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The treatment given to the construction of the set of real numbers in the initial training of mathematics teachers at São Paulo state universities: An analysis based on the syllabi

El tratamiento dado a la construcción del conjunto de los números reales en la formación inicial de los profesores de matemáticas de las universidades públicas paulistas: Un análisis a partir de los planes de enseñanza

Le traitement accordé à la construction de l'ensemble des nombres réels dans la formation initiale des professeurs de Mathématiques des universités publiques de l'État de São Paulo : Une analyse basée sur les plans d'enseignement

O tratamento dado à construção do conjunto dos números reais na formação inicial dos professores de matemática das universidades públicas paulistas: Uma análise a partir dos planos de ensino

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Abstract

The construction of the real numbers should be addressed in the initial training of mathematics teachers in such a way that the ideas related to the structures that make up the set are approached rigorously, yet contextualized within their historical construction process and treated in a manner that allows prospective teachers to understand, both formally and broadly, concepts taught in basic education. Based on this perspective, this article discusses, through the analysis of syllabi, how the construction of the real numbers is addressed in the Foundations of Analysis course in mathematics teaching degree programs at public institutions in the state of São Paulo. For this purpose, the syllabi for the analysis courses at these institutions were analyzed

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using a qualitative approach, through documentary research methodology. The works from the basic bibliographies recommended in the syllabi were also examined through bibliographic research to verify whether the referenced materials cover the construction of the real numbers and how this construction is carried out. The analysis of the syllabi revealed that only four of the 22 mathematics teaching degree programs investigated mention the construction of the real numbers in the Foundations of Analysis component, considering teacher training and the teaching of real numbers in school mathematics. The results indicate the need to broaden reflections on the role of the analysis in the initial training of mathematics teachers, always considering their future professional practice.

Keywords: Construction of real numbers, Teacher training, Teaching practice, Real analysis.

Resumen

La construcción de los números reales debe ser abordada en la formación inicial del profesor de matemáticas de manera que se traten de forma rigurosa las ideas referentes a las estructuras que componen el conjunto, pero contextualizadas en su proceso histórico de construcción y tratadas de modo que permitan al futuro profesor comprender formal y ampliamente los conceptos trabajados en la educación básica. Así, este artículo discute, a partir del análisis de planes de enseñanza, cómo se aborda la construcción de los números reales en la asignatura Fundamentos de Análisis en cursos de licenciatura en matemáticas de instituciones públicas del estado de São Paulo. Con este propósito, los planes de enseñanza de la asignatura de Análisis de estas instituciones fueron analizados bajo un enfoque cualitativo, mediante la metodología de investigación documental. También se examinaron las obras de las bibliografías básicas recomendadas en los planes de enseñanza a través de investigación bibliográfica, con el fin de verificar si las referencias indicadas contemplan la construcción de los números reales y de qué manera se realiza esta construcción. El análisis de los planes de enseñanza reveló que sólo cuatro de los 22 cursos de licenciatura en matemáticas investigados hacen referencia a la construcción de los números reales en la asignatura Fundamentos de Análisis, considerando la formación de profesores y la enseñanza de los números reales en la matemática escolar. Los resultados señalan la necesidad de ampliar las reflexiones sobre el papel del análisis en la formación inicial de los profesores de matemáticas, pensando siempre en su futura práctica profesional.

Palabras-clave: Construcción de los números reales, Formación de profesores, Práctica docente, Análisis real.

Résumé

La construction des nombres réels devrait être abordée dans la formation initiale des enseignants de mathématiques de manière rigoureuse, en traitant des idées relatives aux structures qui composent cet ensemble, tout en les contextualisant dans leur processus historique de construction. Il est essentiel de les présenter de façon à permettre aux futurs enseignants de comprendre formellement et largement les concepts travaillés dans l'enseignement de base. Sur la base de cette perspective, cet article discute, à partir de l'analyse de plans d'enseignement, de la manière dont la construction des nombres réels est traitée dans la discipline Fondements de l'Analyse dans les cursus de Licence en Mathématiques des Institutions Publiques de l'État de São Paulo. Dans ce but, les plans d'enseignement de la discipline d'Analyse de ces institutions ont été analysés selon une approche qualitative, à travers une méthodologie de recherche documentaire. Les ouvrages des bibliographies de base recommandées dans les plans d'enseignement ont également été examinés via une recherche bibliographique, afin de vérifier si les références indiquées incluent la construction des nombres réels et de quelle manière cette construction est réalisée. L'analyse des plans d'enseignement a révélé que seulement quatre des 22 cursus de Licence en Mathématiques étudiés font référence à la construction des nombres réels dans la discipline Fondements de l'Analyse, en considérant la formation des enseignants et l'enseignement des nombres réels dans les mathématiques scolaires. Les résultats indiquent la nécessité d'élargir les réflexions sur le rôle de l'Analyse dans la formation initiale des enseignants de mathématiques, en tenant toujours compte de leur future pratique professionnelle.

Mots-clés: Construction des nombres réels, Formation des enseignants, Pratique enseignante, Analyse réelle.

Resumo

A construção dos números reais deve ser tratada na formação inicial do professor de matemática de forma que sejam abordadas de maneira rigorosa as ideias que se referem às estruturas que compõem o conjunto, porém contextualizadas em seu processo histórico de construção e tratadas de modo que permitam ao futuro professor compreender formal e amplamente conceitos trabalhados no ensino básico.

Fundamentado nessa perspectiva, este artigo discute, a partir da análise de planos de ensino, como a construção dos números reais é tratada na disciplina Fundamentos de Análise em cursos de licenciatura em matemática de instituições públicas do estado de São Paulo. Com esse propósito, os planos de ensino da disciplina de Análise dessas instituições foram analisados em uma abordagem qualitativa, por meio da metodologia de pesquisa documental. Também foram examinadas as obras das bibliografias básicas recomendadas nos planos de ensino por meio de pesquisa bibliográfica, a fim de verificar se as referências indicadas contemplam a construção dos números reais e de que forma essa construção é realizada. A análise dos planos de ensino revelou que apenas quatro, dos 22 cursos de licenciatura em matemática investigados, fazem referência à construção dos números reais na disciplina Fundamentos de Análise, considerando a formação de professores e o ensino dos números reais na matemática escolar. Os resultados apontam a necessidade de ampliar as reflexões sobre o papel da análise na formação inicial dos professores de matemática, pensando sempre em sua futura prática profissional.

Palavras-chave: Construção dos números reais, Formação de professores, Prática docente, Análise real.

The treatment given to the construction of the set of real numbers in the initial training of mathematics teachers at São Paulo state universities: An analysis based on the syllabi

Introduction

Understanding how teachers' notions of concepts influence students' conceptions of the content being developed, as well as how teachers develop their knowledge about a specific concept and carry out its re-elaboration for teaching purposes, are questions present in research on teacher education and professional development, as seen in Shulman (1986), Moreira and David (2007), and Fiorentini and Oliveira (2013).

