

Using data visualizations for reading the word and the world

Uso de visualizaciones de datos para leer la palabra y el mundo

Utiliser la visualisation des données pour lire la parole et le monde

Usando visualizações de dados para ler a palavra e o mundo

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Abstract

People are frequently exposed to a wide variety of data visualization in their daily lives. Though there is a long history of mathematics and statistics educators investigating ways to support people in reading graphs of data the demands of today's diverse data visualizations are different and furthermore sociopolitical. In this paper we present the development process we went through to create a theory rich and empirical explored framework for reading data visualizations by investigating how teachers read data visualization in critical ways. We do this by presenting two iterations of a design

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research project aimed at developing mathematics teachers' critical statistical literacy for doing and teaching data investigations of sociopolitical issues. From our two qualitative studies we found three meaningful conclusions: 1) What questions we ask in a data visualization activity seem to help create opportunities for people to read graphs in different ways. 2) How a data visualization relates to a person's lived experience seems to impact how they read the data visualization. 3) Data visualization activities can serve as a starting point to developing critical statistical literacy connecting people's current reading of the world with reading the word.

Keywords: Statistics education; Data visualization; Critical literacy; Critical statistical literacy.

Resumen

Las personas están expuestas con frecuencia a una gran variedad de visualizaciones de datos en su vida cotidiana. Aunque los educadores en matemáticas y estadística llevan mucho tiempo investigando formas de ayudar a las personas a leer gráficos de datos, las exigencias de las diversas visualizaciones de datos actuales son diferentes y, además, sociopolíticas. En este artículo presentamos el proceso de desarrollo que seguimos para crear un marco teórico y empírico para la lectura de visualizaciones de datos, investigando cómo los maestros leen las visualizaciones de datos de forma crítica. Para ello, presentamos dos iteraciones de un proyecto de investigación de diseño destinado a desarrollar la alfabetización estadística crítica de los profesores de matemáticas para realizar y enseñar investigaciones de datos sobre cuestiones sociopolíticas. A partir de nuestros dos estudios cualitativos, llegamos a tres conclusiones significativas: 1) Las preguntas que hacemos en una actividad de visualización de datos parecen ayudar a crear oportunidades para que las personas lean los gráficos de diferentes maneras. 2) La forma en que una visualización de datos se relaciona con la experiencia vivida por una persona parece influir en cómo la lee. 3) Las actividades de visualización de datos pueden servir como punto de partida para desarrollar una alfabetización estadística crítica que conecte la lectura actual que las personas hacen del mundo con la lectura de la palabra.

Palabras clave: Educación estadística, Visualización de datos, Alfabetización crítica, Alfabetización estadística crítica.

Résumé

Dans leur vie quotidienne, les gens sont fréquemment exposés à une grande variété de visualisations de données. Bien que les enseignants en mathématiques et en statistiques cherchent depuis longtemps des moyens d'aider les gens à lire les graphiques de données, les exigences actuelles en matière de visualisation de données sont différentes et, qui plus est, sociopolitiques. Dans cet article, nous présentons le processus de développement que nous avons suivi pour créer un cadre théorique et empirique riche pour la lecture des visualisations de données, en étudiant comment les enseignants lisent les visualisations de données de manière critique. Pour ce faire, nous présentons deux itérations d'un projet de recherche en conception visant à développer les compétences statistiques critiques des enseignants de mathématiques pour mener et enseigner des enquêtes sur des questions sociopolitiques à partir de données. Nos deux études qualitatives nous ont permis de tirer trois conclusions significatives : 1) Les questions que nous posons dans le cadre d'une activité de visualisation de données semblent contribuer à créer des occasions pour les gens de lire les graphiques de différentes manières. 2) La manière dont une visualisation de données est liée à l'expérience vécue d'une personne semble avoir un impact sur la façon dont elle lit la visualisation de données. 3) Les activités de visualisation des données peuvent servir de point de départ pour développer une culture statistique critique reliant la lecture actuelle du monde par les individus à la lecture du mot.

Mots-clés : Enseignement des statistiques, Visualisation des données, Littératie critique, Littératie statistique critique.

Resumo

As pessoas são frequentemente expostas a uma grande variedade de visualizações de dados em suas vidas diárias. Embora haja uma longa história de educadores de matemática e estatística investigando maneiras de ajudar as pessoas a ler gráficos de dados, as demandas das diversas visualizações de dados atuais são diferentes e, além disso, sociopolíticas. Neste artigo, apresentamos o processo de desenvolvimento pelo qual passamos para criar uma estrutura rica em teoria e explorada empiricamente para a leitura de visualizações de dados, investigando como os professores leem a visualização de dados de maneira crítica. Fazemos isso apresentando duas iterações de um projeto de pesquisa de design que visa desenvolver a alfabetização estatística crítica de professores de matemática para fazer e ensinar investigações de dados sobre questões sociopolíticas. A partir de nossos dois estudos qualitativos, chegamos a três conclusões significativas: 1) As perguntas que fazemos em uma atividade de

visualização de dados parecem ajudar a criar oportunidades para as pessoas lerem gráficos de maneiras diferentes. 2) A forma como uma visualização de dados se relaciona com a experiência de vida de uma pessoa parece impactar a maneira como ela a lê. 3) As atividades de visualização de dados podem servir como ponto de partida para o desenvolvimento de uma alfabetização estatística crítica, conectando a leitura atual que as pessoas fazem do mundo com a leitura da palavra.

Palavras-chave: Ensino de estatística; Visualização de dados; Alfabetização crítica; Alfabetização estatística crítica.

Using data visualizations for reading the word and the world

Introduction

The reading of graphs of data has been often associated with statistical literacy (Gal, 2002), which has been included in the school mathematics curriculum of the U.S. over the past several decades (Scheaffer & Jacobbe, 2014). Past scholarship (i.e., Curcio, 1987; Friel et al., 2001; Shaughnessy, 2007) synthesized different ways that people engage in reading graphs and shaped curriculum and guided scholarship on people's understanding of reading graphs. However, the ways data is visualized have changed significantly over the past few decades due to technological advances and new forms of media such. Wilkerson and Laina (2018) describe data visualizations as those that "use context rich and interactive methods to create narratives and allow users to explore data for themselves" (p. 1). This definition expands beyond traditional graphs to include new forms of data visualization, such as dynamic and interactive spatial displays of data like ArcGIS story maps or scrolly-telling, where data visualizations change as a person scrolls down an article on a device. Data visualization has also emerged as an important skill for data literacy that is highly interrelated with statistical literacy but does have new skills and practices (Ghodoosi et al., 2023; Gould, 2017; Ridsdale et al., 2015). With such advances in how data is visualized and presented to the public, we see a need to revisit old frameworks with new data and lenses to consider the new realities of how people encounter data in the world today.

