

## **Mathematical mindset: a validation study of the impact of a summer camp on brazilian student learning**

*Mentalidades matemáticas: um estudo de validação sobre o impacto do curso de férias na aprendizagem de estudantes brasileiros*

*Mentalidades matemáticas: un estudio de validación sobre el impacto del curso de verano en el aprendizaje de estudiantes brasileños*

*Mentalité mathématique : une étude de validation sur l'impact du cours d'été sur l'apprentissage des élèves brésiliens*

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### **Abstract**

This article investigates the effects on mathematical learning of 68 students from two municipal schools in Cotia - SP who took part in the Mathematical Mindsets summer camp (Curso de Férias Mentalidades Matemáticas -CFMM). The study was guided by the following questions: Did students improve their mathematical skills? Did students

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improve their attitudes and self-beliefs about their mathematical abilities? The results refer to the analysis of data from the MARS assessment and the attitude survey, carried out by the students before and after the implementation of the camp. After 10 days of classes and activities with instructors trained in the MM approach, the effect size was 0.35. No significant differences in performance were found in relation to family income, age or schooling. Another effect was a change in the students' attitude towards the mathematical learning process: greater willingness to deal with errors, greater mastery of argumentative skills and an understanding of mathematics as a more open, visual and creative discipline.

**Keywords:** Mathematics, Equity, Teaching practices, Curriculum.

### Resumen

Este artículo investiga los efectos en el aprendizaje matemático de 68 estudiantes de dos escuelas municipales de Cotia – SP que participaron en el curso de verano **Mentalidades Matemáticas** (Curso de Verano Mentalidades Matemáticas – CFMM). El estudio fue guiado por las siguientes preguntas: ¿Los estudiantes mejoraron sus habilidades matemáticas? ¿Los estudiantes mejoraron sus actitudes y creencias sobre sus propias habilidades matemáticas? Los resultados se refieren al análisis de los datos de la evaluación MARS y de la encuesta de actitud, aplicadas a los estudiantes antes y después de la implementación del curso. Después de 10 días de clases y actividades con instructores formados en el enfoque MM, el tamaño del efecto fue de 0,35. No se encontraron diferencias significativas de desempeño en relación con los ingresos, la edad o el nivel educativo de las familias. Otro efecto observado fue el cambio de actitud de los estudiantes hacia el proceso de aprendizaje matemático: mayor disposición para lidiar con errores, mejor dominio de habilidades argumentativas y una comprensión de la matemática como una disciplina más abierta, visual y creativa.

**Palabras clave:** Matemáticas, Equidad, Prácticas de enseñanza, Currículo.

### Résumé

Cet article examine les effets sur l'apprentissage des mathématiques de 68 élèves de deux écoles municipales de Cotia – SP ayant participé au cours d'été **Mentalidades Matemáticas** (Cours d'Été Mentalidades Matemáticas – CFMM). L'étude a été guidée par les questions suivantes : Les élèves ont-ils amélioré leurs compétences en mathématiques ? Les élèves ont-ils amélioré leurs attitudes et leurs croyances concernant leurs propres capacités en mathématiques ? Les résultats proviennent de

l'analyse des données de l'évaluation MARS et du questionnaire d'attitude, administrés avant et après la mise en œuvre du cours. Après 10 jours de cours et d'activités menés par des instructeurs formés à l'approche MM, la taille de l'effet observée était de 0,35. Aucune différence significative de performance n'a été constatée en fonction du revenu, de l'âge ou du niveau d'instruction des familles. Un autre effet observé a été le changement d'attitude des élèves face au processus d'apprentissage mathématique : une plus grande disposition à affronter les erreurs, une meilleure maîtrise des compétences argumentatives, ainsi qu'une compréhension des mathématiques en tant que discipline plus ouverte, visuelle et créative.

**Mots-clés** : Mathématiques, Équité, Pratiques d'enseignement, Programme scolaire.

### Resumo

Este artigo investiga os efeitos na aprendizagem matemática de 68 estudantes de duas escolas municipais de Cotia – SP que participaram do curso de férias **Mentalidades Matemáticas** (Curso de Férias Mentalidades Matemáticas – CFMM). O estudo foi orientado pelas seguintes questões: Os estudantes melhoraram suas habilidades matemáticas? Os estudantes melhoraram suas atitudes e crenças sobre suas próprias habilidades matemáticas? Os resultados referem-se à análise de dados da avaliação MARS e da pesquisa de atitude, realizadas pelos estudantes antes e depois da implementação do curso. Após 10 dias de aulas e atividades com instrutores formados na abordagem MM, o tamanho de efeito foi de 0,35. Não foram constatadas diferenças significativas de desempenho em relação à renda, idade e escolaridade das famílias. Outro efeito foi a alteração de postura dos estudantes em relação ao processo de aprendizagem matemática: maior disposição para lidar com os erros, maior domínio das habilidades argumentativas e o entendimento da matemática como uma disciplina mais aberta, visual e criativa.

**Palavras-chave:** Matemática; Equidade; Práticas de ensino; Currículo.

# **Mathematical mindset: a validation study of the impact of a summer camp on Brazilian student learning**

## **Introduction**

This study analyzes the application of a systemic intervention carried out during the January school vacations, based on growth mindset concepts proposed by Dweck (2017) and the Mathematical Mindsets approach developed by Boaler (2017). This study takes as a reference a study previously conducted and implemented in the United States (Boaler, 2019, 2021), which has been translated and adapted to the Brazilian educational reality, considering local cultural, curricular and institutional specificities (D’ambrosio, 1995).

Despite the progress in the field of math education research, Brazilian students continue to perform well below international benchmarks. According to PISA data, 70.3% of 15-year-olds in Brazil do not reach the minimum level of mathematical proficiency required to exercise their citizenship (INEP/ MEC, 2016). In addition, the Brazilian Yearbook of Education (2021) reports that only 10.3% of high school graduates demonstrate adequate learning in mathematics. Given the central role played by mathematics in the 21<sup>st</sup>-century economy, it is of utmost urgency to explore effective strategies for improving the teaching and learning of mathematics across all levels of basic education in Brazil.

## **Theoretical Basis of Mathematical Mindsets**

The Mathematical Mindsets (MM) approach is based on three theoretical components: Psychology, Neuroscience and Mathematics Education.

### *Evidence from Psychology*

Dweck (2017) has pioneered a field of research that has shown that people's beliefs about ability and intelligence fall within a spectrum ranging from a fixed mindset - the belief that a subject's ability is innate and cannot be changed - to a growth mindset, which consists of the belief that ability is malleable and can be developed through hard work and effort. While a growth mindset has been shown to be beneficial for academic achievement and performance in mathematics for a wide range of students (Bui et al, 2023), having a growth mindset may be especially important for students who are traditionally marginalized in mathematics and related fields. Altendorff (2012) found that in math classes, female students are more likely to adopt fixed-mindset beliefs about intelligence compared to their male counterparts.