For Shulman (1986), these questions reflect how explanations and representations (such as analogies, examples, counterexamples, and demonstrations) are formulated, as well as the conditions teachers have to clarify students' doubts—that is, how the transition occurs between the concept itself, the teacher's knowledge of that concept, and the concept to be taught.

Also, according to Shulman (1986), teachers must have extensive knowledge of the content to be taught and of teaching processes. To clarify this idea, he divides the knowledge necessary for teachers into three main categories: content knowledge, pedagogical content knowledge, and curriculum knowledge. Content knowledge, as a necessary category of teacher knowledge, goes beyond mere mastery of facts. According to Shulman (1986), teachers must know the structure that makes a given concept valid in the way it is proposed, knowing how to justify the validity of a particular proposition to their students, its importance in the discipline under study, and its relationship with other disciplines and practice. Thus, the author states that teachers must understand the subject they will teach broadly and structurally, knowing how to define central and peripheral topics and in which situations the use (or not) of certain content is appropriate, being able to go beyond a mere definition of truths that must be accepted and memorized by students.

According to Shulman (1986), knowing the pedagogical dimension of a concept means knowing how to teach a particular idea, which requires mastering various forms of representing a subject so that it becomes comprehensible to others, such as illustrations, examples, counterexamples, and demonstrations.

Research by Moreira and David (2007) and Fiorentini and Oliveira (2013) suggests unifying training in mathematical, didactic-pedagogical, and professional practice training. To this end, the mathematical knowledge developed in mathematics teacher education programs must be treated comprehensively, clearly, and precisely, so that the various meanings of mathematics in the different contexts in which it applies are explored. From this perspective, we can infer that mathematics teacher education programs, whose main objective is teacher training, must develop knowledge encompassing mathematical theory across its various dimensions to prepare the prospective teachers for professional life.

Therefore, as Fiorentini and Oliveira (2013) argue, the mathematics addressed in teacher education programs, although having numerous elements in common with scientific mathematics, should be based on school mathematics (Moreira & David, 2007), since the teacher must know the different representations, diverse explanations, and applications in the most varied situations involving the practical world and other sciences.

From this perspective, it does not mean that the specific mathematical knowledge addressed in teacher education programs should be treated superficially or as inferior to that developed in teaching degree programs. In fact, it is the exact opposite: by knowing the definitions and formal proofs, the teacher must be able to represent and base their arguments so that students understand and can build their own justifications, through teacher mediation, to understand mathematics as useful and essential in the construction and development of the world.

The construction of the set of real numbers and the approach used in it have the potential to promote the understanding of the student teacher (Durand-Guerrier, 2016; Bertolucci & Balieiro Filho, 2024), enabling them to base the teaching of real numbers in their future teaching practice, aiming to build meaningful learning with students, where learners can understand mathematics and use it appropriately in interpreting various contexts and solving resulting problems, being able to positively interfere in the construction of society, breaking with the long-persisting idea that mathematics is difficult and of little use.

Various studies — e.g., by Margolinas (1988), Bagni (2000), Bergé (2007), Fischbein, Jehiam, and Cohen (1995), Penteado (2004), Moreira, Cury, and Vianna (2005), Martines (2012), Broetto and Santos-Wagner (2019) — show students' difficulties both in high school and higher education in irrational numbers, infinity, points on a line, density and continuity, and the number line. In this sense, considering that the real analysis course aims to consolidate the knowledge developed throughout the undergraduate program, especially regarding the set of real numbers (Bergé, 2007) — a numerical set widely studied during the final years of elementary school and high school, where the professional trained in the mathematics teacher education program will work — the ideas addressed in this course should foundation the teacher's work.

A formal construction of the real numbers, by Dedekind's, Cantor's, or other methods, for example, is a construction in which each idea is formally and rigorously proven, generalizing the results until concluding that the set of real numbers thus obtained can be described axiomatically as a complete ordered field. Any complete ordered field is necessarily isomorphic to the field of real numbers, as stated by Spivak (1967) or Rudin (1971). Axiomatic approaches assume the existence of a set satisfying the fundamental properties and explore other properties; therefore, in these cases, the construction of the real numbers is not performed, as presented in Lima (2013).

The real numbers can also be partially constructed in formal approaches that summarize the procedures used in the construction, as in Ávila (2001). Similarly, they can be partially constructed in an informal yet rigorous manner, providing an understanding of the ideas related to the expansion of the sets of natural, integer, and rational numbers, exploring the set of irrational numbers in various ways, bringing the various representations of real numbers, working on their properties — which justify many algorithms used in basic education – and their applications for problem solving, as in Paterlini (2012), who also explores the teaching of these contents.

This article discusses how the construction of the set of real numbers is addressed in the syllabi of Foundations of Analysis courses in mathematics teacher education programs at public higher education institutions in the state of São Paulo and its implications for the training of mathematics teachers. To this end, the syllabi for the Analysis courses at these institutions were analyzed using a qualitative approach, through a documentary research methodology. The works from the recommended basic bibliographies in the syllabi were also examined through bibliographic research to verify whether the indicated references address the construction of real numbers and how this construction is carried out.

The motivation for this analysis lies in the hypothesis that whether this construction is carried out or not, and how it is done, may impact the training of mathematics teachers and, consequently, the difficulties presented by basic education students, due to the lack of articulation between the teaching of real numbers at these levels, as pointed out by Bertolucci and Balieiro (2024), Broetto and Santos-Wagner (2019), and Durand-Guerrier (2016).

According to the Area Synthesis Report: Mathematics (Teaching Degree/Teacher Education), 450 mathematics teacher education programs participated in the 2017 National Student Performance Examination (ENADE) of higher education. The document also indicates that most programs were held in public institutions, totaling 307, equivalent to 68.2% of the programs that took the ENADE (Brasil, 2017, p. 24).

Furthermore, the Report indicates that the Southeast region had the highest participation of mathematics teacher education programs in ENADE, with 159 programs, representing 35.3% of the total (Brasil, 2017, p. 24). Given this data, 4587 students from this region were enrolled, representing 34.4% of the country's total (Brasil, 2017, p. 32).

Within the Southeast region, the state of São Paulo had the highest representation in the exam, with 82 programs (Brasil, 2017, p. 29), equivalent to approximately 51.6% of the total programs in the Southeast and 18.2% of all mathematics teacher education programs in Brazil.

Thus, due to its broad representation of mathematics teacher education programs in one of the most important evaluations of higher education programs in Brazil, the state of São Paulo was chosen as the parameter for this investigative research. In the state of São Paulo, almost half of the mathematics teacher education programs are offered by public higher education institutions (Brasil, 2017, p. 24). Furthermore, the decision to analyze the syllabi of public institutions stemmed from the fact that all of them post the course syllabi for the mathematics teacher education program on their websites, which allowed for a quicker, more direct investigation of the syllabi for the real analysis courses (under various designations) at these institutions.

