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Design of a rubric for the evaluation of practices with modelling in the early years

Diseño de una rúbrica para la evaluación de prácticas con modelado en los primeros años

Conception d'une grille d'évaluation des pratiques de modélisation dans les premières années

Desenho de uma rúbrica para avaliação de práticas com modelagem nos anos iniciais

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Abstract

This article aimed to present the design of a rubric for assessing practices with Modelling in the Early Years, focusing on the contributions of this approach to the teaching and learning of Mathematics. The instrument, named the Rubric for Assessing Mathematical Modelling Practices in the Early Years (RAPMAI), was developed based on the Rubric for Assessing Mathematical Modelling Processes (REMMP) and contributions from a Mathematical Modelling perspective, grounded in Burak's work (2010). The results indicate that the RAPMAI highlights the importance of Mathematical Modelling practices, considering the unique characteristics of the Early Years, children's interests, and the crucial role of research, investigation, and collaborative work in this stage of schooling. Additionally, it helps teachers understand assessment as an integral and ongoing part of Modelling practices. This context strengthens children's autonomy, fosters

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emotional connections, and encourages more meaningful and collaborative learning by promoting problem formulation and decision-making. Thus, the developed rubric proved to be an essential tool for understanding and monitoring these advancements, which go beyond mathematical learning and contribute to the development of social, emotional, cognitive, and ethical skills in children, while also promoting a more responsible, sensitive, and committed teaching practice aligned with children's interests.

Keywords: Modelling in mathematical education, Early years, Pedagogical practice, Assessment rubric, Assessment in modelling practices.

Resumen

Este artículo tiene como objetivo principal presentar el diseño de una rúbrica para evaluar prácticas con Modelización en la Educación Primaria, centrada en las contribuciones de este enfoque metodológico para la enseñanza y el aprendizaje de las Matemáticas. El instrumento, denominado Rúbrica para la Evaluación de Prácticas con Modelización en la Educación Primaria (RAPMAI), fue elaborado a partir de la Rúbrica para la Evaluación de Procesos de Modelización Matemática (REMMP) y de aportes de una perspectiva de Modelización Matemática en la Educación Matemática. La RAPMAI resalta la relevancia de las prácticas con Modelización Matemática al considerar las singularidades de la Educación Primaria, los intereses de los niños y el papel fundamental de la investigación, la indagación y el trabajo colaborativo en esta etapa escolar. Asimismo, favorece que el profesorado comprenda la evaluación como parte integral y continua de las prácticas de modelización. De este modo, la rúbrica desarrollada se constituye en un instrumento esencial para comprender y acompañar los avances de los estudiantes, los cuales trascienden el aprendizaje matemático y contribuyen al fortalecimiento de habilidades sociales, emocionales, cognitivas y éticas en la infancia, promoviendo además una práctica docente más responsable, sensible y comprometida con los intereses de los niños.

Palabras clave: Modelización en la educación matemática, Años iniciales, Práctica pedagógica, Rúbrica de evaluación, Evaluación en las prácticas de modelización.

Résumé

Cet article a pour objectif principal de présenter la conception d'une grille d'évaluation des pratiques de modélisation dans l'enseignement primaire, en mettant l'accent sur les contributions de cette approche méthodologique à l'enseignement et à l'apprentissage des mathématiques. L'instrument, dénommé Grille d'Évaluation des Pratiques de Modélisation à l'École Primaire (RAPMAI), a été élaboré à partir de la Grille d'Évaluation des Processus de Modélisation Mathématique (REMMP) et des apports d'une perspective de modélisation mathématique en didactique des mathématiques. La RAPMAI met en évidence l'importance des pratiques de modélisation mathématique en tenant compte des spécificités de l'école primaire, des intérêts des enfants ainsi que du rôle fondamental de la recherche, de l'investigation et du travail collaboratif à ce stade de la scolarité. Elle contribue également à ce que l'enseignant conçoive l'évaluation comme une partie intégrante et continue des pratiques de modélisation. Ainsi, la grille développée s'avère être un instrument essentiel pour comprendre et suivre les progrès des élèves, lesquels dépassent l'apprentissage mathématique et participent au développement de compétences sociales, émotionnelles, cognitives et éthiques chez les enfants, tout en favorisant une pratique enseignante plus responsable, sensible et engagée envers les intérêts de ces derniers.

Mots-clés: Modélisation en didactique des mathématiques, Premières années, Pratique pédagogique, Grille d'évaluation, Évaluation des pratiques de modélisation.

Resumo

Este artigo tem como objetivo principal apresentar o desenho de uma rubrica para avaliar práticas com Modelagem nos Anos Iniciais, com foco nas contribuições dessa abordagem metodológica para o ensino e a aprendizagem da Matemática. O instrumento, denominado Rubrica para Avaliação de Práticas com Modelagem nos Anos Iniciais (RAPMAI), foi desenvolvido com base na Rubrica para Avaliação de Processos de Modelagem Matemática (REMMP) e em contribuições de uma perspectiva de Modelagem Matemática na Educação Matemática. A RAPMAI destaca a importância das práticas com Modelagem Matemática, ao considerar as singularidades dos Anos Iniciais, os interesses das crianças e o papel fundamental da pesquisa, da investigação e do trabalho colaborativo nessa etapa da escolarização. Além disso, contribui para que o professor compreenda a avaliação como parte integrante e contínua das práticas com Modelagem. Assim, a rubrica desenvolvida se revelou um instrumento essencial para compreender e monitorar os avanços dos alunos, os quais transcendem o aprendizado matemático e contribuem para o fortalecimento de habilidades sociais, emocionais, cognitivas e éticas nas crianças, promovendo também uma atuação docente mais responsável, sensível e comprometida com os interesses das crianças.

