B (near) and Question A (immediate). Profile 2: participants who correctly answer Question C (distant) and incorrectly answer Question B (near). Profile 3: participants who correctly answer Question C (distant), incorrectly answer Question A (immediate). Profile 4: participants who correctly answer Question C (distant), correctly answer Question B (near) and Question A (immediate).

Profile 1 (answers Question C correctly, answers Questions A and B incorrectly) was the least frequent, a finding observed in both types of sequences (repetitive pattern: 4.57%, non-repetitive: 1.22%). This indicates that a participant who answered Question C correctly (distant unknown) rarely answered the other questions incorrectly. On the other hand, the percentage of children in Profile 4 (answers Question C correctly, answers Questions A and B correctly) was the highest, both in the repetitive pattern sequence (22.10%) and in the non-repetitive pattern sequence (26.71%). This result shows that children who answered Question C correctly (distant unknown) tended to answer the other questions correctly.

Since the percentages of children from Profile 1 and Profile 4 in the two types of sequences were very close (see Table 4), it is possible to assume that the way of dealing with the location of the unknown is independent of the type of sequence. In other words, when the child demonstrates a sufficient notion of generalization to successfully handle items where the unknown is distant, they apply this notion both in solving

sequences with repetitive patterns and in sequences with non-repetitive patterns, which are the most difficult. It seems that the nature of the pattern has less impact on performance when the child is able to generalize. However, this statement needs to be taken with caution, and it is necessary to investigate a greater diversity of sequence tasks with repetitive and non-repetitive patterns.

Discussion and Conclusions

This study investigated the influence of the type of pattern sequence (repetitive and non-repetitive) and the location of the unknown (immediate, near, and distant) on the performance of students from the 1st to the 5th grade of Elementary school in solving problems that require algebraic generalization. Although the study is not procedural in nature, the discussions are strongly supported by available scientific literature, which highlights the relationship between the location of the unknown and the procedures employed by the students.

Generalization was examined based on what Stacey (1989) termed near and distant generalizations. These generalizations are based on the location of the unknown in problems involving algebraic reasoning. Near and distant generalizations differ in their complexity, since distant generalizations demand a higher level of abstraction from children than near generalizations do. This assertion is supported by studies on problem-solving involving functional reasoning, such as the research by Merlini, Spinillo, and Magina (2025) on linear and affine functions. The authors conclude that another type of unknown must be investigated: the immediate unknown, which is explored in the present research. The immediate unknown is the unknown term positioned immediately after the last known term in the item. The relationship between the position and the term found there can be easily established from the representation supports

provided. This investigation included the immediate unknown to explore its role in children's algebraic thinking.

Considering the theoretical perspective of Stacey (1989) and the evidence presented by Merlini, Spinillo, and Magina (2025), another assumption that served as the basis for this investigation was the idea advocated by Vergnaud (1994; 1997) that, broadly speaking, it is necessary to investigate mathematical reasoning in different situations. This assumption has characterized the methodological design of numerous studies in the area, creating investigative scenarios in which participants solve different types of problems within the same field of mathematical knowledge. In this regard, two types of algebraic problems involving repetitive and non-repetitive sequence patterns were examined. The results obtained are presented and discussed in relation to these aspects, revisiting the raised hypotheses.

Regarding the nature of the sequence, the hypothesis was that children perform better on the repetitive pattern sequence than on the non-repetitive pattern sequence. This hypothesis was confirmed, as observed in relation to each grade level, indicating that the pattern of results remained consistent from the 1st to the 5th grade. However, there was progress between school grades in both types of sequence. This progress was characterized by advances before the 3rd grade and after the 4th grade, indicating that from the 3rd to the 4th grade, performance is relatively stable. Even in the 5th grade, the non-repetitive pattern sequence remained challenging for some students.

The hypothesis that performance is best in items where the unknown is immediately located was confirmed in each school grade. The data showed that student success decreased from the immediate unknown to the near unknown, and then to the distant unknown. These results support the notion that locating the unknown involves distinct levels of

generalization, increasing from the near to the distant unknown, as Stacey (1989) proposed. Nevertheless, Merlini, Spinillo, and Magina (2025), in a study on solving linear and affine function problems, observed that students from 1st to 5th grade performed similarly when the unknown was near or distant. In concluding this investigation, the authors highlight the importance of examining another type of unknown: the immediate unknown. They argue that, despite not involving generalization, the role of the immediate unknown in solving algebraic problems must be examined. Thus, the present study incorporated the immediate unknown into the resolution of repetitive and non-repetitive pattern sequence problems. The results indicated an effect of the location of the unknown on performance for both types of problems.