As how we visualize data has evolved, so too has its use in people's daily lives. For example, consider the prevalence of data dashboards and visualizations during the COVID-19 global pandemic where people were bombarded with complex and live data that they had to make sense of to make educated decisions about their behavior that could have life or death consequences. The pandemic also sparked a flurry of data literacy work (Fernández-Pascual et al., 2024) especially around making sense of data visualization (e.g. da Silva et al., 2021; Engledowl & Weiland, 2021; Kahn et al., 2023; Lim et al., 2023; Rubel et al., 2021). The politized nature of the pandemic also sparked a critical literacy orientation of much of the recent work on data visualization, bringing new considerations to the literature, which previously had primarily taken a positivistic world view on developing students' technical skills for interpreting data visualization. Critical orientations helped to highlight issues such as the formatting, narrating, and framing of data visualizations (Rubel et al., 2021), the role of emotions in considering data visualization (Kahn et al., 2023; Lim et al., 2023), as well as highlighting issues

of mis information in data visualizations for students (Engledowl & Weiland, 2021). These works all point to important considerations for teaching students data visualizations.

Teachers, who are entrusted to enact the mathematics curriculum that students experience directly (Remillard & Heck, 2014), are at the forefront of curricular shifts such as updating how we teach and learn about data visualizations. As a result, in this study, we focus on a design research study of supporting mathematics teachers' in critically reading data visualizations. Studying mathematics teachers reading of data visualizations is particularly important not just because they can shift the types of experiences students have with statistics, but because they themselves have often had few if any prior experiences learning statistics themselves (Shaughnessy, 2007). Additionally, past work has found that mathematics teachers are not confident in their ability to teach the statistics concepts required in their state standards (Lovett & Lee, 2017, 2019). The emergence of data science entering the K-12 curriculum in many parts of the U.S. and other countries such as Germany, Australia, New Zealand, and Canada (Drozda et al., 2022; Sukol, 2024, 2025) has only increased the demand on mathematics teachers to incorporate data visualization into their curriculum.

In this paper, we study the overarching research question: How do mathematics teachers read relevant data visualizations? We address this question by presenting two studies that have been key in our ongoing development of a new framework for reading data visualizations from a critical statistical literacy perspective (Weiland, 2017) building from prior research. Rather than just presenting a polished final framework we think it is important to show key aspects of the development process to help justify the decisions we made in creating the framework and to make our chain of reasoning transparent. Because we are showing the developmental process of the framework, we will use a slightly nontraditional structure beginning with the overall design of the design research project that this work comes from before then presenting the two studies separately with their own methods and results section and a revisions section between the studies. We end by discussing implications for the design of data visualization activities for teacher education and our emerging view of the developmental process of critical statistical literacies drawing from Freire's literacy work but translating it to statistical literacy.

Overarching project theoretical framework

This work is part of a design research project focused on developing mathematics teacher's critical statistical literacy (Weiland, 2017; Weiland & Sundrani, 2022) for doing and teaching statistics and more recently data science. Critical statistical literacy draws upon the work of Paulo Freire (Freire, 1970; Freire & Macedo, 1987) to imagine what reading and writing the word and the world with data looks like intersecting practices from critical literacy and statistical literacy. Weiland and Sundrani (2022) outline 10 practices of a critical statistical literacy that represent macro level practices such as identifying and interrogating social structures that shape and are reinforced by data-based arguments and using statistical investigations to alleviate and resolve sociopolitical issues of injustice. An example of some of these practices in action is that of Santos et al. (2024) and Silva and Souza (2024) in their use of intersectionality to carry out investigations of demographic data including race and sex to consider who has power and who does not in Brazil in elected government positions. We also note that there is synergy between critical statistical literacy (Weiland, 2017) and Tygel and Kirsh (2016) critical data literacy, in particular on data investigation or enquiry. One difference is that critical data literacy does not focus much on the consumption of data arguments, as critical data literacy is focused on the "set of abilities which allows one to use and critically produce data" (Tygel & Kirsh, 2016, p.117).

Though there are examples of the macro practices of critical statistical literacy in practice, they are few. Furthermore, the macro level practices represent a philosophical end goal, which provides limited usefulness in trying to design meaningful learning environments for people to develop these practices. In other words, how does one become literate? The focus of this project is on investigating teacher's development of a critical statistical literacy for doing and teaching statistics guided by a sociopolitical framing of the process of learning where individual's development is mediated by cultural tools and power relations in society (Esmonde, 2016; Lave & Wenger, 1991; Lerman, 2000; Vygotsky, 1930; Wenger et al., 2002).

To describe and explain the process for development we use learning theories for adult literacy, specifically the critical literacy work of Paulo Freire and others (Freire, 1970; Freire & Macedo, 1987) in conjunction with a situated learning perspective (Boaler & Greeno, 2000; Greeno, 1998; Lave & Wenger, 1991). Freire's perspective on the process of learning has been taken up in mathematics education (Brantlinger, 2011; Frankenstein, 1998; Gutstein, 2003, 2006) and has been

considered in the context of teaching statistics (Frankenstein, 1994; Gal, 2002; Lesser, 2007; Weiland, 2017, 2019). Freire believed in the importance of breaking down hierarchies in education with teachers and students learning together in dialogue. This view connects to the assumptions of situated learning theories focusing on people's attunement to the practices and norms of a community of practice (CoP) through being engaged in those practices with a legitimate member of the CoP (Greeno, 1998; Lave & Wenger, 1991; Lerman, 2000). The core of dialogue and literacy are words which include,

"reflection and action, in such radical interaction that if one is sacrificed—even in part—the other immediately suffers. There is no true word that is not at the same time a praxis. Thus, to speak a true word is to transform the world" (Freire, 1970, p. 87).

This highlights Freire's perspective that literacy is transformative and liberating. Freire often talked about literacy as reading and writing the word and the world to highlight the transformative nature of literacy through reflection and action, often referred to as praxis (Darder, 2014). From this perspective, fostering literacy begins with connecting to people's current reading of the world based on their lived experiences. Freire (1970) referred to the initial understandings people have of the world as generative themes, which are salient "things" from people's lived reality. As Freire discusses,

Authentic education is not carried on by "A" for "B" or by "A" about "B," but rather by "A" with "B," mediated by the world—a world which impresses and challenges both parties, giving rise to views or opinions about it. These views, impregnated with anxieties, doubts, hopes, or hopelessness, imply significant themes on the basis of which the program content of education can be built. (p. 93)

