Task management in the teaching of limits of real functions in higher education

Gestión de tareas en la enseñanza de límites de funciones reales en la educación superior

Gestion des tâches dans l'enseignement des limites de fonctions réelles dans l'enseignement supérieur

Gestão de tarefas no ensino de limites de funções reais no ensino superior

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Abstract

The teaching of limits of real-valued functions in higher education presents a challenge due to the complexity of this mathematical concept and the difficulties students face in learning it. In light of this issue, the present study aims to analyze task management in the teaching of limits of real functions, examining how the planning, implementation, and assessment of such tasks influence learning. This qualitative, formative study was conducted with two groups of participants: (1) ten students from the Bachelor's Program in Science and Technology at the Federal University of Maranhão; and (2) three instructors who teach Calculus to non-mathematics majors. The student training program consisted of 120 hours and placed particular emphasis on a specific set of tasks. The instructors

participated in this program as observers and as validators of the tasks with respect to their structure and content. The study was grounded in the Didactic Suitability Criteria (DSC) of the Onto-semiotic Approach. The results indicate that appropriate task management, guided by the DSC and supported by technological resources such as the GeoGebra *software*, contributes to content assimilation, meaning-making, and student engagement. We conclude that careful planning and interactive strategies promote improved learning outcomes, enabling students to overcome common difficulties in understanding the notion of limits.

Keywords: Calculus teaching, Task management, Limit of a real function, Didactic suitability, Mathematics education.

Resumen

La enseñanza de los límites de funciones reales en la educación superior constituye un desafío debido a la complejidad de este objeto matemático y a las dificultades que los estudiantes enfrentan en su proceso de aprendizaje. Ante esta problemática, el presente estudio tiene como objetivo analizar la gestión de tareas en la enseñanza de los límites de funciones reales, examinando cómo la planificación, la implementación y la evaluación de dichas tareas influyen en el aprendizaje. La investigación, de enfoque cualitativo y de carácter formativo, se llevó a cabo con dos grupos de participantes: (1) diez estudiantes del Grado en Ciencia y Tecnología de la Universidade Federal do Maranhão; y (2) tres profesores que imparten Cálculo en titulaciones no matemáticas. La formación dirigida a los estudiantes tuvo una duración de 120 horas y prestó especial atención a un conjunto específico de tareas. Los profesores participaron en dicha formación como observadores y validadores de las tareas en cuanto a su estructura y contenido. El estudio se fundamentó en los Criterios de Idoneidad Didáctica (CID) del Enfoque Ontosemiótico. Los resultados indican que una gestión adecuada de las tareas, orientada por los CID y apoyada en recursos tecnológicos como el software GeoGebra, contribuye a la asimilación de los contenidos, la construcción de significados y la

implicación del alumnado. Concluimos que una planificación cuidadosa y el empleo de estrategias interactivas favorecen un mejor aprendizaje, permitiendo superar las dificultades habituales en la comprensión de la noción de límite.

Palabras clave: Enseñanza del cálculo, Gestión de tareas, Límite de una función real, Idoneidad didáctica, Educación matemática.

Résumé

L'enseignement des limites de fonctions réelles dans l'enseignement supérieur constitue un défi en raison de la complexité de cet objet mathématique et des difficultés que rencontrent les étudiants dans leur processus d'apprentissage. Face à cette problématique, la présente étude vise à analyser la gestion des tâches dans l'enseignement des limites de fonctions réelles, en examinant comment la planification, la mise en œuvre et l'évaluation de ces tâches influencent l'apprentissage. La recherche, de nature qualitative et formative, a été réalisée auprès de deux groupes de participants : (1) dix étudiants du cursus de Licence en Sciences et Technologie de l'Université Fédérale du Maranhão ; et (2) trois enseignants qui dispensent des cours de calcul pour des formations non mathématiques. La formation destinée aux étudiants, d'une durée de 120 heures, a accordé une attention particulière à un ensemble spécifique de tâches. Les enseignants ont pris part à cette formation en tant qu'observateurs et validateurs des tâches quant à leur structure et à leur contenu. L'étude s'appuie sur les Critères d'Adéquation Didactique (CAD) de l'Approche Ontosémiotique. Les résultats indiquent qu'une gestion adéquate des tâches, guidée par les CAD et soutenue par l'utilisation d'outils technologiques tels que le logiciel GeoGebra, contribue à l'assimilation des contenus, à la construction de significations et à l'engagement des étudiants. Nous concluons qu'une planification rigoureuse et l'emploi de stratégies interactives favorisent un meilleur apprentissage, permettant de surmonter les difficultés récurrentes dans la compréhension de la notion de limite.

Resumo

O ensino de limites de funções reais no ensino superior constitui um desafio devido à complexidade desse objeto matemático e às dificuldades enfrentadas pelos estudantes em seu processo de aprendizagem. Diante dessa problemática, o presente estudo tem como objetivo analisar a gestão de tarefas no ensino de limites de funções reais, investigando como o planejamento, a implementação e a avaliação dessas tarefas impactam a aprendizagem. A pesquisa, de abordagem qualitativa e de natureza formativa, foi realizada com dois grupos de participantes: (1) dez estudantes do curso de Bacharelado em Ciências e Tecnologia da Universidade Federal do Maranhão; e (2) três professores que lecionam Cálculo para cursos não matemáticos. A formação destinada aos estudantes teve carga horária de 120 horas, com atenção especial a um conjunto específico de tarefas. Os professores participaram dessa formação na condição de observadores e validadores das tarefas quanto à sua estrutura e conteúdo. O estudo fundamentou-se nos Critérios de Adequação Didática (CAD) da Abordagem Ontossemiótica. Os resultados indicam que a gestão adequada das tarefas, orientada pelos CAD, aliada ao uso de recursos tecnológicos, como o *software* GeoGebra, contribui para a assimilação dos conteúdos, a construção de significados e o engajamento dos estudantes. Concluímos que um planejamento cuidadoso e o emprego de estratégias interativas favorecem a aprendizagem, permitindo a superação de dificuldades recorrentes no processo de compreensão da noção de limite.

Task management in the teaching of limits of real functions in higher education

Introdução

The teaching of limits of real functions constitutes one of the central challenges in research in Mathematics Didactics focused on the teaching and learning of Calculus content, frequently marked by difficulties in understanding abstract concepts and their practical applications. Such difficulties are directly related to the formal and abstract nature of the notion of limit, which requires not only an intuitive understanding but also the development of robust analytical skills, in contrast with the procedural mathematical methods required in numerical or algebraic operations (Araya, 2022; Bertolazi, 2017; Silva et al., 2025). In this context, the way teachers plan and manage tasks is decisive in ensuring that students can overcome these difficulties and build a solid and lasting understanding of the topic (Gusmão, 2019).

The task, according to Gusmão (2019), is understood as a broad set of proposals—such as problems, activities, exercises, projects, and investigations—that aim to promote students' mathematical learning, which may vary in complexity and form of resolution, from simple reproduction of procedures to levels that require critical reflection. Task planning is a reflective and dynamic process in which the teacher defines objectives, actions, and resources, organizing content in a progressive and contextualized manner to foster understanding and the overcoming of difficulties, especially in complex topics such as limits. Task management, in turn, refers to the set of actions carried out by the teacher during its implementation in the classroom, such as preparation, contextualization, formulation of questions, interaction with students, and time organization, being essential for teaching to occur in a structured, meaningful way, adapted to students' needs.

Tasks related to different mathematical objects have been planned and managed based on normative and evaluative principles, such as the Didactic Suitability Criteria (DSC) (Breda et al., 2018; Font et al., 2010) from the Onto-semiotic Approach (Godino, 2024). For example, the research by Mendes Nunes et al. (2025) sought to analyze the competencies[1] developed by teachers when examining mathematical tasks

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[1] Within the Onto-semiotic Approach, the concept of competence is clearly distinguished from mathematical objects. While mathematical objects refer to the entities involved in the mathematical task—such as concepts, procedures, properties, representations, and

related to length measurements. The investigation conducted by Sousa and Gusmão (2023) analyzes the creative competence of (future) teachers to design[2] mathematical tasks or redesign previously existing tasks involving Quadratic Functions, as well as the knowledge and criteria they mobilize in this (re)design process. The study is based on online training that integrated moments of synchronous communication (via Google Meet) and asynchronous communication (in a Virtual Learning Environment). Another example is the research by Pochulu and Font (2025), which shows that integrating Generative Artificial Intelligence tools and the DSC helps future teachers analyze, create, and reformulate mathematical tasks in a critical and reflective manner.