### *Evidence from Neuroscience*

MM leverages a range of studies that have demonstrated the potential of all students to develop and strengthen brain pathways that enable mathematical learning (Abiola; Dhindsa 2011). While traditional mathematics classes provide little opportunity for the development of creativity and reinforce pernicious beliefs about who can and cannot learn it at the highest levels (Anyon, 1980; Oakes, 1985), extensive studies on the brain have demonstrated its malleability and plasticity (Boaler, 2019; Green, Bavalier, 2008). Furthermore, confronting the idea that working with drawings, visualizations and models is a sign of low-level mathematics or mathematics aimed at young children, Boaler, Chen, Williams and Cordero (2016) point out that high-level mathematics can be predominantly visual and draw attention to the fact , for decades, schools have presented mathematics as a "matter of numbers and symbols, ignoring the potential of visual mathematics to transform students' mathematical experiences and develop important brain pathways" (p.4). Boaler *et al.* point out that "the neurobiological basis of mathematical cognition involves a dynamic and complicated communication between the memory, control and detection brain systems, and the visual processing regions of the brain" (p. 2), and that students who learn through visual approaches gain access to new and deeper understandings, which foster the understanding of mathematics. Sowell (1989) points out that the visual processing of mathematical ideas may explain why several research studies indicate that teachers who are more focused on the use of visual mathematics and well-chosen manipulatives stimulate higher student performance at any grade level. Boaler (2024) reports on a study showing that the brains of mathematicians differ from other academics, in their extensive use of visual pathways in the brain, even when working numerically.

### *Evidence from Mathematics Education Literature*

The third component of MM comes from the Mathematics Teaching Literature around cognitive demand, problem solving, and formative assessments. A large body of research in mathematics education has identified effective teaching strategies, such as the implementation of activities with high cognitive demand (Stein, Smith, Henningsen & Silver, 2000), consisting of problems that promote conceptual understanding and the development of thinking, reasoning and problem-solving (Doyle,

1983, 1988; Hiebert, 1993; Stein, Grover & Henningsen, 1996). Onuchic and Allevato (2009) state that problem solving, if worked on through observation, pattern recognition and considered as a teaching methodology creates a context that is very conducive to the construction of mathematical knowledge.

Following this same line of reasoning, Stein and Lane (1996) found that the highest learning gains in a mathematics performance assessment were related to the degree to which activities were organized and implemented in order to engage students in high levels of thinking and cognitive reasoning. According to the North American Council of Teachers of Mathematics (NCTM, 1991), these findings support the position that the nature of the activities to which students are exposed determines what they learn. Another finding that supports this idea comes from the responses of 13 million students around the world to questions about their mathematical strategies in the PISA assessment. The analysis showed that students who adopted a memorization approach were the lowest achieving students in every country (Boaler & Zoido, 2016).

In the field of education, there is a frequent culture of assigning points and grades in tests, and the consequences of this, according to Boaler (2016), are that students begin to see themselves through their grades and scores, using these results as a measure of their own value and identity. Pinto and Santos (2006) see assessment as a way of leveraging learning, in which contexts, the involvement of participants and the social construction of knowledge must be taken into account, as well as the social, cultural and cognitive processes of the classroom. This type of assessment is now called "formative assessment", i.e. interactive assessment, focused on students' cognitive processes and associated with processes of feedback, regulation and self-regulation of learning (Fernandes, 2006). According to Boaler (2016) formative assessment practices, such as rubrics and feedback, should be used to find out where students are in their learning and to help both teacher and student determine what they need to know next. Boaler emphasizes that this method provides a clear view of progress and goals to be achieved and is consistent with a mindset approach to assessment (Boaler, Dance & Woodbury, 2018). In addition, research shows that when teachers change the way they assess students' work, it has a positive impact not only on academic performance and future learning trajectories, but also on students' self-confidence and motivation (Black & William, 2005).

## Components of the Mathematical Mindsets approach

The MM approach lies at the intersection of teacher, student and curriculum learning.

### *Teacher learning*

To understand teacher learning, the MM approach is based on two theories. The first emphasizes that the relationship between teachers' beliefs and practices is complex and should be seen as dialectical (Thompson, 1992). The second argues that knowledge and beliefs are formed through participation in learning communities (Wenger, 1998).

We understand that all teachers were once students themselves and that their journeys as learners were marked by experiences and feelings that shaped their relationship with mathematics and have a direct impact on their teaching practice. Lortie introduced the notion of "apprenticeship of observation" (1975, p. 61) and it has since become synonymous with the phrase: "teachers teach the way they were taught" (Heaton; Michelson, 2002, p. 51). Lortie also suggested that students get generalized ideas from what is "good" and "bad" teaching based on how they have been affected by certain teaching practices.

It is essential that teachers reflect on their own trajectory in mathematics, to connect with their own narrative identity about mathematics (Boaler, 2024). This exercise allows for an understanding of teachers' convictions, which are built on interrelated identity experiences - not only as educators, but also as learners (Drake; Sherin, 2006, p. 158). Not only do teachers learn a lot from this exercise, but they are also able to identify possible points to work on for their development. As teachers acquire knowledge, reformulate their beliefs and adopt new practices, they appropriate new tools and use existing ones in innovative ways (Lave; Wenger, 1991). We believe that the exercise of reflecting on teaching practices should not be an isolated event, but a practice incorporated into initial teacher training, whether it be during a degree in pedagogy or mathematics.

In a study on teacher education programs (*licenciatura*) in Brazil, Gatti (2010) showed that the course designed to prepare future teachers placed a strong emphasis

on theoretical knowledge in their syllabi. While such theories are essential for grounding pedagogical understanding, the lack of practical training leaves prospective teachers ill-prepared to navigate the complexities of real classroom environments.

### *Student learning*

Our position on student learning is in line with our sociocultural perspective on teacher learning (Thompson, 1992; Wenger, 1998). In 1994, Gray and Tall showed that students' approaches to mathematics can lead to different outcomes. Studies indicate that effective teachers have high expectations of their students, form positive relationships with them (Pianta, Hamre, Stuhkman, 2003) and teach with cultural sensitivity (Gay, 2000; Gutiérrez, Rogoff, 2003). However, many teachers do not teach in these ways (Stigles, Hiebert, 2009), and ineffective classroom practices such as timed tests, valuing speed and procedural teaching are among the reasons for the high number of children and adults with math anxiety (Boaler, 2024). This anxiety negatively impacts students' ability to succeed in mathematics and enjoy it (Ashcraft, Krause, 2007), as shown in OECD studies, whose countries with students who have a higher level of math anxiety have a delay in performance of almost one school year (OECD, 2013).