Palavras-chave: Modelagem na educação matemática, Anos iniciais, Prática pedagógica, Rubrica de avaliação, Avaliação em práticas com modelagem.

Design of a rubric for evaluating Modelling practices in the early years

Introduction

In recent years, incorporating Modelling into curricula has gained prominence in different countries, mainly due to its increasingly important role in real-life applications (e.g., engineering, life sciences, and social sciences) and education (Trelles-Zambrano & Alsina, 2017). According to Kaiser (1995), Modelling and the introduction of information technology are probably two of the most common features in mathematics curricula around the world in recent decades.

The National Common Core Curriculum (Base Nacional Comum Curricular- BNCC) for Elementary Education, approved in 2017, provides guidelines for constructing a curriculum that can facilitate learning at each stage of basic education. According to the document, this requires making decisions involving various actions, including: "Contextualizing the contents of the curriculum components, identifying strategies to present, represent, exemplify, and connect them, making them meaningful based on the reality of the place and time in which learning takes place" (Brazil, 2018, p. 529).

Furthermore, the BNCC points out that.

> [...] students must develop skills related to research processes, model building, and problem solving. To this end, they must mobilize their own ways of reasoning, representing, communicating, and arguing, and, through joint discussions and validations, learn concepts and develop increasingly sophisticated representations and procedures. (Brazil, 2018, p. 529)

These recommendations, drawn from official documents, imply adopting teaching methodologies that go beyond the traditional view of students as passive recipients, professors as knowledge transmitters, and content as an end in itself, divorced from sociocultural contexts. One possibility is Modelling from the perspective of Mathematics Education.

Currently, studies on mathematical Modelling stand out in Brazil (Almeida & Brito, 2005; Almeida & Dias, 2004; Almeida, Silva, & Vertuan, 2012; Barbosa, 2001, 2004; Bassanezi, 2002, 2011; Biembengut, 2009; Biembengut & Hein, 2003; Burak, 1992, 2004, 2010; Burak & Klüber, 2008; Caldeira, 2009). The authors have diverse backgrounds, enabling them to conceptualize and approach mathematical Modelling in various ways in the classroom.

Based on these authors, we seek to elucidate the underlying concepts of teaching and mathematics in each approach. Table 1, inspired by Klüber and Burak (2008, p. 31), summarizes these understandings and highlights the differences in Modelling conceptions between those focused on applied mathematics and those focused on mathematics education.

Table 1

Understanding Modelling: a summary of our understanding (Jocoski, 2024, p. 39)

Authors	Modelling Design
Bassanezi (2002)	Modelling is a teaching and learning strategy that can be understood as the art of transforming real-world problems into mathematical problems and solving them by interpreting their solutions in real-world language. [...] It presupposes multidisciplinarity. And, in this sense, it aligns with new trends toward the removal of boundaries between different areas of research. [...] it is a dynamic process used to obtain and validate mathematical models (Bassanezi, 2002, p. 16).
Barbosa (2004)	Modelling is a learning environment in which students are invited to inquire and/or investigate, through mathematics, situations arising from other areas of reality" (Barbosa, 2001, p. 6).
Meyer, Caldeira and Malheiros (2011)	Modelling is an approach that aims to transform the teaching and learning of mathematics, enabling students to address mathematical problems without definite answers (Meyer, Caldeira & Malheiros, 2011, p. 23).
Almeida, Silva, and Vertuan (2012)	Modelling is a pedagogical alternative in which we use mathematics to approach a problem situation that is not essentially mathematical (Almeida, Silva & Vertuan, 2012, p. 17).
Biembengut (2014)	Modelling is a process used to develop models in any field of knowledge. It is a research process (Biembengut, 2014, p. 21).

Burak (2010)	Modelling is a teaching methodology that "consists of a set of procedures whose objective is to construct a model to attempt to explain, mathematically, the phenomena present in human daily life, helping people to make predictions and decisions (Burak, 2010, p. 18).
Alsina and Salgado (2021)	Initial Mathematical Modelling is a process or cycle that, in the context of solving real problems, helps to create the first models for analyzing, explaining, and understanding reality, based on a process of reflection that involves constant back-and-forth between real contexts and the mathematics that early-age students mobilize (Alsina & Salgado, 2021b, p. 2).

Authors	**Theoretical basis**
Bassanezi (2002)	It does not explicitly state its understanding of the theoretical basis, but it does indicate elements that converge with perspectives from applied mathematics.
Barbosa (2004)	Critical Mathematics Education (Skovsmose, 2001)
Meyer, Caldeira and Malheiros (2011)	Critical Mathematics Education. (Skovsmose, 2001)
Almeida, Silva, and Vertuan (2012)	It does not explicitly state its understanding of the theoretical basis, but it does indicate elements that converge with the perspectives of Mathematics Education.
Biembengut (2014)	It does not explicitly state its understanding of the theoretical basis, but it does indicate elements that converge with the perspectives of Applied Mathematics.
Burak (2010)	Cognitivist orientation: constructivist (Piaget), meaningful learning (Ausubel), and socio-interactionist (Vygotsky). Based on Higginson (1980).
Alsina e Salgado (2021)	Realistic Mathematics Education (Freudenthal, 1991).

Based on our analysis of various understandings of Modelling, we observed different approaches and theoretical foundations reflecting the authors' distinct perspectives. Each author or group of authors presents Modelling with specific emphases and objectives. In this article, we focus on Modelling from a mathematics education perspective.