Considering the importance of task analysis and management according to the DSC carried out by teachers, it is understood that this approach becomes urgent in the teaching of limits of real functions, especially given the specific challenges faced by students in assimilating the formal definition and understanding the meanings of limits in different mathematical and physical contexts.

Based on the above, this study aims to analyze task management in the teaching of limits of real functions, investigating how the planning, implementation, and evaluation of tasks, guided by the Didactic Suitability Criteria, impact learning. In particular, the principles of epistemic, cognitive, affective, interactional, mediational, and ecological suitability are considered, in order to ensure that tasks are appropriate to students' level of knowledge, promote critical reasoning, and foster interaction between teacher and student. This integrated approach seeks not only to facilitate the formal understanding of the notion of limit but also to build a deeper relationship between students and Mathematics.

The research also considers mediational suitability, especially regarding the role of technological tools in promoting more meaningful and interactive learning, connecting the concept of limit to real and practical applications, similarly to what was done by Galindo Illanes et al. (2023) and Galindo Illanes and Breda (2024) for the teaching of the derivative. Through

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arguments—competence concerns the subject's capacity to mobilize and articulate these objects in a pertinent manner to interpret, solve problems, and make didactic or mathematical decisions, in accordance with the Didactic Suitability Criteria.

[2] Design, also understood here as conception, is a process that involves actions of creating, devising, and producing authentic and original learning situations and resources. Redesign presents characteristics of creativity but not of originality. It involves actions of adjustments and modifications of data in already existing situations and resources, in order to adapt, improve, and make them suitable for learning processes.

planned didactic interventions, the aim is to address the most frequent difficulties faced by students, among which the following stand out: difficulty in understanding the objective of the proposed task; obstacles in manipulating mathematical sentences across different registers; limitations arising from the didactic process itself; and aspects related to the cognitive nature of learners, who may not yet have developed the cognitive structures necessary to assimilate the complexity inherent to Calculus knowledge, particularly the notion of limit. Furthermore, the aim is to promote not only theoretical understanding of the topic but also its application in contextualized problems.

Limits and teaching

The notion of limit is present in various situations experienced by students during Basic Education, such as inscribing regular figures in a circle, approximating points, dividing line segments, analyzing the behavior of numerical sequences, comparing areas, and approximating values through variations, among others. Thus, it is expected that, upon entering Higher Education, students already possess intuitive notions of limit, even without having been formally exposed to this specific explanation about the behavior of functions in the neighborhood of a point (Silva, 2017). Differential and Integral Calculus (DIC), like all mathematical knowledge, is built upon concepts that derive both from other areas of knowledge and from Mathematics itself, giving it a complex and interconnected structure (Silva, 2017).

According to Eves (2004) and Boyer (2012), the initial development of Calculus was marked by the absence of coherent justifications for some of the procedures used, such as the acceptance or exclusion of infinitesimals[3]. Despite the technical efficiency of these approaches, the lack of mathematical rigor generated resistance to their acceptance. Only with the emergence of the formal definition of limit was it possible to consolidate the technique of derivation (or differentiation) and integration, providing Calculus with the rigor necessary for its full acceptance. This approach not only brought mathematical consistency to the operations but also became the central axis of the discipline. Thus, the language of limits, introduced in the nineteenth century, definitively replaced the logic of infinitesimals.

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[3] Leibniz called infinitesimals the small portions to be summed in the integration process, while at the same time imagining curves as being constituted of infinitesimal parts, as if they were small line segments.

The formal definition of limit of real functions of a real variable is based on the analysis presented in the book "Cálculo A," in which the notion of limit is initially approached intuitively. According to Flemming and Gonçalves (2007, p. 66), "Intuitively, we say that a function has limit L when x tends to a if it is possible to make f(x) arbitrarily close to L, provided we choose values of x, with x ≠ a, sufficiently close to a." To formalize this idea, we consider a function f(x) defined on an open interval I containing the point a, except possibly at a itself. Thus, we state that the limit of f(x) as x approaches a is L, and we represent this relationship as: lim f(x) = L, if, for every $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$. This definition describes the behavior of the function f(x) as it approaches a value L, considering that the function assumes values of x that tend to a, but not necessarily a itself.

In DIC courses, the teaching process usually begins with examples of numerical sequences, addressing the concepts of domain, codomain, and their variations, so that students can identify the formation rule of a function, with or without the aid of graphical resources. Both in the classroom and in textbooks, there is a need for students to perceive how, when comparing two sequences, one of them converges to a limit number as the values of the other approach a specific number. From this initial perception, the formal definition of the limit of functions is presented (Silva, 2017).

Subsequently, the notion of derivative function is introduced, based on the limit applied to the slope of a line that progressively approaches the tangent of a curve. Finally, the concept of integral is developed based on the limit, considering the approximation of the area under a curve in the Cartesian plane, through partitioning and summation of areas of plane figures (Silva, 2017).

In practice, many students face difficulties in understanding lateral limits and the formal definition of limit, since these concepts demand a more detailed and abstract analysis of the behavior of functions. The main difficulty lies in assimilating that, for the limit to exist at a point, the values of the function, when analyzed from both the right side and the left side, must converge to the same value. This requirement of uniformity in lateral limits may seem counterintuitive at first, especially for students who are taking their first steps in the formal study of Calculus (Feitosa & Rodrigues, 2023).

The formal definition of limit, which involves the idea of arbitrary approximation (the concepts of "epsilon" and "delta"), adds another level of complexity, as it requires a rigorous and precise interpretation of how the function behaves as it approaches a specific point. These obstacles, combined with students' limited experience with more advanced mathematical abstractions, make this stage of learning require more detailed didactic support and differentiated tasks that help build and solidify these concepts (Dos Santos, 2013).

To reduce the difficulties that students encounter in understanding limits, various studies have been proposing diverse pedagogical methods. Among them, the use of visual tools stands out, such as the GeoGebra software, which allows for graphically exploring the behavior of functions and visualizing the concept of approximation in real time (Silva, 2017; Silva et al., 2025a). Although the emotional dimension remains little investigated in studies dedicated to the topic (Silva et al., 2025b), collaborative learning has also been widely investigated, promoting group discussions that encourage students to reflect on the concepts and construct them actively with their peers. Furthermore, according to Gusmão and Font (2021), the focus on tasks constitutes an approach that has been gaining ground, as it draws teachers' attention to the necessary care from the selection to the implementation of tasks in the classroom, contributing to the enrichment of their didactic and mathematical knowledge.

Task planning and management

A task is understood, according to Gusmão (2019, p. 1), as "a broad set of proposals, encompassing problems, activities, exercises, projects, games, experiments, investigations, etc., that the teacher brings to the classroom with the aim of promoting the mathematical learning of their students." It should be added that, according to this author, the level of competence required to solve a task varies according to the degree of thinking involved, and can be classified into three levels: reproduction, where there is repetition of routine practices and procedures; connection, which involves the integration and connection between mathematical concepts; and reflection, which refers to the capacity to reflect on the processes used in problem solving. Furthermore, the nature of tasks can be divided into closed tasks, which admit a limited number of answers, such

as traditional exercises, and open tasks, which allow multiple answers and representations, typical in more complex problems. Regarding the types of tasks, there are exercises, which focus on the repetition of procedures and calculations, and problems, which require the search for unknown elements and the understanding of relationships between concepts.

Task planning is understood as the reflective and dynamic process in which the teacher defines objectives, goals, actions, and resources necessary to promote student learning. The planning of mathematical tasks, combined with their management in the classroom, has an important impact both on students' experience with mathematical knowledge and on the depth of their learning (Pereira & Gusmão, 2020). For Gusmão (2019), planning constitutes an important stage of teaching activity. It is through planning that the teacher establishes the objectives, goals, actions, and resources necessary to produce results that converge toward student learning.