The importance of real analysis for mathematics teacher training

Various research studies have as their central objective to uncover the role of the Real Analysis course in the education of mathematics teachers who will work in basic education, that is, the final years of elementary school and high school.

Reis's research (2001), for example, questioned teacher-researchers who are notable as textbook authors, seeking to understand how the relationship between rigor and intuition is established in the teaching of differential and integral calculus and real analysis, which are necessary for a meaningful understanding of the concepts covered in these courses. Meanwhile, studies by Moreira, Cury, and Vianna (2005) involved interviews with teacher-researchers from the main universities and research institutions in the country, given their influence on curricula and their frequent participation in the faculty of mathematics teacher education programs. Furthermore, Martines (2012) interviewed real analysis instructors and coordinators of mathematics teacher education programs at public higher education institutions in the São Paulo hinterland, aiming to understand the importance of this course in teacher education programs.

Bolognezi (2006) investigated the importance of the real analysis course for teacher education, focusing on teaching in high school, and found that most student teachers reported difficulties understanding the content and perceiving the utility of the real analysis course for their future teaching practice. The course instructors attributed undergraduates' difficulties to a lack of prior preparation. The high school teachers indicated that they did not consider the real analysis course fundamental to their classes because it did not demonstrate the pedagogical nature of the concepts covered, and because of excessive rigor, which is not applicable to their practices.

In common, the authors drew conclusions regarding the approaches proposed in the real analysis course. It is possible to conclude that an axiomatic approach to the content with excessive formalism and rigor does not favor teacher education, as it does not provide a multiple and comprehensive understanding of the content that will be taught in the prospective teacher's practice. In this sense, it hinders the synthesis and representation necessary for teaching basic education content, following the ideas of Shulman (1986), harming the knowledge of the concept and its pedagogical aspects, as it makes it difficult to perceive these contents as part of the work object of school mathematics.

A coherent approach, therefore, according to Reis's conception (2001), values students' intuitions, that is, their initial perceptions about the concepts studied and the ideas they construct throughout their education. In this process, according to the author, the necessary rigor for a solid mathematical foundation of the studied concepts is incorporated, promoting in students a broad and well-founded understanding of real analysis.

Following this idea, real analysis would indeed have the role of consolidating and formalizing knowledge developed throughout the mathematics teacher education program, as identified by Moreira, Cury, and Vianna (2005) and Martines (2012), but without failing to provide broad mathematical knowledge about the curriculum that underpins the teacher's work, thus addressing difficulties pointed out by Bolognezi (2006) and Martines (2012). In this sense, we understand that the real analysis course has the responsibility to promote and concretize the knowledge previously addressed in the mathematics teacher education program, with emphasis on numerical sets, a topic covered in various aspects throughout basic education. Therefore, we should not overlook that the studied concepts should be perceived as teaching objects for teachers' future professional practice.

It is worth emphasizing that this does not mean reducing the mathematical education of the prospective teacher, as these authors also stress. What is proposed as the function of the mathematical analysis course for teacher education programs is, in fact, the opposite: to perfect the prospective teacher's mathematical knowledge, so that they can develop a flexibility of thought that provides a well-elaborated and founded synthesis regarding, especially, the set of real numbers, thus facilitating its re-elaboration for teaching according to the numerous problem-situations proposed by the basic education curricula with which they will work and by the multiple learning conditions they will encounter during their professional practice.

This perspective requires a teaching that enables significant progress, aiming to conceive the set of real numbers as a complete ordered field in a purely axiomatic approach, following an essentially formalist approach, as highlighted by Martines (2012). It is about grounding knowledge of this content in the curriculum and the work of school mathematics, so that the student teacher can assimilate and reformulate it in response to the challenges they will encounter in their professional practice.

The treatment given to the set of real numbers in teacher education

In agreement with the authors Moreira, Cury, and Vianna (2005), Bolognezi (2006), and Martines (2012), it falls to mathematics teacher education programs to develop in undergraduates a broad and structural conception regarding the set of real numbers, so that they conceive it as a teaching object, developing the ability to justify, represent, and relate real numbers, their properties, and applications, based on a meaningful teaching-learning process for the student teachers.

This aspect reiterates the importance of carrying out work in the real analysis course that is oriented towards teacher education, since it is the role of this course to formalize the knowledge developed previously, especially concerning the set of real numbers, in a coherent approach, as considered by Reis (2001) and Martines (2012), which provides training in the three categories of knowledge established by Shulman (1986).

To better understand the training needs of mathematics teachers, especially regarding knowledge about real numbers, the studies conducted by Igliori and Silva (1999), Penteado (2004), and Corbo (2012) were considered.

The studies by Igliori and Silva (1999), Penteado (2004), and Corbo (2012) revealed difficulties with the definitions of rational and irrational numbers, with set expansions, with the representation of real numbers on the number line, the order relation in the set of real numbers, and various representations of these numbers. Furthermore, doubts regarding the concept of the density of rational and irrational numbers in the set of real numbers were explicit, and there was also evidence of difficulty with diverse interpretations, such as understanding statements, reformulating and recording one's own reasoning, and interpreting students' conceptions.

From this perspective, this indicates that content knowledge and pedagogical content knowledge grounded in curriculum knowledge were not adequately developed during the initial training of these teachers regarding numerical sets. The results found by Corbo (2012) confirm this statement when analyzing the possible approaches indicated by the investigated teachers, showing that they had some notion but could not establish clear objectives with the suggested proposals, revealing gaps in the understanding of concepts and also difficulties in the pedagogical re-elaboration of the contents to be taught. Corbo (2012) and Penteado (2004) conducted interesting interventions for the continuing education of these teachers, which indicated success in overcoming doubts and internalized erroneous concepts.

Given this, we consider it important to discuss the various representations of real numbers and their properties, considering the diverse meanings of these ideas, and their relationships with other mathematical concepts and with the practical world within the mathematics teacher education program. Furthermore, we understand that an in-depth understanding of these concepts allows them to be re-elaborated as teaching objects, work that must be carried out during the teacher's initial training. Thus, doubts about irrational numbers, infinity, points on a line, density and continuity, among others, can be addressed, especially in the real analysis course, considering its role of consolidating and formalizing the student teachers' knowledge, particularly regarding real numbers. Additionally, there is also insistence on the need to consider teaching practice, including, according to Reis (2001), rigor in accordance with the (initial and transitional) intuitions of the student teachers, formalizing the concepts in a way that fosters the understanding of the various meanings of operations and properties, in a multifaceted manner, according to Martines (2012).