From this perspective, Modelling is understood as a learning environment, a teaching methodology, or a pedagogical alternative that improves the teaching and learning of mathematics across different educational levels. Regardless of the terminology used, this approach promotes the investigation and critical understanding of mathematical and extramathematical knowledge. It enables social discussions and reflection on the entire Modelling process (Almeida, Silva, & Vertuan, 2012; Alsina & Salgado, 2021; Barbosa, 2004; Burak, 2010; Meyer, Caldeira, & Malheiros, 2011).

Modelling from the perspective of mathematics education aims to overcome the rigid, compartmentalized treatment of mathematical content. It offers a different approach by not following a linear sequence when addressing content, constituting a methodology that "favors understanding fundamental ideas and contributes significantly to perceiving the importance of mathematics in daily life, whether or not one is a mathematician" (Burak & Martins, 2015, p. 102). Similarly, it considers mathematics and its teaching to be social practices. It develops a set of actions that expand the classroom space, guided by principles involving interest, an anthropological vision, and the construction of mathematical and interdisciplinary knowledge (Burak & Klüber, 2016).

This approach is based on the studies and research of Brazilian author Dionísio Burak. He understands it as a teaching methodology that begins with themes or problem situations related to students' daily lives. This methodology provides teaching that is closer to students' experiences, from early childhood education through the early years of schooling. It promotes meaningful mathematics that favors learning (Burak, 2010) and the possibility of moving beyond workbooks and didactic textbooks (Silva & Klüber, 2012). This approach breaks with decontextualized teaching.

Burak (2010) presents two principles for Modelling practices in the classroom: (1) start with the group's interests and (2) obtain information and data in the group's environment. Pedagogical actions based on these principles begin with students' interests in a Modelling practice, because "interest in the activity is directly related to intrinsic motivation and gains strength in a context that nurtures both interest and motivation" (Burak & Klüber, 2013, p. 36).

Modelling practices in the classroom follow a dynamic, cyclical process consisting of five interconnected stages, as proposed by Burak (2010, 2017) and Burak and Aragão (2012). The first stage is choosing a theme based on the students' interests and their reality. The professor plays an essential role in supporting this choice by considering cultural, social, and current aspects, such as sports, technology, or global events.

The second stage is exploratory research, which involves gathering information on the chosen theme using various sources such as books, magazines, websites, interviews, and lectures with experts. This research provides the basis for understanding the theme and identifying problems.

In the third stage, problem identification, children formulate questions based on the collected material, thereby developing autonomy, creativity, and critical thinking skills. This collaborative, interactive process encourages reflection and hypothesis building.

The fourth stage, problem solving and mathematical content development, reverses the traditional logic of mathematics education. Rather than learning concepts first and then applying them, students are presented with problems that drive the need for new knowledge, which they construct with the professor's guidance. Thus, students explore mathematical and non-mathematical concepts in meaningful, contextualized ways.

Finally, critically analyzing the solutions promotes reflection on the feasibility and adequacy of the answers. In addition to mathematical consistency, we consider the applicability of the solutions to the context under study. This process enables a deeper understanding and discussion of different approaches and justifications for procedures, thereby consolidating learning in a meaningful way.

Thus, we reaffirm that the process developed through Modelling practice, as proposed by Burak (2010, 2017) and Burak and Aragão (2012), is cyclical and dynamic. After the critical analysis of solutions, the cycle can restart with the selection of a new topic or the further exploration of the initial one. This restart occurs as new information is acquired or different perspectives emerge throughout the process. Therefore, Modelling is a flexible, interactive process that allows for constant back-and-forth between stages, favoring continuous knowledge construction.

Modelling cycles for mathematics education in the early years

Some studies, including those by Alsina and Salgado (2020), Alsina et al. (2021), and Toalongo-Guamba et al. (2021), examine whether young children in early childhood education and the early years can develop Modelling processes to solve real-world problems.

Based on the principles of Realistic Mathematics Education (Freudenthal, 1991) and the National Council of Teachers of Mathematics (NCTM) competency-based approach, Alsina and Salgado (2021a, 2021b), Alsina et al. (2021b), and Toalongo-Guamba et al. (2021) sought to identify the elements intervening in the Modelling process and the characteristics of the mathematical models young children could create. Assuming that Modelling is a nonlinear, interactive process, these studies are developed through cycles (Carreira et al., 2011; Geiger, 2011; Girnat & Eichler, 2011; Greefrath, 2011; Kaiser, 1995).

Alsina and Salgado (2021) emphasize that only a limited number of studies analyze Modelling processes from this cyclical perspective.

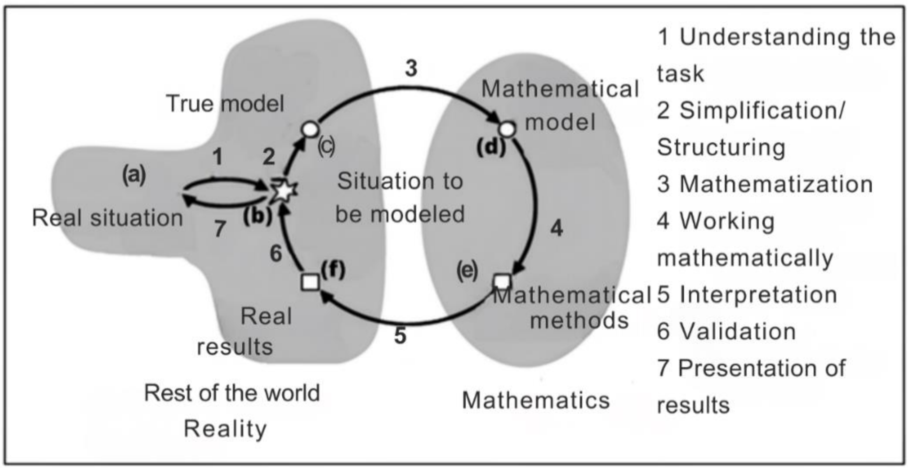
Furthermore, early mathematics curricula do not approach the different fields of mathematics (numbers, geometry, and algebra) in a cross-curricular manner, nor do they involve reflection in the translation process between real-world contexts and mathematics (Trelles-Zambrano & Alsina, 2017). According to the authors, this situation negatively affects the mathematical education of young children because they do not have the opportunity to develop Modelling processes through the solution of real problems.