The teaching of complex mathematical objects, such as limits, requires careful and structured planning that allows students to develop a gradual and in-depth understanding. In this sense, it is important for the teacher to organize content progressively, beginning with intuitive approaches that establish a solid foundation before introducing formal definitions and more elaborate mathematical abstractions. It is essential, for example, that planning includes varied and contextualized tasks that connect concepts to students' practical reality, facilitating their comprehension. Moments of reflection and discussion in the classroom also play a fundamental role, as they allow students to question, debate, and collectively construct understanding of the topic. The use of technological tools and interactive activities that enable visualization for understanding concepts and their practical application is another strategy that can enrich planning and reinforce learning (Bruno, 2019). Thus, by carefully planning the teaching of limits, it is possible to propose strategies that help overcome common difficulties related to the topic.

Task management in Mathematics Education plays a fundamental role in the organization and efficient development of classroom activities, fostering an environment in which the teaching process of certain mathematical objects takes place in a structured and meaningful way. Task management is understood as "a set of actions developed by the teacher to efficiently explore tasks in the classroom, aiming to achieve the established objectives" (Pereira & Gusmão, 2020, p. 147). According to Gusmão (2019), task management comprises the actions mobilized by the teacher when conducting and implementing tasks in the classroom context, encompassing initial preparation, contextualization, interaction between teacher and students and among students themselves, as well as the elaboration of questions, problematizations, time management, and other necessary pedagogical arrangements.

According to Pereira and Gusmão (2020), the concept of management has been widely adapted and applied in the educational field over the years, gaining prominence in various dimensions, including the pedagogical dimension. Several authors have presented studies related to pedagogical management: Ponte (2005) addresses curriculum management; Vasconcelos (2012) and Weinstein and Novodvorsky (2015) deal with classroom management; and Gauthier (2013) discusses subject matter management and class management.

Sousa et al. (2019, p. 43) define task management as "all aspects of task application and conduction—initial preparation, contextualization, questions, provocations and problematization made, time distribution, teacher-student and student-student interaction, among other arrangements." This focus on pedagogical management allows the teacher to plan, organize, and evaluate tasks in a way that meets the needs and level of understanding of students.

In the teaching of limits, good task management allows the teacher to create didactic sequences that lead students to a gradual construction of knowledge, respecting the pace of learning and providing regular feedback that assists in content consolidation. The term didactic sequences is

understood here as a structured organization of progressively articulated didactic situations, designed to favor the appropriation of a given mathematical knowledge through the interaction between teacher, student, and environment (Suleiman, 2015). Thus, task management becomes a tool to optimize teaching and learning processes in mathematics, organizing time, resources, and pedagogical practices in order to enhance the development of students' competencies (Gusmão, 2019).

Mathematics Education, as a field of technical-scientific nature, offers a variety of theoretical-methodological approaches that assist in describing and explaining how mathematical knowledge is constructed, how teaching and learning processes develop, and how they can be improved (Breda et al., 2018; Godino, 2024). One of these approaches is the Onto-semiotic Approach to Mathematical Cognition and Instruction (OSA), characterized as an inclusive theoretical framework that, by articulating and integrating existing theories in the field of Mathematics Education (Breda et al., 2021), allows for a unified treatment of epistemological, ontological, semiotic-cognitive, educational-instructional, ecological, instructional optimization, and teacher education problems (Font et al., 2010; Godino et al., 2007; Godino, 2024).

The OSA provides a series of theoretical-analytical tools, among which the Didactic Suitability Criteria (DSC) stand out, constituting an instrument for global reflection on the practice of teaching and learning mathematics, its evaluation, and its continuous improvement (Font et al., 2010). The DSC are defined as follows (Breda et al., 2015, p. 9, translated from Font et al., 2010):

> Epistemic: refers to ensuring that the Mathematics taught is "good mathematics." For this purpose, in addition to taking the prescribed curriculum as a reference, it involves taking as a reference the institutional Mathematics that is transposed into the curriculum.
> Cognitive: expresses the degree to which the intended/implemented learning is within the students' zone of potential development, as well as the proximity of the acquired learning to that which was intended or implemented.

Affective: temporal distribution of the affective states (attitudes, emotions, affects, motivations) of each student in relation to mathematical objects and the study process followed.

Interactional: degree to which the modes of interaction allow for the identification and resolution of meaning conflicts and favor learning autonomy.

Mediational: degree of availability and adequacy of the material and temporal resources necessary for the development of the teaching-learning process.

Ecological: degree of adaptation of the study process to the educational project of the institution, curricular guidelines, and conditions of the social environment.

To represent the global evaluation of teachers' practices, a hexagonal scheme is frequently used (Figure 1), in which the regular hexagon corresponds to the ideal suitability of a mathematics teaching and learning process, while the irregular hexagon represents the actual suitability achieved during its implementation (Godino et al., 2008; Gusmão & Font, 2021).

Figure 1[4]

Didactic Suitability Criteria (Sousa & Gusmão, 2023)

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[4] Note to Figure 1: As the original figure is not accessible, the following explanation is provided for non-Portuguese speaking readers. The hexagonal diagram represents the six Didactic Suitability Criteria (DSC) from the Onto-semiotic Approach. Each vertex of the hexagon corresponds to one dimension:
INTERACIONAL (Negociação) – Interactional (Negotiation): concerns dialogue, interaction, and communication in the classroom.; ECOLÓGICA (Adaptação) – Ecological (Adaptation): relates to alignment with society, curriculum, and school context.; MEIOS (Disponibilidade) – Mediational (Availability): refers to temporal, material, and technological resources.; EPISTÊMICA (Representatividade) – Epistemic (Representativeness): concerns the institutional meaning of mathematical objects.; COGNITIVA (Proximidade) – Cognitive (Proximity): relates to the personal meaning of objects for students, involving coupling, participation, and appropriation.; AFETIVA/EMOCIONAL (Implicação) – Affective/Emotional (Implication): encompasses attitudes, affects, and motivations. The concentric hexagons indicate levels of suitability: Baixa (Low), Média (Medium), and Alta (High). The regular (outer) hexagon represents ideal suitability, while the irregular (shaded) hexagon represents the actual suitability achieved during implementation.
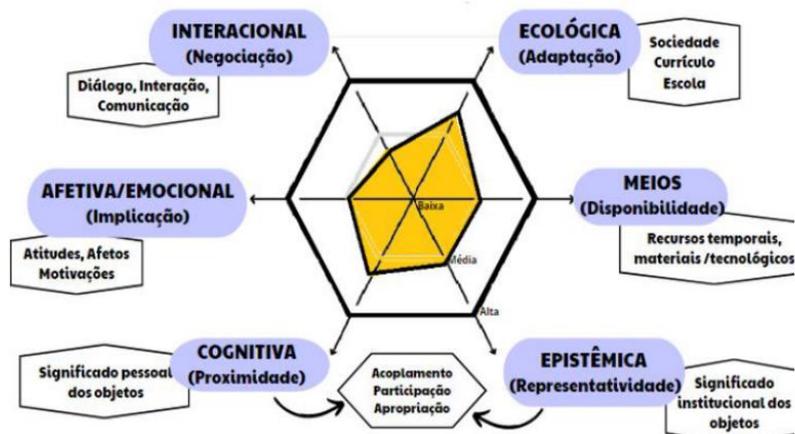
In this research, all necessary suitability dimensions were considered; however, this study emphasizes the affective dimension, which encompasses knowledge related to students' affective, emotional, attitudinal aspects and beliefs regarding mathematical objects and the instructional process adopted. This dimension refers to the degree of engagement—such as interest, motivation, and predisposition—demonstrated by students during the learning process. Furthermore, it is associated with factors that depend both on instruction and on the students themselves and their prior school history (Godino et al., 2008).