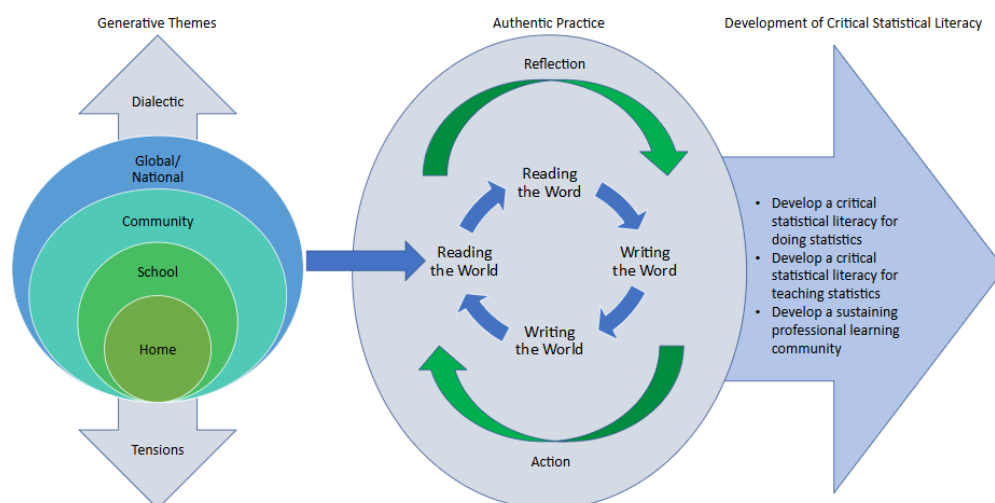
Therefore, learning occurs in authentic dialogue, mediated by the world, building from generative themes. A starting point for development is reflecting on tensions in a community/system (i.e. generative themes). These tensions are discussed in different ways in the literature. Freire (1970) for example talks about dialectic tensions, which he prompts groups to reflect on through generative themes. A common theme employed by Freire was domination/liberation, drawing from people's lived experiences relative to this theme, to reflect on the tensions between these opposites that cannot exist without each other, to find a balance between them or in other words to navigate such tensions (Darder, 2014). Learning can then be seen through actions taken because

of reflecting on such tensions. For example, Vygotsky's (1930) theory of the zone of proximal development discusses how individuals can learn through dialogue with more knowledgeable others to incorporate things they can do with the support of others into what they are able to do on their own. Reinterpreting this perspective through situated learning in communities of practice (CoP), people move from being periphery legitimate members of a CoP to legitimate members by becoming attuned to the affordances and constraints of the CoP through engaging in the practices of that community with legitimate members (Lave & Wenger, 1991).

To summarize, change occurs through individuals being engaged in the practices of a community in cycles of reflection and action (i.e. praxis) to become attuned to new practices. The process of learning Freire described is often encapsulated in the phrase "reading and writing the word and the world" (Darder, 2014; Freire & Macedo, 1987). Beginning with people's current reading of the world (i.e. generative themes), Freire worked to build technical understandings of reading the written word, becoming attuned to the practices of the community, beginning with easier practices. From learning how to read the word, people can reflect on their reading of the world. Reflection comes with action though, and that action initially takes the form of writing the word, building from peoples' reading of the word. Action in literacy also leads to people's writing the world transforming it for a more just future connecting to their agency and its role in the learning process. To consider the developmental process we created a hypothetical theory of change based on sociocultural and sociopolitical theories of learning shown in Figure 1.

Figure 1

Theory of change for design research project



Source: The authors.

Pilot study reading data visualization framework

The theoretical framing of critical statistical literacy (Weiland, 2017), which theorizes the intersection of critical literacies draws heavily from Freire's (1970) literacy work and the idea of reading and writing the word and the world and statistical literacy drawing heavily from Gal's (2002) descriptions of reading in statistics and Wild and Pfannkuch's (1999) work on statistical enquiry serves as the learning goal of this project. We decided in our design work to begin by focusing on reading data visualizations which is one of the most common ways we read statistics/data in our daily lives and parallels Freire's literacy work beginning with people's reading of the world. To start we tried to just use the practices described by Weiland and Sundrani (2022) for reading in critical statistical literacy, which include:

- R1. Making sense of language and statistical symbols systems and critiquing statistical information and data-based arguments encountered in diverse contexts to gain an awareness of the systemic structures at play in society.
- R2. Identifying and interrogating social structures which shape and are reinforced by data-based arguments.
- R3. Understanding one's social location, subjectivity, political context and having a sociohistorical and political knowledge of self and understanding how it influences one's interpretation of information.
- R4. Evaluating the source, operationalization, collection and reporting of statistical information and how they are influenced by the author's social position, and sociopolitical and historical lens.
- R5. Interrogating the epistemological and historical underpinnings of statistical practice and how it has shaped data-based discourses and beyond.

However, these practices were written at a macro level and so we found them challenging to use on a meso-level to operationalize in a classroom setting. Another issue we encountered was that our framework did not adequately capture teachers' progress toward critical statistical literacy but instead mostly just captured their reading of the graphs in a technical disciplinary sense. Therefore, we went back to the literature to look for insights into how to create a coherent and operationalized framework for reading the word and the world. When using the term *reading the word*, we are describing reading the technical language of statistics, whereas when we use the term

reading the world, we are referring to people's understanding of the world, societies, and communities in which they live and experience directly.

A crucial aspect of the development of statistical literacy is understanding how to *read the word*. Embedded within this goal is the recognition that multiple skills in reading data visualizations need to be targeted in the classroom as students engage with different representations. Past scholarship has emphasized an explicit focus on reading graphs to support graph comprehension (Curcio, 1987; Friel et al., 2001; Shaughnessy, 2007). For example, to understand how people read graphs, Curcio (1987) created three hierarchical levels for educators to consider as they formulate tasks and questions aimed at improving students' graph comprehension – reading the data, reading between the data, and reading beyond the data. The descriptions of these can be found in the first column of Table 1. Shaughnessy (2007) extended these reading levels to incorporate a fourth level – reading behind the data – to highlight the causes for variation in data represented in graphs and to make connections between the context of the data and the graph. Since then, numerous studies have used the reading levels framework to analyze news stories and student work with graphs, statistical tables, and maps (da Silva et al., 2021; Rubel et al., 2016). There is also important work from statistical literacy on reading in statistics such as considering how data is collected to determine what type of claims are warranted, when common forms of bias are present in the methods and other common methodological issues such as sample size (Bailey & McCulloch, 2023; Gal, 2002; Utts, 2003).

Reading the world facilitates an individual's understanding and analysis of power, oppression, and structural inequities (Freire, 1970; Gutstein, 2006). For example, Rubel et al. (2016) discuss how students often try to locate themselves in the data they are investigating. Rubel et al. (2021) extend their previous work by including the practices of renarrating, reformatting, and reframing. These practices describe considering the author's message in a data visualization and reimagining it for more just purposes. Bailey and McCulloch (2023) also discuss practices such as acknowledging alternate explanations of the data and recognizing gaps in one's knowledge of the context being explored that is needed to interpret the statistical message. To acknowledge the subjectivity involved in data visualizations that are created by someone to serve a purpose as well as the power dynamics that are always inherently involved in social interactions reality described in the literature, we expand upon the original graph reading frameworks described earlier (i.e. Friel et al. 2001; Shaughnessy, 2007),

which focused on objective readings of graphs from a positivistic worldview. Our work draws from sociopolitical framings (Gutiérrez, 2013).