The authors describe Modelling cycles in various ways depending on different approaches and understandings of Modelling, as well as whether complex or non-complex tasks are used. The structuring of Modelling cycles indicates possible stages in the development of Modelling practices and highlights the nonlinearity of modelers' actions. This means the cycles include back-and-forth movements between stages, which are recurrent and relevant to the development of Modelling practices (Alsina & Salgado, 2021).

Blum and Lei (2007) proposed a Modelling cycle from a cognitive perspective, as illustrated in Figure 1.

Figure 1

Modelling Cycle, adapted and translated from Blum and Leiβ (2007)



The Blum and Leiβ Modelling Cycle (2007) consists of stages such as identifying and formulating mathematical problems, collecting and analyzing data, constructing and validating models, and presenting and interpreting results. This cycle offers children the opportunity to develop crucial skills, including applying mathematics to real-world situations, working in teams, communicating mathematical ideas clearly, and making data- and model-based decisions.

According to the cycle proposed by Blum and Leiβ (2007), based on a real-world situation, students work to understand the problem, which generates a conceptual model in their minds. The next step, simplifying and structuring (2), involves identifying variables and/or conditions that lead to a real model (c). Mathematization (3) leads to the mathematical model (d), a formal mathematical expression of the relationships between the variables without losing sight of the problem's conditions. Then, the mathematical work is carried out until the mathematical results are obtained. These results are interpreted in light of real-world validation.

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These results are interpreted (5) in terms of real results (f) to validate them (6).

Decisions are communicated throughout the Modelling process, and the resulting concrete model is applied to the real context (7). This model is then compared with the conceptual model (b). It is important to note that this process is nonlinear, and students can go from one point to another in the framework without following an established order. This back-and-forth path allows them to refine the desired model. Finally, students should share the mathematical model with their peers, collect relevant observations, and make adjustments to improve the model. This is particularly relevant for younger students, as processes such as interaction, negotiation, dialogue, and knowledge construction are prevalent at this age (Alsina et al., 2021a).

According to Borromeo Ferri (2007), the cycle is illustrative but should not constrain Modelling practices, since students develop their own Modelling routes when carrying them out, implying random shifts between stages. Thus, one of the main characteristics of the Modelling Cycle is that children can start at any point without following an established order.

In a study conducted by Jocoski (2024), we propose analyzing the representations of mathematical concepts, problem-solving actions, and interactions experienced by children in Early Years Modelling practices in mathematics education in terms of assessment tools for Modelling. To this end, we adapted the Modelling Cycle of Blum and Leiβ (2007) to align with the theoretical principles of this approach.

Our adaptation considers that the real world and mathematics are interconnected and that social actions and aspects of children's daily lives can be incorporated into Modelling practice, in addition to the mathematics discussed.

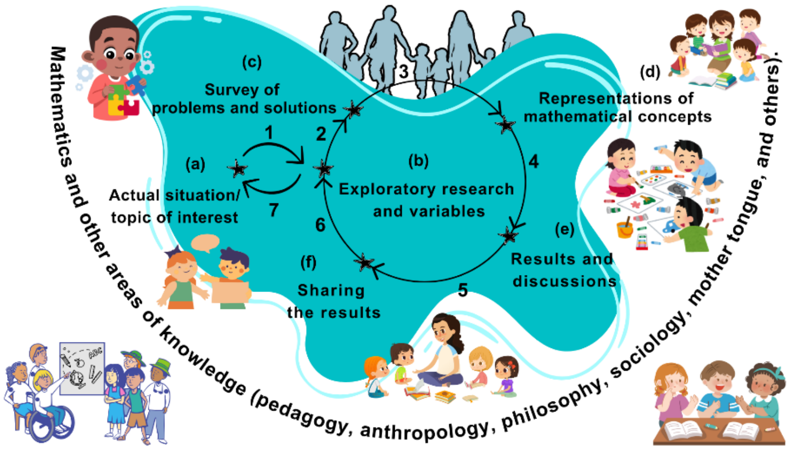
This approach is represented by a dynamic scenario marked by children's active participation, family involvement, and teacher-led mediation and problematization. These elements demonstrate the interactions between mathematics and everyday life, thereby enriching the learning process. The learning environment becomes a living space where ideas intertwine and evolve, providing a rich experience for all participants. Integrating these elements enriches the learning experience and enables children to develop fundamental skills, such as critical thinking, motor and cognitive abilities, problem-solving, representation of mathematical concepts, communication, and socialization of results. Additionally, it promotes more conscious actions in everyday life, facilitates decision-making, and strengthens emotional connections.

We have adapted the Modelling Cycle of Blum and Leiβ (2007) to meet the specific needs of children. This approach is sensitive to their needs and interests, promoting active participation and engagement through topics that interest them, research, and problem-solving. Interacting with topics of interest to children expands opportunities for collaborative, contextualized learning.

Thus, our proposal considers not only mathematical aspects but also the integral development of children in the early years. We value their experiences and prior knowledge and recognize the importance of integrating these elements into the learning process. Figure 2 illustrates our adaptation of the Modelling Cycle of Blum and Leiβ (2007), which guided our research on mathematical Modelling (Jocoski, 2024). It highlights how we adjusted this cycle to meet the specific needs of young children. In this adaptation, we approached Modelling in mathematics education from Burak's (2010) perspective.