Methodology

The research adopts a qualitative approach and is part of a research-training process, understood as a collective process aimed at promoting socialization, exchange, and sharing of experiences (Silva, 2020). The investigation was conducted virtually, through the Google Meet platform, and had the following participant groups: (i) students from the Interdisciplinary Bachelor's Degree in Science and Technology (BICT) at the Federal University of Maranhão (UFMA), a program that integrates initial training in technological and scientific careers, and (ii) professors who teach Differential and Integral Calculus (DIC) in non-mathematics career programs at the same institution, such as engineering and related courses. The broader research from which this study derives was submitted to and approved by the Research Ethics Committee (REC), under opinion number 5.689.171, and included among its planned actions an online training course

focused on DIC teaching, with emphasis on the design of mathematical tasks.

This investigation is also characterized as interventional (Teixeira & Megid Neto, 2017), as it was not limited to the results obtained, but sought to understand the training process in its dynamics, considering behaviors, interventions, interactions, and productions of the participants, as recommended by Bogdan and Biklen (1994).

Regarding the participants, the training course had 21 enrollees and consisted of six meetings. Of these, 18 were students, of whom 11 participated in at least one meeting, and 3 were professors: one responsible for teaching the subject to these students and two guests, who also teach Calculus for programs in areas other than mathematics. For this study, the tasks developed in the fourth meeting of the training were analyzed. In order to preserve the identity of the participants, the following codes were used: P1, P2, and P3 for professors; A1, A2, A3, ..., A10 for students; and PE for the researcher.

The methodology of this study was structured according to the Task Study and Design Cycle (TSDC) by Gusmão and Font (2021), which comprises 8 stages: Diagnosis, Study, Analysis, Planning and Selection, Design/conception, Implementation, Evaluation, and Redesign. These were grouped in this study into three: planning management (diagnosis, study, analysis, selection, and redesign of tasks); implementation management (implementation); and evaluation management (evaluation), with a focus on developing strategies for teaching limits, according to the understandings of Pereira (2019) and Gusmão (2019). Considering the moments of direct interaction with the participants, it was only in the fourth meeting of the training that we specifically analyzed these stages, seeking to understand how each of them contributes to the effectiveness of the teaching process.

Regarding Planning, the aforementioned authors highlight the importance of carrying out two fundamental steps: the study and selection of tasks, both essential for teaching effectiveness. In the Planning phase,

for the first stage, the training course was planned and documented in detailed forms containing objectives, content, and strategies, with group tasks in online format distributed across six meetings. The mathematical objects related to each task of meeting four were also analyzed. In the second stage, three tasks selected for the fourth meeting were applied, following the planning, respecting the time allocated for activities, and promoting interaction among participants, with close monitoring to address difficulties in real time.

Finally, for the analysis of planning results, the evaluation of the process was carried out based on the DSC adapted for task design, according to Table 1.

Tabela 1

Critérios de análise de desenho de tarefas (adaptado de Sousa & Gusmão, 2023, p. 8)

Task design analysis criteria (adapted from Sousa & Gusmão, 2023, p. 8)

TDI	Task Design Indicators
Epistemic Adequacy	
EP1	Is the statement presented in clear, correct language appropriate to the educational level?
EP2	Do they use different languages and forms of mathematical expression (verbal, graphic, symbolic, pictorial, etc.)?
EP3	Is the selection of tasks representative and varied, including both closed-ended and open-ended tasks?
EP4	Are the tasks of different types?
EP5	Do they promote hypothesis formulation, open-minded thinking (reversible, flexible, decentralized thinking), and encourage the use of argumentation and justification processes?
Cognitive Adequacy	
IC1	Do they build upon the students' prior knowledge?
IC2	Do they expand, reinforce, and systematize knowledge?
IC3	Do they respect the cognitive development level of the students?
C4	Do they encourage the use of different, creative, and original problem-solving strategies?
IC5	Do they cater to different learning objectives and lead the problem solver to develop different cognitive and metacognitive skills?
Interactional Suitability	
IN1	Does it include opportunities for dialogue and discussion among students or between teacher and students?
IN2	Do they encourage individual, paired, or group problem-solving?

IN3	Do they allow for the generation of cognitive conflict (in the Piagetian sense) and the negotiation of meanings?
IN4	Do they encourage responsibility for the study (exploration, formulation, and validation)?

Adequacy of Means

ME1	Do they provide or suggest the use of manipulable and/or technological materials to assist in the task?
ME2	Do they allocate sufficient time for its completion and for maintaining concentration and interest?
ME3	Are the timeframes appropriate for the types of tasks (reproduction, connection, reflection, etc.)?
ME4	Do they foresee adequate spaces for its implementation?
ME5	Do they include opportunities for hands-on experimentation to aid in understanding concepts and their applicability?

Affective Adequacy

EM1	Do they promote interactivity, attraction, fun, and inclusion, raising self-esteem, a sense of belonging, openness to subjectivity, and a love of mathematics?
EM2	Do they value different types of reasoning and responses?
EM3	Do they encourage participation and interest?
EM4	Do they promote the perception of the usefulness of mathematics in life and at work?
EM5	Do they promote student involvement in solving tasks (returning learning in Brousseau's sense)?
EM6	Do they present achievable challenges, triggering increasingly complex levels of thinking?
EM7	Do they showcase the application and beauty of mathematics?

Ecological Suitability

EC1	Do they include the official curriculum documents (national and local)?
EC2	Are they seeking connections between different areas of mathematics and between different fields of knowledge?
EC3	Are the tasks contextualized within the social and cultural environment?
EC4	Are the task contents useful for social and work life?

These criteria were applied to analyze whether the tasks, resources, and course conduction met the proposed objectives. We evaluated the adequacy of the content, the pedagogical relevance of the activities, the clarity of explanations, and the interactivity provided by digital resources. The use of criteria adapted for task design allowed for the identification of strengths and aspects to be improved in task management, ensuring a

critical and grounded analysis of the course's effectiveness in promoting meaningful student learning.

In the implementation phase, students were organized into two groups, using separate rooms in Google Meet. It should also be noted that the online meetings were transformed into spaces conducive to learning, promoting the exchange of ideas, dialogue, and collaborative task resolution. Social interactions between participants and the researcher, according to Chizzotti (2003), are essential and inherent to the research process. Thus, for data production and analysis, the instruments used were videos recorded during the online meetings and the tasks proposed by the researcher, with the objective of verifying the management of tasks on limits. The analyses were conducted in light of the theoretical framework, with special attention to the affective dimension, seeking to identify aspects that contributed to or limited the promotion of meaningful student learning.

Results and discussion

In this section, the results are discussed in three moments: planning management, implementation management, and evaluation management, presenting the characteristics of the three tasks carried out in the fourth meeting of the research-training and their relationship with the didactic suitability criteria.

Gestão do planejamento

Planning is fundamental in the management process and represents an important stage of teaching activity. Thus, in the planning management of the study, the first author conducted bibliographic reviews, selected content, spoke with professors, and jointly planned the training with the other authors, as described in Table 2.

Table 2.
Study Planning Management

STUDY	DESCRIPTION
- Systematic reviews, production of an article	- In-depth study based on two literature reviews addressing difficulties and challenges related to the topic;

on PPP and content selection.	- Production of an article on Political Pedagogical Projects (PPP) for non-mathematical career courses in order to enable a better design of tasks addressing specific aspects of the necessary knowledge; - The selection of content was based on the first systematic review, which identified the topic that students had the most difficulty understanding: limits.
- Conversation with teachers and training planning	- Meeting between the researcher and her advisors to define the target audience for the course; - Meeting with the subject professor to adjust the lesson plan; - The planning stemmed from the researcher's work experience, focusing on improving the tasks since they comprised the tools used in their work as a teacher.

The systematic literature review on Tasks for the Teaching of Differential and Integral Calculus (Silva et al., 2025b) aimed to identify contributions, institutions of origin, content addressed, emotional aspects, and possible new research problems for further investigation. As a result, "Limit of a real function of a real variable" was highlighted, mainly considering the difficulties that students present during the teaching and learning processes of this mathematical object. Based on this, we developed a new systematic review study (Silva et al., 2025a), seeking to critically analyze existing knowledge about tasks in the teaching of limits of real functions. This new review explored the difficulties and challenges faced by students, as well as evaluated the implementation of tasks according to the Task Design Criteria (TDC).

Both systematic reviews were fundamental in defining the content to be addressed, objectives, mathematical knowledge to be further explored, and in conceiving tasks that would engage students in the topic while helping them perceive the mathematical object addressed through theoretical and practical applications.