Research shows the importance of facing challenge for brain growth and the importance of making mistakes (Moser et al., 2011), findings that are in line with the research of Steuer, Rosentritt-Brunn and Dresel (2013). According to these scholars, when students see their mistakes in the classroom in a welcoming way, they are more committed to their work. It is essential to emphasize the importance of messages that promote the development of a growth mindset, such as valuing difficulties and mistakes as constituent elements of the learning process. The role of teachers in this context is fundamental, as it is they who can welcome students and help them deal with with potentially negative feelings, which can contribute to the development of a growth mindset. Fonseca (2016) states that emotions are an essential source of learning, as people seek out activities and occupations that bring them well-being, and tend to avoid activities or situations that bring them discomfort. Furthermore, in the area of socio-emotional learning and "belonging", Walton and Cohen (2007, 2011) found in their

randomized controlled experiment that a brief intervention, aimed at helping students deal with social adversity and stereotypes, leveraged learning outcomes which resulted in improved health and well-being three years after the intervention.

There are many interactions between teachers and students in math classes that can help or hinder the development of their mindsets (Boaler, 2016; 2024; Sun, 2018). It is not enough to post posters on the walls with growth mindset messages and expect students to change, it is important to bring mindset into the culture of schools and classrooms (Hecht et al, 2023). Teachers need to convey messages about mindset in a variety of ways. The education field has become more aware that changing students' mindsets occurs through teaching practices, as well as through the way teachers express themselves verbally (Boaler, 2016; Suh, Graham, Ferranone, Kopeinig, Bertholet, 2001; Sun, 2018; Hecht et al, 2023). In this context, instead of only praising correct answers, teachers can recognize students' effort, progress in learning from mistakes, the use of effective strategies or making relevant connections between mathematical ideas. In this way, the learning process is emphasized, not just performance (Boaler, Dance & Woodbury, 2018).

### *Curriculum*

An educational curriculum supports content development and pedagogical content knowledge on the part of the teacher (Shulman, 1986). It also gives visibility to pedagogical judgments (Ball; Cohen, 1996) and the design principles underlying each activity (Davis; Krajcik, 2005). Such a perspective helps teachers to make decisions about how to adapt other curriculum materials and to apply their knowledge more flexibly in other contexts (Davis; Krajcik, 2005). However, as mentioned by Valle, the math curricula we commonly see "tend to fragment the learning of the subject into lists of numerous concepts and methods that teachers need to teach. This makes it difficult for many students and teachers to see the relationships between mathematical concepts" (2019, p. 72).

An important step towards seeing these relationships and connections is getting to know the big ideas in mathematics. With these in mind, they become more attentive

to the choice of activities and the connections to highlight through conversations and teaching (Boaler, Munson, Williams, 2017; Boaler, 2024). One way to promote this approach is through open-ended activities that are challenging but accessible, so-called "low floor, high ceiling" activities. According to Boaler (2016), these activities allow all students to understand mathematical concepts and develop them to advanced levels. As well as being engaging and stimulating, their value goes beyond the fact that they work for students with different levels of prior attainment. They also teach relevant mathematics, arouses students' interest and encourage creativity.

### **Indicators of the effectiveness of the Mathematical Mindsets approach**

Our research, specifically on the effectiveness of MM, is supported by several studies. The first, *"Changing Students Minds and Achievement in Mathematics: The Impact of a Free Online Student Course"* (Boaler, Dieckmann, Pérez-Núñez, Sun, Williams, 2018), showed with an online course that students' mindsets and mathematics achievement could be changed with the intervention of messages about mindset and the sharing of mathematical strategies and ways to approach mathematics delivered directly to students.

The second study, *"Achieving Elusive Teacher Change through Challenging Myths about Learning: A Blended Approach. Education Sciences"* (Anderson, Boaler, Dieckmann, 2018) reflects the results of a blended approach on a broader scale. Youcubed, the Stanford University Research Center whose main objective is to encourage, train and empower mathematics teachers by transforming the latest research on learning the subject into practical and accessible content, adapted the intervention to a network of teachers in different school districts who studied and implemented the Mathematical Mindsets approach over the course of an entire year and again found positive results. In particular, this approach improved the performance of girls, students whose first language was not English, and low-income students.

*"Prove it to Me! Mathematics Teaching in the Middle School"* (Boaler, 2019) is the article most relevant to this study. It details the design and results of a targeted, integrated intervention, an 18-day summer camp that took place at Stanford University, involving the curriculum and teaching practices as part of the controlled

environment created for the study. The intervention was deemed successful as "the average gain score for participating students across all sites was 0.52 standard deviation (SD) units" (Boaler et al., 2021, p. 7), regardless of gender and ethnic group.

In a follow up study of the camp taught by teachers in ten districts, using the curriculum materials developed at youcubed, (Boaler et al., 2021). In addition to these studies Bui et al. in 2023 conducted a meta-analysis to consider the times when mindset interventions do and do not work. They conclude that mindset interventions are effective when they also involve a change in the ways mathematics is taught – in essence, the use of a “mathematical mindset” approach, and are not effective when they are delivered without a change in teaching approach. The different studies show the positive impact that teachers have when they share mindset ideas, change their approach to teaching, and share a mathematical mindset curriculum, which improves students' performance in mathematics, as well as their beliefs and attitudes.

This study shares the results when a mathematical mindset approach was delivered in a different cultural context (D’ambrosio, 1995) and investigates whether students from Brazil would also show an improvement in their math achievement and a change in their attitude and beliefs towards mathematics. The research also served to assess whether the results are correlated with the students' sociodemographic profile and school performance.

### **Research questions**

**Research question 1:** Do students who participate in the Mathematical Mindsets Summer Camp show improvement in the mathematical skills targeted by the program? Does the effect vary according to the student's characteristics?

**Research question 2:** Do students who participate in the Mathematical Mindsets Summer Camp improve in terms of the attitudes and beliefs targeted by the program? Does the effect vary according to the student's characteristics?

## **Study design**

### *Program description*

The 10-day MM summer camp implemented in Brazil strictly followed the version piloted in California in 2015 (Boaler, 2019). This model was expanded to 10 different locations during the 2019 summer camp in the United States and served as the basis for replication (Boaler et al, 2021). The project's intent in Brazil arose from the youcubed team's initiative to establish the guidelines for this math summer program in a different context, as a local proof of concept from which a portfolio of evidence of the MM approach in Brazilian schools would be developed.

The program proposes interactive, visual and investigative mathematical experiences to promote the exploration of mathematical ideas in greater depth and at the same time cultivate a growth mindset in students. The program in Brazil used the approved math curriculum activities, adapted for the 4th and 5th grades, and met the minimum requirements of 30 hours of class time. Camp sites had the freedom to adjust the order and flow of activities to meet specific needs. The implementation team at the Sidarta Institute, a non-profit organization whose mission is to promote quality education for all, developed significant adaptations, such as a 100-hour preparatory camp for teachers hired specifically for this program and an afternoon curriculum based on games. This curriculum aims to reinforce the Mathematical Mindset (MM) messages and promote a greater sense of community throughout the camp.