Figure 2

Modelling Cycle in the Early Years from a Mathematics Education Perspective (Jocoski, 2024, p. 58)



1 Understanding the task
2 Simplification/ Structuring
3 Mathematization
4 Working mathematically
5 Interpretation
6 Validation
7 Presentation of results

Our Modelling Cycle indicates that the initial organization of the classroom into small groups of three to five members, along with the first interactions between children and professors, will lead to the selection of one or more real-life situations or topics that interest the children. In other words, the children will have the freedom to choose their research topic(s) and will work to understand them using their prior knowledge. This will require the children to conduct research. The organization and research phase involves identifying the variables to be researched, raising problems, and seeking solutions. Through mathematization, the first representations of mathematical concepts will be created to depict real-world situations. At this stage of schooling, these situations can be depicted through drawings, graphs, games, tables, and manipulatives. In this dynamic, mathematical, and extramathematical work, advances are made with new propositions to problems. If necessary, new research and conceptual deepening may emerge until the groups initiate the results and discussions (e) through the interpretation and evaluation phase (5). This phase marks the start of socializing the results (f). Through the professor's mediation, children begin the process of critical dialogue and deepening (6) and communicate decisions made throughout the Modelling practice. They can also socialize their results with the larger group, school, or school community by exhibiting and sharing them (7) through physical or digital resources such as folders, poster displays, videos, and artistic presentations.

This structure can support learning in mathematics by considering the interrelationships between mathematics and other areas of knowledge. It promotes comprehensive training and the development of children in their early years. This approach encourages collaboration among children, stimulates research and problem-solving, and fosters creativity and strong mathematical communication. It also works to democratize access to education.

Active involvement from families and the school community is essential to this process, creating a support network that enriches the educational environment. Family presence and participation not only strengthen emotional bonds and trust but also contribute to contextualized learning by integrating experiences from home and the community into the educational process.

Through this approach, children, teachers, families, and the school community have the opportunity to collectively and collaboratively build knowledge and negotiate meanings. Additionally, this approach helps develop fundamental social and emotional skills for living in society, such as empathy, resilience, and cooperation. Thus, Modelling becomes an ongoing process of constructing and reconstructing knowledge with a focus on children's comprehensive education.

Assessment rubrics for modelling practices in the early years

According to Montgomery (2010), rubrics are assessment tools that use clearly defined criteria and proficiency levels to evaluate student performance. Rubrics are used for both summative and formative assessments. Summative assessments assign grades, while formative assessments help understand student performance.

According to Andrade (2000), instructional rubrics are tools that evaluate student work and support the development of thinking skills, ranging from simple to complex. Most rubrics have two characteristics in common: a list of "what counts" in a project or task and quality ratings for each criterion. These gradations help students understand what constitutes excellent, good, or poor work. There are many benefits to using instructional rubrics: they are easy to use and understand, they clarify professors' expectations, they provide students with useful feedback, and they support students in developing skills (e.g., writing) and a deeper understanding (Andrade, 2020).

The Rubric for the Evaluation of Mathematical Modelling Processes (REMMP) (Toalongo-Guamba et al., 2020; Alsina & Salgado, 2020) is an assessment tool designed for use from early education (ages 3–5) through high school (ages 15–18). It covers a wide age range, so users can easily relate the main characteristics of a Modelling cycle to previous or subsequent educational levels. Regarding assessment, it is important to note that REMMP primarily evaluates students' teamwork since working in small groups is a key characteristic of Modelling practices. However, using REMMP with students individually is not ruled out, although this approach is less common. The validation process for this instrument was carried out by eight Modelling experts from Spain and the United States. The results were analyzed using the Content Validity Ratio (CVR) index, which was first proposed by Lawshe (1975) and later modified by Tristán-López (2008) into the CVR'. (Toalongo-Guamba et al., 2020; Alsina & Salgado, 2020).

The REMMP instrument comprises seven elements corresponding to the phases of the Modelling Cycle proposed by Blum and Leiβ (2007). This cycle allows children to move back and forth without following an established order, enabling them to refine the desired mathematical model (Alsina & Salgado, 2020).

Alsina and Salgado (2020) indicate that the REMMP instrument aims to enable children to relate the content of the problem to their prior knowledge (comprehension); identify the important data in the problem (structuring); present some difficulties in replacing the elements of the real context with mathematical objects (mathematization); progressively use mathematical objects and strategies to propose solutions to the problem (working mathematically); compare the solution with the initial problem (interpretation); justify the proposed mathematical model via valid arguments (validation); communicate the decisions made throughout the Modelling process and the mathematical model obtained applied to the real context (presentation).

Table 2 presents the components and indicators of REMMP for Early Childhood Education (3 to 6 years of age) and Early Years (6 to 12 years of age) according to Toalongo-Guamba et al. (2020).

Table 2

REMMP components and indicators for Early Childhood Education and Early Years (Toalongo-Guamba et al., 2020)

Components	Early Years (a) (3-6 Years old)	Early Years (b) (6-12 Years old)
1. Comprehension	1.1.a. The content of the problem is related to prior knowledge.	1.1.b. Explains the problem to classmates and the professors, showing how the content relates to their prior knowledge.
	1.2.ab. Asks and/or answers questions about the problem.	
	1.3.a. Indicate the type of solution that the problem would generate, for example, a pattern, a number, a graph, etc.	1.3.b. Indicate the type of solution that the problem would generate, for example, a number, a range of values, a set of values, a graph, a formula, a table, etc.