Still within the scope of planning, meetings were held between the researcher and her advisors to define the target audience of the course. As

previously mentioned, the training participants were BICT students, the professor responsible for these students, and two invited professors. There were also meetings with the course professor to understand how the subject would be taught and what content would be covered, allowing for the planning and selection of tasks, as well as the definition of the training implementation period. Additionally, a registration form was created containing information such as: name, email, phone number, category (professor or student), undergraduate program, most suitable time for the training (9 AM or 2 PM), and CPF [Brazilian individual taxpayer registry number].

In the task selection stage, the planning of selection and adjustment of tasks was carried out, as well as the planning of time, space, and resources (Table 3).

Table 3
Task Selection Planning Management

TASK SELECTION	DESCRIPTION
-Planning the Selection and Adjusting of Tasks	- Planning the selection of tasks that would be covered in the training; - Preparation of worksheets for each task; - Planning task adjustments with participants in mind; Note: The tasks were designed to be worked on in groups, aiming to maximize interaction and collaborative learning.
-Planning time, space and resources	Approach: format*on-line* - Number of meetings: six (total for the training) - Teaching Resources:*slides*for interactive lecture classes,*software*how*GeoGebra*using*applet*s, *Excel*.

The selected tasks had already been developed and used by the first author in the context of her work in the classes where she teaches in undergraduate Natural Sciences Teaching degree programs. However, all of them underwent adjustments (redesigns), taking into account the student profile and difficulties identified in the reviewed studies and in conversations with the professors.

Figures 2, 3, and 4 present a synthesis of the objectives of each task. As an example of redesign, noteworthy is the case in which the mathematical object, initially a simple quadratic function, was replaced, in the training context, by a piecewise-defined function. This modification aimed to expand the possibilities of analysis from a single function.

Task 1 was developed to enable understanding of the meaning of lateral limits, through algebraic procedures and graphical visualization using, as technological tools, applets developed by the authors.

Figura 2[5]

Tarefa 1. Elaboração própria

TAREFA 1		
Conteúdo abordado: Limites Laterais	**Descrição:** Calcular os limites laterais de uma função algebricamente e em seguida verificar as respostas no *apllet* do GeoGebra.	

Dada a Função

$$f(x) = \begin{cases} x^2 - x - 1 & : x < 2 \\ 1 & : 2 \leq x < 4 \\ \dfrac{1}{x-7} - 1 & : \text{caso contrário} \end{cases}$$

Encontre:

a) $\lim\limits_{x \to 1^-} f(x)$ b) $\lim\limits_{x \to 1^+} f(x)$ c) $\lim\limits_{x \to 1} f(x)$

d) $\lim\limits_{x \to 4^-} f(x)$ e) $\lim\limits_{x \to 4^+} f(x)$ f) $\lim\limits_{x \to 4} f(x)$

Para casa

g) $\lim\limits_{x \to 7^-} f(x)$ h) $\lim\limits_{x \to 7^+} f(x)$ i) $\lim\limits_{x \to 7} f(x)$

Primeiramente calcule os limites em uma folha de papel e em seguida use o applet para verificar as respostas.

Nível da tarefa	Tipo de tarefa	Natureza da Tarefa
1° momento: nível de reprodução 2° momento: nível de conexão	1° momento: Tipo exercício 2° momento: Tipo Problema	Fechada

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Task 2 (Figure 3) was proposed to be discussed during class as a deepening of the previous task and to bring concrete meaning to lateral limits, using problems involving notions of physics.

Figura 3[6]

Tarefa 2

$$f(x) = \begin{cases} x^2 - x - 1 : x < 2 \\ 1 \quad : 2 \le x < 4 \\ \dfrac{1}{x - 7} - 1 : \text{otherwise} \end{cases}$$

Find:

- a) $\lim\limits_{x \to 1^-} f(x)$ b) $\lim\limits_{x \to 1^+} f(x)$ c) $\lim\limits_{x \to 1} f(x)$ d) $\lim\limits_{x \to 4^-} f(x)$ e) $\lim\limits_{x \to 4^+} f(x)$ f) $\lim\limits_{x \to 4} f(x)$

Homework: g) $\lim\limits_{x \to 7^-} f(x)$ h) $\lim\limits_{x \to 7^+} f(x)$ i) $\lim\limits_{x \to 7} f(x)$

Instruction: First calculate the limits on a sheet of paper and then use the applet to verify the answers.

The figure includes a GeoGebra applet screenshot showing the graph of the piecewise function with interactive sliders to explore lateral limits visually.

Task Level	Task Type	Task Nature
1st moment: reproduction level	1st moment: Exercise type	Closed
2nd moment: connection level	2nd moment: Problem type	

[6] **Note to Figure 3 (Task 2):** As the original figure is not accessible, the following explanation is provided for non-Portuguese speaking readers.
TASK 2
Content addressed: Lateral Limits (*Limites Laterais*)
Description: Interpret the applications and state the result by analyzing the graphs.
Problem 1: A gas (water vapor) is maintained at constant temperature. As the gas is compressed, the volume V decreases until it reaches a certain critical pressure (P). Beyond this pressure, the gas assumes liquid form. Observing the figure below, determine:
a) $\lim\limits_{p \to 100^-} V$ b) $\lim\limits_{p \to 100^+} V$ c) $\lim\limits_{p \to 100} V$
The accompanying graph shows Volume V (liters) on the vertical axis versus Pressure P on the horizontal axis, with values ranging from approximately 0.4 to 0.6 liters. The graph

Task 3 (Figure 4) was developed to address the concept of limits with delta conditions. The task is open-ended, as the solution is not limited to a single value of delta; any value smaller than the delta found satisfies the condition for the proof.

Figura 4[7]

Tarefa 3.

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shows a curve decreasing as pressure increases, with a discontinuity at P = 100, where the gas transitions to liquid state (*Líquido*).

Problem 2: A patient in a hospital receives an initial dose of 200 milligrams of a medication. Every 4 hours, they receive an additional dose of 100 mg. The quantity f(t) of the medication present in the bloodstream after t hours is displayed in the figure below. Determine and interpret:

a) $\lim_{t \to 4^-} f(t)$ b) $\lim_{t \to 4^+} f(t)$

The accompanying graph shows the medication quantity f(t) in milligrams on the vertical axis (ranging from approximately 100 to 300 mg) versus time t in hours on the horizontal axis. The graph displays a sawtooth pattern with discontinuous jumps at t = 4, 8, 12 hours, representing the additional doses administered.

[7] **Note to Figure 4 (Task 3):** As the original figure is not accessible, the following explanation is provided for non-Portuguese speaking readers.

TASK 3

Content addressed: Formal Definition of Limit (*Definição formal de Limite*)

Description: Show the limit algebraically using the definition and then use the applet to verify that, from that delta, the definition is satisfied.

Problem 5: Show, using the limit definition, that $\lim_{x \to 1} -2x + 5 = 3$.

Problem 6: Verify if the δ found satisfies the definition in the GeoGebra applet.

The figure includes a GeoGebra applet screenshot titled "Exercitando a definição" (*Practicing the definition*), which displays:

- The formal epsilon-delta definition of limit in Portuguese: "Let f(x) be defined on an open interval I containing x_0, except possibly at x_0 itself. We say that the limit of f(x) as x approaches x_0 is L, written $\lim_{x \to x_0} f(x) = L$, if for every ε > 0, there exists a δ > 0 such that |f(x) − L| < ε whenever 0 < |x − x_0| < δ."
- A graphical visualization showing the function f(x) = -2x + 5, with interactive sliders for ε (epsilon) and δ (delta), allowing students to visually verify the epsilon-delta relationship.

Task Level Task Type Task Nature

Connection Problem Open

TAREFA 3	
Conteúdo abordado: Definição formal de Limite.	**Descrição:** Mostrar algebricamente limite usando a definição e em seguida usar o *apllet* para ver que a partir daquele delta a definição é satisfeita.

5) Mostre usando a definição de limite que $\lim_{x \to 1} -2x + 5 = 3$.