The students from the two participating schools were randomly distributed into four classrooms, balanced with 50% from each of the two schools. Their daily routine consisted of three hours of math lessons in the morning, followed by two hours of recreational activities in the afternoon aimed at reinforcing key concepts. For the morning session, each group was assigned two lead teachers, a generalist and a math specialist. In the afternoon, student rotated through the recreational activities conducted by different teachers. Appendices A, B and C show a typical day of the summer camp program and give examples of activities contained in the curriculum.

This program was transformed into an Educational Technology: Mathematical Mindsets Summer Camp, available for free through the Itau Foundation portal<sup>5</sup>.

Youcubed's research director, Jack Dieckmann, remotely delivered 25 hours of training directly to the summer camp teachers during the months of October to December 2019. These sessions focused on the mathematical concepts present in each of the summer camp activities, and the essential MM teaching practices developed at youcubed. He was also present in person during the first week of the summer camp, when he observed classes, met with teams of teachers and helped them with daily reflections and adjustments to lesson planning and activities, ensuring a high level of quality and fidelity in its implementation.

### *Sample selection and description*

The criteria for selecting the schools were: schools with similar profiles in dimensions such as sociodemographic composition of the population, IDEB performance and school size, in which the administration would be willing to facilitate the collection of data needed for the project. Two elementary schools (School 1 and School 2), located near a rural area in the municipality of Cotia, in the state of São Paulo, were selected. The camp itself took place at School 2. The sample consisted of students who were moving from 4th to 5th grade at both schools. Enrollment was open to both schools and students were invited to participate regardless of their mathematical performance. The camp was offered free of charge to the students, but transportation was the responsibility of each family. A total of 98 students enrolled, but only 74 showed up on the first day. The condition for staying was at least 80% attendance. As a result, only 68 completed the camp. Overall, the student sample was diverse in terms of gender, race, socioeconomic status and previous performance, and allowed us to test the effects on specific groups of students. Table 1 below provides descriptive data on the students who completed the program.

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<sup>5</sup> <https://fundacaoitau.org.br/escola/autoformativos/tecnologia-educacional-curso-de-ferias-mentalidades-matematicas>

**Table 1***Characterization of the students who completed the camp*

		<b>No.</b>	<b>%</b>
Gender	Female	34	50
	Male	34	50
Race/Ethnicity	White/Caucasian	21	31
	Asian/Eastern	1	2
	Brown	28	41
	Black/Afro-descendant	7	10
	Not declared	1	2
	Not informed	10	15
GPA (previous year)	Low GPA	8	12
	Middle GPA	32	47
	High GPA	28	41
Highest level of education in the family	Completed High School	18	27
	Completed Higher Education	12	18
	Completed Postgraduate degree	3	4
Average family income	No family income	3	4
	Less than one minimum salary	5	7
	1-2 minimum salary	27	40
	2-5 minimum salary	18	27
	5-10 minimum salary	4	6
	10-20 minimum salary	1	2
	Not informed	10	15

*Measuring mathematics learning*

As with the Stanford model, all students received a pre/post assessment consisting of 4 activities from MARS (Math Assessment Resource Service)<sup>6</sup> with the same activities used in the pre/post camp assessment. MARS offers a wide range of performance assessment tasks, rubrics and tools to measure and enhance learning. These mathematical tasks are considered a rigorous measurement, as students must reason within a context, formulate a strategy and explain their mathematical

<sup>6</sup> <https://www.map.mathshell.org/>

reasoning, not just choose the right answer. The sites participating in the camp were given a protocol for administering the same type of test following the recommendations of the technical manuals of the Silicon Valley Math Initiative (SVMI)<sup>7</sup> - the organization that curates and scores the MAC/ MARS assessments in the United States. The students had 20 minutes to work individually on each activity, and these were grouped into two sessions of 40 minutes each.

The assessments were scored remotely by an external group of assessors trained in SVMI, who used double-checking procedures to ensure their reliability. The MM teachers were not involved in the correction, nor did they have access to the assessments before the summer camp. Similar assessments are freely available to the public at <https://www.map.mathshell.org/>.

### *Measuring students' beliefs and attitudes about mathematics*

Along with improving performance in mathematics, one of the key aims of the summer camp intervention is to influence students' attitudes and beliefs towards the subject, promote a growth mindset and change outdated beliefs around the importance of speed and memorization. Indeed, a significant part of the curriculum and teaching practices consists of giving students explicit messages in these key areas.

To capture changes in these ideas pre/post-camp, a survey was administered using a 4-point Likert scale of agreement, where 1 means totally disagree and 4 totally agree (we indicate the "reverse code", whose items have negative words) with the statements presented. This survey was created and adapted by youcubed, which has been administering similar surveys for the past 5 years in the US and abroad. Below are some of the statements that made up the survey. Appendix D contains the full list of statements.

P1. I like math.

P2. When confronted with a math problem, I give up easily (reverse code).

P12. Struggle is important to math learning.

P17. I get a sinking feeling when I try hard math problems (reverse code).

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<sup>7</sup> <https://svmimac.org/>

## *Contextual data from classroom observations and interviews*

A professional film crew was hired to film five days of classes to capture qualitative data on the students' approaches and engagement to help future research and evaluation. On the last day of the summer camp, interviews were conducted with all the students to get a first-hand understanding of their experiences on the camp. The questions asked are listed below:

1. How would you describe your experience with mathematics to a friend who didn't take part in the summer camp?
2. How is the summer Math camp the same as your math class during the year? How is it different?
3. What was the best part of the Math camp for you? Why?
4. What helped you do well in the summer Math camp?
5. What kind of help would you like?

## **Results**

### *MARS evaluation results*

The scores for the assessments were assigned remotely by an external group of assessors trained in SVMII, who used double-checking procedures to ensure their reliability. A total of 33 possible points were awarded for the activities, distributed as follows: Boxes (10 points), Buttons (8 points), Hexagons in a row (8 points), How many cubes (7 points). For this analysis, we created a composite score by adding up the scores of the four activities and analyzed the scores of each activity separately.

The assessment activities covered various mathematical competencies set out in the Common National Curriculum Base (BNCC). These include identifying regularities, establishing and using rules for forming numerical or figurative sequences (skills **EF04MA05** and **EF05MA06**), understanding the concepts of volume and its relationship with multiplication and addition operations (skills **EF05MA19** and **EF05MA20**), and algebraic reasoning aimed at solving equations and inequalities of one variable (skills **EF06MA07**, **EF06MA08**, **EF07MA05**, **EF07MA07**), with a focus on constructing algebraic expressions and solving problems involving relationships between quantities. The activities also covered skills related to the analysis of patterns,

the generalization of regularities and the identification of mathematical structures, fundamental skills for the development of algebraic thinking.