To create an initial framework we chose to draw upon the levels described by Friel et al., (2001) and elaborated upon by Shaughnessy (2007) as one of the dimensions and then to create a second dimension that highlighted reading the word practices and reading the word practices drawing upon two decades of statistical literacy research to elaborate on what practices could look like. In Table 1 we present the initial framework and practices. It is important to note that though we developed categories in our framework that we differentiate, we see them as deeply interrelated, where expertise consists of coordinating between different types of reading of data visualizations to read the word and the world.

Table 1

Reading the Word and World with Data Visualizations Framework

	Description	Reading the Word Practices	Reading the World Practices
Reading the data	Lifting information from the graph to answer explicit questions for which the obvious answer is in the graph Extract information from the data (Friel et al., 2001)	• Recognizing the components of graphs and the relationship among the components • Speaking the language of specific graphs when discussing information displayed in graphical form	• Looking for oneself in the data

Reading between the data	Interpretation and integration of information that is presented in a graph – the reader completes at least one step of logical or pragmatic inferring to get from the question to the answer (Friel et al., 2001) Find relationships in the data (Friel et al., 2001)	• To understand the relationships among multiple representations of data • Finding relationships or trends in the data visualized beyond simply lifting information from the graph • Making sense of a graph beyond simply lifting information from the graph • Recognizing the types of relationships (correlational or causal) that could be claimed based on the data collection methods • Identifying how the author has highlighted particular relationships in the graph	• Making sense of the data visualization in relation to one's own personal experiences • Questioning how and why the author has highlighted particular relationships in the graph • Reimagining other ways that relationships could be highlighted or visualized
Reading beyond the data	Extending, predicting, or inferring from the representation to answer questions – the reader gives an answer that requires prior knowledge about a question that is related to the graph (Friel et al., 2001) Move beyond the data (Friel et al., 2001)	• Interpreting information in a graph and answering questions about it beyond initial sense making • Predicting or extrapolating based on claims made from the graph • Making claims/inferences based on patterns and trends in the data to a population beyond what is represented in the data	• Making predictions/claims/inferences from the data visualization by drawing upon one's own personal experiences • Acknowledging possible alternate explanations of the data represented • Recognizing the story, the author is trying to tell with a data visualization • Questioning the author's motives for telling the story they are telling in the data • Identifying possible inequities while interpreting the data visualization • Recognizing one's bias and its impact on interpreting the data

Reading behind the data	Making connections between the context and the data (Shaughnessy, 2007), including questioning how the data was collected and how the data visualization represents the context	• Looking for possible causes of variation in the data, based on the context being measured and the way the data was collected • Recognizing appropriate graphs for a given data set and its context • Questioning methods (e.g. sample size, collection methods, etc.) and their impacts on inferences (e.g. practical significance vs. statistical significant; effect vs. no effect) • Recognizing when common sources of bias are present in the data collection • Questioning the methods used to represent/display the data • Recognizing and questioning the source of the data including what is quantified and how it was measured	• Using your knowledge of the context of the data to interpret why particular patterns exist in the data and data generation process • Using knowledge of one's community to interpret why particular patterns exist in the data to question aspects of the data generation process • Questioning the investigative process undertaken based on personal experiences/identity • Recognizing that one needs more contextual knowledge to interpret the statistical message in the data visualization
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Source: The authors.

Pilot study methods

To begin to explore how teachers read data visualizations we started by using graphs from the New York Times What's going on in this Graph column that uses data visualizations from the newspaper as a discussion point for students. We kept the questions typically asked in the NYTs column and added a couple of our own which we then assigned to teachers in a graduate course on teaching statistics. This was a qualitative study that was part of a larger design-based research project (Bakker, 2018; Cobb et al., 2003) studying mathematics teacher's development of critical statistical literacies for doing and teaching statistics.

The participants were a convenience sample of students of a master's level course on teaching statistics and probability at a Hispanic- and Asian American and Pacific Islander-serving institution in the southwestern region of the U.S. 8 of 12 students consented to be a part of the study, all of which identify as female, with four identifying as white; two as white and Mexican, Mexican American, or Chicano; one as white and Black or African American; and one as Black or African American. We chose to use teachers as participants to refine our reading framework as they have experience

reading data visualizations both as professionals and in regard to the content they teach, meaning they would likely read data visualizations in a variety of ways to help us capture wide variation in ways of reading data visualizations.

The data consisted of participants' responses to three discussion board assignments involving reading data visualizations from *The New York Times*' "What's Going on in This Graph?" We have included the original assignment in Appendix A that includes all questions we asked the participants to respond to. Here, we only report on responses to the common questions asked on all three of the data visualizations, which can be seen in the first column of Table 2. The responses to each question were coded based on the reading levels framework in Table 1 by two of the authors and compared to ensure 100% agreement.

Pilot study results

Overall, we saw evidence of all of the ways of reading data visualizations in our sample (see Table 3). Additionally, we were able to code all participants' responses to the reading data visualization questions. This provides evidence that our initial framework was capturing the variation in ways of reading data visualizations in the sample we collected. We also found evidence of the need for *reading the world* practices to be described beyond prior frameworks in line with the growing body of literature in critical data literacies described earlier. For example, here is a statement from one participant that is an example of reading the data (world) as they are seeing their own personal story played out in the data:

This data is representative of the trends that I have experienced in higher education myself. I completed my undergraduate education as a traditional-aged undergraduate student. Most of my course work was in person; very seldomly did I have the chance to take an online class or opted into taking an online class. Whereas my graduate education has primarily had some education component in each class.

Connecting one's personal experiences to the broader trends in a visualization is important for readers when interpreting data. This practice is an aspect of what experts do in moving between views of data including individual, case and aggregate (Konold et al., 2015). The participant showed evidence of this in an earlier statement in her response to the question of, what do you notice?

This graph is being used to represent how chief online officers expect students to experience their classes. The data is being represented in a horizontal bar graph.

A trend that I observed in the graph is that by 2025 less than 5% of classes are expected to be experienced in a fully on campus or little to no online matter. Most classes are expected to be experienced in a balance of online and on campus format.

In this statement, we see the participant reading the data by stating elements of the data visualization she observes in stating the type of graph is being used. She then goes on to read between by stating a trend in the data she is observing of few experiences for any group to experience little to no online educational components. She then ends with reading beyond the data by making a claim/inference that most students will experience a balance of online and on-campus educational experiences.

From looking at the frequency at which our participants are reading graphs in different ways, it is clear that overall participants are reading the graphs in typical, statistical kinds of ways, which makes sense given the participants are mathematics teachers and have had many experiences in developing quantitative reasoning over their educational experiences. Furthermore, much of the focus is on the original three ways of reading described by Friel et al. (2001) in their framework. Reading behind the data occurred but was not very prevalent as compared to the other types of reading, which is important to note as this type of reading encapsulates making connections between the context and the data (Shaughnessy, 2007). Shuttling between context and data is a crucial aspect of statistical inquiry (Wild & Pfannkuch, 1999), so it is troubling that it is the least common type of reading occurring.