6) Verifique se o δ encontrado satisfaz a definição lá no applat do geogebra

Exercitando a definição
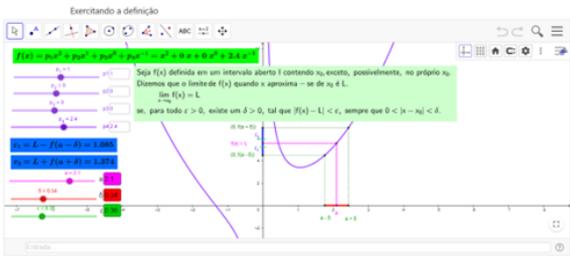
Nível da tarefa	Tipo de tarefa	Natureza da Tarefa
Conexão	Problema	Aberta

The three proposed tasks clearly meet the DSC in various dimensions. Regarding Epistemic suitability, all tasks have statements with clear language appropriate to the level of education (EP1), using different forms of mathematical expression, such as graphical and symbolic representations (EP2). They are varied, encompassing both closed and open problems, which enables a greater range of reasoning and methods (EP3, EP4). Furthermore, they promote hypothesis generation and encourage argumentative processes, contributing to the development of students' critical thinking (EP5).

Regarding Cognitive suitability, the tasks start from students' prior knowledge, expanding and systematizing new concepts (IC1, IC2). Additionally, they are appropriate to the cognitive level of participants, encouraging the use of creative and original strategies for problem solving (IC3, IC4). The variety of learning objectives addressed allows students to develop diverse cognitive and metacognitive competencies (IC5).

In terms of Interactional suitability, the tasks provide moments for dialogue and argumentation among students and between students and teacher (IN1), stimulating the resolution of activities with students organized in groups and allowing the generation of cognitive conflicts, which are essential for the construction and negotiation of meanings (IN2, IN3). In this way, the tasks encourage students' responsibility for their learning, promoting the exploration and validation of mathematical concepts (IN4).

The tasks are also consistent with the Mediational suitability criteria, providing technological resources, such as the use of applets developed in GeoGebra, to assist in understanding the concepts addressed (ME1). The time allocated for carrying out the activities was carefully planned to ensure students' concentration and interest (ME2), with appropriate times for different types of tasks, such as reflection and connection (ME3). Additionally, there are specific moments for practical experimentation, which assists in understanding and applicability of concepts (ME5).

Regarding Affective suitability, the tasks promote interactivity, inclusion, and diversity, raising students' self-esteem and stimulating interest in Mathematics (EM1, EM3). They value different types of reasoning and responses, promoting active student participation (EM2). The proposed challenges are accessible and, at the same time, require increasing levels of complexity in mathematical thinking (EM6), in addition to highlighting the beauty and applicability of Mathematics in everyday life (EM7).

Finally, in terms of Ecological suitability, the tasks were validated by professors, are aligned with official curricular documents, and are contextualized according to the level of education (EC1, EC3). They integrate different areas of mathematical knowledge, as well as establish connections with other areas, promoting broader learning (EC2). The contextualization of content is also in tune with students' social and work life, ensuring practical relevance for what is taught (EC4).

The planning moment proves to be of paramount importance in the context of research-training and in the context of teaching and learning processes of a given mathematical object, since the study and selection of good tasks depends directly on deepening knowledge about students' difficulties and needs, as well as best practices in teaching specific content (Pereira & Gusmão, 2020). It is through this study that the teacher becomes equipped to develop didactic tasks that not only meet the suitability criteria but are also capable of engaging students, promoting understanding of the concept of limits of real functions. The knowledge constructed in this phase allows the teacher to develop tasks that are not only appropriate to

students' learning level but also challenge them to apply theoretical and practical concepts in relevant contexts, facilitating the overcoming of difficulties encountered (Gusmão, 2019).

We also sought to propose interactive tasks that would spark student engagement, making the most of the virtual environment as a space for knowledge construction, as proposed by Galindo Illanes et al. (2023). Interactive tasks are especially suitable in online environments, as they allow students to explore concepts in a dynamic and participatory manner, rather than simply consuming information passively. The potential of the online format lies in its capacity to offer a variety of multimedia resources, such as videos, simulations, and applets, as in the case of this training, which can enrich the understanding of mathematical concepts (Ramos et al., 2014).

Finally, it should be noted that time management is an essential aspect for the success of any pedagogical meeting, especially in virtual environments, where maintaining students' attention and engagement can be more challenging. In the fourth meeting, the careful distribution of time for each stage—from the review of concepts to task resolution and presentation of results—was fundamental to ensure that students had the opportunity to explore and understand the content, favoring the construction of knowledge about the proposed mathematical object.

Efficient time management enabled interactions to be rich and productive, facilitating the progressive development of tasks, which were planned to advance from simpler to more complex concepts. Furthermore, the alternation between tasks of different natures—some requiring connection between meanings and others demanding greater reflection—was essential to maintain a balance between challenge and comprehension. In virtual environments, this organization becomes even more important, as it ensures that all students can keep up with the pace of the class, participate actively, and construct their knowledge (Pereira & Gusmão, 2020).

In this context, adequate task design is decisive for promoting effective learning, especially in complex topics, such as the notion of limits, involving the condition of delta as a function of epsilon, for example. Proposing diversified tasks that address different levels of abstraction allows students to gradually advance in their understanding, starting from more accessible concepts and progressing to more complex ideas (Gusmão, 2021).

Implementation management

According to Gusmão (2019), Implementation Management refers to the practical execution of what was planned, the moment of putting mathematical tasks into practice in the classroom to ensure that learning objectives are achieved. The author emphasizes that the conduction of tasks must be done carefully by the teacher, who has the role of guiding the completion of activities, encouraging students to offer explanations and justifications for their answers. Implementing the plan goes beyond initial intentions, requiring the teacher to put tasks into practice in a planned and structured manner. To achieve educational objectives, it is essential that tasks be proposed and conducted with attention to detail, ensuring that each stage contributes effectively to student learning.

Table 4 highlights the main actions that occurred during implementation.

Table 4

Implementation Management. Developed in-house.

IMPLEMENTATION MANAGEMENT	
Part 1: Review and explanation	Review of the concepts from the previous lesson; - Expository section with dialogue
Part 2: Task 1	- First task of the day: calculate the lateral limits of a function defined by several sentences and use a GeoGebra application to visualize the results. -Develop the tasks using the *applet*s -Division by teams Return to the main room for discussion.
Part 3: Task 2	-Second task of the day: practical applications of limits through problems;

	- Interpreting the problems together with the whole class, through a dialogic lesson. - Students completing the tasks.
Part 4: Task 3	The final task of the meeting was the formal definition of a limit. - Use of *applet*s -Division by teams Return to the main room for discussion.

The fourth meeting of the course was a moment full of practical tasks and discussions about the notions of limits. From the beginning, there was intentional care with the review of prior knowledge, a fundamental aspect of the cognitive criterion (IC1), by reviewing the content covered in the previous meeting, in which limits had been explored intuitively using approximations with tables of values and graphs.

In the development of the first task, the lateral limits of a piecewise-defined function were addressed. Attention was paid to the epistemic criterion, with the explanation of algebraic calculation from a specific point of the function, highlighting the distinction between limits from the left and from the right. Next, a GeoGebra application, previously configured, was used, favoring the mediational criteria ME1 and ME5. The dynamic visualization of limits from different sides also contributed to the affective criteria EM1 and EM3, promoting greater engagement and confidence among students regarding the content.

The students were then divided into two groups, using separate rooms in Google Meet. Although the logistics of the division required more time, which highlights the mediational criterion (time management), the task was successful. The students were able to manipulate the application and understand the proposed concepts. The group dynamics also favored the interactional criterion, by stimulating the exchange of ideas and mutual support among participants during the activity. At the end, everyone returned to the main room, and each group explained the limit for a different point of the function. A task was assigned to be completed at home, including a bonus question that required research on infinite limits,

with discussion planned for the following meeting. This proposal favored the interactional criterion, by promoting students' autonomy in actively seeking new knowledge, encouraging protagonism in the learning process.