Branchetti (2017) points out that the relationship between the continuum and real numbers is one of the most complex issues in the foundations of mathematics, as it involves significant challenges. For example, the author notes that differential calculus deals with continuous magnitudes, but an explanation of this continuity is not provided in purely arithmetic terms. In the construction of the set of real numbers done by Dedekind as cuts of sequences of rationals, emphasizing that the new numbers, the irrationals, were necessary creations to identify the points on a line and the numbers.

Branchetti (2017) points out that research on the teaching and learning of real numbers and continuity in High School and university has been conducted in several countries and that most of it concerns the difficulties faced by high school and university students and prospective teachers. The main difficulties reported involve understanding irrational numbers, infinity, points on a line, density, continuity, and the number line, indicating that students cannot correctly define the concepts of rational and irrational numbers. The main implications of her study, conducted with a high school teacher with a Ph.D. in mathematics, are that, even with a doctorate in analysis, in his transition to the teaching profession, the teacher does not utilize his mathematical knowledge in his teaching practice because he lacks didactic and epistemological knowledge about the teaching of real numbers. Furthermore, the author argues that to develop effective learning activities on real numbers, teachers (even mathematicians) must receive training that goes beyond content knowledge and involves epistemological and didactic aspects of the content. Thus, we can consider that the construction of the real numbers, although important for teacher education, is not sufficient for a prospective teacher to effectively address key aspects of the real numbers in the classroom.

Aiming to discuss students' difficulties in understanding the concept of continuity in the transition from the study of calculus to the study of analysis, Bergé (2007) argues that, considering the historical development of mathematics, completeness emerges as a tool for constructing proofs in a context where the existence of numbers is not guaranteed. "This genesis distinguishes completeness from other concepts in analysis" (Bergé, 2007, p. 232). From this perspective, the author points out that it makes no sense to include completeness as a topic of study in an introductory differential calculus course, since this discipline does not prioritize theoretical justification. However, completeness takes on meaning when students must prove results that are obvious to them, as expected in analysis courses. At this moment, reflection and discussion about what a proof is, the need for an axiomatic system for constructing proofs, and, consequently, the need to move away from intuition, become possible. Therefore, the real analysis course provides an opportunity to construct the real numbers, addressing didactic and epistemological aspects that will contribute to teacher education.

In agreement with Durand-Guerrier (2016), we understand that the construction of the real numbers and the way this construction is carried out in the teacher education program impact students' understanding of real analysis concepts and, consequently, the teaching and learning of the content. The author, based on her research into university students' knowledge of real numbers, corroborates her hypothesis that targeted work with university students on the construction of real numbers is necessary.

Thus, constructing real numbers in the analysis course is an opportunity for the prospective mathematics teacher to develop a broad understanding of algebraic operations, the order of operations, and the completeness of the set of real numbers.

The National Common Base for the Initial Training of Basic Education Teachers (BNC-Formação), defined in Resolution No. 2/2019 (Brasil, 2019), when introducing the idea of specialized content knowledge, emphasizes that the teacher must have deep content knowledge, pedagogical knowledge (how to teach the content), and knowledge of students (how students learn the content), seeking a connection with basic education.

Therefore, in accordance with the BNC-Formação, in teacher education, it is not enough to adopt a purely formal and axiomatic approach in the construction of real numbers, in which ideas are addressed in a generalized way. Instead, the construction of real numbers must help prospective teachers understand the operations used as algorithms in basic education, knowing their meanings in order to use them in problem-solving and knowing how to justify their validity, to help understand ideas such as the density of real numbers, commensurability and incommensurability, for example, so that they can explore these notions in their future professional practice.

The construction of real numbers in mathematics teacher education programs

Among the various questions that motivated this research, the two main questions, which can be considered guiding principles for the study, are: Do the syllabi indicate that the construction of real numbers is carried out in the real analysis course? Do the syllabi for these courses specify how this construction is performed?

For the investigation and interpretation of the obtained data, the documentary research methodology embedded in a predominantly qualitative approach was used. The documents selected as data sources were the syllabi for the real analysis courses in mathematics teacher education programs at public higher education institutions in the state of São Paulo.

According to Castro, Tucunduva, and Arns (2008), the planning of teaching activities should function as a guide for practice, autonomously directing the teacher to achieve their objectives in educating their students as citizens, revealing the importance of this act.

The teaching plan for the real analysis course should therefore outline the objectives, content, and methodologies that will guide teaching when delivering the course. For Libâneo (2013), the teaching plan should present a program to be followed during a specific course, containing "justification of the course in relation to the school's objectives; general objectives; specific objectives; content (with the thematic division of each unit); probable timeframe and methodological development (activities of the teacher and students)." (Libâneo, 2013, p. 257).

Considering the role of planning and the teaching plan for the teaching of Mathematical Analysis, we consulted the documents with the objective of investigating whether there are indications of how the construction of the set of real numbers is done in this course, if it is performed. To this end, the research pays attention to the content described in the syllabi and the objectives expressed in these documents (explicit data), with the intention of reflecting on the basic bibliography indicated in the syllabi (implicit data), in order to broadly understand the analyzed documents, considering the questions that drove the investigation. Thus, the information contained in the syllabi revealed the need to also investigate the bibliographies mentioned in these documents.

The research used syllabi as primary data sources (primary data sources), which were provided by the institutions on their websites or, when not available, requested via email from these institutions. The pre-analysis of these documents revealed the need to understand how the construction of the set of real numbers was presented in the basic bibliographies indicated in these documents (secondary data sources). Thus, it is possible to state that the documentary research methodology with a qualitative character was predominantly used, but bibliographic research was also conducted to refine the exploration of the collected materials.

To conduct this investigation, the syllabi for the Real Analysis course (among various other designations) of the Mathematics Teacher Education programs at the following public Higher Education institutions in the State of São Paulo were consulted:

i. Federal Institute of São Paulo (IFSP) in the cities of Araraquara, Birigui, Bragança Paulista, Campos do Jordão, Caraguatatuba, Cubatão, Guarulhos, Hortolândia, Itapetininga, São José dos Campos, and São Paulo;

ii. Federal University of ABC (UFABC) in Santo André;

iii. Federal University of São Carlos (UFSCar) in the cities of São Carlos and Sorocaba;

iv. São Paulo State University "Júlio de Mesquita Filho" (UNESP) in the cities of Bauru, Guaratinguetá, Ilha Solteira, Presidente Prudente, Rio Claro, and São José do Rio Preto;

v. State University of Campinas (UNICAMP);

vi. University of São Paulo (USP) in the cities of São Paulo and São Carlos.

The Mathematics Teacher Education program at UNESP in Guaratinguetá, at the time of the research, did not include the Real Analysis course. According to information received via email, the course was part of the old program structure, valid until 2014, returning in 2020 in the new restructuring, which had not been regularized by the date this research was completed. Furthermore, the Bachelor of Exact Sciences with a major in Mathematics program at USP in São Carlos did not have any courses related to Real Analysis in its curriculum.