In addition, these activities also aimed to assess the mathematical practices described in the Common Core State Standards (CCSS)<sup>8</sup>, such as: reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; identifying and using patterns and structures; and, identify and express regularities in repeated reasoning. These practices are in line with General Competence 2 of the BNCC, which emphasizes the importance of exercising intellectual curiosity and resorting to the approach typical of the sciences - such as investigation, critical analysis, developing hypotheses, formulating and solving problems and creating solutions.

We analyzed this data by subtracting the pre-score from the post-score and conducting a paired t-test. We calculated the main effect sizes used to quantify growth by dividing this difference by the standard deviation of the summed scores in the pre-test administration. This follows the same method as the comparative study conducted at Stanford University in 2015 and 2021. The results for the full sample are shown in Table 2. One way to interpret the effect size for the general public is to consider 1 standard deviation (SD) as equivalent to the progress a student can be expected to make in 3 years of schooling. With this conversion estimate, an effect size of 0.35 was found for specific mathematical concepts covered by the assessments – which is considered equivalent to a little over one year of schooling.

**Table 2**

*Pre/post improvement in MARS for all students*

<b>Obs.</b>	<b>Pre-Score</b>	<b>Post-Score</b>	<b>Difference (Post-Pre)</b>	<b>Effect size</b>
68	6,7	8,4	1,65**	0,35**

\*\*\*p<0.01, \*\*p<0.05, \*p<0.1

<sup>8</sup> The Common Core State Standards is a set of curriculum standards that defines what students should learn in math and English each level, guaranteeing a national curriculum base for education in the United States.

As a basis for comparison, the effect size of the original Stanford summer camp was 0.96 SD. However, there were some important differences between the Stanford and Brazilian courses: the program developer, Jo Boaler, taught the Stanford camp, which was held at the university itself and lasted three weeks. In Brazil, the camp took place in a public school for two weeks. In addition, the Stanford camp consisted of 6th and 7th graders, while Brazil worked with students entering 5th grade. Taking these factors into account, we consider the results of the Brazilian camp to be quite robust.

When we disaggregated the MARS data by school, we found a school effect on the MARS results, shown in Table 3. Both schools increased their scores after the camp, but only one of them showed statistically significant results. Using the same conversion as before, a substantial effect size of 0.57 for School 2 was found for the specific mathematical concepts covered by the assessments. The School 2 campus was where the summer camp was held and we speculate there may be a “home court” advantage for those students.

**Table 3**

*Pre/post improvement in MARS by school*

	<b>Obs.</b>	<b>Pre-Score</b>	<b>Post-Score</b>	<b>Difference (Post-Pre)</b>	<b>Effect size</b>
School 1	34	6,9	7,4	0,41	0,11
School 2	34	6,5	9,4	2,88***	0,57***

\*\*\*p<0.01, \*\*p<0.05, \*p<0.1

To explore variability in learning outcomes, we used Ordinary Least Squares (OLS) regression analysis to link student characteristics to MARS score increases, which allowed us to test for varying effects. Appendix E provides the full results of the regressions on total MARS scores and activities. Our regression analysis of MARS composite assessment results included student demographic variables such as race and gender, parental education levels and family income. We found a statistical difference of -4.7 for black students compared to white students, but this difference was at the 0.10 level, not the 0.05 level, which we used as our significance threshold. What is even more striking is that we found no statistical differences in the students in relation to their parents' education, nor in relation to their previous performance. These

characteristics are usually strong predictors of student performance, but in the case of this summer camp, they didn't have the same predictive power. This is evidence that the low floor high ceiling approach to mathematics that was used, is helpful for both low and high achieving students.

Regarding Research Question 1, in which we investigated whether the students who participated in the MM program showed improvement in the mathematical skills targeted by the program, the answer was positive. The students showed an improvement in math learning with an average effect size of 0.35 standard deviations.

#### *Student opinion survey results - Comparison at item level*

Using a paired t-test at the item level, the students showed a significant improvement in more than half of the survey items, pre/post-camp (at the 0.01 or 0.05 significance level), as shown below. Appendix F shows the full results. Students seem to have understood the messages about the importance of mistakes (P7) and placing less value on speed (P5, P10). The students' level of anxiety about math seems to have decreased (P4, P21) and their enjoyment of the challenges (P18) increased.

In the analysis carried out, it was not possible to identify a consistent pattern relating the answers in the questionnaire to the characteristics including gender, race, family income and family education level. However, when considering previous academic performance, a pattern was observed in which students with higher GPA in mathematics obtained lower scores for questions 5, 11, 12 and 19. This result suggests that, in this sample, students with higher GPA in mathematics tend to have a more fixed mindset.

#### *Student opinion survey results - Principal Component Analysis (PCA)*

A principal component analysis (PCA) was carried out on the pre-/post-camp increment responses in the student opinion survey, which generated five initial components, only four of which were conceptually coherent. These four components are listed below, along with the items associated with each of them. We have also included the Cronbach's alpha reliability coefficient for each component. The first,

second and fourth components are in the moderate range, and the performance of the third is less reliable.

**Table 4**

*Results of principal component analysis*

<b>Research component</b>	<b>Associated Survey Item</b>	<b>Cronbach's Alpha</b>
Component 1: Beliefs about effort in mathematics	P12. Struggle is important to math learning.  P18. I think I can solve very hard/complex math problems  P19. If I put in enough effort, I can succeed in math.	0,698
Component 2: Attitudes/feelings towards mathematics	P2. When confronted with a math, I give up easily (reverse code).  P15. I like to solve complex problems.  P8. Math is my favorite topic.  P4. Sometimes math makes me feel afraid (reverse code).	0,705
Component 3: Attitudes towards error in mathematics	P7. Mistakes are a good thing and part of math.  P21. I feel ashamed when I make a mistake solving a a math problem (reverse code).	0,451
Component 4: Beliefs about behavior in mathematics	P10. It is important in math to be fast (reverse code).  P11. Creativity is really important in mathematics.  P9. It's really good to do math exercises in groups and discuss them with my friends	0,674

To further investigate the research data, we regressed these four components against the characteristics of the students in our dataset. The full results are provided in Appendix G. Students falling into the less than 1 minimum wage category scored higher on Component 3 than those with no income, but no other patterns detected were related to income. We found no effects in terms of school, gender, parents' schooling or previous academic achievement.

**Table 5***Summary of Research Question*

<b>Question</b>	<b>Answer</b>
Do students who participate in the MM summer camp improve the attitudes and beliefs targeted by the summer camp?	Yes, the students improved in seeking challenges, reduced anxiety, less emphasis on speed and positive thinking about mistakes in math.
Does the effect vary according to the student's characteristics?	We found no effects in terms of school, gender, parents' schooling or academic achievement. Effects relating to race and income are inconclusive.