Table 2

Participants' Reading of Data Visualizations by Question

Question	WORD				WORLD			
	Reading the Data	Reading Between the Data	Reading Beyond the Data	Reading Behind the Data	Reading the Data	Reading Between the Data	Reading Beyond the Data	Reading Behind the Data
What do you notice?	10	11	3	0	0	0	3	0
What do you wonder?	0	1	0	10	1	5	4	6
Create a catchy headline that captures the graph's main idea.	1	3	13	0	0	0	7	0

What relationship(s) is the author highlighting?	3	16	7	0	0	0	2	0
What is a claim you could make from this data visualization?	2	7	14	0	0	0	2	0
What evidence are you using to make this claim?	13	14	2	0	0	0	2	0
How does this relate to you and/or your community?	0	0	2	0	4	4	9	1

Source: The authors.

Another pattern we noticed is that the questioned posed seemed to be related to the type of reading that the participant engaged in responding to the question. For example, most responses to the question, what do you notice, led to the participant reading the data or reading between the data (word) with very little reading the world. Responses to the prompt “what do you wonder” account for all but one of the responses where participants read behind the data. Responses to “how does this relate to your community” also lead predominantly to participants reading the world. This is an important insight as this instructional route is very common in mathematics education but seems to provide only a narrow range of reading types in response meaning we may need to rethink this instructional routine to include more targeted follow up questions to go beyond merely, noticing and wondering to get at deeper readings of data visualizations.

Table 3

Participants reading of the data visualizations

Participant	WORD				WORLD			
	Reading the Data	Reading Between the Data	Reading Beyond the Data	Reading Behind the Data	Reading the Data	Reading Between the Data	Reading Beyond the Data	Reading Behind the Data
Haya	5	8	6	2	0	1	2	1
Alice	3	9	6	3	1	0	1	0
Julie	4	4	6	2	1	0	5	1

Danielle	4	7	2	1	0	1	6	1
Jennifer	6	8	2	0	0	2	3	1
Michelle	5	6	6	0	1	1	5	2
Nicole	1	5	11	1	1	3	1	1
Valerie	7	5	3	1	1	1	3	0
TOTAL	35	52	42	10	5	9	26	7

Source: The authors.

Revisions

We learned a lot from our initial pilot study in terms of the importance of what questions we ask. This made us start to think about what questions would help support different types of reading that we could then provide to teachers as a starting point for designing their own activities while also driving our design with teachers. In other words, this finding helped us think about how to design our activities differently for the next iteration of our work. This led us to eventually create a tool for teachers to use about how to create data visualization activities to support students in learning in different ways which is now freely available for use (Weiland et al., 2024).

We also started to identify some issues in our framework though in terms of how we had operationalized the practices. In particular, we struggled with how to code some of the statements made where participants talked about their own personal experiences in relation to the data visualization but in ways that did not seem related to identifying underlying systems and structures of injustice. It was important to capture these statements as they do represent participants readings of the world in relation to the data visualization but we wanted to differentiate those readings of the world from more structural reading that are more indicative of critical consciousness. To address this issue we drew from Lee et al.'s (2021) *Call for a Humanistic Stance in Data Science Education*, where they put forth three layers of such an education including personal, cultural, and sociopolitical layers. We used this idea to then split reading the world into two layers with one layer focusing on personal/community practices and the other is focused on sociopolitical factors (see Table 4). We also liken this perspective to the key aspects of culturally relevant pedagogy, where academic excellence maps to reading the word, cultural competence maps to reading the world with personal/cultural practices, and sociopolitical consciousness maps to reading the world sociopolitical practices (Ladson-Billings, 1995). As a reminder though we developed categories in our framework that we differentiate we see them as deeply interrelated where expertise

consists of coordinating between different types of reading of data visualizations to read the word and the world.

Table 4

Revised Framework for Critically Reading Data Visualizations

Reading Types	Reading the Word Practices	Reading the World Personal/Community Practices	Reading the World Sociopolitical Practices
<i>Reading the data</i> Extract information from the data (Friel et al., 2001)	To recognize the components of graphs, the interrelationships among these components, and the effect of these components on the presentation of information in graphs (Friel et al., 2001) To speak the language of specific graphs when reasoning about information displayed in graphical form (Friel et al., 2001)	Looking for oneself in the data (Rubel et al., 2016)	Look for source of data Look for author/affiliation of visualization Questioning why an author has highlighted particular aspects of a graph or left them absent
<i>Reading between the data</i> Find relationships in the data (Friel et al., 2001)	To understand the relationships among a table, a graph, and the data being analyzed (Friel et al., 2001). Finding relationships or trends in the data visualized Recognizing the types of relationships (correlational or causal) can be	Making sense of the data visualization in relation to one's own personal experiences	Questioning how and why the author has highlighted particular relationships in the graph (Rubel et al. 2021)

	claimed based on the data collection methods (Utts, 2003)		
	Identifying the relationships highlighted in the graph (Framing; Rubel et al. 2021)		
Reading beyond the data	Interpreting information in a graph and answering questions about it (Shaughnessy, 2007)	Making predictions/ claims/ inferences from the data visualizations by drawing upon one's own personal experiences	Making connections to alternate explanations of others (Bailey & McCulloch, 2023)
Move beyond the data (Friel et al., 2001) to consider interpretations, inferences, and predictions/ extrapolations	Predicting outcomes based on reasonable claims made from the graph (Shaughnessy, 2007)	Recognition of one's bias and its impact on interpreting data (Bailey & McCulloch, 2023; Author, 2017)	Recognizing the story, the author is trying to tell with this data (Rubel, 2021)
	Making claims/inferences based on patterns and trends in the data to a population beyond what is represented in the data	Acknowledging possible Alternate Explanations (Bailey & McCulloch, 2023)	Questioning the author's motives for telling this story (Rubel et al., 2021)
		Drawing upon personal experiences facing inequities in the interpretation of the data visualization (Bailey & McCulloch, 2023)	Identifying structural inequities at play in the interpretation of the data visualization (Bailey & McCulloch, 2023)
		Connecting to one's feelings/emotions related to the data visualization (Kahn et al. 2022)	
Reading behind the data	Looking for possible causes of variation (Shaughnessy, 2007), based on the context being measured and the	Using your knowledge of the context of the data to interpret why particular patterns exist in data as well as data generation process	Questioning sample size and methods (Bailey & McCulloch, 2023) and their impacts on inferences (i.e.
Making connections between the context and			

the data (Shaughnessy, 2007)	way the data was collected	Using knowledge of one's community to interpret why particular patterns exist in the data to question aspects of the data generations process	practical significance vs. statistical significant; effect vs. no effect) (Utts, 2003)
	Looking for relationships between variables based on the context		
	Recognizing appropriate graphs for a given data set and its context (Shaughnessy, 2007)	Questioning the investigative process undertaken based on personal experiences/identity	Recognizing when common sources of bias are present in the data collection (Utts, 2003)
	Recognizing Appropriate Statistics & Appropriate Representations (Bailey & McCulloch, in 2023)	Recognition of the gaps in one's knowledge of the context needed to interpret the statistical message. (Bailey & McCulloch, 2023)	Recognizing and questioning the source of the data including what is quantified and how it was measured (Rubel et al., 2021; Author, 2017)

Source: The authors.