With the syllabi, content analysis was performed following the phases of pre-analysis, material exploration, and result processing (Godoy, 1995) from a predominantly qualitative perspective. The pre-analysis involved organizing the documents, which were stored digitally in the cloud using the Google Drive platform services. These syllabi were printed, and an initial reading was conducted, which allowed for the formulation of initial hypotheses.

Subsequently, these materials were explored, leading to a bibliographic survey of the references indicated as the basic bibliography in the investigated documents. It is important to highlight one of the main difficulties of this research: not all the indicated books were found, and when obtained, they were not always the same edition indicated in the syllabi. However, the indications summarize to 23 references, of which 20 were analyzed, i.e., approximately 87% of the total, which provided a good amount of material for gathering the implicit data.

Thus, this refinement enabled a more comprehensive interpretation of the syllabi, which were subsequently interpreted one by one, generating the results and conclusions of this work.

Results

The construction of the real numbers can be carried out in a strictly formal and abstract manner, with generalized results that are applicable in the pursuit of other results and problem-solving within the established procedural logic, meeting the needs of the future mathematician (a professional trained by Bachelor's degree programs in Mathematics). On the other hand, this construction can be based on the teaching of content related to the set of real numbers—that is, performed with mathematical rigor regarding definitions and proofs, but not in a strictly formal way—using an approach that provides a broad understanding of these concepts, enabling their teaching and use in solving practical problems that Basic Education students may encounter in their daily lives. This allows the prospective teacher (a professional trained by the Mathematics Teacher Education program) to justify results and deal with their students' doubts and errors.

In cases where the approach is strictly axiomatic—in other words, where the set of real numbers is presented as a complete ordered field—the construction of this set is not performed; only its properties are presented to allow for the continuation of work on other content in the course.

In light of this, an initial reading of the syllabi, which served as the primary data for this research, was conducted. This initial contact with the documents provided familiarity with the syllabi, objectives, and content covered in the real analysis courses at the investigated institutions. For some syllabi, this initial reading already allowed inferences about whether the construction of the real numbers was carried out in each mathematics teacher education program. However, before making such claims, preference was given to investigating the implicit data contained in these documents, which, in this case, were the basic bibliographies. This is because, according to Libâneo (2013), the content is indicated by teachers and serves to guide lesson preparation. Therefore, this content is present in the bibliographic references, which also outline the possible approaches to these concepts, whose function is to guide student studies, according to Libâneo (2013).

It is worth reiterating the difficulty in finding the same editions of the indicated bibliographic references, and even the books themselves. Searches were conducted online and in the UNESP library, using the Athena Catalog as a search tool and consulting personal collections.

It is important to note that, in the case of references to articles from the *Revista do Professor de Matemática* (RPM)—cited in two syllabi—articles from *Educação Matemática em Revista* (EMR)—cited in one teaching plan—the journal *Tendências de Matemática Aplicada e Computaciona*l (TEMA)—cited in one teaching plan—and the *Revista Eletrônica Paulista de Matemática*—cited in one teaching plan, the specific articles used were not indicated, which made the analysis of these references unfeasible.

The table below summarizes the editions consulted and those that were not found. It also shows the number of times they were indicated in the investigated syllabi and, in summary, whether the set of real numbers was constructed.

Table 1

Summary of the Core Bibliographies Cited in the Syllabi

Reference consulted	Number of Times Cited in Syllabi	Construction of the Set of Real Numbers
Ávila, G. (2001). Análise Matemática para a Licenciatura. 1st ed. São Paulo: Edgard Blücher.	20	Partially performed, i.e., the author outlines a guide for the construction.
Ávila, G. (1999). Introdução à Análise Matemática. 2nd ed. São Paulo: Edgard Blücher.	8	Partially performed, i.e., the author outlines a guide for the construction.

Bartle, R. G. (1983). Elementos de Análise Real. Trad. Alfredo A. de Farias. Rio de Janeiro: Campus.	1	No.
Bourchtein, A.; Bourchtein, L. (2010). Análise real: funções de uma variável real. São Paulo: Ciência Moderna.	1	Book not found.
Ferreira, J. A (2013). Construção dos Números. 3rd ed. Rio de Janeiro: SBM.	1	Partially performed, meaning it does not demonstrate all the necessary properties for the construction of the reals.
Figueiredo, D. G. (2008). Análise I. 2nd ed. Rio de Janeiro: LTC, 2008.	11	No.
Guidorizzi, H. L. (2001). Um Curso de Cálculo (Vol. 1). Rio de Janeiro: LTC, 2001.	2	No.
Lima, E. L. (2013). Análise Real, vol 1. 12th ed. Rio de Janeiro: IMPA.	14	No.
Lima, E. L. (2017). Curso de Análise. vol. 1. 14th ed. Rio de Janeiro: IMPA.	7	No.
Lima, P. C. (2013). Fundamentos de Análise I [Electronic format]. Belo Horizonte: CAED-UFMG.	1	No.
Muniz Neto, A. C. (2015). Fundamentos de Cálculo – Coleção PROFMAT – Ed. Sociedade Brasileira de Matemática.	1	Book not found.
Niven, I. (2012). Números: Racionais e Irracionais. 1st ed. Rio de Janeiro: SBM.	1	No.
Panonceli, D. M. (2017). Análise Matemática. São Paulo: Pearson.	2	Book not found.
Paterlini, R. R. (2012). A Aritmética dos Números Reais. [Formato eletrônico] São Carlos: UFSCar.	1	Partially performed, i.e., it does not demonstrate all the necessary properties for the construction of the real numbers.
Golderg, R. R. (1963). Methods of Real Analysis. New York.	1	No.
Rudin, W. (1976). Principles of Mathematical Analysis. 3ª ed. New York: McGraw-Hill.	1	Yes.
Rudin, W. (1971). Princípios de Análise Matemática. São Paulo: Ao livro técnico.	3	Yes.
Simmons, G. F. (1988). Cálculo com Geometria Analítica. Vol. 1. Trad. Seiji Hariki. São Paulo: McGrawHill.	2	No.

Spivak, B. (1967). Calculus. New York: W. A. Benjamin Inc.	2	Yes.
Stromberg, K. R. (1981). An Introduction to Classical Real Analysis. Belmont: Wadsworth Inc.	1	No.
Táboas, P. Z. (2008). Cálculo em uma variável real. São Paulo: Editora da Universidade de São Paulo.	1	No.
Villanueva, D. A. Z. (2014). Princípios de Análise e Exercícios de Cálculo. 1st ed. São Paulo: Editora Livraria da Física	1	Partially performed, i.e., it does not demonstrate all the necessary properties for the construction of the reals.
White, A. J. (1975). Análise real: uma introdução. Trad. Elza F. Gomide. São Paulo: Edgard Blücher, EDUSP.	1	No.