Finally, as part of the investigation into the relationship between the two main results, the same four components of the student opinion survey were regressed on the overall MARS scores and the MARS sub-activity scores. In the overall analysis, no significant differences were found apart from the race factor. When analyzing student performance, those with high or average academic performance showed lower score gains than students with low academic performance, which may indicate a small marginal benefit of acquiring additional knowledge for those with high and average performance. Thus, although we found significant changes in students' beliefs and attitudes in the desired direction, the empirical link between the two is not proven by this data. Our research continues to refine its conceptual model by identifying mediating variables, such as changes in attitudes and beliefs, to explain variability in mathematical learning outcomes, in this case MARS assessment scores.

**Discussion**

This study sought to investigate the impact of a 10-day Mathematical Mindset intervention on two elementary schools located near a rural area in the municipality of Cotia, in the state of São Paulo (refer to the Sample selection and Description section for more detailed information on student profile). It is interesting to note that, although not commonly practiced in Brazil, the idea of studying in a summer camp was well received by students and parents. Compared to the low performance shown by

annual standardized state tests, this study demonstrates that students can make significant learning gains and change their mindset in a short space of time.

There are three noteworthy aspects to the formulation of the program. First, the context and sample of the program took place in a typical public school, with a typical distribution of students from the surrounding area. This is very encouraging, as it shows that the Mathematical Mindsets promotes equity goals for a wide range of students.

The second aspect is related to the mathematical assessment selected for this program. The formulation of the MARS tests allowed us to observe not only correct mathematical answers, but also the quality of the students' reasoning and explanations. These answers provided us with additional, direct evidence of the students' mathematical reasoning, unlike typical multiple-choice questions. Although the overall study demonstrated significant learning gains, it is important to mention that more research is needed to better understand the effects observed in schools in relation to the MARS assessments.

Thirdly, pre- and post-camp surveys on mentality show that the program succeeded in influencing students' beliefs about the nature of mathematics and about themselves as learners of the subject.

### **Conclusions and recommendations**

Considering that this is the first implementation of the Mathematical Mindsets summer camp in Brazil, we conclude that the results of this intervention are extremely encouraging. The Sidarta Institute team was able to reproduce statistically significant gains in both math learning and attitudes towards the subject, similar to the camps at Stanford and other schools in the USA. We emphasize the achievements of the teaching team because the students, who came from typical Brazilian public schools, were not familiar with the performance assessments used, and some were not fully literate. We also point out that although the curriculum is conceptually aligned with the pre/post- course assessments, the teaching was not geared to the format of the assessments.

Given the results, we see the following possible lines of work:

- a) **Research:** Maintain the expansion of the summer courses to other

regions, study the students of the MM summer camp to test the improvement of continued learning and school engagement in the school year following the camp. Involve other Brazilian researchers to formulate their own questions around the MM summer camp. Use qualitative frameworks to understand the role of the afternoon curriculum based on games and play, and its relationship to the mathematics curriculum in the classroom;

b) **Materials Development:** Use data sets collected during the summer camp (classroom videos, samples of student work, interviews with students and teachers) for ongoing research and as raw material for teacher training in the MM approach;

c) **Delivery models:** Given the huge distances in Brazil's geography, we recommend developing synchronous and asynchronous teacher training models so that this teaching approach can reach more schools across the country;

d) **Learning Communities:** Continue to promote the MM approach with learning communities, making use of social media and mass communication to connect teachers, teacher trainers and allied school managers; and

e) **Integration into National Systems:** Establish deeper links between the BNCC and the summer camp curriculum so that the MM approach continues to respect the Brazilian context and is rooted in it, given its great variety.

f) **Integration in regular math classes:** use the results of the summer course to plan ongoing and lasting implementation throughout a semester or a full school year, to ensure the consistency and sustainability of the new approach.

Since the end of the summer camp in 2020, the MM program has expanded its reach to several municipalities and states. One of the interventions was even a new round of application of the summer camp curriculum conducted by the teachers themselves from the Municipal Education Network for students from 5th to 9th grade in the Municipality of Vespasiano. The data is currently being analyzed, and we will soon have a new article on this experience.

## Special thanks

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## Appendix A

### Sample Daily Routine

Time	Activity
8:00 – 8:30	Lanche
8:30 - 12:00	MM Videos: "Speed is not important" and "The importance of struggle" <a href="https://www.youcubed.org/pt-br/resources/rapidez-nao-e-importante/">https://www.youcubed.org/pt-br/resources/rapidez-nao-e-importante/</a> <a href="https://www.youcubed.org/pt-br/wim/a-importancia-do-esforco/">https://www.youcubed.org/pt-br/wim/a-importancia-do-esforco/</a> Number Talks (42 - 19) Graph talks Inspiring Mathematicians: Florence Nightingale Activity: Painted Cubes (see Appendix B)
12:00 - 13:00	Lunch
13:00 - 14:00	Strategy Games (Othello, Forms and Colors, Da Vinci Code, Mancala)
14:00 - 15:00	Construction Games: Piling Boxes (see Appendix C)
15:00 - 15:30	Daily journaling/Evaluation
15:30 - 16:00	Snack
16:00 - 17:00	Teachers Daily Debrief and Planning

## Appendix B

### Activity: Painted Cubes

This activity is all about connecting geometric thinking—such as surface area and volume—with the discovery of patterns and the generalization of constructions, forming a solid foundation for algebraic thinking. There are so many patterns in this problem that students will notice them both numerically and visually. Encourage students to use justifications that include both visual and numerical reasoning.

#### Planning

Activity	Duration	Description/Facilitation	Materials
Groupwork	30 min.	Explore the number of cubes with three, two, one, and zero painted faces in a <b>3×3×3 cube</b> .  Question: <i>What about a 4×4×4 cube?</i>	<ul style="list-style-type: none"><li>• 150 Sugar Cubes (for each group of 4 students)</li><li>• Chalk</li><li>• Activity Sheet</li><li>• Pencil/Pen</li></ul>
Sharing/ Whole-Class Discussion	30 min.	Create a table of the class's discoveries. Work toward agreement on each entry. Share patterns that you notice.	<ul style="list-style-type: none"><li>• ChalkBoard</li></ul>

## Activity

We use this activity as an open exploration for students to experiment with and investigate geometric patterns, focusing on the use of tables to organize discoveries and make generalizations. There are so many patterns to think about when understanding this problem that it becomes a good opportunity to establish reasons for recording what students see in a chart.



Using cubes for the task allows students to explore surface area and volume of cubes of different sizes, color coding, building and dismantling cubes. When teams are exploring the cubes, we give them space to approach the problem from any direction that makes sense to them. With middle school students, we observe and record their discoveries.

We begin with the question:

“If we took a **4×4×4 cube** and dipped it in paint, how many cubes would have **three painted faces**, **two painted faces**, **one painted face**, and **no painted faces**?”

As they explore, we notice that students want to use a table to organize the numbers they find. We do not direct them on what labels to use for the columns, because the layout of the table is not important during the exploration phase. We plan to create a class table later during the whole-class discussion so that we can organize the students’ ideas in a format that supports pattern finding.