The class continued with practical applications of limits, through functions that modeled situations such as the pressure of a gas, the time a medication remains in the bloodstream, the temperature of a part when leaving an industrial furnace, and the velocity of a particle in classical physics and in the theory of relativity, evidencing the epistemic criterion EP3. Furthermore, by integrating different mathematical content and areas of knowledge, with a strong relationship to everyday life and science, the presence of ecological criteria EC2 and EC3 is observed. One female student stood out by answering several questions correctly. Students were asked to respond with the values of the lateral limits when analyzing the graph. It is worth noting that whenever students made errors, the researcher re-explained the notions.

The last task of the meeting was about the formal definition of limit. The researcher explained the definition succinctly—epistemic criterion—gave an algebraic example, and demonstrated how it worked in the GeoGebra application. The students were then divided into groups again. Both groups were able to respond to the problem following all the guidelines; however, only one group fully understood the integration of all meanings. This difference only became clear when the researcher reviewed the recordings of the students' discussions, as during class, no one expressed significant doubts.

The management of content and class is fundamental for student learning, especially in complex topics such as those covered in the course on limits. According to Gauthier (2013), effective class management involves keeping students engaged and motivated, which is crucial for creating a positive learning environment where students feel encouraged to participate and explore new concepts.

Evaluation management

Evaluation Management involves the continuous analysis and evaluation of teaching and learning practices, aiming to improve these processes. According to Gusmão (2019), this management can be understood as a cyclical movement that includes reflection and self-knowledge, in which the teacher is able to analyze and evaluate their own practice in a critical and reflective manner.

Evaluation management includes the identification and analysis of the teacher's evaluative practices in the context of mathematical tasks, ensuring that these practices are fair and contribute to the development of students' knowledge. These elements form a continuous reflective spiral of planning, implementation, and evaluation, which is fundamental to ensure that students achieve meaningful learning.

An evaluative process was carried out for both planning and implementation, around three aspects: what worked from what was proposed, what did not work and could have been approached differently, and what was not achieved and could be modified/redesigned for future applications, as shown in Table 5.

Table 5
Evaluation Management. Own elaboration.

EVALUATION MANAGEMENT	
What worked	Initial Review; -Use of technological tools; Practical application; Active participation: One student stood out by answering several questions, bringing a positive dynamic to the class.
What didn't work	- Group Logistics; - The teachers' lack of participation in all meetings; - Assimilation of more complex concepts.
Changes planned	- In-person meetings; - Monitors for assistance; Another trainer

An analysis of evaluation management according to the suitability criteria was also carried out, as described in Table 6.

Table 6.
Analysis of Task Evaluation Management according to Adjustments. Own elaboration.

Epistemic	•The tasks present clear and accessible language, appropriate to the students' educational level. •The tasks cover relevant mathematical concepts, such as the concept of function limits and their interpretations in different contexts (gas pressure, medication dosage, part cooling). •They offer a moderate challenge.
Cognitive	•The tasks build upon the students' prior knowledge, reinforcing concepts already learned and expanding their understanding of function limits. •They are appropriate to the participants' level of cognitive development, but could more explicitly address the individual differences between students. • They encourage the use of varied strategies for problem-solving.
Interactional	•The tasks encourage interaction among students, with planned times for group work and for dialogue between students and the teacher. •They allow us to identify and resolve cognitive conflicts, such as different interpretations of lateral boundaries or the concept of a non-existent boundary. • Dialogue and argumentation, especially in the context of solving group tasks, are encouraged, but there could be greater incentive to promote student autonomy in the learning process.
of Means	•The tasks utilize appropriate resources, such as the use of*applets* no *GeoGebra* and the *Excel*, to aid in understanding the concepts discussed. •The time allocated for completing the tasks was planned sufficiently, allowing for the proper execution of the activities.
Affective	•During the completion of the tasks, the students' engagement was observed, indicating positive attitudes and motivation in the face of the proposed challenges, especially when they were able to advance in their understanding of the concepts of limits. •The interactions within the groups revealed an environment of mutual support, contributing to the building of affective bonds and to the reduction of anxiety related to mathematical content.
Ecological	•They make use of interdisciplinarity, associating limits with concepts from physics, as advocated in the curricula of non-mathematical careers. •The tasks are contextualized appropriately to the students' social environment, especially with practical examples applicable to everyday life.

Good task management was achieved, with a well-structured approach that solidly integrated the epistemic dimension. This was duly represented through an in-depth study in which we sought to identify students' main difficulties, complemented by a systematic literature review. Additionally, Pedagogical Political Projects (PPP) of non-mathematics careers were studied, specifically regarding the content of limits, with the objective of analyzing the "representativeness of the mathematical object" (Gusmão, 2021, p. 680). This study provided a solid foundation for understanding the challenges and proposing more appropriate pedagogical strategies for teaching limits of real functions.

We sought to explore the affective dimension, which proved to be scarce in our first systematic review. However, students' statements during the process revealed a deeper understanding of the content, demonstrating that learning impacted not only the cognitive dimension but also began to touch the affective dimension, as can be inferred from the dialogue fragment presented in Figure 5.

Figura 5

Atividade no GeoGebra e Diálogo com participante. Elaboração própria
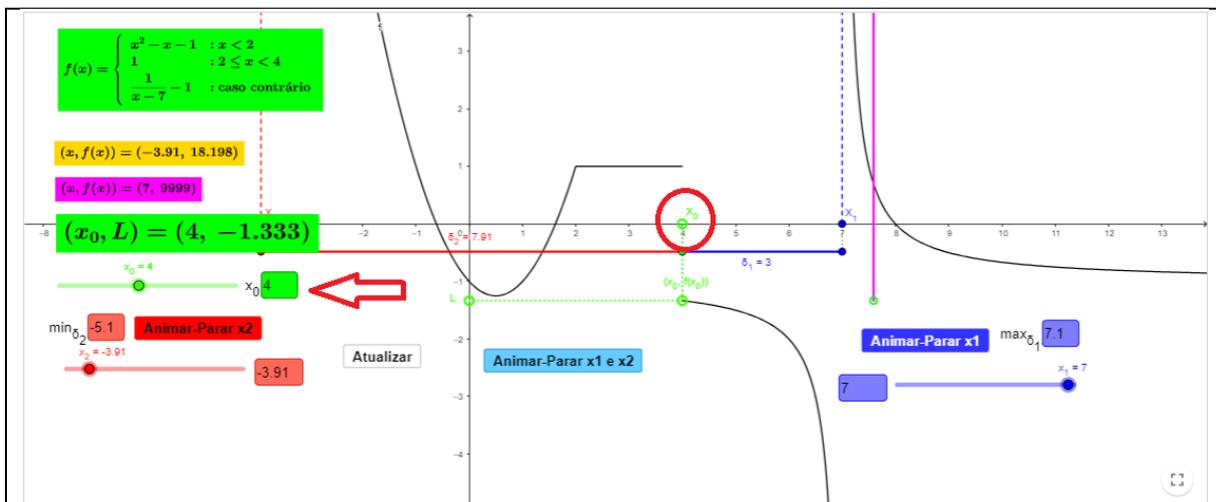
T (Teacher): Right. Were you able to do number four?
S5 (Student 5): I tried, but I'm getting different values for the one-sided limits.
T: Spot on! This one is indeed different. See how at 4, the graph has a jump. It's not continuous. There is a discontinuity here. So, the one-sided limits really are different. What did you get from the left?
S5: From the left, it's... It's 1, right?

> T: Nailed it! It's 1. And from the right?
> S5: Minus four-thirds.
> T: Spot on.
> S5: So, in this case, the limit at 4 doesn't exist, right?
> T: Which one?
> S5: At 4. Because they are different.
> T: It doesn't exist, exactly, because the values are different, both from the right and from the left. Exactly. Very well done.
> S1: Wow, I finally get it, both in the calculations and visually.

By focusing efforts on reducing the difficulties faced by students, the affective dimension is brought closer, promoting an environment of greater confidence and emotional involvement with the content.

The affective dimension of students manifested itself subtly throughout the interaction. The statements indicate progress in cognitive understanding, especially when they began to recognize discontinuities and distinct lateral limits. The exchange of praise reinforces students' emotional security, motivating them to continue exploring the content and favoring their affective engagement with mathematics. For example, A1's statement, "Wow, I finally understood," evidences a moment in which cognitive understanding connects directly with emotional relief, reflecting personal satisfaction for overcoming previous difficulties.