In Ávila (2001), the constructions by Dedekind and Cantor are cited (Ávila, 2001, pp. 29-32), pointing the reader to the books by Rudin and Spivak for a complete understanding (Ávila, 2001, p. 31). Cantor's construction is also outlined so the reader comprehends the underlying idea, but without extensive detail (Ávila, 2001, pp. 71-73).

In Ávila (1999), the author presents an interesting historical approach to the construction of the set of real numbers, although he does not perform this construction in the book. He discusses the ideas of important mathematicians on the subject, such as Eudoxus and Dedekind, highlighting Dedekind's idea for constructing the real numbers (Ávila, 1999, pp. 12-15), but refers the reader to the books by Dedekind and Spivak to find the complete construction of the real numbers via cuts.

In the work of Bartle (1983), the author mentions that it is possible to construct the real numbers starting from the sets of natural, integer, and rational numbers or from the set of rational numbers alone; however, he does not carry out this construction, favoring instead the presentation of a list of properties (algebraic properties, order properties, and the completeness property) related to the system of real numbers (Bartle, 1983, p. 38).

In Ferreira (2013), the construction of the set of real numbers is performed by starting from the set of rational numbers with their algebraic and arithmetic properties, and the author chooses Rudin’s model as the guide for presenting this subject.

In Figueiredo (2008), the construction of the real numbers is not rigorous. However, the author provides a brief notion of Dedekind cuts and also signals the possibility of constructing the real numbers using Cauchy sequences (Figueiredo, 2008, pp. 3-13).

In Lima (2017), the author alerts the reader to the possibility of constructing the real numbers from the natural numbers by successive extensions of the concept of number. Furthermore, he emphasizes that this can be done in several ways and that the crucial step in this construction is the passage from rational numbers to real numbers, which can be achieved by Dedekind cut method or by Cantor’s process via Cauchy sequences. Lima (2017) defines a complete ordered field K and, based on preceding expositions, adopts the fundamental axiom of mathematical analysis: there exists a complete ordered field, R, called the field of real numbers. Finally, he examines some properties of the real numbers by virtue of being a complete ordered field.

In Lima (2013), the properties of addition, multiplication, order relation, and decimal representation of rational numbers are explored. Irrational numbers are introduced through exercises and examples. The set of real numbers, however, is defined as an ordered field, and its properties are explored. Nevertheless, the construction of this numerical set is not performed.

Niven’s (2012) book does not present a formal construction of the real numbers, but rather develops interesting work on rational and irrational numbers that can undoubtedly contribute to the education and practice of the prospective mathematics teacher.

Paterlini (2012) offers a very interesting approach to real numbers for the education of prospective teachers, grounded in historical context, problem solving, proposed investigations, and activities for both prospective and in-service teachers. The author explores natural numbers, integers, and rational numbers, with important ideas such as successor and predecessor in the set of naturals, prime numbers, multiples, the greatest common divisor, decimal and fractional representations of rational numbers, and equivalent fractions. Rational numbers are introduced through the example of √2, and the author presents various ways to demonstrate their irrationality. Subsequently, he explored the ideas of commensurability and incommensurability. The author presents the models of Eudoxus and Dedekind, giving an idea of the construction of real numbers performed by these mathematicians, followed by a summary of the properties of this numerical set. We can infer that the author performs a construction of the real numbers via Dedekind cuts in an informal manner (stating definitions, propositions, and theorems without proving them), but with mathematical rigor, albeit not in a formal way.

The work by Goldberg (1963) presents the decimal representation of real numbers (decimal expansion), demonstrates that this set is uncountable, and uses real numbers for various examples; however, the construction of this number set is not performed.

Rudin's (1976) work, cited in other books, performs the classic construction of the real numbers by means of Dedekind cuts and obtains a complete ordered field whose elements are collections of rational numbers (Rudin, 1976, pp. 17-21). The same occurs in Rudin (1971).

Simmons (1988) develops studies centered on the properties of real numbers in the development of differential and integral calculus and analytic geometry. However, the author indicates the sets (natural, integer, rational) through numerical examples, presents some examples of irrational numbers, and represents some numbers (natural, integer, rational, and irrational) on the real line, emphasizing that the above describes a one-to-one correspondence between all real numbers and all points on the line. Furthermore, in the additional topics, section A.1, he demonstrates the irrationality of √2. However, the construction of the real numbers is not performed.

Spivak (1967) uses Dedekind cuts to construct the real numbers. The author demonstrates properties of the real numbers and, in Chapter 29, demonstrates their uniqueness (as the unique complete ordered field, up to isomorphism).

In Villanueva (2014), the author formally explores the sets of real numbers, integers, rationals, and irrationals, giving an idea of set expansion and the construction of the set of real numbers based on the idea of Dedekind cuts, partially performing this construction in a formal approach.

In White (1975), the author emphasizes the importance of proofs in mathematics and introduces real numbers and their properties through 13 axioms (addition, multiplication, distributivity, order, and completeness), as he intends to explore other related ideas in this set. Thus, he does not perform the construction of the real numbers.

In Guidorizzi (2001), Lima (2013), Stromberg (1981), and Táboas (2008), the construction of the real numbers is not addressed.

Our analysis shows that most works discussing the construction of the real numbers to some extent are based on Dedekind's construction in the 19th century, as presented by Rudin (1971, pp. 3-11). In this approach to constructing the real numbers, the set of rational numbers is assumed to be known, and the construction proceeds formally and rigorously, building the structures until a definition of irrational numbers is reached, thereby consolidating the construction of the reals. We note that the concepts are treated abstractly, aiming to characterize this set, focusing more on its structures. The reasoning required to carry out the proofs, either constructively or by contradiction, is noteworthy, demanding from the reader refinement and care in executing the procedure, which is done in a very well-articulated manner for elaborating the proofs.

However, it must be emphasized that the works of Rudin (1971) or Spivak (1967), which present a formal construction of the set of real numbers, are necessary for mathematical rigor but become insufficient when adopted in isolation, without adequate discussion about the didactic-pedagogical dimension of the addressed contents. The abstract form, although valuable for guaranteeing the validity and consistency of the results, is not sufficient to support the education of the teacher who will work in basic education. Indeed, the way real numbers are handled in this construction does not directly contribute to the teacher's work. It is clear that this approach allows for understanding the notions of order, the operations, their properties, and, for example, establishing the idea of the density of rationals and irrationals. However, the way real numbers are treated, as cuts or the separation between them, does not constitute in itself a teaching object for the future basic education teacher.