After clarifying questions about what they were preparing to explore, we gave each team a box of sugar cubes to model what would happen with a **4×4×4 cube dipped in paint**. Teams were given enough time to color, count the cubes, and record their discoveries in a chart in their notebook. We brought the class together once each team finished investigating the **4×4×4** and began thinking about a **3×3×3** cube.

We wrote the table we wanted to use for the discussion on the board. Then we gathered the class and asked students to share their discoveries and record them in the table. As students added their results, we encouraged them to include everything they found and told them they were free to add columns or rows and disagree if they noticed something different from another student.

We continued discussing each entry until there was agreement. We left the bottom row without a title. Our goal was for someone to notice the connection to the **volume of a cube**. If you wait long enough, it will happen. It always has for us. It was worth the wait!

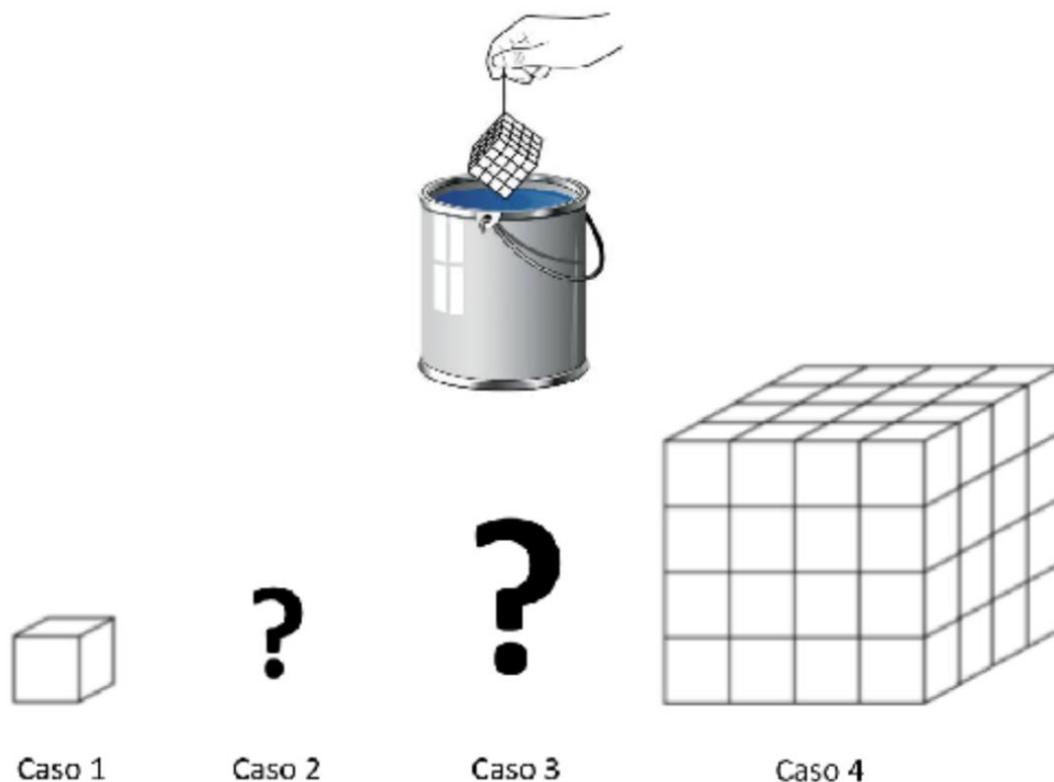
<b>Cube Dimension</b>						
# painted faces	1x1x1	2x2x2	3x3x3	4x4x4	5x5x5	$n \times n \times n$
0						
1						
2						
3						
4						
5						
6						

### Activity Extension

- Create your own investigation using other shapes.

## Painted Cubes

### Activity Sheet



Imagine that we paint a **4×4×4 cube** on all sides.

- How many small cubes have **3 blue faces**?
- How many have **2 blue faces**?
- How many have **1 blue face**?
- How many have **not been painted**?

How many small cubes would have **3, 2, 1, and 0 painted faces** in a **10×10×10 cube**?

**Think visually!**

## Appendix C

### Activity: Piling Boxes

#### Materials Needed

- Cardboard boxes of different sizes
- 8 photos of constructions made with the available boxes
- The book *The Man Who Loved Boxes* by **Stephen Michael King**, available for reading



#### Objective

Recreate the constructions shown in the photos using cardboard boxes.

#### How to Play

Each group of four chooses an image to reproduce using the cardboard boxes.

#### For the Instructor

Encourage students to work collaboratively. If you notice a student not participating, ask who the **Harmonizer** is and whether they are ensuring everyone's participation. Encourage students to **think aloud** about what they are doing, valuing respectful communication and the exchange of ideas. If necessary, say:

- "So-and-so is giving an idea that might help. Did you hear it? What do you think about that idea? Why?"
- "You are persisting. That is important when we want to achieve a goal."
- "When you interact and talk, you learn from each other and learn more."
- "The strategy you used \_\_\_\_\_ was excellent. Who can explain why it worked?"
- 

#### Suggested Mathematical Mindset Practices

→ **PMM 3:** Challenge and Effort

→ **PMM 5:** Connections and Collaboration

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## Appendix D

### List of Survey Statements

- P1. I like math.
- P2. When confronted with a math problem, I give up easily (**reverse coded**).
- P3. I feel encouraged by other people to efforts to do math
- P4. Sometimes mathematics makes me feel afraid (**reverse coded**)
- P5. People who really understand math will get an answer quickly (**reverse coded**)
- P6. I have enough time in my math classes to solve the questions the teacher provides
- P7. Mistakes are a good thing and part of mathematics.
- P8. Mathematics is my favorite subject.
- P9. It is really good to do math exercises in group and discuss them with my friends.
- P10. In mathematics, it is important to be fast (**reverse coded**).
- P11. Creativity is really important in mathematics.
- P12. Struggle is important for learning mathematics.
- P13. I am unable to think clearly when working on math (**reverse coded**).
- P14. The teacher is the only one who knows if I understand the content (**reverse coded**).
- P15. I like to solve complex problems.
- P16. My math class moves at the right pace for me.
- P17. I get a sinking feeling when I try to solve difficult math problems (**reverse coded**).
- P18. I think I can solve really hard/complex math problems.
- P19. If I put enough effort I can succeed in mathematics.
- P20. In my math classes, I have opportunities to work together with others.
- P21. I feel shame when I make a mistake while solving a math problem (**reverse coded**).

## Appendix E

### Regression Analysis : MARS (post-pre) score gains

Each column of this table represents a different statistical regression, where the top of the column is the dependent variable. The coefficients are interpreted as score gains with respect the default category of each covariate. For example, for the "School" covariate, the coefficients show the score gains of "School 1" relative to "School 2".