The results obtained here diverge from those reported by Merlini, Spinillo and Magina (2025) regarding the role of the location of the unknown in solving algebraic thinking problems. One possible explanation is that linear and affine function problems are more difficult than repetitive and non-repetitive sequence problems. In that study, students had significant difficulty solving the problems, so performance was equally limited regardless of the unknown's location. In the present study, the repetitive and non-repetitive pattern sequences were less complex, allowing the effect of the unknown's location to become evident. The effect of the location seems to emerge when algebraic problems are not beyond the children's ability to solve them.

In terms of the development of algebraic thinking, the analysis of the participants' profiles revealed that those who performed well with the distant unknown (a higher level of generalization) could successfully solve questions with immediate and near unknowns. However, the reverse was not true. This result reveals a progression that can be explored didactically, as discussed below. Furthermore, as previously mentioned, the impact of

whether the pattern is repetitive or non-repetitive on performance diminishes when a child achieves a high level of generalization, as they seem capable of making generalizations about a wide variety of tasks. However, this statement needs to be taken with caution, as it is necessary to conduct investigations examining a greater diversity of repetitive and non-repetitive pattern sequences than the present study examined.

Another aspect that deserves to be addressed is the role of the immediate unknown in algebraic thinking. As mentioned, this unknown element does not require generalization, since it is positioned immediately after the last known term presented in a given sequence. In this case, the unknown can be determined simply from the representation supports presented, not requiring a rule that encompasses all possible cases. However, according to our analysis, it can play an important role in developing algebraic thinking in instructional situations, especially with students in the early years of Elementary School, serving as a starting point for near and distant generalization. Assigning a relevant role to the immediate unknown theoretically broadens the perspective that emphasizes only near and distant generalization (see Vale, 2013; Stacey, 2013) and expands the educational implications in applied terms.

Kieran (2022) argues that interest in developing algebraic thinking in the early years began to emerge in the 1990s. Since then, there has been a steady growth in research dedicated to exploring ways to promote this thinking. Initially, the interest focused primarily on addressing the question of which types of algebraic thinking would be feasible for students at the beginning of their schooling. Over the last 30 years, the author continues, research has evolved to encompass a wide range of truly multidimensional activities, with generalization as the common thread. The present research reiterates the importance of generalization associated with another facet of algebraic thinking: the sequence of patterns. This reiterates the role played

by the location of the unknown, which materializes generalization in this investigative scenario.

From an applied point of view, it would be interesting to propose didactic situations in which:

(i) the location of the unknown was manipulated so that students were presented with situations involving immediate, near, and distant unknowns. Carraher, Schliemann, and Brizuela (2001) comment that manipulating unknowns is fundamental to developing the ability to make algebraic generalizations.

(ii) initially, easy situations were presented, such as repetitive pattern sequences involving immediate, near, and distant unknowns, in that order. Then, more challenging situations involving non-repetitive pattern sequences with the three types of unknowns in that order.

(iii) in addition to activities in which they had to continue the sequence by completing what is missing, students could also be asked to produce the patterns themselves, as done by Rodrigues and Serra (2015).

(iv) the teacher explained the structure of the sequence to the students, highlighting the configuration of the pattern, the terms, and their relationships. In other words, the sequence was the subject of the students' reflection and analysis, as suggested by Warren (2005), Mulligan and Mitchelmore (2009), and Du Plessis (2018).

(v) lead students to discover the general rule that would apply to all presented cases, as discussed by Carraher and Earnest (2003) when reporting a classroom experiment in which students were asked to discover the underlying rule of each situation.

The points mentioned above require, as can be noted, an active role on the part of the teacher, as well as consideration of the limits and possibilities of the students, according to the results obtained in this study.

Finally, the importance of two key aspects of algebraic thinking is reiterated: generalization and pattern identification. Generalization is both an instructional objective and a means to achieve more sophisticated reasoning. In turn, pattern identification and the discovery of the logic that governs these patterns are fundamental to arriving at a generalization.

The close association between generalizing and identifying patterns is highlighted by Mason et al. (1985, p. 8) when they state that "Algebra is the language through which generality is expressed. To learn the language of Algebra, it is necessary to have something to say. It is necessary to identify some pattern or regularity, and then try to express it succinctly, so that it is possible to communicate a given perspective to someone, and use it to answer specific questions." Though this statement was made four decades ago, it remains relevant and serves as a premise for conducting research with children about algebraic thinking.

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