	1.4.a. Represents the main characteristics of the problem through drawings.	1.4.b. Express what the solution to the problem would bring to the environment.
2. Structures	2.1.a. Identifies the main elements of the problem.	2.1.b. Identifies the data that is known, that can be known, and that is unknown in the problem.
	2.2.ab. Proposes ideas and/or assumptions that contribute to simplifying the problem	
3. Mathematical	3.1.ab. Replaces real elements with mathematical objects	
	3.2.a. Explains the use of mathematical objects.	3.2.b. Justifies the use of mathematical objects based on the characteristics of the problem.
		3.3.b. Identifies all the mathematical parameters present in the problem and the different relationships between them
4. Mathematical work	4.1.ab. Uses age-appropriate strategies to propose solutions to the problem.	
	4.2.a. Uses age-appropriate mathematical objects to solve the problem.	4.2.b. Uses mathematical objects and operates with them to solve the problem.
	4.3.ab. Obtains an initial mathematical model as a result of previous work.	
5. Interpretation	5.1.a. Compare the solution with the initial problem.	5.1.b Verifies the consistency of the mathematical solution applied to the initial real-world context.
	5.2.a. Defends the validity of the results obtained.	5.2.b. Identifies possible limitations or restrictions of the mathematical solution in the initial real-world context
6. Validation	6.1.ab. Justify the proposed model using valid arguments.	

	6.2.ab. Evaluate whether the model obtained provides a partial or total solution to the initial problem.	
		6.3.b. Identify whether the model is always valid or whether changes are needed to make it generalizable to new situations.
7. Exhibition/Presentation	7.1.ab. Explains the rationale behind the decisions made during each stage of the process.	
	7.2.ab. Explains the model obtained and applied in the real context, its scope, and limitations in age-appropriate language.	
	7.3.ab. Uses different types of examples, representations, diagrams, drawings, graphs, tables of values, symbolic language, etc.	
	7.4.ab. If the technology is used in one or more stages of the process, clearly indicate at what point, how, and for what purpose it was used.	
	7.5.ab. Listen to comments and/or suggestions made by classmates and/or professors.	
	7.6.ab. Responds to comments and/or suggestions from classmates and professors using age-appropriate language.	
		7.7.b. If approaches were used in the process that did not lead to a solution, reflect on them and share their main aspects.
		7.8.b. Critically analyzes presentations made by classmates.

In childhood Modelling practices using REMMP, children socialized their mathematical models with their peers during the final phase of the cycle. Studies such as those by Toalongo-Guamba et al. (2020) and Alsina and Salgado (2022) have shown that children gather relevant observations and make necessary adjustments to improve their models by sharing their ideas. This exchange of information fosters a deeper understanding and a more collaborative learning environment.

The Modelling Cycle by Blum and Lei (2007) and the REMMP rubric are interconnected and serve as guides for professors and for assessing children's learning. Both provide a clear structure for implementing Modelling, helping professors plan tasks that encourage students to construct mathematical models and solve problems.

Therefore, we consider the REMMP rubric effective for assessing Modelling processes because it provides a clear, detailed structure that facilitates assessing various aspects of children's learning throughout the Modelling cycle. However, when analyzing the rubric, we identified some limitations and areas for improvement to better meet specific educational needs, especially in the early years.

Since we believe that Modelling in mathematics education, as conceived by Burak (2010), brings mathematical concepts closer to interactive relationships between children, we adapted REMMP to incorporate this perspective. This adaptation reinforces the importance of social and collaborative interactions while promoting the development of problem-solving skills and the representation of mathematical concepts. We understand that children are still developing abstraction and generalization skills at this stage of schooling, which are crucial during childhood.

In this sense, Silva and Tortola (2025) demonstrate that teaching decisions directly influence the mathematical referrals children produce when discussing mathematical knowledge for teaching in Modelling contexts in mathematics education. According to the authors, choosing less abstract representations does not reduce learning complexity, but rather organizes it progressively. This requires professors to intervene in ways that broaden and deepen constructed meanings.

Gomes and Silva (2025) support this understanding by analyzing the production of diagrams by first-grade children in Modelling practices. They show that gestures, speech, and manipulable materials act as means of externalizing mathematical thinking and reveal knowledge such as counting, numerical organization, and the multiplicative principle.

Therefore, in the early years, representations of mathematical concepts should be simpler and more intuitive, using resources such as drawings, graphs, tables, and manipulatives to depict real-world situations rather than resorting to abstract and overly complex mathematical models.

The goal is for children to understand mathematical concepts meaningfully by constructing and manipulating these representations. This approach facilitates understanding of concepts and stimulates critical thinking and problem-solving in a practical, contextualized way. It prepares children to face mathematical challenges and everyday situations and contributes to a more complete, conscious civic education.

However, we believe that the REMMP rubric, in its original form, may not fully address some specific aspects of Modelling practices in childhood, especially in the early years.

Interest: Issues related to interest, rather than motivation, are fundamental. According to Sass and Liba (2011), who discuss interest in John Dewey's (1859–1952) theory, it is defined as an intrinsic relationship between an individual, their environment, and an object. This relationship leads to the recognition of the child's "self" in the process, making it a conscious action. Sass and Liba emphasize that interest results from the interaction between the subject and the object, the individual and society, and the child and the school. Additionally, they acknowledge that Dewey's theory considers students' individuality in relation to their aptitudes, needs, and preferences, without assuming everyone functions the same way.

Research Opportunity: Children's curiosity leads them to explore and question the world around them. They seek answers about cultural, geographical, natural, social, technological, and other phenomena. This search process is often associated with children's incessant use of the question "Why?"— a practice that adults often undervalue. When children ask "Why?" they are not just looking for ready-made answers; rather, they are seeking support and encouragement to conduct their investigations and construct meaning in relation to the elements they wish to understand (Rinaldi, 2016). Children's research activities stimulate the collection and analysis of information and support evidence-based decision-making. These are fundamental skills for developing scientific and mathematical thinking.