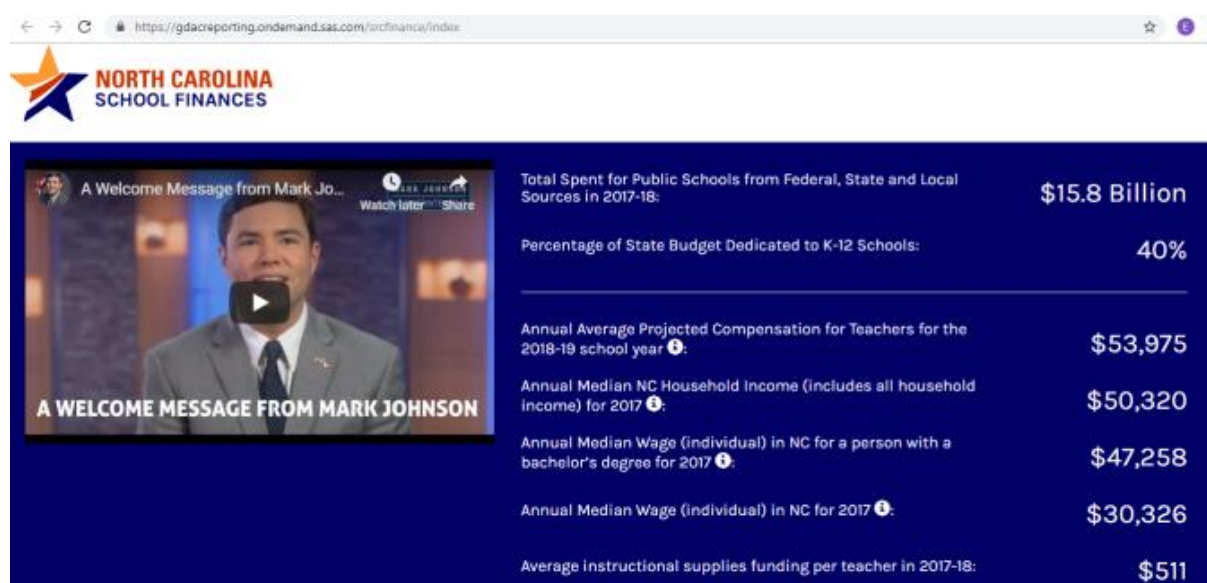
Professional learning community study methods

Using what we learned during our pilot study we designed a larger project that involved building a professional learning community of mathematics teachers as a community of practice based on our larger theoretical framework described earlier. Here we present data from one of the initial activities of the larger project where the teachers were first introduced to data visualization activities during a two-week summer workshop in the summer of 2023. The seven participants in our study were high school math teachers recruited from a school district in the southeastern U.S. One of our participants was a district administrator. Six participants identified as a woman, and one identified as a man. Five participants identified as Black or African American, and two identified as white. The teachers' years of experience include: two people with 4-7 years of experience; one person with 8-10 years of experience; and four people with 16+ years of experience.

Using the redesigned framework (see Table 4), we redesigned the types of activities we were using to engage teachers in reading data visualizations which we then analyzed data from using the new framework to test. In redesigning our activities, we now view data visualizations like Freire's picture cards he used in his community circles. Therefore, we intentionally chose a data visualization of an issue that our participants would have some close learned experiences around to draw upon. Figure 2 shows the first data visualization we use in our sequence of activities which was created by the Department of Public Instruction in North Carolina where the participants were teachers. It was related to their daily lived experience, but was also designed with a political message in mind, which the teachers picked up on immediately and sparked discussions. The political message was that teachers are paid well and do not need raises in salary, which they are not by many measure and they are well aware of. The statistical concept that was related to this message is how measures of center like the mean and median are impacted differently by outliers and skewed distributions. Basically, you cannot compare the mean of one heavily skewed distribution to the median of another. We also purposefully asked questions that were related to different reading types and then explicitly described those types of reading to the teachers.

Figure 2

Screenshot of the data visualization we used in the first data visualization activity participants engaged in for the professional learning community



Data sources included video recordings of the professional development session, daily written reflections by participants, and ongoing work samples. This comprehensive approach allowed us to capture the nuances and variations in how teachers engage

with and interpret data visualizations. This study focuses on the first 15 minutes of a single introductory activity where the participants are given three questions to consider about a data visualization (see Figure 2): What do you notice? What do you wonder? How does this impact your community?, which is most directly related to the findings we presented in the pilot study. Three members of the research team were present and helped facilitate the professional development and took on a researcher/participant role during the activity.

Participant and researcher utterances during the session were coded based on the critical reading data visualizations framework outlined in Table 4. Three of the authors independently coded the responses, and a comparison was conducted. Differences in coding were discussed until the coders reached 100% agreement. This rigorous coding process enhances the trustworthiness and dependability of our analysis, providing a solid foundation for understanding the diverse ways in which teachers read and interpret data visualizations (Lincoln & Guba, 1985). After coding, we then looked at the frequency of each reading type and layer represented in the data. We identified the patterns in the frequencies for further exploration. We arrived at our findings by looking at themes in the data analysis across reading type, layer, and participant. Once we identified patterns across the themes to develop our findings, described in further detail below.

We identified two main findings that answer our research question: How do mathematics teachers read a relevant data visualization? In our first finding, we discovered that teachers exhibited engagement across all reading types, with a noticeable emphasis on reading behind the data. Additionally, our analysis revealed that teachers engaged with all three layers of reading, with a predominant focus on the sociopolitical layer. For our second finding, we noticed that different participants engaged in different frequencies.

Table 5 demonstrates the ways teachers engaged with the reading types and layers. Some reading types were taken up less often. For example, reading between the data was underrepresented at the word and personal layers, and no teachers engaged in the reading beyond the data at the word layer.

Table 5

Counts and percentages of code occurrences for each dimension of the framework for reading data visualization

	Reading The data	Reading Between the Data	Reading Beyond the Data	Reading Behind the Data	Row Total
Word	9 (16%)	1 (2%)	0 (0%)	3 (5%)	13 (23%)
Personal	1 (2%)	1 (2%)	5 (9%)	5 (9%)	12 (21%)
Sociopolitical	3 (5%)	9 (16%)	10 (18%)	10 (18%)	32 (56%)
Column Total	13 (23%)	11 (19%)	15 (26%)	18 (32%)	57 (100%)

Note: Percentage values have been rounded to the nearest whole number for clarity and ease of interpretation. The values in parentheses are the counts of each code's occurrence. All percentages are out of the total of 57 occurrences.