According to Rocha (2016, p. 35), "emotions, attitudes, and beliefs act as driving or resistance forces to learning," highlighting the importance of considering the affective dimension in the educational process.

Below are other fragments of students' statements that evidence how understanding the content impacted their perceptions and emotions.

> A7: I really enjoyed it. I was able to understand more about lateral limits.
> A11: Today's class was important for clarifying doubts regarding delta and epsilon, since understanding these questions about intuitive notion was a very big doubt I had. In addition to better understanding the graph with lateral limits.
> A9: Even though I arrived late, I was able to absorb the content well.

The statements recorded by students in the attendance form reveal a progressive assimilation of the content, especially regarding lateral limits and the formal definition of limit, evidencing learning under the cognitive criterion. It was observed that the use of technological resources was fundamental for graphical visualization, facilitating intuitive understanding

(ME5 and IC2). Furthermore, group activities played an important role in the collaborative construction of knowledge, allowing for the exchange of ideas and the resolution of doubts in a more dynamic manner.

Additionally, the mention of solving contextualized questions (ecological DSC) demonstrates that aspects of lateral limits and infinite limits (epistemic DSC) were successfully addressed. See Figure 6.

Figura 6[8]

Questão sobre limites laterais e limites infinitos e Diálogo com participante.

Elaboração própria

Seja $T = f(t)$ a temperatura de uma peça t minutos depois de retirada de um forno industrial. A figura abaixo mostra a curva da temperatura versus tempo para a peça, onde r denota a temperatura ambiente. Pergunta-se:

a) Qual é o significado físico de $\lim_{t \to 0^+} f(t)$?

b) Qual é o significado físico de $\lim_{t \to \infty} f(t)$?
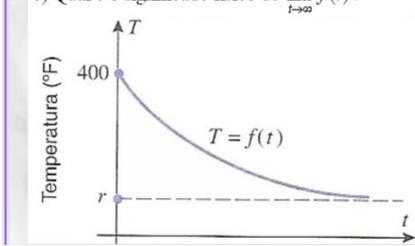
PE: Let T equal f of t be the temperature of the part in minutes after removal from the industrial furnace. The figure below shows the temperature versus time curve of the part, where R denotes the ambient temperature. So, what happens? (...) What is the physical meaning of the limit of f of t as t approaches zero from the right? (...)

---

[8] Note to Figure 6: As the original figure is not accessible, the following explanation is provided for non-Portuguese speaking readers.

Limits (Limites)

Let T = f(t) be the temperature of a part t minutes after removal from an industrial furnace. The figure below shows the temperature versus time curve for the part, where r denotes the ambient temperature.

Questions:

a) What is the physical meaning of $\lim_{t \to 0^+} f(t)$?

b) What is the physical meaning of $\lim_{t \to +\infty} f(t)$?

The accompanying graph shows:

Vertical axis: Temperature (°F), with a marked point at 400

Horizontal axis: time t

A decreasing exponential curve labeled T = f(t), starting at 400°F when t = 0

A horizontal dashed line at level r representing the ambient temperature

The curve approaches the ambient temperature r asymptotically as time increases.

This task illustrates the physical application of limits: as t approaches 0 from the right, the limit represents the initial temperature of the part immediately after removal from the furnace (400°F); as t approaches infinity, the limit represents the final equilibrium temperature, which equals the ambient temperature r.

> A2: Would it be 400 in this case?
> A5: 400
> PE: That's right! So, the physical meaning is that it is as hot as possible. In this case, it is the maximum heat, because that's how it was when removed. So, it is at 400, an extremely high temperature.
> And for item b), when time tends to infinity, what is happening?
> A2: It is approaching R here, which is the ambient temperature.
> PE: Exactly, it is approaching R here, which is the ambient temperature. It cools down completely and reaches the temperature of the environment.
> A3: These applications are really cool.

The positive feedback evidences the importance of the DSC, such as the appropriate choice of resources and tasks that connect intuition to formalization, promoting a more suitable and welcoming learning environment. Furthermore, the discussion present in the statements clearly illustrates the ecological dimension of teaching limits, by contextualizing the mathematical concept through a concrete everyday situation, such as the cooling of a part after being removed from an industrial furnace.

This approach promotes interdisciplinarity, integrating knowledge from physics, specifically about heat transfer, with mathematics, expanding students' understanding of the behavior of the temperature function in relation to time. The contextualization of the problem allows students to perceive the practical application of limits, bringing them closer to real situations and awakening greater interest and engagement with the content, as evidenced by A3's comment: "These applications are really cool." In this way, learning is strengthened, demonstrating that mathematical objects have relevance in various areas of knowledge (EC4).

It is believed that there was good management of activities, reflected in appropriate didactics and use of resources. The careful choice of tools, such as GeoGebra for the visualization of lateral limits, facilitated students' understanding of the meanings of limit, while the organization of tasks enabled doubts to be resolved throughout the process, promoting gradual and structured learning. Additionally, well-executed planning ensured that the class objectives were achieved, providing a teaching experience that favored both the cognitive and affective dimensions of students.

Evaluation plays a fundamental role in the educational process, as it allows for critical feedback on the planning and execution of classes, offering a comprehensive view of what worked and what can be improved. Through evaluation, it is possible to review and adjust theoretical, methodological,

and attitudinal aspects, increasing teaching effectiveness. This process of continuous reflection ensures that teaching activities are aligned with learning objectives and meet students' needs. As Gauthier (2013) emphasizes, all teaching activity requires systematic and continuous evaluation, encompassing practice, the practice of practice, and practice about practice. This evaluation cycle allows for the identification of areas for improvement and the implementation of strategies that promote more effective learning, ensuring that the educational experience is constantly enhanced.

Finally, the results achieved can be considered satisfactory, since participants' statements evidence active involvement in the learning process, as well as manifestations of engagement and satisfaction upon understanding the concepts addressed. Expressions of relief, enthusiasm, and recognition of their own learning indicate that students not only advanced cognitively but also experienced positive feelings related to overcoming difficulties in the study of limits, reinforcing the effectiveness of the intervention carried out.

Final considerations

The study achieved its central objective by investigating task management in the teaching of limits of real functions, evidencing that a didactic organization grounded in the Didactic Suitability Criteria contributes significantly to overcoming the difficulties presented by students in understanding this mathematical object. The analysis carried out demonstrated that the articulation between planning, implementation, and evaluation of tasks enabled the progressive construction of knowledge, favoring the understanding of concepts such as lateral limits, discontinuities, and physical meanings of limit, with support from technological resources such as GeoGebra and collaborative activities.

The results reveal that attention to the epistemic and cognitive dimensions had a direct impact on the affective dimension, since students began to express greater confidence, involvement, and satisfaction upon perceiving their advances in learning. The recorded statements indicate that the training process not only promoted conceptual understanding but also contributed to strengthening participants' confidence, evidencing that adequate task management can positively impact both students' performance and their emotional relationship with mathematical content.

Thus, it is concluded that the approach adopted in this investigation reaffirms the relevance of task management as a powerful strategy for teaching limits in non-mathematics career programs, by coherently integrating theoretical aspects, didactic resources, and pedagogical interaction. The analyzed experience demonstrates that care in the design and conduction of tasks expands the possibilities for meaningful learning, while promoting a more welcoming, participatory, and cognitively productive environment.

Based on the study's conclusions, future research can explore different task management strategies in diverse contexts, investigating how didactic organization, grounded in the Didactic Suitability Criteria, influences the learning of limits in subjects or courses from different areas, including comparisons between face-to-face, hybrid, and fully virtual environments. Additionally, studies can deepen the analysis of the affective and emotional dimension in mathematics teaching, systematically examining how task management contributes to students' confidence, motivation, and satisfaction, as well as its impact on knowledge retention and application over time. Finally, the integration of technological resources and collaborative activities deserves continued investigation, evaluating how different digital tools, simulations, and interactive strategies enhance conceptual understanding, favor meaningful learning, and promote more inclusive and engaging educational experiences.

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