Social Interaction and Collaboration: Emphasizing social interaction and collaboration among children is fundamental to learning. From selecting a topic of interest or a real-life research situation to presenting and sharing results, collaboration plays a vital role. This collaborative process promotes the exchange of ideas, development of communication skills, and the ability to work in teams—all of which are essential for effective learning and social development.

Vygotsky (1978) emphasizes the importance of interaction and collaboration among children as central elements of diverse teaching methodologies. He argues that social interaction is a vital driver of knowledge construction, enabling children to engage in richer, more contextualized learning. Through collaboration, children share ideas and perspectives and develop social skills such as empathy and cooperation. They also strengthen their ability to construct knowledge collectively.

Strategies such as group work, classroom discussions, and collaborative projects facilitate this social interaction. These strategies provide opportunities for children to actively engage, solve problems together, and deepen their understanding of the concepts under study.

These practices also help develop social-emotional competencies and interpersonal skills, preparing children for effective, collaborative participation in various social and school contexts.

Emotional and Social Development: It is fundamental to consider children's emotional and social development as an essential part of the Modelling process. Model practices should foster a safe, welcoming environment where children can express their emotions, learn to resolve conflicts, and cultivate empathy and respect for others. This supportive environment is crucial for children's emotional growth.

According to Rueda and Paz-Alonso (2013, p. 1), emotional development "involves increasing the ability to feel, understand, and differentiate increasingly complex emotions, as well as the ability to self-regulate them so that the individual can adapt to the social environment or achieve present or future goals." Integrating these aspects into Modelling practices contributes to children's overall well-being, promoting more holistic, balanced learning that encompasses not only mastery of school knowledge but also emotional and social growth.

By adapting the REMMP rubric to include these aspects, we can create a more robust and appropriate tool for evaluating the Modelling process in the early years. This will promote a more inclusive, collaborative, and reflective learning environment. Based on these considerations, we adapted the REMMP instrument according to the cycle proposed in Figure 2. We incorporated contributions from a mathematics education perspective to Modelling, thus creating the Rubric for the Evaluation of Modelling Practices in the Early Years (RAPMAI). This adaptation process is based on research from a doctoral thesis by one of the article's authors, conducted in a school setting and approved by the Human Research Ethics Committee at the Federal University of Paraná (UFPR) under CAAE No. 58695922.5.0000.0214, in accordance with Opinion No. 5469973.

Table 3 presents the RAPMAI components and indicators, reflecting this expanded, integrated approach. The goal is to construct an evaluation process that respects children's uniqueness and the dynamics of classroom interaction.

Table 3

Components and indicators of the rubric for evaluating practices with Modelling in the Early Years (RAPMAI) (Jocoski, 2024, p. 73)

Components	**Early Years (a) (6-12 years old)**
1. Selection and understanding of one or more real situations or themes	1.1. Meets with colleagues, participating in small working groups; 1.2. Presents their views on a real situation or topic to be investigated; 1.3. Agrees with the decision of the small group or the class in general regarding the real situation(s) or chosen topic(s).
2. Organization and investigation	2.1. Relates the real situation or chosen topic to school content appropriate for their age group, based on prior knowledge; 2.2. Organizes work in small groups, researching or suggesting courses of action based on the decision made; 2.3. Raises possible problems based on the research carried out and begins to resolve them.
3. Mathematization	3.1. Justifies the use of mathematical objects; 3.2. Uses one or more representations to elucidate mathematical concepts and solutions to the proposed problems; 3.3. Seeks to identify the mathematical parameters present in the collected data and the different relationships between them.
4. Mathematical and extra-mathematical work	4.1. Uses various age-appropriate strategies to propose solutions to the problem(s); 4.2. Adapts or develops new representations of the mathematical concepts discussed throughout the process; 4.3. Advances to new discussions, deepening mathematical and extramathematical concepts.
5. Interpretation and evaluation	5.1. Interprets the results found in the context of the real situation or chosen topic; 5.2. Identifies possible limitations or restrictions of the representations of the mathematical concepts developed; 5.3. Evaluates the adequacy and effectiveness of the proposed solutions, considering different perspectives and contexts.
6. Critical dialogue and	6.1. Explains the reasons behind the decisions made during each stage of the process;

further exploration	6.2. Listens, responds, and critically analyzes presentations made by classmates, as well as comments and/or suggestions raised by classmates and/or the professor. 6.3. Explores new questions and challenges related to the topic, using the knowledge acquired to solve additional problems or expand their understanding of the concepts covered.
7. Presentation and sharing of results	7.1. Uses different types of examples, representations, diagrams, drawings, graphs, tables of values, and symbolic language to illustrate the results; 7.2. Clearly indicates where, how, and for what purpose the technology was used in one or more stages of the process; 7.3. Shares their results with the class, the school, and/or the school community.

When adapting the instrument for the early years (ages 6-12), we considered not only mathematical aspects, but also children's overall education. This broader approach is based on Burak's (2010) stages of work and aims to prepare children for academic, life, and citizenship challenges. Rather than focusing exclusively on mathematical content, the adapted REMMP instrument (Toalongo-Guamba et al., 2020), now known as RAPMAI, aims to contribute significantly to children's overall development. It promotes an integrative approach that develops skills that transcend mathematics and encompasses social, emotional, and ethical competencies essential for forming citizens prepared to live in society.

With the adaptation, we seek an approximation with the educational concepts of Burak (2010) from both the Modelling Cycle of Blum and Leiβ (2007) and the REMMP instrument. In this sense, the seven components of RAPMAI are closely linked to the seven phases of the Modelling Cycle in the early years from a mathematics education perspective that aligns with Burak's (2010) principles. This approach begins with the students' interests and the importance of gathering information and data directly from the environment that sparks the group's interest.