Source: The authors.

In analyzing the various reading types, regardless of the layer (considering one dimension of our framework), we observed that teachers notably engaged in reading behind the data (18/57; 32%), surpassing the percentages of other reading types such as reading the data (13/57; 23%), reading between the data (11/57; 19%), and reading beyond the data (15/57; 26%). For example, Leona's statement, "you see that important. It's also the vocabulary of median and average because if the community doesn't understand the differences," serves as one instance of reading behind data (from sociopolitical perspective) and that reflects her acknowledgment of the significance of statistical terminology and the importance of understanding small differences in data analysis. By emphasizing the need for clarity and comprehension among community, she is making a connection between the data and its relevance to the community's knowledge and understanding in a way that it is important for everyone in the community to understand the numbers, so they know how that affects them.

In examining the different layers, irrespective of the reading types (considering the alternate dimension of our framework), we found that teachers engaged in reading the data visualization in sociopolitical ways more than any other layer (32/57; 56%

overall), compared to the personal layer (12/57; 21%) and the word layer (13/57; 23%). The reading practices in the sociopolitical layer focus on considering the sociocultural context of the data visualization, which the teachers have firsthand experience and background knowledge of. To illustrate this, consider the following statement from Nancy where she questions the relationships the author of the data visualization is highlight, “But glossing over the fact that we have the small print that says average and then, yeah, median and you're wanting me to compare those two.” Nancy goes on to read the data in a sociopolitical way combining her knowledge of the content with her knowledge of her community to point out that this approach is taking advantage of a common misunderstanding of the differences of means and medians and how the shape of a distribution impacts them, “they say the middle income the middle of this and they don't realize that, that middle is usually the median because we know the distribution is not going to be a symmetric or roughly symmetric right.”

Teachers also read beyond the data unpacking the story they thought the author of the data visualization was trying to make and questioning the authors motives. For example, Anna stated, “I think this is designed to show that teachers are making more than most North Carolina incomes.” Some participants also began to make connections between the data visualization they were reading and how they could use it in their own teaching. For example, Melody said, "So I would have used this as a perfect example to my students of how we can make statistics say anything we want," where she is connecting issues of the story being presented how she could use this in her teaching.

We also found that the teachers engaged in reading the data visualization in different ways. Three of the teachers did not engage in verbally describing their reading of the data visualization throughout the fifteen-minute activity. Interestingly, these were the teachers with the least prior teaching experience in the group, though all three had at least four years of prior experience so they were by no means novices. Of the five teachers that did verbalize their reading of the data visualization, three out of the five used sociopolitical more than any other reading type. Table 6 provides a breakdown of each participant’s reading type and layer.

Table 6

Instances of teacher engagement with data visualization activity

	Nancy	Anna	Leona	Natalie	Melody
Reading the data	2	3	2	2	0

Reading the Word	Reading Between the data	1	0	0	0	0
	Reading Beyond the data	0	0	0	0	0
	Reading Behind the Data	1	0	0	1	1
Reading the World Personal	Reading the data	1	0	0	0	0
	Reading Between the data	1	0	0	0	0
	Reading Beyond the data	2	0	1	1	1
	Reading Behind the Data	3	0	0	1	1
Reading the World Sociopolitical	Reading the data	1	1	0	1	0
	Reading Between the data	3	4	0	2	0
	Reading Beyond the data	2	3	1	2	2
	Reading Behind the data	0	0	3	2	5

Note. The columns in this table exclusively represent data for teachers who actively engaged in verbal communication during the activity. There were also three teachers who did not engage in verbal discourse during the specified period.

Out of all the participants, Nancy used the most variety of reading types throughout the entire activity. Nancy's engagement is significant because she engaged in almost every type of reading at every layer, which was not typical, as evidenced in Table 6. For example, she was the only participant who engaged in reading the data and reading between the data at the personal layer, as evidenced in her statement:

That's the question: does the data represent you? So you subscribe to an identity or something of that nature, but when you look for the data on that identity or whatever it is that you're representing it in that, like does it represent you, It's supposed to represent population or something like that.

Nancy's ability to clearly verbalize her reading of the data visualization was found across the activity and contributed to why so many of her utterances were coded from our framework. This also points to a limitation of our study in that we don't know how the participants who were not communicating their reading of the graph or chose not to communicate at all were engaged in reading the data visualization. This has implications for our design and pedagogy, which we discuss in the next section.

Conclusions

In this paper, we sought to explore how mathematics teachers read relevant data visualizations. Drawing from data collected from a larger design research project, we

were able to begin to investigate our question and further refine a framework for reading data visualizations across multiple iterations. We purposefully present the evolution of our framework here to make transparent our thinking and to show how the framework was developed through strong interaction with theory and empirical research in practice. We think this strengthens our arguments and provides clarity for others who wish to use the framework in their own future work. We also present multiple iterations of the development of this framework as we learned different things from each study, all of which are useful for informing statistics, data science, and mathematics education. The three main conclusions we present in this section are:

- 1) What questions we ask in a data visualization activity seem to help create opportunities for people to read graphs in different ways.
- 2) How a data visualization relates to a person's lived experience seems to impact how they read the data visualization
- 3) Data visualization activities can serve as a starting point to developing critical statistical literacy, connecting people's current reading of the world with reading the word.

I note that we have written these claims softly as this work is initial with small groups of teachers in a particular context. However, we think our initial findings are promising and worth sharing and further consideration by others in other settings. Now we will unpack these conclusions further.

From our pilot study it became apparent that just asking questions about noticing and wondering limited the types of reading that people engaged in. In connecting back to our theoretical framework questions are important in statistical literacy throughout the enquiry process (Wild & Pfannkuch, 1999) and for reading/consuming statistical information. Take for example Gal's (2002) worry questions which he connected to critical evaluation connected to Freire's work. Though our finding is not necessarily surprising, it is an important one to point out as just asking what do you notice and what do you wonder questions is a very common instructional routine in mathematics education in the U.S. These questions are meant to provide a low entry for learners into a mathematics task and to engage learners. We agree that these questions in our experience do what they are meant to. However, we have often seen tasks around data visualization stop there and do not go farther. This we believe is problematic as it may limit the opportunities that learners have to read data visualizations. Another instructional routine we have seen interest in and from a design perspective lends itself to asking a variety of questions to push learners to read in different ways is slow reveal

graphs (Laib, n.d). This routine involves starting with a data visualization where multiple components of it are hidden from learners and then slowly revealed over time general using a slide deck. For example, you might start with a scatterplot but with the labels of the axes hidden from view on the first slide paired with some initial questions such as what do you notice and what do you wonder. Then on the next slide you might reveal one axis label or perhaps just the quantities for the scale and then ask more questions slowly revealing more information on each slide. Though we have found no systematic research on this instructional routine, anecdotally we have seen differences in responses when we use them, and the teachers we work with all report having much more meaningful conversations with students using the slow reveal routine. More research is needed here.