Among the analyzed works, Paterlini (2012) proposes an approach to real numbers in mathematical analysis that enables an understanding of basic education mathematics through higher education mathematics, addressed in teacher education programs. The author uses a historical approach, starting with the human need to count, introducing the ideas of successor and predecessor of natural numbers, and commenting on different numeral systems. Indeed, this construction brings a historical approach to the set of natural numbers, addressing ideas such as the successor of a number in this set, the notion of order, operations and properties, as well as notions widely worked on in basic education, such as the concepts of multiple and divisor, greatest common divisor, prime numbers, and parity, also clarifying the idea of the number zero. The expansion of the set of integers and their properties is completed, and a historical approach to the set of rational numbers is also presented, with clarification of operations with fractions and irreducible fractions — ideas much studied in basic education — and of the decimal representation of rational numbers (as finite or repeating decimals). To introduce the idea of irrational numbers, again through a historical approach, the number $\sqrt{2}$ is explored, with various types of proofs of its irrationality. Thus, the ideas of commensurability and incommensurability are approached and situated historically, and Thales' theorem is addressed, presenting, synthetically, the properties of the set of real numbers.

It is noted that the proposed construction of R by Paterlini (2012) brings rigorous definitions and demonstrations, but not in a strictly formal way, so that the student teacher can understand the idea of algorithms and concepts that are widely covered in the final years of elementary school and in high school, such as the idea of division, the sum of infinite terms of a geometric progression, "rules of signs," and cross multiplication, for example.

The way definitions are presented and proofs are conducted can prevent prospective teachers from internalizing misconceptions and, consequently, from sharing them with their future students, as found by Corbo (2012), Penteado (2004), and Igliori and Silva (1999). The construction of R by Paterlini (2012) even addresses difficulties of practicing teachers and prospetive teachers, pointed out by those authors, such as, for example, the definition of the sets of natural numbers, integers, rationals, and irrationals, decimal representation, and the idea of successor in the set of natural numbers and density of sets.

The construction of the set of real numbers by Paterlini (2012) is based on school mathematics (Moreira & David, 2007), promoting discussions through problem-solving, themes for investigation, and activities aimed at student teachers and teachers about the teaching of ideas related to those numerical sets. It is worth noting that the rigorous yet not strictly formal approach provides a structural yet broad understanding of the developed concepts. Work conducted in this way is pointed out as essential by Martines (2012), as it articulates the mathematical education of the prospective teacher with the knowledge necessary for teaching this subject in the final years of elementary school and in high school.

After analyzing the books referenced in the syllabi, the explorations carried out enabled categorization of the produced data, as follows in items 1 to 8 below. It is important to emphasize that this categorization serves only to summarize the investigations conducted in the previous section, which are essential for a correct interpretation of the results.

1. The teaching plan indicates the formal construction of the set of real numbers, demonstrating concern with teaching and teaching practice.

i. IFSP Araraquara;

ii. UFABC;

iii. UNESP Presidente Prudente;

iv. USP São Paulo;

v. USP São Carlos;

vi. UNICAMP (course: Introduction to Real Analysis).

2. The teaching plan indicates the formal construction of the set of real numbers but gives few indications regarding teaching practice and instruction.

i. UFSCar Sorocaba;

ii. UNESP Ilha Solteira;

iii. UNICAMP (course: Analysis I).

3. The teaching plan points to the use of various approaches for the construction of the set of real numbers, demonstrating concern with teaching and teaching practice.

i. IFSP Campos do Jordão.

4. The teaching plan shows that the construction of the set of real numbers is partially performed in a formal approach, also revealing concern with teaching and teaching practice.

i. IFSP Cubatão;

ii. IFSP Guarulhos;

iii. IFSP Hortolândia;

iv. IFSP Itapetininga.

5. The teaching plan indicates that the construction of the set of real numbers is partially performed, in a formal approach. However, there are few indications regarding teacher education and instruction.

i. IFSP Caraguatatuba;

ii. UNESP Bauru;

iii. UNESP Rio Claro.

6. An axiomatic approach to the real numbers is used; however, the teaching plan demonstrates concern with the future teaching practice of the student teacher and with instruction.

i. IFSP Birigui;

ii. IFSP Bragança Paulista.

7. An axiomatic approach to the real numbers is used, and the teaching plan provides few indications regarding teacher education and instruction in school mathematics.

i. IFSP São José dos Campos;

ii. UNESP São José do Rio Preto (course: Introduction to Mathematical Analysis);

iii. UNESP São José do Rio Preto (course: Analysis on the Real Line).

8. The teaching plan shows that the construction of the set of real numbers is not performed; however, the course explores this set extensively, revealing significant concern with the education of the prospective teacher and instruction in school mathematics.

i. IFSP São Paulo;

ii. UFSCar São Carlos.

It is now important to revisit the knowledge essential for the education of the prospective teacher, according to Shulman (1986), to refine the analysis of the results: content knowledge, pedagogical content knowledge, and curriculum knowledge.

Categories 2, 5, and 7 include syllabi that demonstrate little concern with the dimensions of content knowledge and pedagogical content knowledge. Indeed, Category 2 encompasses syllabi that indicate the formal construction of the set of real numbers; however, there are few indications regarding the teaching of contents related to this construction in basic education, which seems more appropriate, according to Moreira and David (2007), for the education of the licenciate in mathematics. The same occurs with Category 5, where the syllabi do not indicate how the knowledge developed in the course (in this case, regarding the real numbers, which are partially constructed) is articulated with school mathematics, or how the knowledge of the worked contents aids the teaching of mathematics in basic education. The syllabi covered by Category 7 demonstrate an axiomatic approach to the set of real numbers, and the documents do not show a concern with teaching practice or the teaching of school mathematics.

Approaches such as those indicated by the syllabi belonging to Categories 2, 5, and 7 fail to explore, formalize, and structurally deepen relevant concepts and properties of the set of real numbers, which are constantly addressed in basic education, fundamental for understanding algorithms and statements, to assist in solving practical problems that students may encounter and have doubts about. Moreira and David (2007) consider it fundamental that the prospective teacher understands mathematical concepts in various contexts, also knowing possible difficulties in learning such content and how to deal with them. Initial education must prepare the prospective teacher for the pedagogical action of teaching mathematics, which may not occur in approaches such as those indicated by the syllabi belonging to Categories 2, 5, and 7. Furthermore, the approaches indicated in these categories may suggest little adherence of these syllabi to the current guidelines for the training of basic education teachers, such as the BNC-Formação (Brasil, 2019), which emphasize that specific contents should be aligned with curricular components of the National Common Curriculum Base — BNCC (Brasil, 2018), that they should be developed in a way that ensures a solid and profound education, but in articulation with pedagogical training, seeking to transpose scientific knowledge into school knowledge.

Fiorentini and Oliveira (2013) emphasize that the teacher must know the multiple meanings and uses of mathematics, learning to conceive it in its conceptual and didactic-pedagogical aspects, knowing how to demonstrate results, solve exercises and problems, and justify these procedures for students. Thus, in the case of the set of real numbers, it is necessary, therefore, that a construction of this set be carried out, so that the student teacher knows and knows how to work with its structures, its properties, and its operations, and also that the prospective teacher knows this numerical set, whose study permeates all of elementary and high school, from the perspective of school mathematics, i.e., knowing how the contents related to real numbers are presented at these levels of education, what the possible approaches are for teaching these concepts, and, for example, how to deal with possible student difficulties.