We also highlight factors that allow for contemplation of mathematical concept representations, problem-solving, and the actions and interactions experienced by children in the Early Years context.

For instance, the dynamics of group work, as expressed in RAPMAI items 1.1 and 2.2, enable children to develop collaboration and communication skills based on real-life topics, situations, and interests. Interactions between participants and constant dialogue (items 4.3 and 6.2) help children develop social and emotional skills fundamental to life in society, such as empathy and solidarity. Similarly, expressing their ideas and listening to others' ideas (items 1.2 and 7.1) helps children learn to respect different points of view and resolve conflicts constructively.

Representing mathematical concepts (item 3.2) involves using various forms of expression, such as graphs, tables, drawings, symbols, manipulatives, and games. These forms contribute to understanding and communicating mathematical ideas. This approach allows children to explore concepts from different perspectives, enriching their understanding and facilitating the construction of knowledge.

Problem solving (item 2.3) involves more than finding the correct answer. It requires identifying and analyzing questions based on the investigation of one or more real situations or topics of interest. This process fosters critical thinking and investigative skills, encouraging children to explore and deeply understand the proposed problems.

Another important point is the development of cognitive skills, such as critical thinking and creativity. These skills are stimulated by the need to solve the proposed problems (items 4.1 and 6.3). Using technology (item 7.2) makes learning more interesting and prepares children for the responsible and efficient use of digital resources. Working with mathematical objects and constructing representations of mathematical concepts (items 3.1 and 3.2) contributes to motor coordination development, especially when manipulatives are used. These examples demonstrate that learning can be a multidimensional process that goes beyond the classroom.

We realize that adaptation not only draws attention to the development of mathematical skills but also to critical thinking, communication, cooperation, and children's socioemotional development. Therefore, investing in improving and disseminating Modelling evaluation tools is essential to enriching pedagogical practice and providing meaningful learning experiences. These experiences prepare children for school and life in society by developing essential skills such as empathy, resilience, and teamwork.

It is important to note that the adaptations that now constitute the RAPMAI instrument are flexible. This allows for the inclusion of new components as the process evolves. This flexibility prevents any future limitations or rigidity. This approach provides openness, enabling continuous and dynamic evaluation of the teaching and learning process. Consequently, it becomes possible to accommodate different learning styles and encourage diverse interactions among children in the early years, while continuously adapting to their unique needs and contexts.

The ability to adapt and expand the instrument's components ensures its continued relevance in identifying and supporting the multiple dimensions of child development. This includes consideration of cognitive, emotional, social, and physical aspects, aligning with a comprehensive educational perspective. Ultimately, this approach contributes to a more responsive and inclusive educational environment where diverse learning methods and interactions are continuously evaluated and improved to promote holistic development.

Final considerations

This article presents the design of RAPMAI, an assessment rubric focused on Modelling and on children's specific interests in the early years of elementary school. Developed from an adaptation of the REMMP instrument based on the theoretical principles of Burak (2010) and the Modelling Cycle of Blum and Leiβ (2007), RAPMAI is a proposal that not only assesses cognitive aspects of mathematical learning but also promotes sensitive and inclusive pedagogical practices committed to children's holistic development.

RAPMAI values students' interests, promotes knowledge construction based on real situations, and stimulates critical thinking, problem-solving, communication, and collaboration skills. From this perspective, assessment is not a static moment but a continuous process that accompanies and engages with the stages of the Modelling cycle. This enables professors to observe, intervene, reflect, and make more informed pedagogical decisions.

RAPMAI's design accounts for childhood diversity, recognizing that young children require visual, concrete, and interactive approaches given their developmental stage. For this reason, the rubric includes indicators related not only to mathematization and working with concepts but also to fundamental aspects such as choosing topics of interest, organizing groups, conducting active research, teacher mediation, socializing results, and engaging in critical dialogue. Thus, the instrument promotes the integration of mathematical and extramathematical knowledge by connecting school with children's daily lives, cultural experiences, and family life.

Another relevant point is recognizing the importance of social interactions and emotions in teaching and learning. RAPMAI emphasizes moments of exchange between children, active listening, collaborative problem-solving, and sharing discoveries with the school community. These activities not only enhance mathematical learning but also the development of social and emotional skills, such as empathy, cooperation, responsibility, and resilience. These skills are fundamental for forming critical and participatory individuals who are aware of their role in society.

The adaptation of REMMP for early childhood education has resulted in a more flexible, contextualized instrument that respects childhood's unique characteristics and broadens the understanding of assessment as a dialogical, formative process. RAPMAI's seven components are directly linked to the phases of the Modelling cycle and Burak's (2010) stages, offering professors a clear yet flexible structure for monitoring children's progress during Modelling practices and recognizing their achievements, challenges, advances, and contributions.

It is important to emphasize that RAPMAI is not a closed or definitive model. Rather, it is an evolving instrument that can (and should) be adapted to the needs of each educational context. Its flexibility allows new components to be incorporated, further expanding the possibilities for assessment and pedagogical intervention. Thus, it promotes meaningful, equitable, and humane practices aligned with national curriculum guidelines and children's learning rights.

We believe RAPMAI significantly contributes to mathematics education in the early years by broadening perspectives on assessment and recognizing that teaching and learning mathematics go far beyond memorizing formulas and procedures. It involves listening, participation, investigation, affection, collaboration, and commitment to the comprehensive education of children. However, more research is needed to evaluate Modelling experiences in a school context. Investing in improving assessment tools like this one is an important step toward creating a school that welcomes and respects each child's unique learning pace and style and believes in the transformative potential of education.

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