From our analysis using our redesigned framework we saw significant shifts in terms of the types of reading teachers were engaging in with the focus predominantly around reading the world (see Table 4), particularly in sociopolitical ways, whereas in our pilot study much of the reading was more focused on reading the word. In returning to our theoretical framework, we see data visualizations as the way of introducing generative themes much in the same way as Freire used picture cards in cultural circles in his literacy work. In the professional learning community study the teachers were engaged in a modified notice and wonder activity with a data visualization we specifically selected because it's sociopolitical relevance to the teachers in our study. This seems to have created the opportunity for the teachers to engage more in sociopolitical readings of the world through the data visualization. This finding has implications for future design in that we hypothesize the reason for the participants' reading predominantly through the sociopolitical layer was because of the relevance of the data visualization itself. The teachers demonstrated a desire to critically examine data beyond its surface level and show interest in uncovering underlying patterns, causes of variation, identifying biases in data collection methods, and understanding contextual factors. What we are working on now is coding the data to look for shifts across all of the data visualization activities to look for connections between the design of the activity and teachers reading of types to see if we can find profiles of reading and also look for underlying design principles specific to reading data visualizations. This is also an area that needs more research.

As data visualizations have become increasingly common in the media today, we see an increased need for teachers to engage in such data visualization activities and to translate them into meaningful experiences for their students as well. We believe

our framework can be useful in helping researchers not only design activities in the future but to analyze the ways in which people engage in reading data visualizations. The work we present here is initial and ongoing and we hope further analysis will provide more refined results. However, we also encourage others to take up this work in other settings. Freire's literacy work continues to be a guiding light for us in this work. Though Freire did not focus on statistical literacy his ideas do translate to it in various ways. As data science education takes off, we have found more and more researchers interested in ideas around data visualization, however we have not seen much work that is deeply situated in educational theory. We hope this work begins to provide some basis in theory for ways that educational researchers can think about how learners learn to read the word and the world with data through theory rich research based approaches with the goal of creating learning experiences for learners to become empowered to take on the many injustices in the world and use data as a tool for identify and addressing them for transforming society.

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APPENDIX A: Reading Data Visualizations Assignments

Reading Data Visualizations Assignment 1

What's Going on in this Graph is a joint venture between the New York Times and the American Statistical Association designed to help support classroom teachers in having conversations about data visualizations in the media about current issues (<https://www.nytimes.com/column/whats-going-on-in-this-graph>). You can also check out the resources ASA provides associated with the postings (see <https://www.amstat.org/ASA/Whats-Going-on-in-this-Graph.aspx>).

For this assignment you will post your response to the questions below related to the data visualization on a discussion board. After posting you will then read and respond to at least 3 peer's responses. Initial responses are due by Sunday at noon. Responses to peers' posts are due by Tuesday by 4pm. <https://www.nytimes.com/2023/01/12/learning/whats-going-on-in-this-graph-jan-18-2023.html>

- What do you notice?
- What relationship(s) is the author highlighting?
- Create a catchy headline that captures the graph's main idea.
- What is a claim you could make from this data visualization?
- What evidence are you using to make this claim?

- What do you wonder?
- How does this relate to you and/or your community?
- What concepts or ideas could this data visualization help you teach?
- What do you think your students would notice?
 - What would you like them to notice?

Reading Data Visualizations Assignment 2

What's Going on in this Graph is a joint venture between the New York Times and the American Statistical Association designed to help support classroom teachers in having conversations about data visualizations in the media about current issues (<https://www.nytimes.com/column/whats-going-on-in-this-graph>). You can also check out the resources ASA provides associated with the postings (see <https://www.amstat.org/ASA/Whats-Going-on-in-this-Graph.aspx>).

For this assignment you will post your response to the questions below related to the data visualization on a discussion board. After posting you will then read and respond to at least 3 peer's responses. Initial responses are due by Sunday at noon. Responses to peers' posts are due by Tuesday by 4pm. <https://www.nytimes.com/2022/10/27/learning/whats-going-on-in-this-graph-nov-9-2022.html>

1. What do you notice?
2. What do you wonder?
3. Create a catchy headline that captures the graph's main idea.
4. What claim(s) could you make from this data visualization?
5. What evidence are you using to make this claim(s)?
6. What does the author include in the data visualization?
7. What relationships is the author highlighting?
8. What story is the author telling with this data visualization?
9. How does this relate to you and your community?
10. What concepts or ideas would this data visualization help you teach?
11. How would you introduce this visualization to your students?

12. Each week the NYTs has a reveal that they do with these data visualization. You can use the link above to see the entire reveal. Here are two student responses that were share in the reveal. "The Inequality of "Equal Representation" by Gwen of Pennsylvania and "Wyoming Voters Have More Power Than You. Here's Why" by Virginia of Eaglecrest High School in Aurora, Colo. How would you evaluate these responses?
13. An additional question they also provide is, "You can calculate the Senate voter representation for your state. Obtain the population [here](#) and the ethnic and racial breakdown [here](#). With two senators per state, how many voters per senator does your state have? To calculate the number of voters by ethnic or racial group, multiply this number by the ethnic or racial group's share of the state's population to determine the representation for each group and round to the nearest integer. What do you think of the difference in representation between Wyoming and your state?" Respond to this question for your state.

Reading Data Visualizations Assignment 3

What's Going on in this Graph is a joint venture between the New York Times and the American Statistical Association designed to help support classroom teachers in having conversations about data visualizations in the media about current issues (<https://www.nytimes.com/column/whats-going-on-in-this-graph>). You can also check out the resources ASA provides associated with the postings (see <https://www.amstat.org/ASA/Whats-Going-on-in-this-Graph.aspx>).

For this assignment you will post your response to the questions below related to the data visualization on a discussion board. After posting you will then read and respond to at least 3 peer's responses. Initial responses are due by Sunday at noon. Responses to peers' posts are due by Tuesday by 4pm.

<https://www.nytimes.com/2022/05/05/learning/whats-going-on-in-this-graph-may-11-2022.html>

Look closely at this graph.

Source: Centers for Disease Control and Prevention High School Youth Risk Behavior Survey

1. What do you notice?
2. What parts/elements/pieces of the graph does the author include?

3. What relationships is the author highlighting?
4. What other relationships in the data do you see?
- a. How could the data visualization be changed to highlight this relationship instead?
5. What concepts or ideas would this data visualization help you teach?
6. How would you introduce this visualization to your students?
7. What story is the author telling with this data visualization?
8. What claim(s) could you make from this data visualization?
- a. What evidence are you using to make this claim(s)?

Space for inserting, in strictly necessary cases, questionnaires and/or other elements complementary to the article of conception and elaboration by the author (s).