<b>MARS (post-pre) score gains</b>					
dependent variable:	<b>Total</b>	<b>Buttons</b>	<b>Hexagons</b>	<b>Cubes</b>	<b>Boxes</b>
	<b>score gain</b>				
<b>School</b> (default category: School 2)					
School 1	1,905	0,331	0,290	-0,608	1.339***
<b>Gender</b> (default category: Male)					
Female	2,573	1,652	0,128	0,420	0,373
<b>Race-ethnicity</b> (default category: Whites)					
Asian	-3,974	-1,653	-3,407	0,376	0,710
Brown	-2,610	-0,611	-0,630	-0,943	-0,427
Black	-4.773*	-1,916	-1,053	-0,504	-1.299*
<b>Income bracket</b> (default: no-salary/unemployed)					
< minimum salary	1,222	3,179	-2,624	0,386	0,281
1-2 minimum salary	1,254	-0,269	-0,174	1,659	0,039
2-5 minimum salary	3,709	0,900	0,639	1,830	0,339
5-10 minimum salary	-1,754	-2,085	-1,902	2,621	-0,388
10-20 minimum salary	-1,807	-3,380	-1,281	2,280	0,574
<b>Parents' education</b> (default category: Incomplete primary school)					
Complete primary school	1,316	0,997	0,941	-0,140	-0,483
Incomplete high school	-1,612	0,910	0,720	-1,190	-2,052
Complete high school	-0,778	2,179	-1,624	-0,614	-0,719
Complete technical education	-2,727	1,979	-3,145	-1,277	-0,285
Incomplete higher education	0,215	3,372	-1,903	-1,558	0,303
Complete higher education	-2,402	2,917	-2,658	-1,794	-0,868
Incomplete graduate	1,569	4,698	-2,871	-1,043	0,784
Complete graduate	-2,487	1,684	-2,671	-0,673	-0,826
Constant	2,665	-1,970	3,023	0,344	1,269
Observations	56	56	56	56	56
R-squared	0,272	0,305	0,266	0,226	0,462

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

# Appendix F

## Survey Questions

Survey question	PRE-survey		POST-survey		Obs	Difference	
	Mean	Std. Dev.	Mean	Std. Dev.		(POST - PRE)	paired t-test
1 I like Math	3,520	0,700	3,790	0,723	52	0,269 *	2,00
2 When confronted with a math problem, I give up easily. (Reverse-code)	3,160	1,076	3,360	1,064	50	0,200	1,09
3 I feel encouraged by other people to make efforts to do math	3,440	0,916	3,420	0,977	52	-0,019	-0,13
4 Sometimes math makes me feel afraid (Reverse-code)	2,620	1,140	3,040	1,188	52	0,423 **	2,02
5 People who really understand math will get an answer quickly (Reverse-code)	1,270	0,603	2,530	1,270	51	1,255 ***	6,28
6 I have enough time in my math classes to solve the questions the teacher provides	3,550	0,757	3,200	1,167	51	-0,353	-2,27
7 Mistakes are a good thing and are part of math	3,460	0,874	3,850	0,697	52	0,385 ***	2,91
8 Math is my favorite topic	3,120	1,078	3,420	0,893	52	0,308 *	2,00
9 It is really good to do math exercises in group and discuss them with my friends	3,740	0,675	3,790	0,778	47	0,043	0,28
10 It is important in math to be fast (Reverse-code)	2,520	1,111	3,830	0,663	48	1,313 ***	8,03
11 Creativity is really important in mathematics	3,600	0,736	3,750	0,700	48	0,146	1,23
12 Struggle is important to math learning	3,920	0,454	3,710	0,798	48	-0,208	-1,53
13 I am unable to think clearly when working on math (Reverse-code)	2,790	1,051	2,850	1,185	48	0,063	0,35
14 The teacher is the only one who knows if I understand or not (Reverse-code)	2,720	1,330	2,550	1,316	47	-0,170	-0,72
15 I like to solve complex problems	3,130	0,866	3,420	0,895	48	0,292 *	2,00
16 My math class is at the right pace for me	3,560	0,741	3,500	0,989	48	-0,062 *	-0,36
17 I get a sinking feeling when I try hard math problems (Reverse-code)	2,740	1,188	3,130	1,172	47	0,383 *	1,77
18 I think I can solve really hard/complex math problems	2,940	0,987	3,510	0,882	47	0,574 ***	3,35
19 If I put in enough effort I can succeed in mathematic	3,850	0,510	3,770	0,786	47	-0,085	-0,63
20 In my math class I have opportunities to work together with others	3,300	0,976	3,060	1,292	47	-0,234	-1,18
21 I feel shame when I make a mistake solving a math problem (Reverse-code)	2,400	1,155	3,150	1,179	47	0,745 ***	3,71

\*\*\* p<0.01, \*\* p<0.05, \* p<0.1

Sample consists of students who had MARS scores and answered the survey questions.

Answer scale: 1-Strongly disagree, 2-Disagree, 3-Agree, 4-Strongly agree. Questions 2, 4, 5, 10, 13, 14, 17, and 21 were reverse coded.

## Appendix G

### Regression of PCA Components on Student Characteristics

	Component 1		Component 2		Component 3		Component 4	
	coeff	stderr	coeff	stderr	coeff	stderr	coeff	stderr
<b>School</b> (default category: School 2)								
School 1	-1,474	1,022	-1,084	1,445	-0,775	0,719	-0,084	1,052
<b>Gender</b> (default category: Male)								
Female	0,346	0,989	0,392	1,398	0,745	0,695	0,005	1,015
<b>Race-ethnicity</b> (default category: Whites)								
Asian	-0,615	2,794	1,857	3,95	0,395	1,965		
Brown	0,357	1,005	-1,855	1,421	0,104	0,707	-0,674	1,035
Black	-0,314	1,481	-1,13	2,093	-0,048	1,041	0,529	1,522
<b>Income bracket</b> (default: no-salary/unemployed)								
< minimum salary	1,775	2,316	-1,24	3,274	3,871*	1,629	1,044	2,383
1-2 minimum salary	2,128	1,973	-3,274	2,788	0,755	1,387	0,467	2,028
2-5 minimum salary	0,884	2,134	-5,153	3,017	-0,239	1,501	-0,739	2,189
5-10 minimum salary	2,272	2,53	-3,701	3,576	0,601	1,779	1,222	2,594
10-20 minimum salary	2,986	3,426	-6,333	4,843	-0,076	2,409	0,255	3,514
<b>Parents' education</b> (default category: incomplete higher education or less)								
Higher education degree	0,543	1,209	-0,301	1,708	3,578	2,272	0,453	1,241
<b>High school GPA</b> (default category: Low GPA)								
Middle GPA	-1,951	1,559	-0,888	2,203	0,088	2,93	-1,162	1,791
High GPA	-3,082	1,557	-1,239	2,201	0,197	2,927	-0,939	1,761
<b>Constant</b>								
	2,697	3,039	8,349	4,295	1,601	2,137	2,848	3,167
Observations	42		42		42		40	
R-squared	0,281		0,277		0,411		0,125	