Thus, according to Fiorentini and Oliveira (2013), the mathematics worked on in teacher education programs should encompass the concepts, the procedures necessary to prove and understand them, and the possible approaches to teaching these ideas. The teaching plan Categories 1, 3, 4, 6, and 8 are those that most closely approximate this conception.

However, the approaches indicated by the syllabi that fall into Categories 6 and 8 do not include the construction of the set of real numbers. Indeed, in Category 6, this numerical set is treated axiomatically. Various properties, representations, and algorithms are left unexplored when real numbers are treated only axiomatically, dimensions that can be important for teaching related content and for addressing basic education students' doubts and misconceptions. Thus, the concern with teaching and teaching practice, as demonstrated by the syllabi in Category 6, should also encompass the various dimensions involved in working with real numbers.

The syllabi included in Category 8 do not construct the set of real numbers; however, they demonstrate extensive work with this numerical set, highlighting the importance of understanding and teaching these concepts in the prospective teachers' future teaching practice.

For Moreira, Cury, and Vianna (2005) and Branchetti (2017), work in real analysis that considers only mathematics in its formal, logico-deductive presentation will not be sufficient for the education of the professional who will work in teaching content related to those addressed in the course. These authors consider it essential that initial teacher education also address issues arising from teaching practice in basic education, particularly in the teaching of school mathematics. The syllabi that most closely approximate this conception are those encompassed by Categories 1, 3, and 4.

The syllabi included in Category 1 demonstrate the performance of the formal construction of the set of real numbers encompassing training directed towards the teaching of real numbers and their properties, as well as the syllabi included in Categories 3 and 4 (which perform the partial construction of this set), which represent only ten of the twenty-four syllabi investigated.

It is worth emphasizing that merely an axiomatic exposition of the set of real numbers, in an approach that does not consider the intuitions of the student teachers, their initial and transitional perceptions, in a way that does not provide a broad understanding of the structures of this set, is not sufficient for teacher education. Indeed, according to Reis (2001), the prospective teacher must understand the concepts to use them adequately in problem solving, and to understand their pedagogical and curricular dimensions, since, in their professional practice, they will work with teaching contents based on these concepts.

When constructing the set of real numbers, the notions of set expansion must be addressed, so that the student teacher can understand the definitions of each of the sets of natural numbers, integers, rationals, and irrationals, and the properties that are valid (or not) in each of them. Furthermore, according to Cunha (2014), issues presented in basic education should be addressed, such as density, infinity, incommensurability, understanding of properties and operations, representations, and functions in the set of real numbers, so that only the formal construction is also not sufficient, being more adequate, according to Moreira and David (2007), to academic mathematics.

The research conducted allows us to state that the syllabi for the real analysis courses that are most adequate regarding the performance of the construction of the set of real numbers aimed at the mathematics teacher education program (teacher education for basic education) are the plans from IFSP Araraquara, UFABC, USP São Paulo, and USP São Carlos — which represents four of the 22 institutions investigated, approximately 18% of these institutions.

Conclusion

To better understand the knowledge developed in the initial education of mathematics teachers regarding real numbers, this work investigated how the performance of constructing this numerical set is reflected in the syllabi of public institutions in the State of São Paulo. Public institutions in the state of São Paulo were chosen because, in most cases, they provide access to syllabi on their websites.

The main result found is that only four of the twenty-two investigated institutions express, in the syllabi for real analysis courses, the construction of the set of real numbers in approaches that allow prospective teachers to understand this numerical set structurally and broadly, as a teaching object in their future professional practice.

The analysis of the books referenced in the syllabi allowed us to observe that it is possible to address the set of real numbers in a purely axiomatic way, as a complete ordered field, and to construct this set in formal approaches that are more abstract, as done by Rudin (1971), or based on the historical construction, privileging the understanding of the ideas encompassed by the real numbers, as performed by Paterlini (2012).

We consider that constructions of R like the one performed by Paterlini (2012) are more adequate for teaching this concept in the real analysis courses of mathematics teacher education programs, as it treats the ideas rigorously regarding the structures that compose the set, yet contextualized in their historical construction process and treated in a way that allows the prospective teacher to understand formally and broadly concepts worked on in basic education, but without an abstract and strictly formal treatment of these ideas. Furthermore, Paterlini (2012) proposes an approach that includes problem solving and discussions on how to teach the concepts covered in the final years of elementary school and in high school, and suggests that prospective teachers deepen their knowledge through themes for investigation.

From this perspective, the concepts are treated broadly and structurally, enabling the prospective teacher to know the justifications for the ideas worked on and the importance of the content for teaching mathematics, stimulating them to reflect on how to re-elaborate or approach the set of real numbers to help their future students build their knowledge on this topic, provide in their education the three categories of knowledge necessary for the teacher outlined by Shulman (1986). By considering the concept and its forms of representation, providing an understanding of definitions and proofs through examples and illustrations, with the intention of being concerned with the historical and concrete construction of these concepts, stimulating reflection on the conceptual, didactic, and pedagogical aspects of the worked content, this approach to the construction of real numbers is in conformity with the ideas of Fiorentini and Oliveira (2013) for teaching mathematical concepts to the prospective teacher.

The discussion in this article was limited to the analysis of the syllabi. However, we consider that there may be teaching practices that incorporate the role of analysis in teacher education, as well as the implications of the construction of real numbers for the understanding of the content developed in high school. Nevertheless, the analyzed syllabi do not allow us to discuss such practices. Still, we consider it important that syllabi can reflect the teacher's intentions, since these documents should be articulated with the pedagogical project of the course and are considered in curriculum restructurings and course recognition renewals submitted, for example, to the State Council of Education (CEE), in the case of state HEIs.

We believe that the results of the analyses of the syllabi from each institution, together with the theoretical study that explicated the importance of a coherent approach for teaching real analysis in mathematics teacher education programs, add ideas to the reflection that is continuously established about the improvement of these programs, with the intention of enhancing the initial education of mathematics teachers, which we believe will have direct repercussions on basic education.

Indeed, the studies carried out in this work show that it is necessary to continually reflect on teachers' initial education, always with their future professional practice in mind. Thus, the mathematics taught in these programs must ground the teaching of mathematics in basic education, which does not mean attenuating the mathematics taught for teacher education programs, but rather formalizing and deepening the ideas so that the prospective teacher understands and can adequately reformulate them to proceed with their teaching. This must occur, especially in real analysis, consolidating and expanding knowledge of real numbers, a subject continuously studied in elementary and high school, and the professional field of the prospective teacher.

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